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Trapped surfaces in a hadronic fluid

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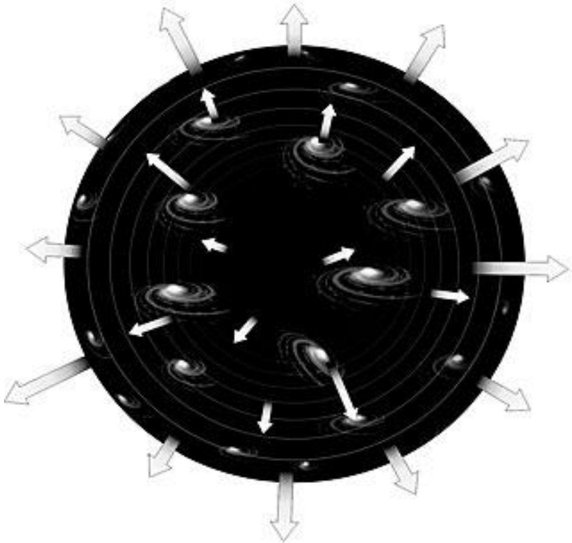
This talk is based on

- Neven Bilić and D. T., Phys. Lett. B 718 223 (2012)
  Analogue surface gravity near the QCD chiral phase transition
- Neven Bilić and D. T., arXiv:1210.3824
  Trapped surfaces in a hadronic fluid
- Neven Bilić, Class. Quantum Grav. 16 (1999)
  Relativistic Acoustic Geometry

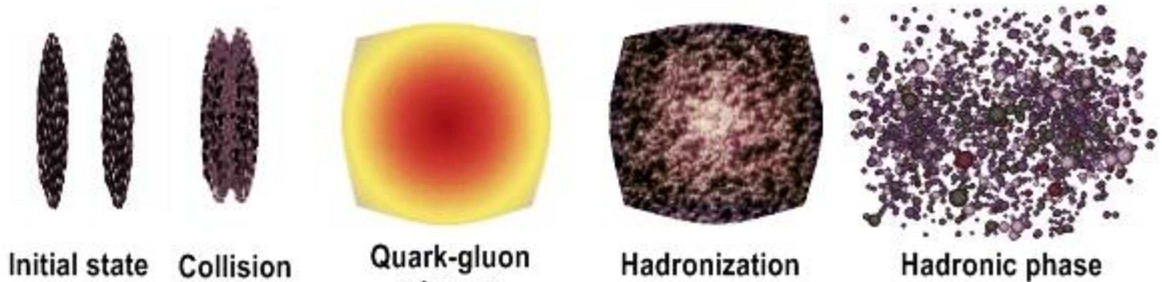
Outline

- Introduction
- Analogue gravity
- Chiral geometry
- Trapped surfaces
- Hawking radiation
- Speculations

