Primordial perturbation spectra in Renormalisation Group driven cosmology

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Outline:

Exact Renormalisation Group

Flow in the theory Space

$$\Gamma_k(\phi) = \sum \tilde{g}_i(k) k^{d_i} \mathcal{O}_i(\phi)$$

present work based on RG *à la* Wilson

EFT at scale $k \quad \Rightarrow \quad$ integration of modes $p > k$

$\mathcal{S} \rightarrow \mathcal{S} + \Delta\mathcal{S}_k$ mass term for $p < k$

define generating functional $W_k(J) \rightarrow$ effective action $\Gamma_k(\phi)$

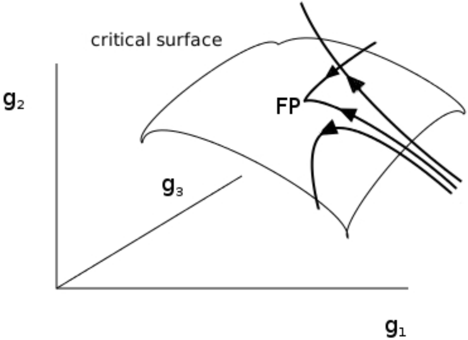
obeying ERGE $\quad k\frac{d\Gamma_k}{dk} = \frac{1}{2}\mathrm{Tr}\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\frac{dR_k}{dk}$

where $k\frac{d\Gamma_k}{dk} = \sum \beta_i \mathcal{O}_i(\phi)$

[Wetterich '91]

Asymptotic safety

- $\exists$ UV fixed point $\lim_{k\to\infty} \tilde{g}_i(k) = \tilde{g}_i^*$
- $M_{ij} = \left.\frac{\partial \beta_{\tilde{g}_i}}{\partial \tilde{g}_j}\right|_*$ has finite number of negative eigenvalues



$\Rightarrow$ **PREDICTIVE THEORY**

Inclusion of matter

Similar results for gravity coupled to scalar field ϕ

$$\Gamma_k[g,\phi] = \Gamma_{\mathrm{EH}} + \Gamma_\phi = \int d^4x\sqrt{-g}\left[-\frac{R}{16\pi G_k} + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + V_k(\phi)\right]$$

How to determine k? Take event $A \quad \Rightarrow \quad k = k_A$

$$G_{\mu\nu} = 8\pi G_A T^A_{\mu\nu}$$
$$\Box\phi = V'_A(\phi)$$

then take event B, with $k_B \neq k_A$

$$G_{\mu\nu} = 8\pi G_B T^B_{\mu\nu}$$
$$\Box\phi = V'_B(\phi)$$

then iterate for every event $x \quad \Rightarrow \quad k \equiv k(x)$

Diffeomorphism invariance of *each* action gives conservation laws

$$\nabla_\mu G^{\mu\nu} = 0$$

$$\nabla_\mu \left. T^{\mu\nu}\right|_\lambda = 0$$

while EOM give the overall conservation

$$\nabla_\mu \left(G T^{\mu\nu} \right) = 0 \qquad \Rightarrow \qquad \nabla_\mu G T^{\mu\nu} - G \left. \nabla^\nu V(\phi)\right|_\phi = 0$$

New constraint $\Rightarrow$ equation for $k(x)$.

Last write $V(\phi) = \sum \lambda_{2i} \phi^{2i}$

$$G = \tilde{G} k^{-2} \text{ and } \lambda_i = \tilde{\lambda}_i k^{4-i}$$

Fixed point regime

Defined by $\beta_{\tilde{G}} \simeq 0$ and $\beta_{\tilde{\lambda}_i} \simeq 0$

Cosmological solutions in the form of power laws:

$$H = \alpha/t \qquad \phi = \varphi/t \qquad k = \chi/t$$

Monomial potential $V = \lambda_n \phi^n$ (n even integer) $\Rightarrow$

$$\begin{aligned}
\alpha &= \frac{4-n}{3\,(2-n)} \\
\varphi^2 &= \frac{1}{(2-n)\tilde{\lambda}_n^*}\left(\frac{4-n}{12(2-n)\pi\tilde{G}_*}\right)^{\frac{4-n}{2}} \\
\chi^2 &= \frac{1}{(2-n)\tilde{\lambda}_n^*}\left(\frac{4-n}{12(2-n)\pi\tilde{G}_*}\right)^{\frac{2-n}{2}}
\end{aligned}$$

