Does Dark Energy Exist?

Timothy Clifton
(Queen Mary, University of London)

Based on the following work with Philip Bull:

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Nobel Prize in Physics 2011

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

Perlmutter

Schmidt

Riess

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What does this mean?

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

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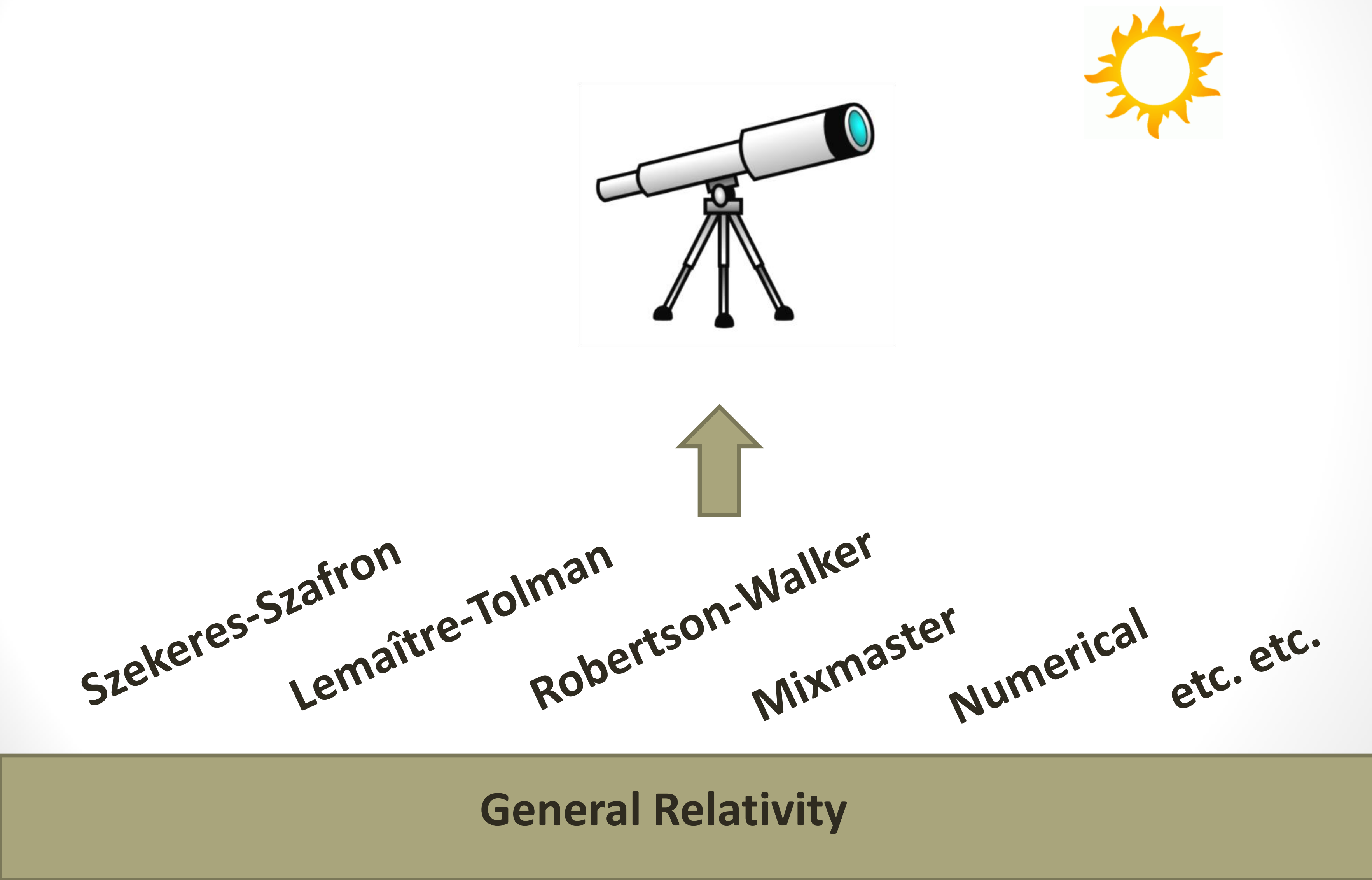
Bottom-up approach to Cosmology

Bottom-up approach to Cosmology

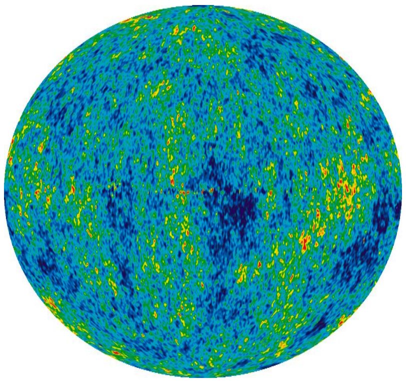
Szekeres-Szafron
Lemaître-Tolman
Robertson-Walker
Mixmaster
Numerical
etc. etc.