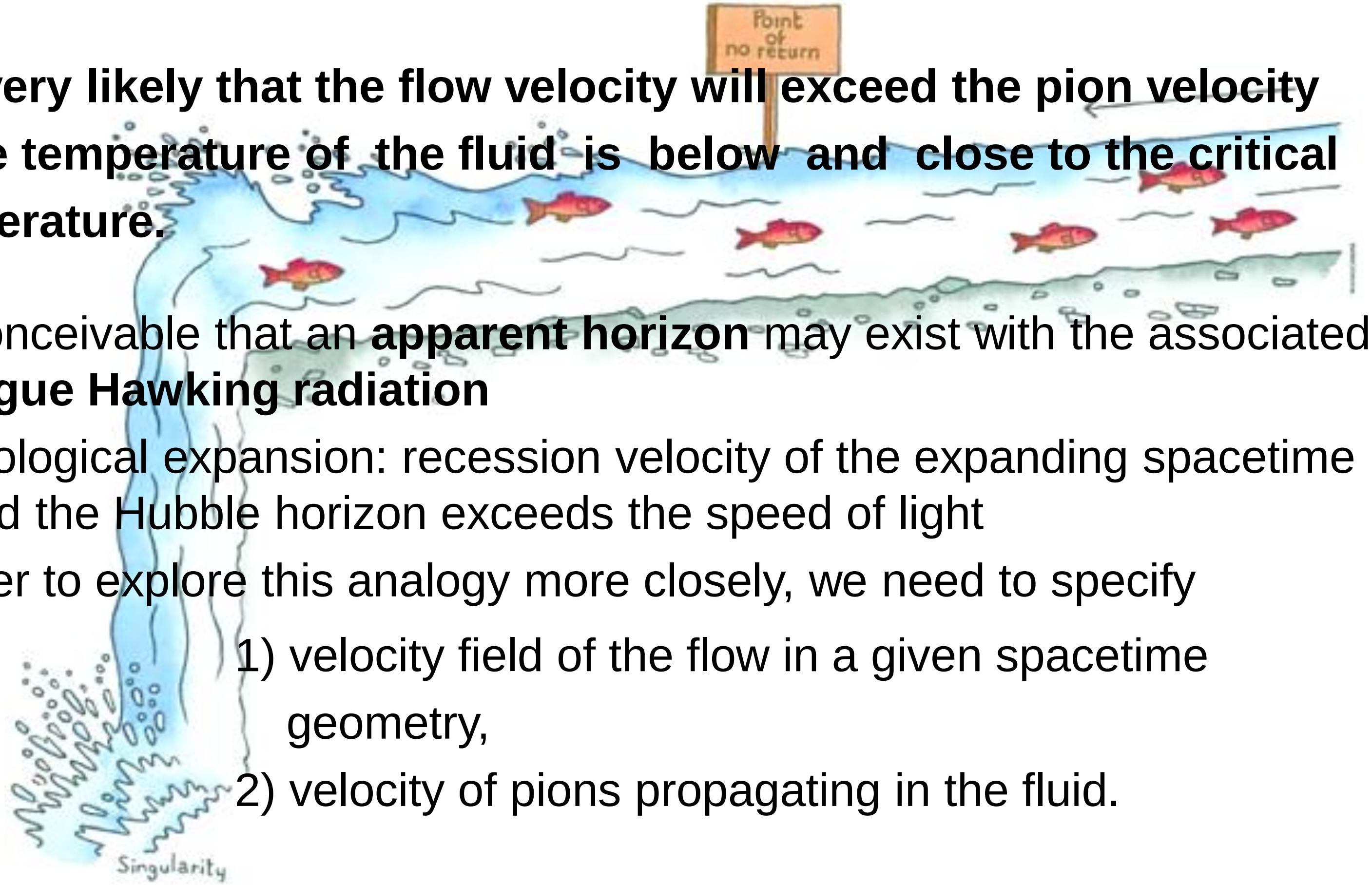
Introduction



- High-energy collisions: matter behaves as a nearly perfect expanding fluid
- Particle physicists: heavy ion collisions create mini Big Bangs
- Analogy: hydrodynamic expansion of a hadronic fluid and cosmological expansion where gravity plays the essential role
- Framework: effective analogue gravity with curved geometry
- In high energy collisions the produced particles are predominantly pions
- Propagation of massles pions provides a setting for a geometric analogue of expanding spacetime



- The velocity of the expanding hadronic fluid may be comparable with the speed of light
- Pions propagate slower than light, with velocity approaching zero at the critical temperature



It is very likely that the flow velocity will exceed the pion velocity if the temperature of the fluid is below and close to the critical temperature.

- It is conceivable that an **apparent horizon** may exist with the associated **analogue Hawking radiation**
- Cosmological expansion: recession velocity of the expanding spacetime beyond the Hubble horizon exceeds the speed of light
- In order to explore this analogy more closely, we need to specify

  1) velocity field of the flow in a given spacetime geometry,

  2) velocity of pions propagating in the fluid.

