Engineering and Physical Sciences Research Council

The University of Nottingham

Shaking Entanglement
Teleportation in Motion

9^{th} Vienna Central European Seminar

University of Vienna, November 30^{th} - December 2^{nd}, 2012

Nicolai Friis - University of Nottingham

work in collaboration with

A. R. Lee, K. Truong, C. Sabín, E. Solano, G. Johansson and *I. Fuentes*

Outline

Express to Alpha Centauri ... now boarding

- Non-uniform cavity motion:

  construction of generic trajectories

- Bogoliubov transformations and entanglement

  Comparison of procedures for Fock states and Gaussian states

- teleportation with non-uniformly moving cavities

  simulation with superconducting circuits

D. E. Bruschi, I. Fuentes and J. Louko, Phys. Rev. D **85**, 061701(R) (2012),
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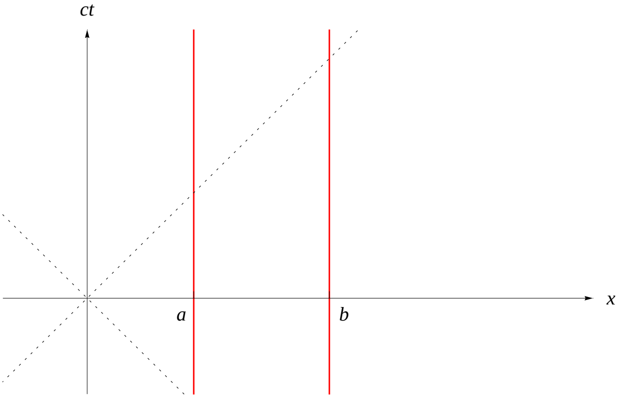
Inertial cavity of width $\delta = b - a$



Uncharged scalar field ϕ

Klein-Gordon Equation: $(\partial^{\mu}\partial_{\mu} + m^2)\,\phi(x) = 0$

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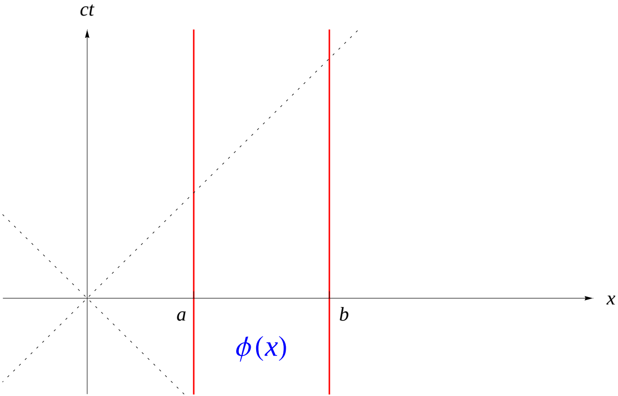

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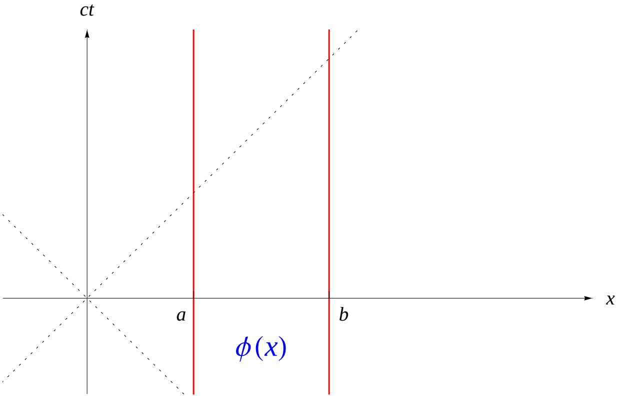

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boundary conditions: *Dirichlet* $\phi(t, \mathbf{a}) = \phi(t, \mathbf{b}) = 0$