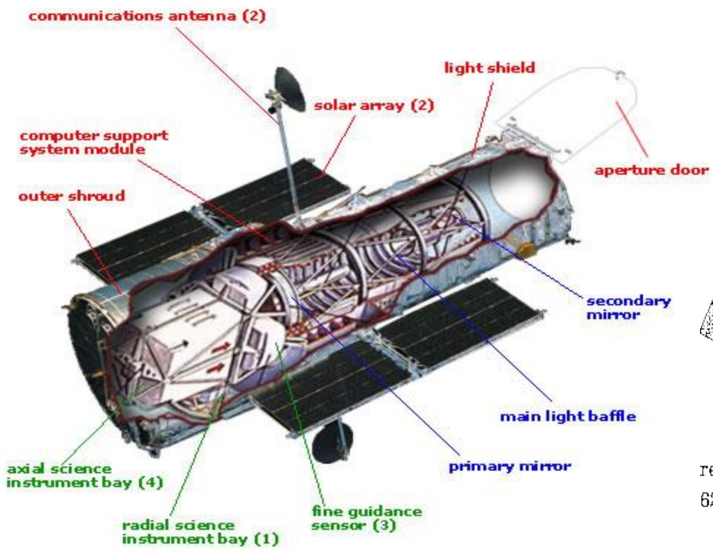
General Relativity

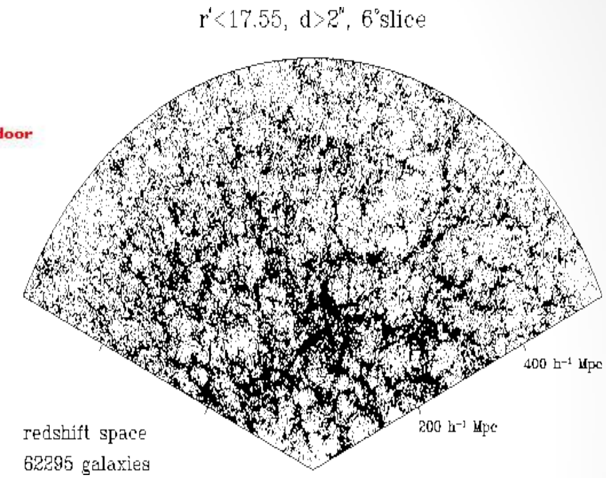
Bottom-up approach to Cosmology



Top-down approach to Cosmology







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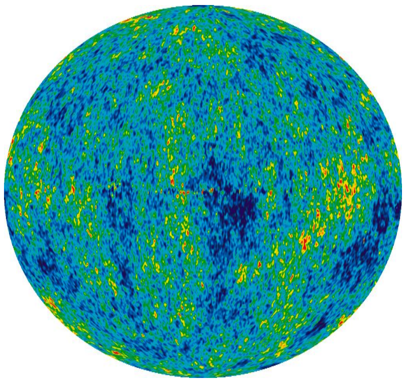
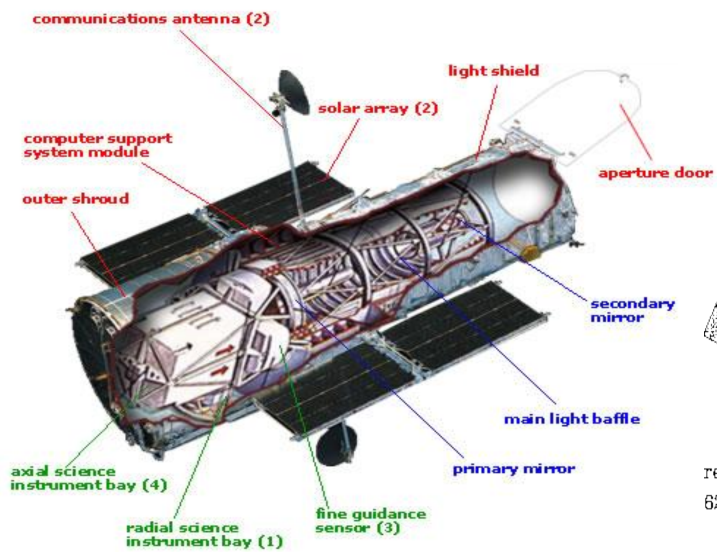
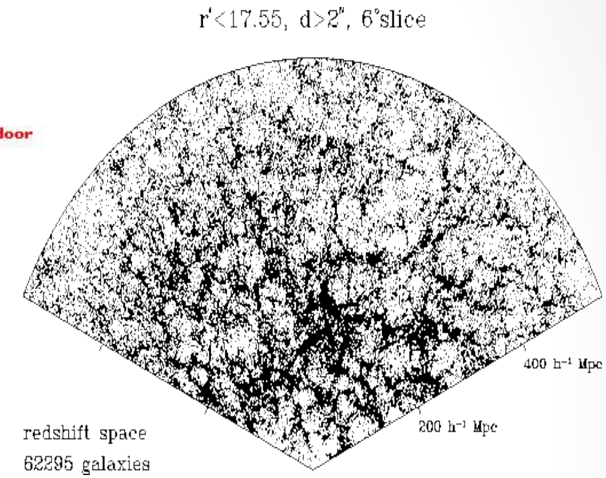
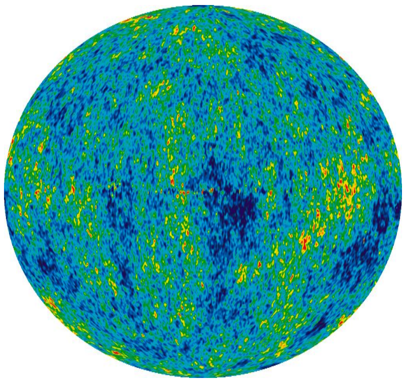


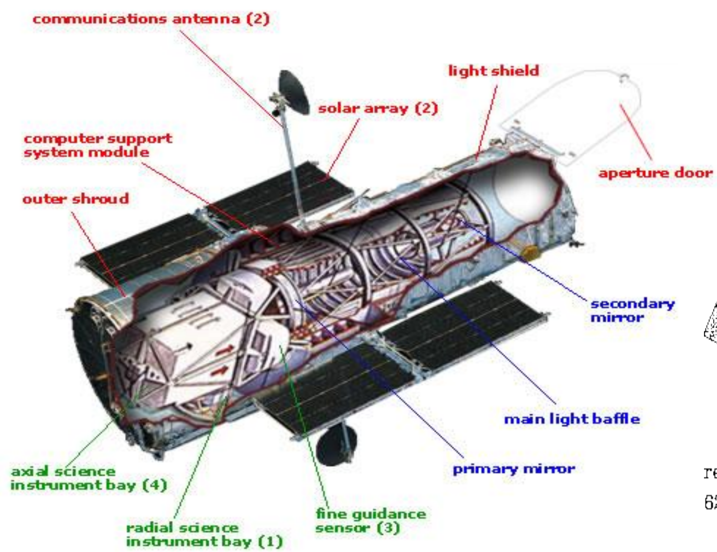


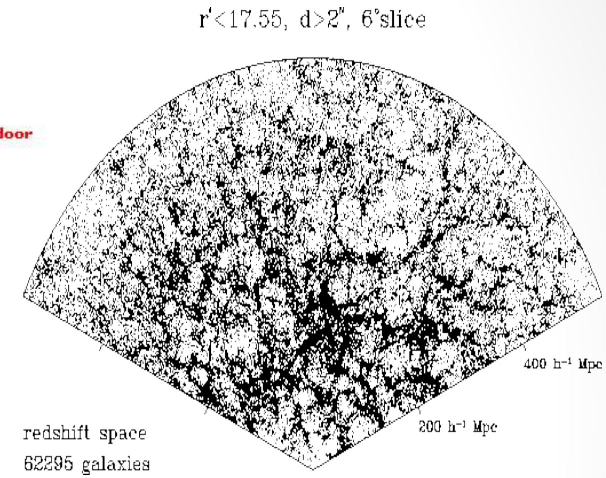


Robertson-Walker

Top-down approach to Cosmology



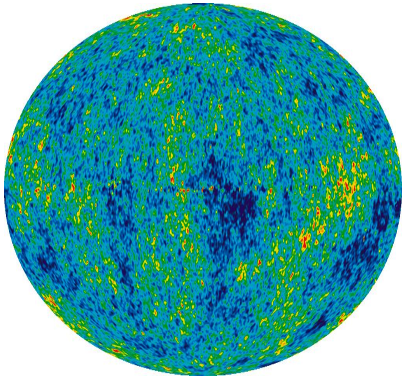


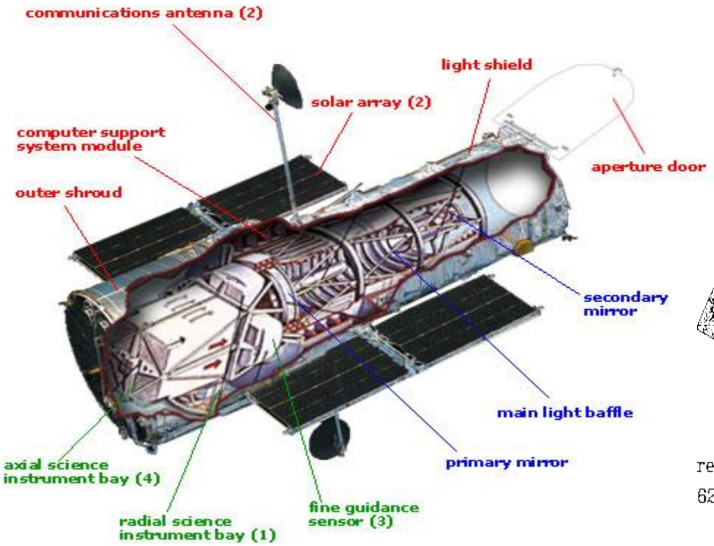


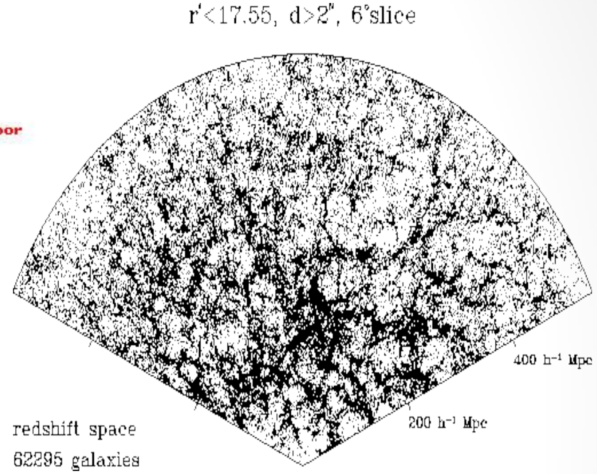
Robertson-Walker

GR?