Analogue gravity

- Geometry and evolution of analogue spacetimes are determined by the equations of fluid mechanics, in contrast to general relativity where they are determined by the Einstein equations
- The basic equations of relativistic fluid dynamics are linearized introducing the small acoustic disturbances around average bulk motion
- The wave equation in such a fluid may be recast in the form of a Klein-Gordon equation in curved spacetime

$$\frac{1}{\sqrt{-G}}\partial_\mu\left(\sqrt{-G}G^{\mu\nu}\right)\partial_\nu\varphi = 0$$

with the acoustic metric tensor given by

$$G_{\mu\nu} = k\left(g_{\mu\nu} - (1 - c_s^2)u_\mu u_\nu\right)$$

N. Bilic, *Relativistic Acoustic Geometry,* Class. Quantum Grav. 16 (1999)

- In the simplest case of a homogenous flow in flat spacetime we get the wave equation

$$\left(\partial_t^2 - c_s^2\nabla^2\right)\varphi = 0.$$

- Horizons and Hawking radiation are generic features of curved spacetimes and QFT in curved spacetime, not necessarily related with gravity per se

Chiral geometry

- Chirally symmetric Lagrangian at finite temperature in curved spacetime

$$\mathcal{L} = \frac{1}{2}\left(a g^{\mu\nu} + b u^\mu u^\nu\right)\partial_\mu \varphi \partial_\nu \varphi - \frac{m_0^2}{2}\varphi^2 - \frac{\lambda}{4}(\varphi^2)^2$$

Linear sigma model

- Parameters a, b depend on the local temperature T, and the parameters of the model m_0 and λ, and may be calculated in perturbation theory

R. D. Pisarski , M. Tytgat, Phys. Rev. D 54 (1996)
D. T. Son, M. A. Stephanov, Phys. Rev. Lett. 88 (2002)
N. Bilic and H. Nikolic, Phys. Rev. D 68 (2003)

- In the chirally broken phase (below T_c) the pions are massless, whereas the quarks and sigma meson acquire a nonzero mass proportional to the chiral condensate $\langle\sigma\rangle$
- In the chirally restored phase (above T_c) the condensate $\langle\sigma\rangle$ vanishes and the pions acquire mass
- The dynamics of pions below T_c may be written in the form

$$\frac{1}{\sqrt{-G}}\partial_\mu\left(\sqrt{-G}G^{\mu\nu}\right)\partial_\nu \pi + \frac{c_\pi^2}{a}V\left(\sigma,\pi\right)\pi = 0$$

where the quantity c_π is the pion velocity: $c_\pi^2 = \left(1 + \frac{b}{a}\right)^{-1}$

- The analog metric tensor is given by

$$G_{\mu\nu} = \frac{a}{c_\pi}\left(g_{\mu\nu} - (1-c_\pi^2)u_\mu u_\nu\right)$$

Relativistic acoustic geometry

- Depends locally on the 4-velocity of the fluid and the pion velocity
- The background metric is (spacially) flat Minkowski spacetime

Bjorken expansion

- We consider a boost invariant spherically symmetric Bjorken type expansion

$$u^\mu = \left(t/\tau, r/\tau, 0, 0\right)$$

where $\tau = \sqrt{t^2 - r^2}$; in coordinates $\left(t, r, \theta, \varphi\right)$

- With the substitution $t = \tau\cosh y, \quad r = \tau\sinh y$

the velocity components become $u^\mu = \left(1, 0, 0, 0\right)$

- In coordinates $\left(\tau, y, \theta, \varphi\right)$ the flat background metric takes the form

$$g^{\mu\nu} = diag\left(1, -\tau^2, -\tau^2\sinh^2 y, -\tau^2\sinh^2 y\sin^2\theta\right)$$

Expanding spacetime!

- Spacially flat Minkowski spacetime is mapped to the expanding Milne universe (still flat)

- The functional dependence of T on τ follows from the energy-momentum conservation $T^{\mu\nu}{}_{;\nu} = 0$
- For $T^{\mu\nu} = (p+\rho)u_\mu u_\nu - pg_{\mu\nu}$ we find $\rho = \rho_0 \left(\frac{\tau_0}{\tau}\right)^4$
- This expression combined with the density of massless boson gas

$$\rho = g\frac{\pi^2}{30}T^4 \Rightarrow T = T_0\frac{\tau_0}{\tau}$$

- Analogue metric tensor depends on the spacetime coordinates through the temperature dependence of the parameters of the chiral model
- The analogue metric tensor is

$$G_{\mu\nu} = \frac{a}{c_\pi} diag\left(c_\pi^2, -\tau^2, -\tau^2 \sinh^2 y, -\tau^2 \sinh^2 y \sin^2\theta\right)$$