No inflationary solution ($\alpha < 1 \quad \forall\, n$)

Trinomial potential including all perturbatively renormalizable terms

$$V(\phi) = \lambda_0 + \lambda_2\phi^2 + \lambda_4\phi^4$$

defining $\tilde{\phi} = \phi/k$ cosmological solution reads

$$\begin{aligned} \alpha &= \frac{2\tilde{\lambda}_0^* + \tilde{\lambda}_2^*\tilde{\phi}^2}{3(\tilde{\lambda}_0^* - \tilde{\lambda}_4^*\tilde{\phi}^4)} \\ \varphi^2 &= \frac{\tilde{\phi}^4}{2(\tilde{\lambda}_0^* - \tilde{\lambda}_4^*\tilde{\phi}^4)} \\ \chi^2 &= \frac{\tilde{\phi}^2}{2(\tilde{\lambda}_0^* - \tilde{\lambda}_4^*\tilde{\phi}^4)} \end{aligned}$$

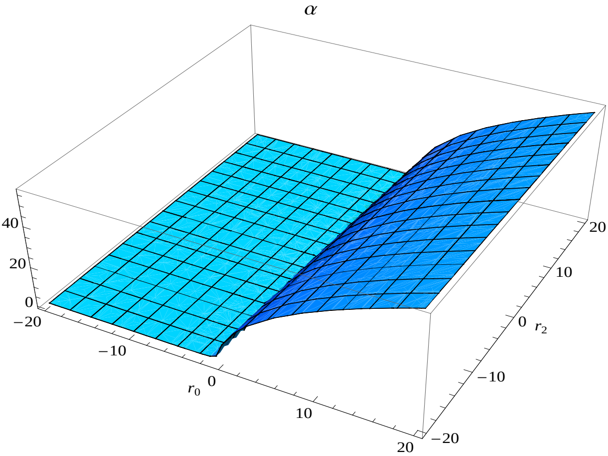
and the value of $\tilde{\phi}$ is given by

$$\tilde{\lambda}_0^* - \tilde{\lambda}_4^*\tilde{\phi}^4 = \frac{1}{12\pi\tilde{G}_*}\left(\frac{2\tilde{\lambda}_0^*}{\tilde{\phi}^2} + \tilde{\lambda}_2^*\right)$$

Redefining FP values of couplings into

$r_0 = \tilde{\lambda}_0^* / \tilde{\lambda}_4^*$ and $r_2 = \tilde{\lambda}_2^* / \tilde{\lambda}_4^*$

find a solution for $\alpha(r_0, r_2, \tilde{G}_*)$



as computations generally give $\tilde{G}_* = \mathcal{O}(1)$

Quasi-classical regime

UV fixed point $\Rightarrow$ for $k < k_{\text{exit}}$ trajectory exits FP and falls towards Gaussian FP ($\tilde{G} = \tilde{\lambda}_i = 0$)

β-functions can be linearised and the flux reads ($\Lambda = 8\pi G \lambda_0$)

$$G(k) = \bar{G} \qquad \Lambda(k) = \bar{\Lambda} + \frac{3}{16\pi}\,\bar{G}\,k^4$$

$$\lambda_2(k) = \bar{\lambda}_2 - \tfrac{3}{16\pi^2}\,\bar{\lambda}_4\,k^2 \qquad \lambda_4(k) = \bar{\lambda}_4$$

constraint equation implies $k(t) = 2\sqrt{\bar{\lambda}_4}\,\phi(t)$

so field EOM reads

$$\ddot{\phi} + 2\sqrt{6\pi\bar{G}\left(\tfrac{1}{2}\,\dot{\phi}^2 + \tfrac{\bar{\Lambda}}{8\pi\bar{G}} + \bar{\lambda}_2\,\phi^2 + \left(1 - \tfrac{3\bar{\lambda}_4}{8\pi^2}\right)\bar{\lambda}_4\,\phi^4\right)}\;\dot{\phi} = -2\left(\bar{\lambda}_2 + 2\left(1 - \tfrac{3\bar{\lambda}_4}{8\pi^2}\right)\bar{\lambda}_4\,\phi^2\right)\phi$$