Ideal approach to Cosmology







Szekeres-Szafron **Lemaître-Tolman** **Robertson-Walker** **Mixmaster** **Numerical** **etc. etc.**

General Relativity

Cosmology without Friedmann

Possible approaches:

1. Explicitly constructing inhomogeneous cosmological models sophisticated enough to model aspects of the real Universe
2. Constructing frameworks within which the large-scale behaviour of space can be interpreted (Buchert)
3. Developing techniques for averaging the geometry of spacetime over cosmological scales (Zalaletdinov)

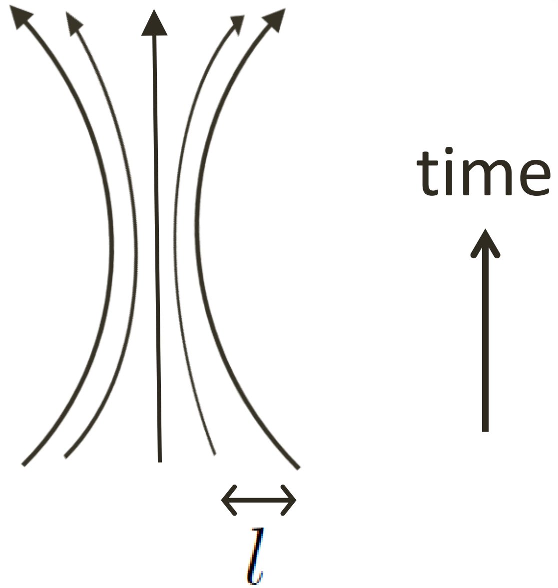
Cosmology without Friedmann

Possible difficulties:

1. Standard cosmological observables that are uniquely defined in FLRW models may not have unique values in general
2. The large-scale, or average, behaviour of spacetime may not correspond to the average of observables
3. Relations between observables and the matter content of the Universe may not be straightforward

Local expansion and acceleration

- Consider the world-lines of a number of neighbouring particles:



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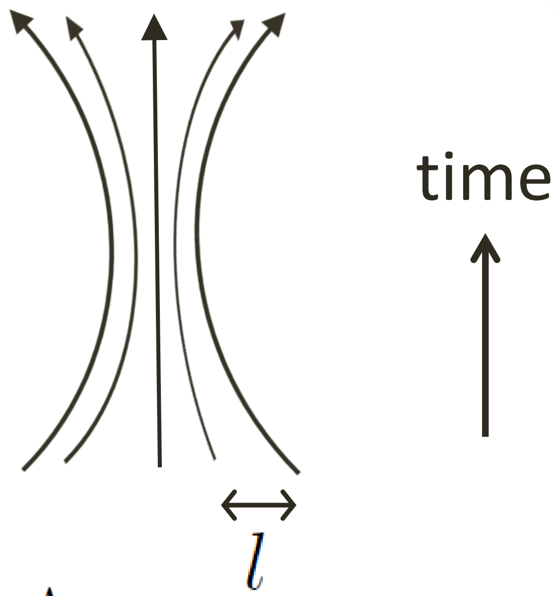
- The expansion and acceleration of the distance between curves are:



$$H^2 \equiv \frac{\dot{l}^2}{l^2} = \frac{8\pi G}{3}(\rho + \sigma^2) - \frac{1}{6}\,{}^{(3)}R + \frac{\Lambda}{3}$$

and

$$\frac{\ddot{l}}{l} = -\frac{4\pi G}{3}(\rho + 3p + 4\sigma^2) + \frac{\Lambda}{3}$$

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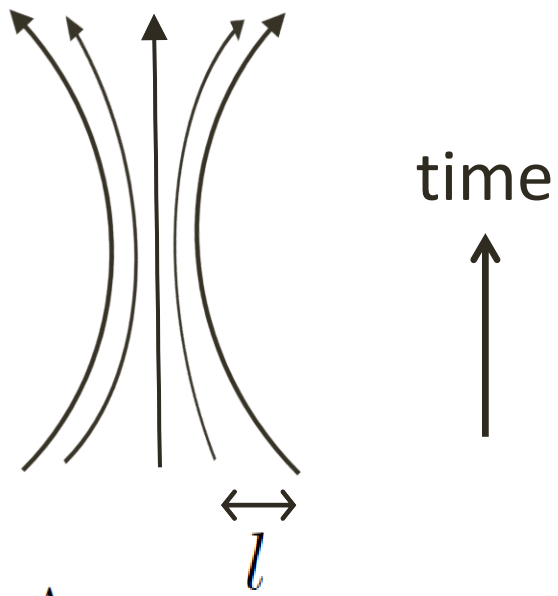

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negative, for normal matter

Non-local expansion and acceleration

- Define the scale factor of an extended region of space:

$$a(t) \equiv \left(\frac{\int d^3x \sqrt{^{(3)}g(t, \bar{x})}}{\int d^3x \sqrt{^{(3)}g(t_0, \bar{x})}} \right)^{\frac{1}{3}}$$

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- Now consider a model universe with 2 different types of region. The expansion of an extended region of this model is:

$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^3}{a_1^3 + a_2^3} H_1 + \frac{a_2^3}{a_1^3 + a_2^3} H_2 = \nu_1 H_1 + \nu_2 H_2$$

The acceleration, however, is:

$$\frac{\ddot{a}}{a} = \nu_1 \frac{\ddot{a}_1}{a} + \nu_2 \frac{\ddot{a}_2}{a} + 2\nu_1\nu_2 (H_1 - H_2)^2$$

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positive!

Problems

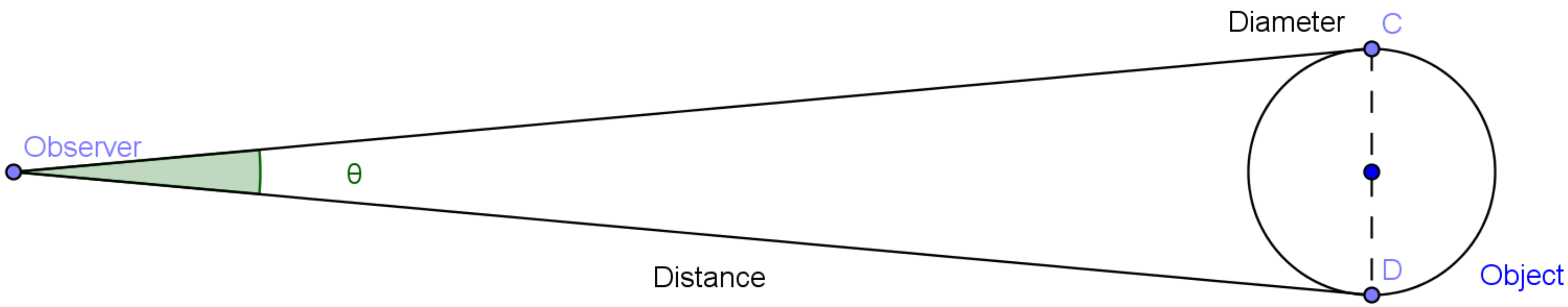
1. Acceleration of finite regions ≠ average acceleration of points within