Analogue cosmology

- Pions propagate in **expanding curved** (3+1) spacetime!
- For a relativistic flow in curved spacetime the apparent and trapping horizons may be defined in the same way as in general relativity
- In expanding fluid the flow is essentially time dependent
- The acoustic geometry formalism must be adapted to a non-stationary spacetime

Trapped surfaces

- A **trapped region** may form with the boundary resembling a black hole horizon if there exists a region where the flow velocity exceeds the pion velocity
- The key element in the study of trapped surfaces is the expansion parameter of null-geodesics

$$\varepsilon_{\pm} \equiv \nabla_{\mu} l_{\pm}^{\mu}$$

  which meassures how light rays expand or converge.
- A 2-dimensional surface $S \subset \Sigma$ with spherical topology is called a *trapped surface* on Σ if the families of ingoing and outgoing null geodesics normal to the surface are both converging or both diverging.
- Equivalently**,** the null expansions on S should satisfy $\varepsilon_+ \varepsilon_- > 0$
  - Past trapped surface: both ε_+ and ε_- are positive
  - Future trapped surface: both ε_+ and ε_- are negative
- A 2-dimensional surface H is said to be *marginally trapped* if one of the null expansions vanishes on H: either $\varepsilon_+|_H = 0$ or $\varepsilon_-|_H = 0$

Apparent horizon!

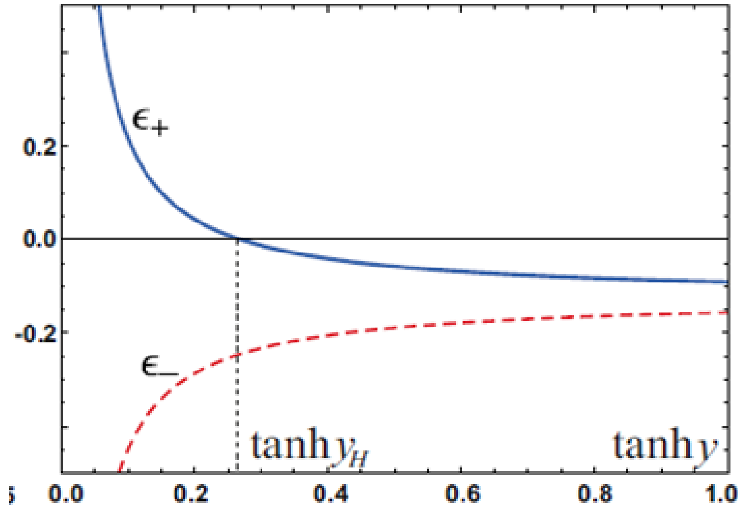
- **Future outer** marginally trapped surface is the outer boundary of a future trapped region and is typical of a black hole.
- **Past outer** marginally trapped surface is the outer boundary of a past trapped region and is typical of a white hole.
- **Past inner** marginally trapped surface is the inner boundary of a past trapped region and represents an "outer" black hole. This situation is physically relevant in the context of an expanding FRW universe.
- **Future inner** marginally trapped surface is the inner boundary of a future trapped region and represents an "outer" white hole. This situation is physically relevant in the cosmological context for a contracting FRW universe.

- Geodesic equation $l^\mu \nabla_\mu l^\nu = 0$ gives future directed in/outgoing radial null geodesics:

$$l^\mu_\pm = \frac{1}{a\tau}\left(1, \pm\frac{c_\pi}{\tau}, 0, 0\right)$$

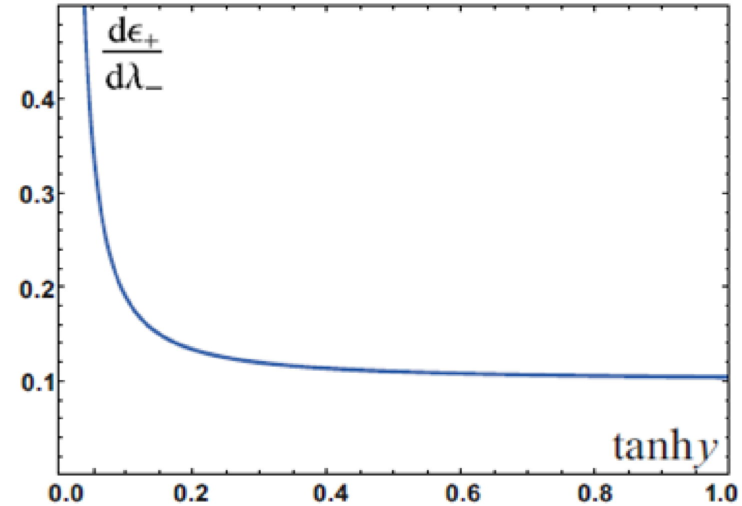
- Null expansions

$$\varepsilon_\pm = \frac{2}{a\tau^2}\left(\frac{\partial_\tau\left(\tau\sqrt{a/c_\pi}\right)}{\sqrt{a/c_\pi}} \pm \frac{c_\pi}{v}\right)$$



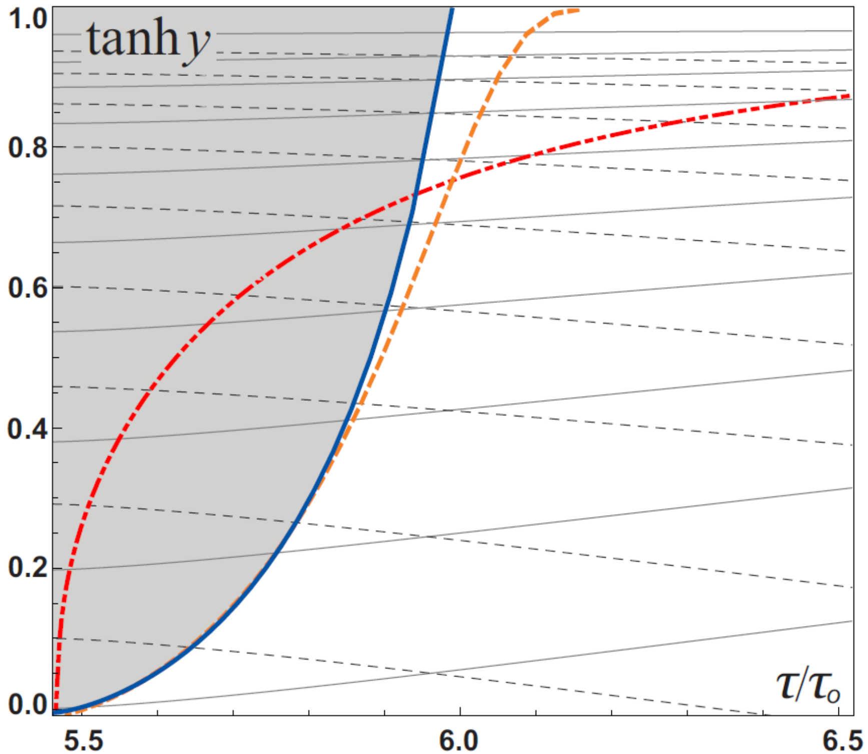
- The condition for the apparent horizon is

$$\frac{c_\pi}{v} \pm \frac{\partial_\tau\left(\tau\sqrt{a/c_\pi}\right)}{\sqrt{a/c_\pi}} = 0$$



- Analog marginally trapped surface
- Future trapped region: a contracting FRW spacetime with k = -1

- Spacetime diagram



Apparent horizon, Hubble horizon, naive horizon

$$v = c_\pi$$

$$v_{rec} = y\frac{\partial_\tau\left(\sqrt{\frac{a}{c_\pi}}\right)}{\sqrt{ac_\pi}}$$

$$\coth y \pm \frac{\partial_\tau\left(\tau\sqrt{a/c_\pi}\right)}{c_\pi\sqrt{a/c_\pi}} = 0$$