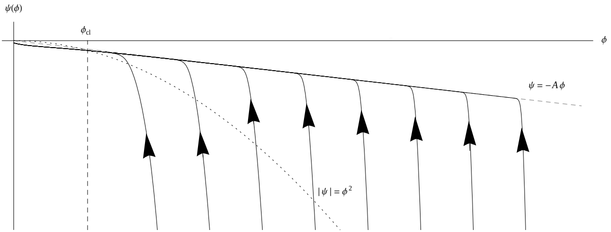
[Narain & Percacci '10]

Kinetic-dominated phase:

$$\dot{\phi}(\phi) \simeq \Phi\, e^{-\sqrt{12\pi \bar{G}}\,\phi}$$

Potential-dominated phase:

$$\dot{\phi}(\phi) \simeq -\sqrt{\frac{2}{3\pi \bar{G}}\left(1-\frac{3\,\bar{\lambda}_4}{8\pi^2}\right)\bar{\lambda}_4}\;\phi$$



Attractor solution $\dot{\phi}(\phi) = -A\phi$

Scalar perturbations

Study evolution of modes within the RG length scale, i.e.

$$|\mathbf{p}| \gg ak \quad \Rightarrow \quad \delta G = \delta\lambda_i = 0$$

defining the variable $v_\mathbf{p} = a\frac{\dot{\phi}}{H}\mathcal{R}_\mathbf{p}$

$$v''_\mathbf{p} + \left(p^2 - \frac{2}{\tau^2}\right) v_\mathbf{p} = 0$$

as in standard de Sitter, and power spectrum (assuming $\alpha \gg 1$)

$$\mathcal{P}_\mathcal{R} \simeq \frac{32}{3}\pi \tilde{G}_*^3 \tilde{\phi}^2 (2\tilde{\lambda}_0^* + \tilde{\lambda}_2^* \tilde{\phi}^2) \quad \Rightarrow \quad n_s = 1$$

$\mathcal{P}_\mathcal{R} \ll 1$ can only be achieved if $r_2 \simeq -2\sqrt{r_0}$

smallness parameter $\delta \quad \Rightarrow \quad r_2 = -2\sqrt{r_0} + \delta/\tilde{G}_*$

$$\Rightarrow \qquad \mathcal{P}_\mathcal{R} = \frac{4}{3}\,\tilde{G}_*\tilde{\Lambda}_*\,\delta$$

Tensor perturbations

Same approximation $\delta G = \delta\lambda_i = 0$

polarisation decomposition of tensor modes $h_{ij} = \sum_{\pi=+,\times} h_\pi \, {e^\pi}_{ij}$

$$h''_{\pi,\mathbf{p}} + \left(p^2 - \frac{\nu^2 - 1/4}{\tau^2}\right) h_{\pi,\mathbf{p}} = 0 \qquad , \quad \nu = \frac{3}{2} + \frac{1}{\alpha - 1}$$

as in standard power law, and power spectrum

$$\mathcal{P}_h \simeq \frac{32}{3} \tilde{G}_*^2 (2\tilde{\lambda}_0^* + \tilde{\lambda}_2^* \tilde{\phi}^2) \, (-p\tau)^{n_T}$$

where $n_T = -\frac{2}{\alpha - 1}$, and using the smallness parameter

$$\mathcal{P}_h \simeq \frac{16}{3} \sqrt{\frac{\tilde{G}_* \tilde{\Lambda}_* \tilde{\lambda}_4^*}{2\pi}} \, \delta \, (-p\tau)^{n_T}$$

so that tensor-to-scalar ratio

$$r = \frac{\mathcal{P}_h(p)}{\mathcal{P}_\mathcal{R}(p)} \simeq \sqrt{\frac{8\tilde{\lambda}_4^*}{\pi \tilde{G}_* \tilde{\Lambda}_*}} \, (-p\tau)^{n_T}$$

Breaking scale invariance

exact FP regime implies $n_s = 1$

maybe too strong approximation. . .

what if we consider linearised flow?

$$k \frac{d\tilde{g}_i}{dk} = M_{ij}(\tilde{g}_j - \tilde{g}_j^*)$$

extremely simplified toy model:

- ⋆ GRAVITY + SCALAR FIELD minimally coupled
- ⋆ GMFP with $\tilde{\lambda}_2^* = \lambda_4^* = 0$
- ⋆ negative critical exponents θ_2 and θ_4

[Narain & Percacci '10]

$$\mathcal{S}[g,\phi;k] = \int d^4x\sqrt{-g}\left[\frac{R-2\Lambda(k)}{16\pi G(k)} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi\right]$$

other assumptions:

⋆ imaginary part of θ neglected (assuming spiralling effect averaged to zero over several oscillations)

$$G(k) = k^{-2}\left[\tilde{G}_* + (\tilde{G}_{\rm f} - \tilde{G}_*)\left(\frac{k}{k_{\rm f}}\right)^{-\theta}\right]$$

$$\Lambda(k) = k^{2}\left[\tilde{\Lambda}_* + (\tilde{\Lambda}_{\rm f} - \tilde{\Lambda}_*)\left(\frac{k}{k_{\rm f}}\right)^{-\theta}\right]$$

⋆ a wise choice for boundary conditions $k_{\rm f}$ and $\tilde{g}_{\rm f} \equiv \tilde{g}(k_{\rm f})$

$$k_{\rm f} \to k_{\rm exit} \quad \Rightarrow \quad \tilde{g}_{\rm f} - \tilde{g}_* \simeq -\tilde{g}_*$$

$$
\begin{aligned}
\dot{H} &= \Lambda - 3H^2 \\
\Lambda' &= \frac{G'}{G}\left(\Lambda - 3H^2\right) \\
\dot{\phi}^2 &= -\dot{H}/4\pi G
\end{aligned}
$$

in the *exact* FP regime

$$H_0(t) = \alpha/t \qquad \phi_0(t) = \varphi/t \qquad k_0(t) = \chi/t$$

$$\alpha = \frac{2}{3} \qquad \varphi = \frac{1}{3\sqrt{\pi \tilde{G}_* \tilde{\Lambda}_*}} \qquad \chi = \sqrt{\frac{2}{3\tilde{\Lambda}_*}}$$

with the linearised flow

$$
\begin{aligned}
H_1(t) &= H_0(t) + \delta H(t) \\
k_1(t) &= k_0(t) + \delta k(t) \\
\phi_1(t) &= \phi_0(t) + \delta \phi(t)
\end{aligned}
$$

making the approximation $k \gg k_{\mathrm{f}}$ solutions read

$$\delta H(t) = -\frac{2^{\theta/2} 3^{\frac{\theta-2}{2}} \theta \, \tilde{\Lambda}_*^{\theta/2} k_{\mathrm{f}}^{\theta}}{1+\theta} \, t^{\theta-1}$$

$$\delta k(t) = -\frac{2^{-\frac{\theta+3}{2}} \, 3^{\frac{\theta-1}{2}} \, (\theta^2+\theta-2) \, \tilde{\Lambda}_*^{\frac{\theta-1}{2}} \, k_{\mathrm{f}}^{\theta}}{1+\theta} \, t^{\theta-1}$$

$$\delta \phi(t) = -\frac{2^{-\theta/2} 3^{\frac{\theta-2}{2}} \, \tilde{\Lambda}_*^{\frac{\theta-1}{2}} \, k_{\mathrm{f}}^{\theta}}{\sqrt{\pi G} \, (\theta^2-1)} \, t^{\theta-1}$$

remember $v_{\mathbf{p}}'' + \left(p^2 - \mathcal{C}(\tau)\right) v_{\mathbf{p}} = 0$

$$\mathcal{C}(\tau) = \frac{P_3\left(\tau^{3\theta}\right)}{\tau^2\, Q_3\left(\tau^{3\theta}\right)}$$

$$n_s - 1 = 2\left(3 - 2\sqrt{c(\tau) + 1/4}\right)$$

defining dimensionless parameter $\eta = (k_{\mathrm{f}} t)^{\theta}$

$$n_s \simeq 1 - \frac{2^{\frac{2-\theta}{2}}\, 3^{\frac{2+\theta}{2}}\, \theta\left(\theta^2 + \theta - 2\right) \tilde{\Lambda}_*^{\theta/2}}{1+\theta}\, \eta$$

spectral index decreases with time
being equal to unity only *at the singularity*

Conclusions & Perspectives

- Exact Renormalisation Group technique indicates that gravity may be asymptotically safe, even with the inclusion of matter fields, like the scalar field considered here
- Fixed point regime of the RG trajectory triggers a phase of power law inflation in the early universe dynamics, and then smoothly approaches classical dynamics at later times
- perturbations lying inside the RG length scale can be treated in the standard way and give predictions for the primordial power spectra, as functions of the fixed point values of the couplings

$\sim$

- Widening of the truncation and predictions based on the actual fixed points
- Effect of linearisation about the fixed point on the shape of power spectra
- Numerical analysis using the full β functions