2. The strong energy condition ($\rho+3p\geq0$) implies *only* that the local expansion rate of space is decelerating

3. Neither of these measures of acceleration are necessarily what is inferred from supernova data

Observations without Friedmann

Angular diameter distance: $$d_A \equiv \frac{CD}{\Theta}$$



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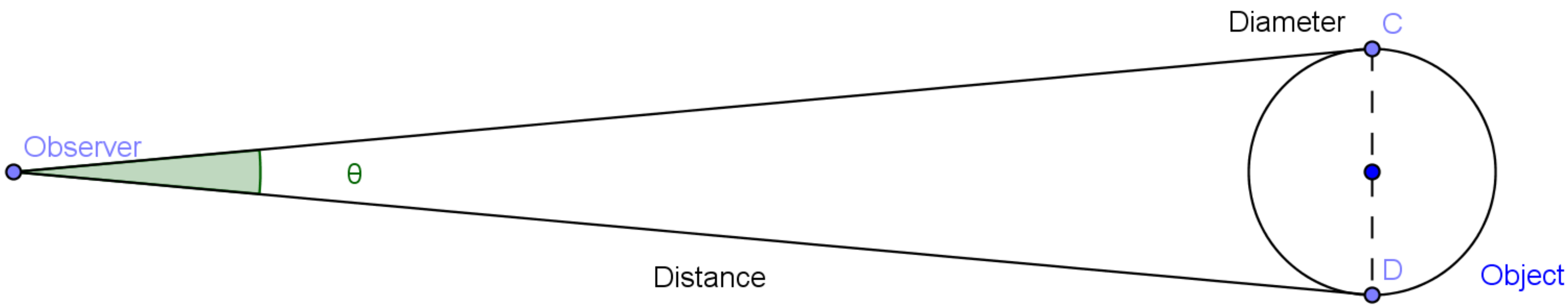
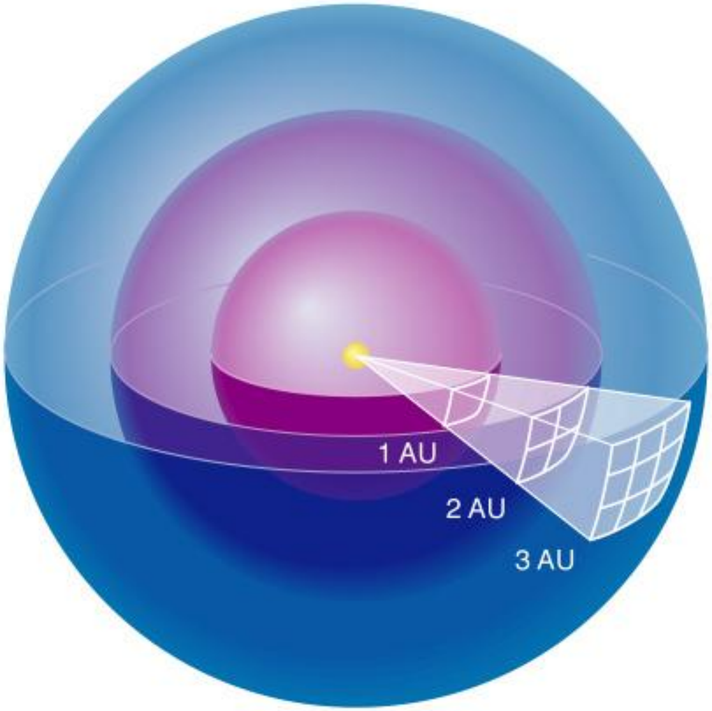


$$\frac{d^2(d_A)}{d\lambda^2} = -d_A\left(|\hat{\sigma}|^2 + \frac{1}{2}R_{ab}k^a k^b\right)$$

Redshift: $$\frac{dz}{d\lambda} = -(1+z)^2 H_{||}(z)$$

Observations without Friedmann

Luminosity distance: $$d_L \equiv \sqrt{\frac{L}{4\pi B}}$$



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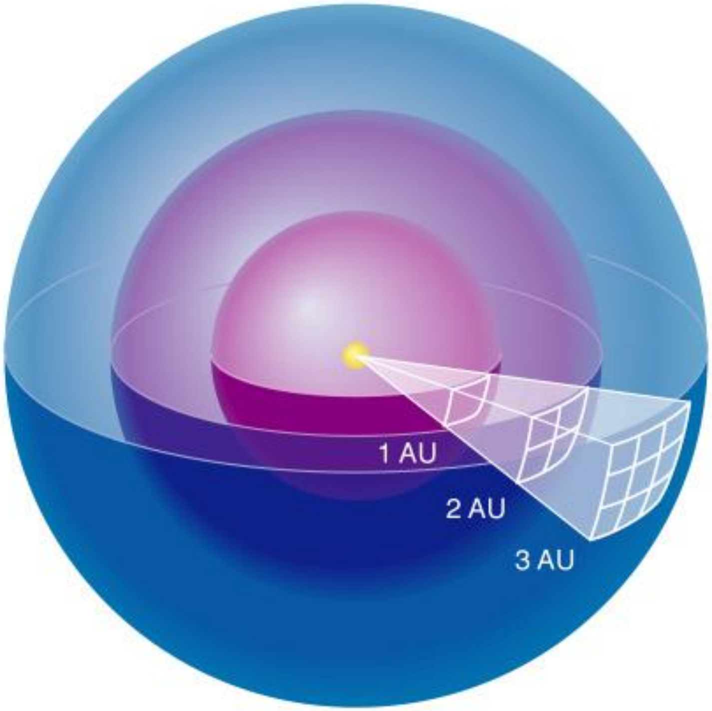

$$d_L = (1+z)^2 d_A$$

Observations without Friedmann

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First, expand the FLRW distance relations:

$$d_A(z) = \frac{cz}{H_0}\left(1 - \frac{1}{2}(3 + q_0)z + \mathcal{O}(z^2)\right),$$

where $q_0 \equiv -(a\ddot{a}/\dot{a}^2)|_0$.

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$$q_0 = -\left.\frac{d_A''}{d_A'}\right|_0 - 3\,.$$

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$$q_0 = -\left.\frac{d''_A}{d'_A}\right|_0 - 3\,.$$

is this still measuring deceleration?

Local observations

Expanding the expressions above around a point gives:

$$d_A = \frac{z}{[K^a K^b \nabla_a u_b]_0} \left(1 - \left[\frac{K^a K^b K^c \nabla_a \nabla_b u_c}{2(K^d K^e \nabla_d u_e)^2}\right]_0 z + \mathcal{O}(z^2)\right)$$

where $K^a = \frac{k^a}{[u_b k^b]_0} = -u^a + e^a$, and where u^a and e^a are unit vectors.

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where $K^a = \frac{k^a}{[u_b k^b]_0} = -u^a + e^a$, and where u^a and e^a are unit vectors.

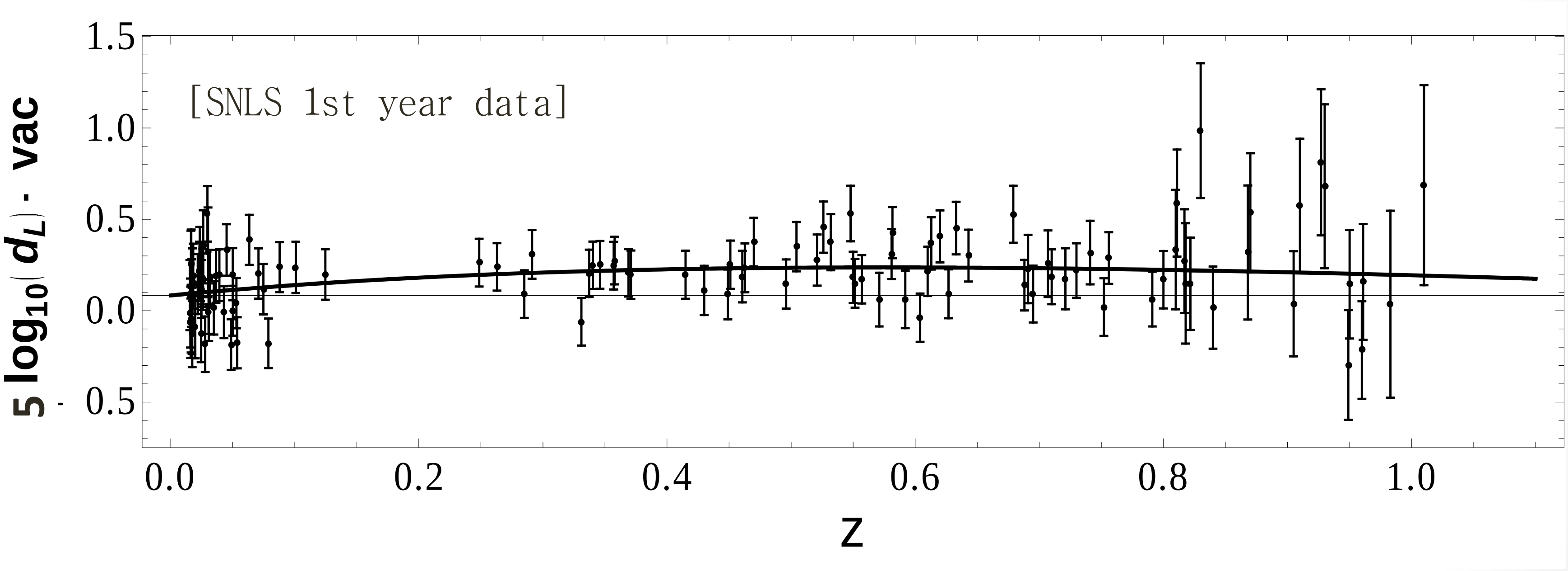
Taking the monopole gives:

$$q_0 = \frac{1}{H_0^2} \left[\frac{4\pi G}{3}(\rho + 3p + 12\sigma^2) - \frac{\Lambda}{3}\right]_0$$

This is the deceleration parameter inferred by taking derivatives of distance measures at the location of the observer.