- Analogue apparent horizon may be regarded as an „outer" **white hole**: the ingoing pions freely cross the apparent horizon whereas the outgoing cannot cross it
- This is opposite to an expanding FRW universe where the inner marginally trapped surface acts as a **black hole**

Hawking radiation

- One immediate effect related to the apparent horizon is the Hawking radiation
- In the case of non-stationary spacetime, the definition of surface gravity is not unique *G. Fodor, K. Nakamura, Y. Oshiro, and A. Tomimatsu, Phys. Rev. D 54 (1996)*
  S. Mukohyama and S. Hayward, Class. Quant. Grav. 17 (2000)
  A. Nielsen and J. H. Yoon, Class. Quant. Grav. 25 (2008)
- We use the Hayward-Kodama prescription which we have adapted to analog gravity *S. A. Hayward, Class. Quant. Grav. 15 (1998)*

$$K^\alpha \nabla_{[\alpha} K_{\beta]} = \kappa K_\beta$$

- Kodama vector generalizes the concept of the time translation Killing vector to a non-stationary spacetime

$$K^\alpha = k\epsilon^{\alpha\beta} n_\beta$$

$\epsilon^{\alpha\beta}$ - *2D Levi Civita tensor in the space normal to the surface of spherical symmetry*

n_α - *vector normal to that surface*

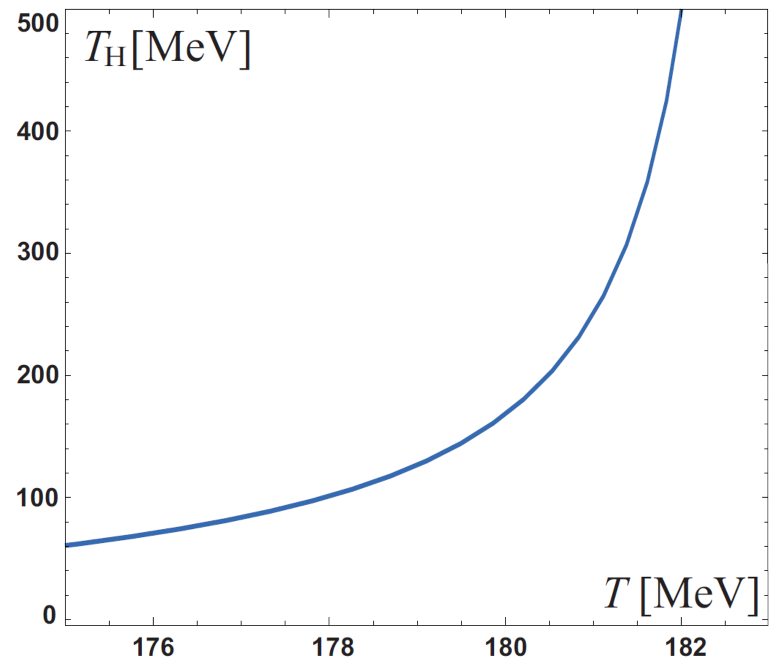
- Works only for spherical symmetry
- We introduced a normalization factor k in order to meet the requirement that K should coincide with the time translation Killing vector ξ for a stationary geometry

$$\xi^\nu \nabla_\nu \xi_\mu = \kappa \xi_\mu$$

- Our result: The surface gravity diverges at the critical point at which the pion velocity vanishes!
- The analog Hawking temperature of thermal pions emitted at the apparent horizon

$$T_H = \frac{\kappa}{2\pi}$$

- Arbitrary large in the limit when the analogue horizon approaches the critical point
- The usual general relativistic Hawking effect: the Hawking temperature is tiny compared with the background temperature
- In principle, one could measure the signals for the analog Hawking effect in a hadronic fluid