Non-local observations

Rather than taking the deceleration inferred from $d_A(z)$ at every point in space, we can instead fit a smooth curve to all available observations:



The value of q_0 can then be derived from the gradient of this curve at z=0.

This is what observers do.

Summary so far

- We have 4 different measures of acceleration:
    1. Locally measured acceleration of particles away from each other
    2. Non-local expansion of a finite region of space
    3. Acceleration inferred from distance relations at a point
    4. Acceleration inferred from averaging observations made of over large scales (what is actually done in the case of supernova observations)
- In a single space-time it is possible to infer 4 different values for the acceleration of the expansion
- It matters (in all cases) if we consider the 'average of the acceleration', or the 'acceleration of the average'. These two concepts are *not* the same
- In only two of these measures does there exist any direct relation between the Strong Energy Condition and acceleration (the locally defined ones)
- In a Friedmann universe all of these measures coincide

Example cosmological models

1. Spherical Collapse Model.
   - (i) Prescribe alternating regions of dust (k=+1) and vacuum (k=-1).
   - (ii) Let each region have FLRW geometry:

$$ds^2 = dt^2 - \frac{a^2\left(dX^2 + dY^2 + dZ^2\right)}{[1 + \frac{k}{4}(X^2 + Y^2 + Z^2)]^2} \ .$$

2. Plane Symmetric Model.
   - (i) Prescribe alternating regions of dust dominated (k=0) FLRW and vacuum anisotropic.
   - (ii) Let the vacuum regions have Kasner geometry:

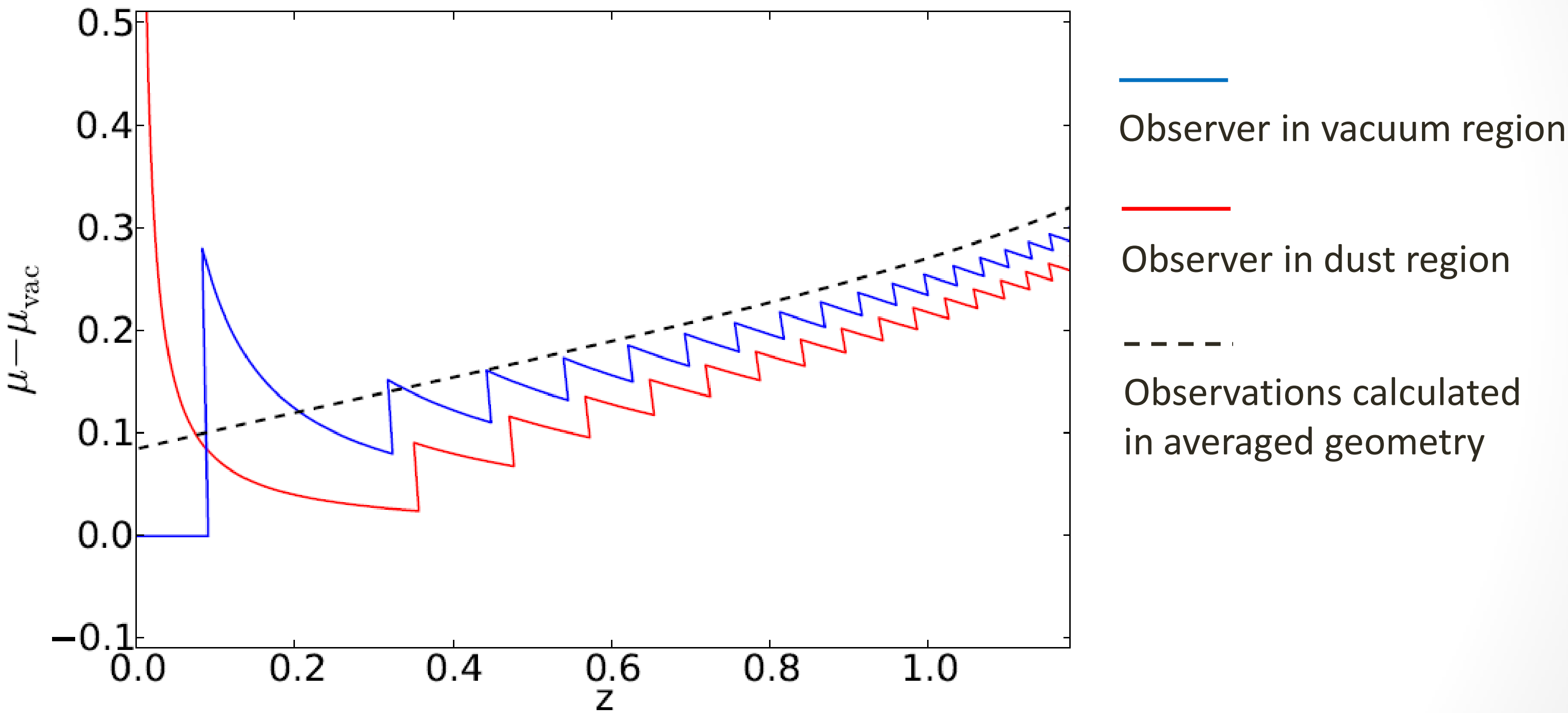
$$ds^2 = -d\hat{t}^2 + b_1^2(\hat{t})d\hat{X}^2 + b_2^2(\hat{t})\left(d\hat{Y}^2 + d\hat{Z}^2\right), \qquad b_1 = a^{-1/2}, \quad b_2 = a\,.$$

3. Spherically Symmetric Model.
   - (i) Let the entire space-time be filled with dust.
   - (ii) The geometry is LTB:

$$ds^2 = dt^2 - \frac{a_2^2(t,r)}{(1 - k(r)r^2)}dr^2 - a_1^2(t,r)r^2 d\Omega^2 \ , \qquad a_2 = (a_1 r)'.$$

Spherical Collapse Model

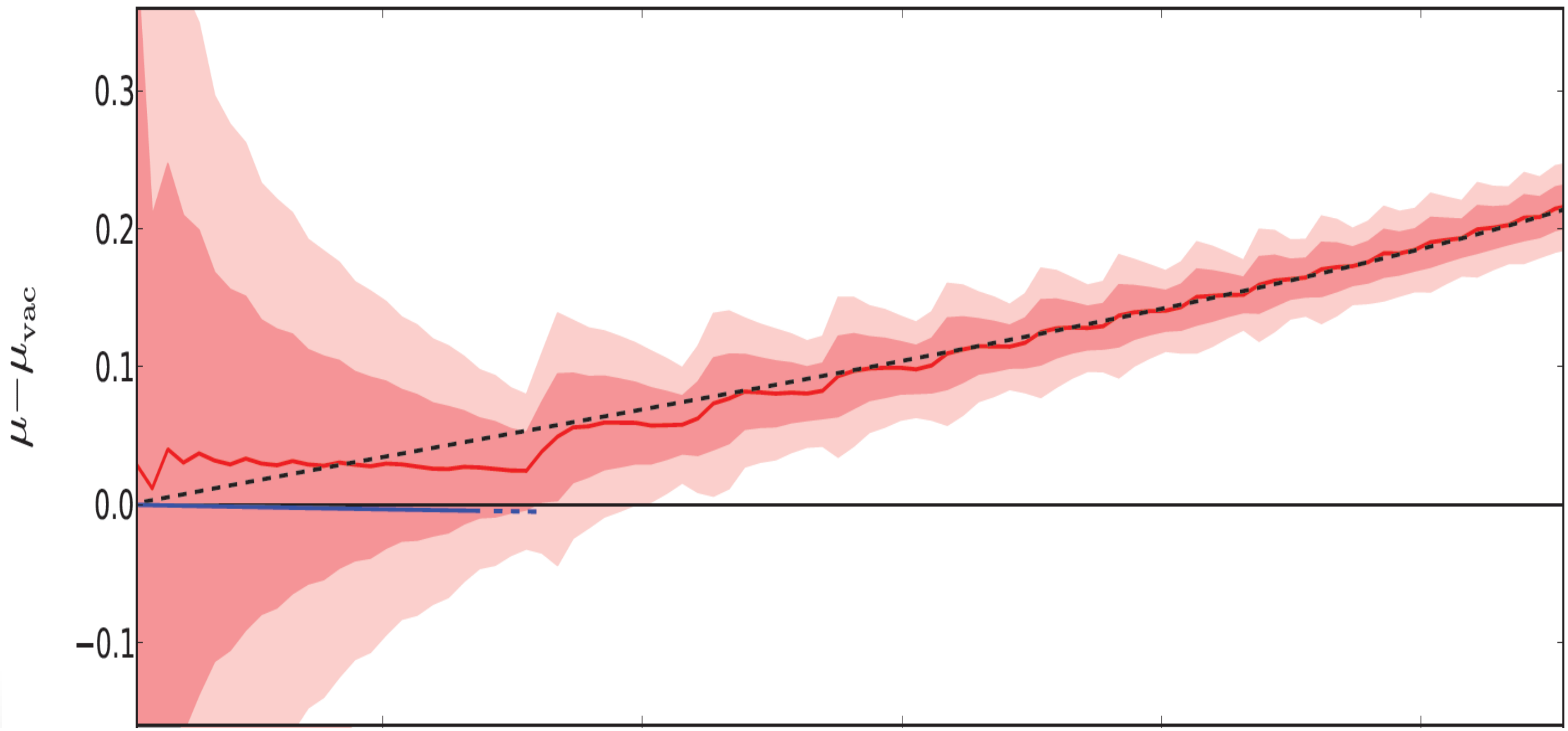
Consider the line-of-sight observations made by single observers, in one direction:



Here:
i. The vacuum and dust regions are 80Mpc and 15Mpc wide, respectively
ii. The vacuum regions have h=0.7, and the dust regions have h=-0.2
iii. The dust regions are 80% over-dense

Spherical Collapse Model

Averaging over many different lines-of-sight:



$$q_{non-local} \approx -0.167 \quad \& \quad q_{local} \approx 0.017$$

Plane Symmetric Model

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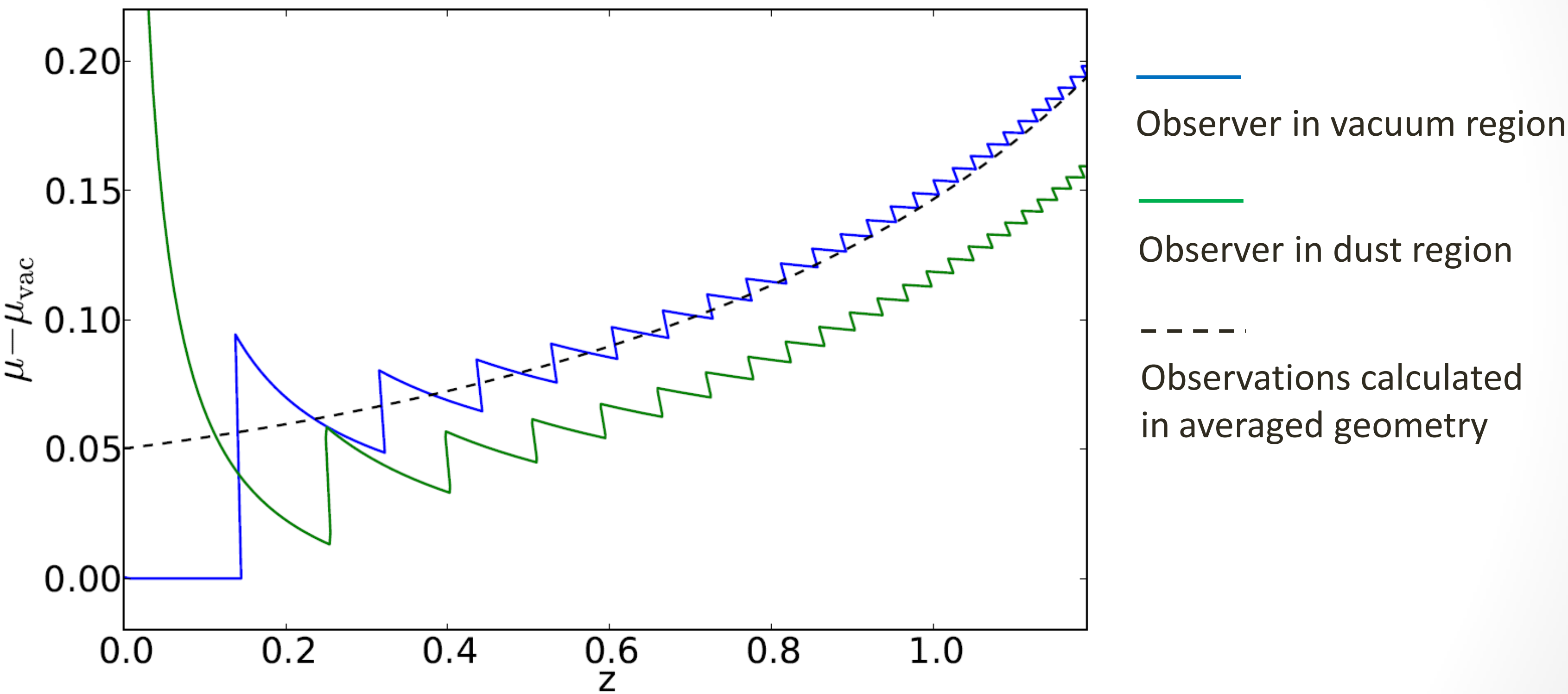
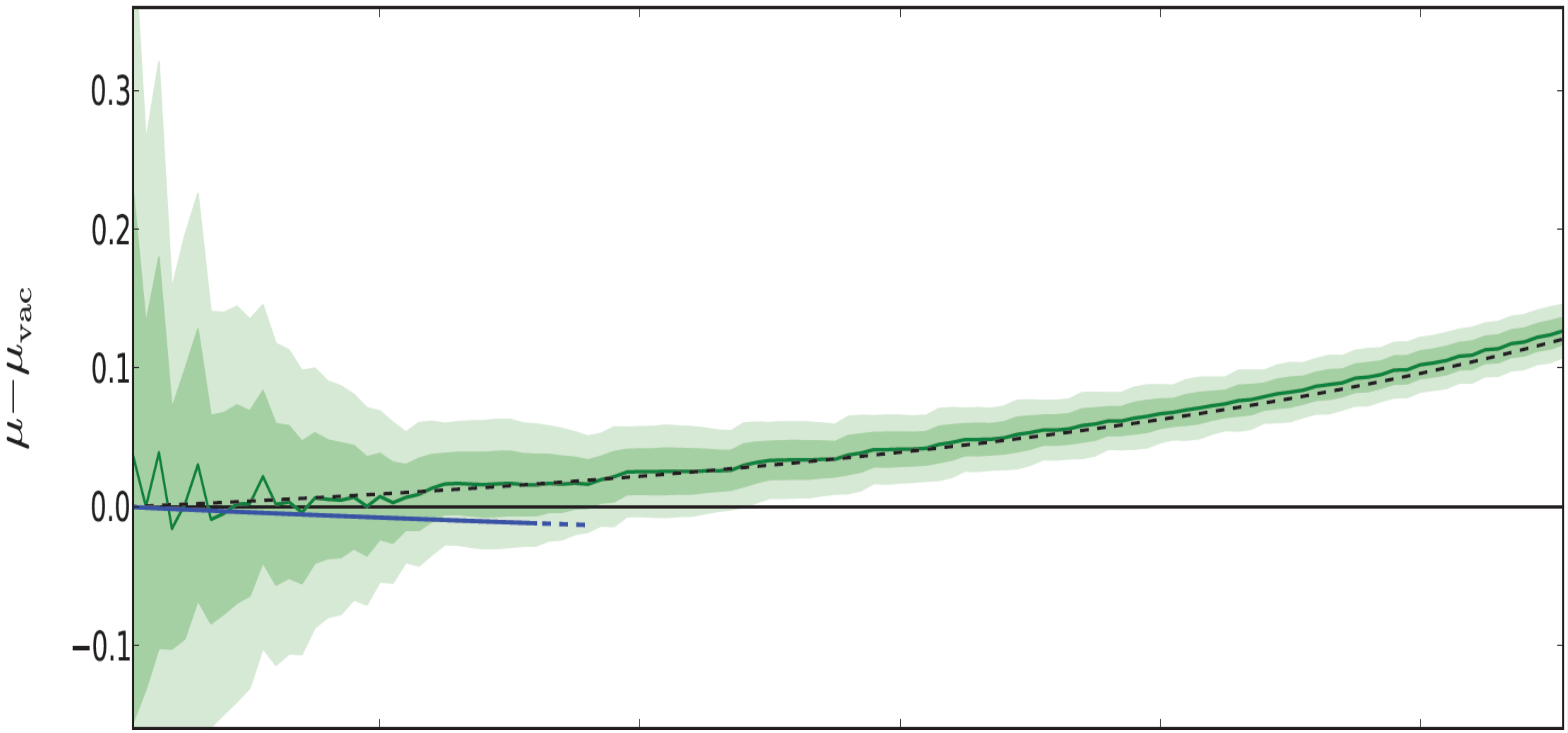