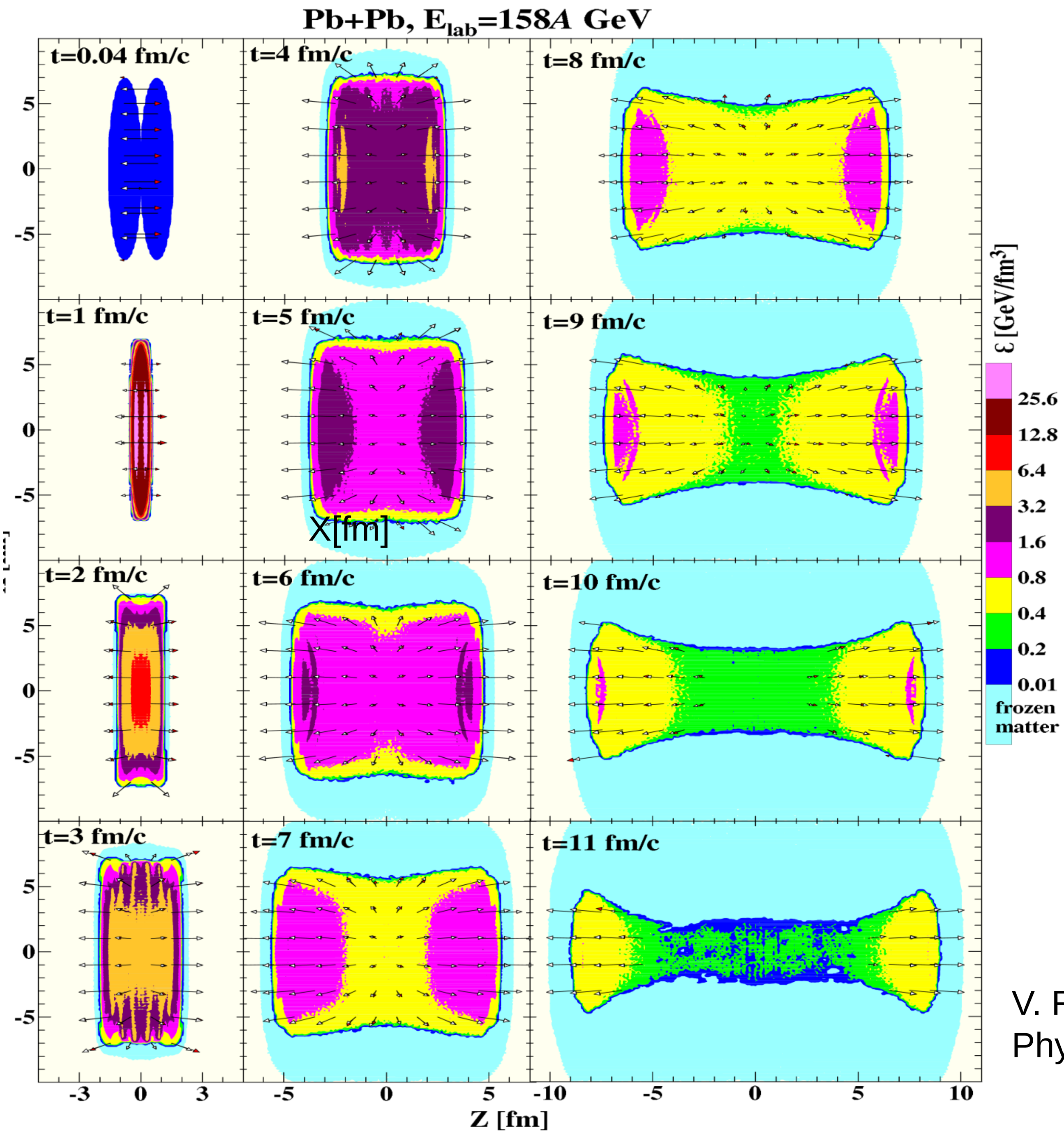


Analog Hawking temperature as a function of the fluid temperature T

Speculations?

- A spherically symmetric expansion model considered here is not realistic for high energy heavy ion collisions
- A more realistic hydrodynamic model would involve a transverse expansion superimposed on a longitudinal boost invariant expansion
- In this case the calculations become rather involved as the formalism for general non-spherical spacetimes is not yet fully developed
- This work is in progress

Thank you!



V. Russkikh, Yu. Ivanov
Phys.Rev.C76 (2007)