Here:
i. The vacuum and dust regions are 1500Mpc and 15Mpc wide, respectively
ii. The vacuum regions have h=0.7 along the line-of-sight, and in the dust region h=-1.4
iii. The dust collapse completely in 5 billion years

Plane Symmetric Model

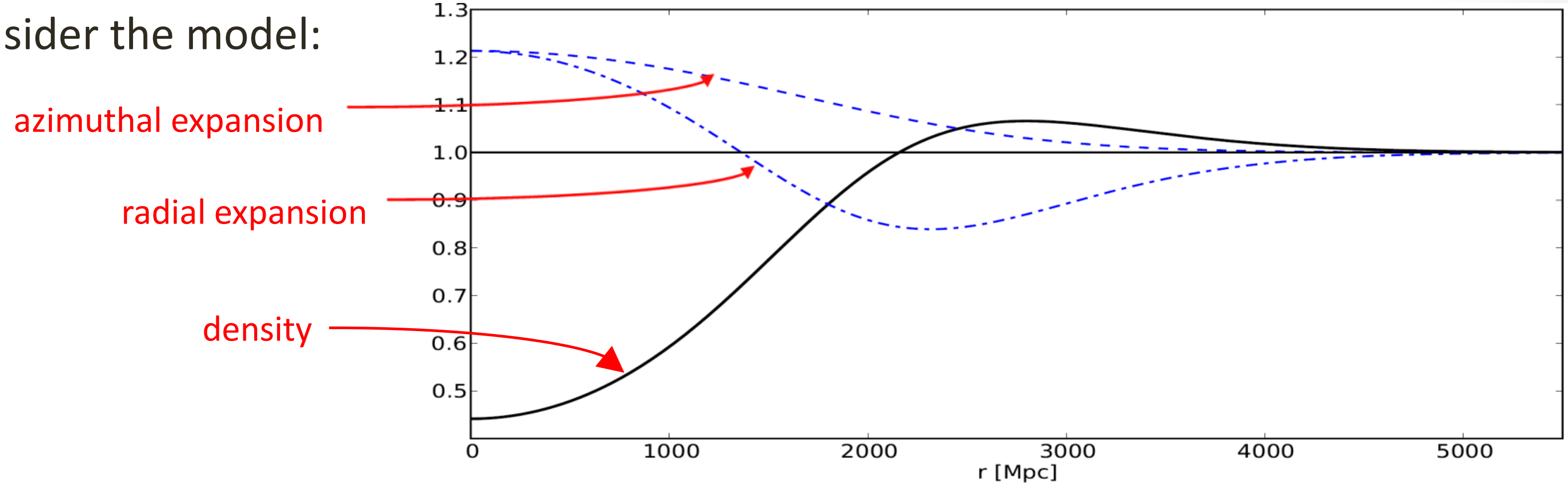
Averaging over many different lines-of-sight:



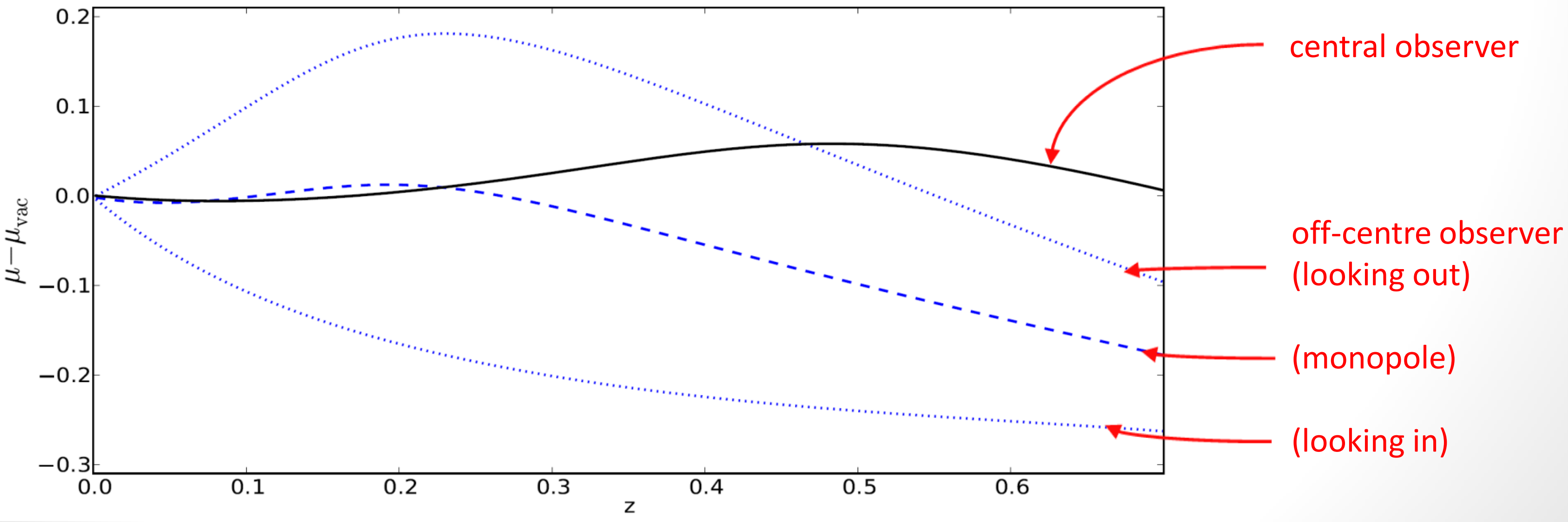
$$q_{non\text{–}local} \approx -7.18 \qquad \& \qquad q_{local} \approx -3.96$$

Spherically Symmetric Model

Consider the model:



Observations are given by:



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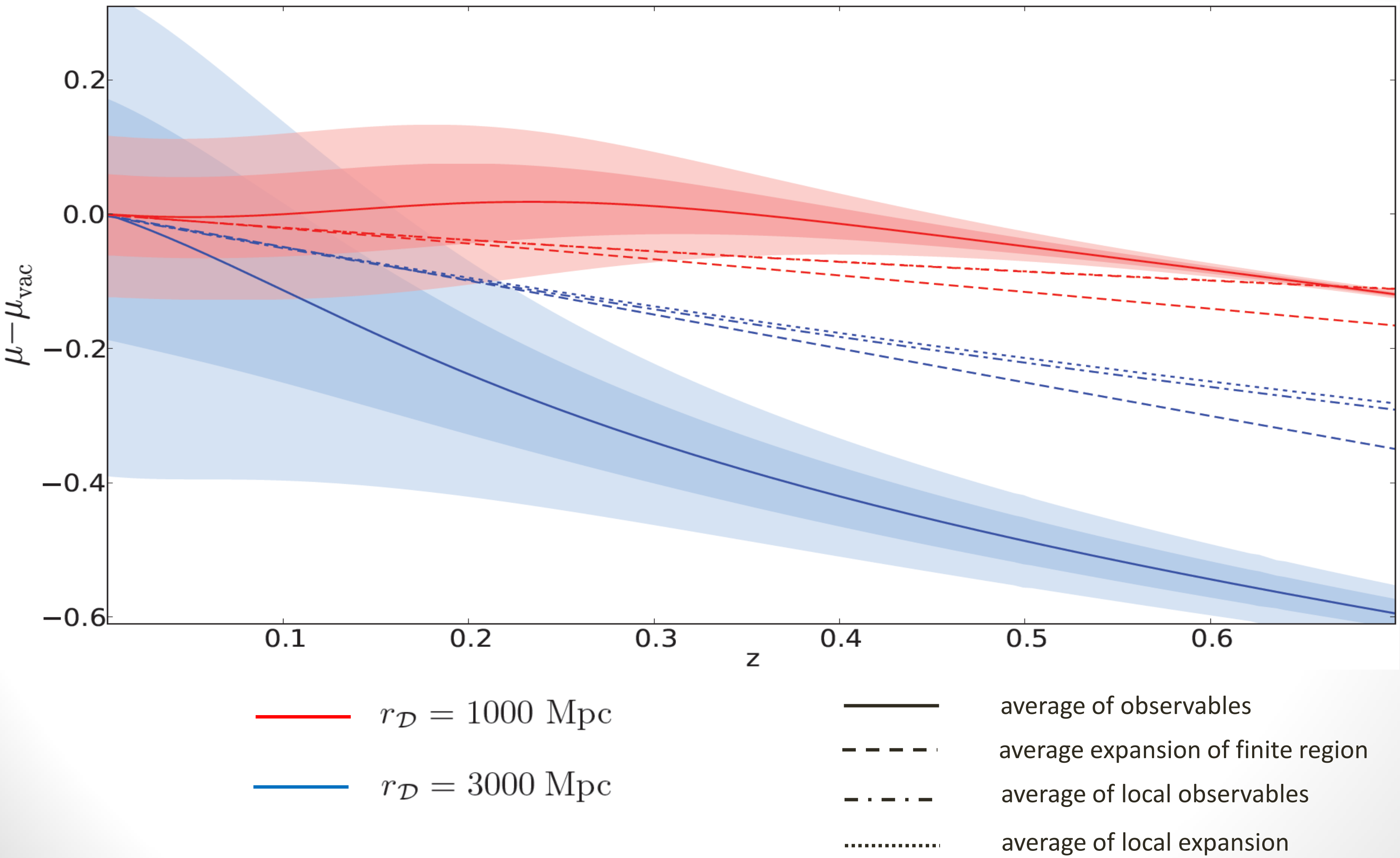
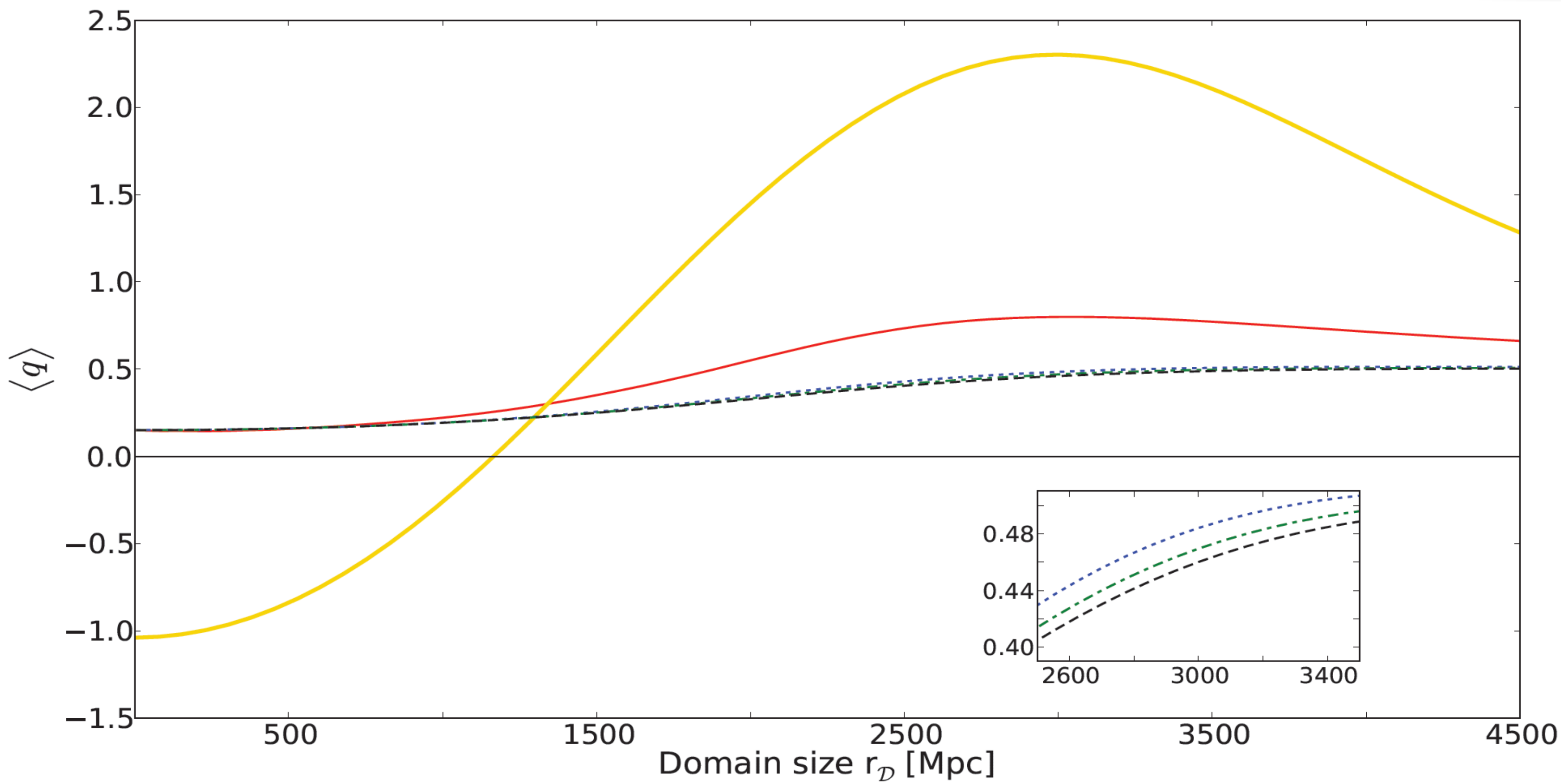


Spherically Symmetric Model

Different deceleration parameters:



average acceleration of finite region

average acceleration from local observables

average acceleration of local geometry

acceleration from average of observables (with z<1)

acceleration from average of observables (with z<0.1)

Conclusions

- In an inhomogeneous universe there are multiple different possible measures of acceleration. These can be very different from each other.
- In statistically homogeneous space-times it appears that observations made over large scales are more closely related to the acceleration of the finite regions, rather than the average of the acceleration of all the points in that region.
- Inferring local acceleration of space requires observations to be made locally, rather than on cosmological scales. This is what is required for a straightforward link between observables and matter content.
- Inferring the existence of Λ requires an understanding of how to relate local and non-local measures of acceleration.
- In space-time without a homogeneity scale, observations made over large scales can appear unrelated to both local and non-local measures of the acceleration of space.
- Acceleration due to ‘back-reaction’ would appear to predict that observables made over small scales should be different to observations made over large scales.

Thank you