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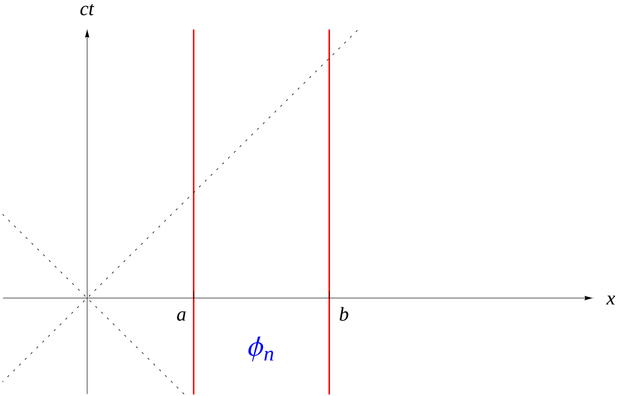
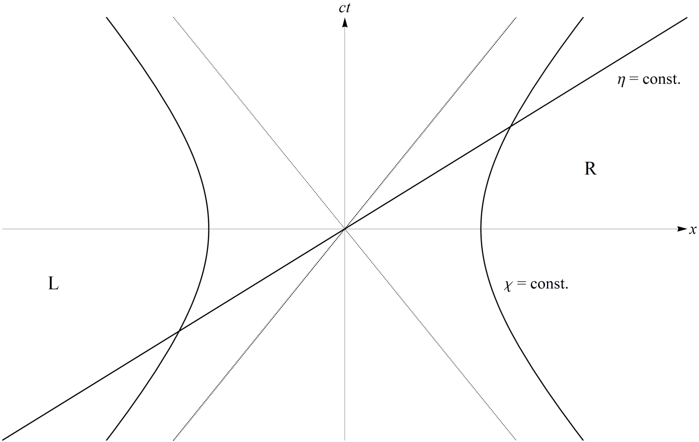


Uncharged scalar field ϕ

solutions: $\{\phi_n\}$: $\phi = \sum_n (\phi_n \, a_n + \phi_n^* \, a_n^\dagger)$, $[\, a_m, a_n^\dagger \,] = \delta_{mn}$

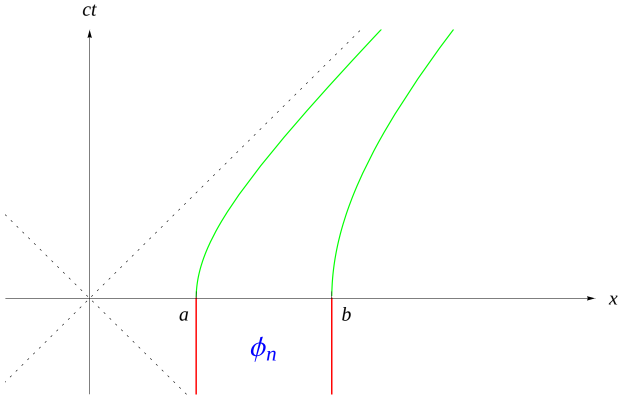
Uniform acceleration in Minkowski spacetime

Rindler coordinates (η, χ)



$$ct = \pm\chi \sinh \eta, \qquad x = \pm\chi \cosh \eta$$

Quantum fields in accelerated cavities



Repeat quantisation procedure in Rindler coordinates

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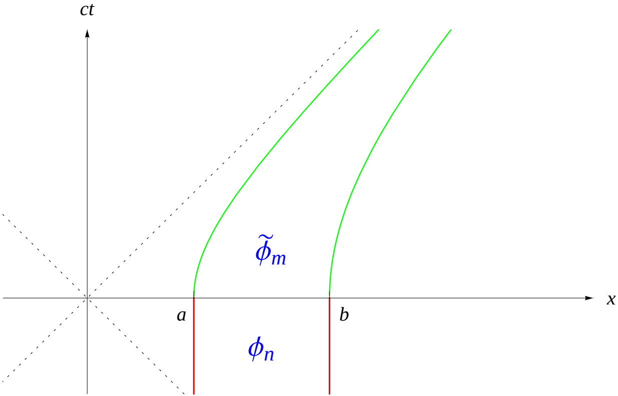

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solutions: $\{\tilde{\phi}_n\}$: $\phi = \sum_n (\tilde{\phi}_n\, \tilde{a}_n + \tilde{\phi}_n^*\, \tilde{a}_n^\dagger)$, $[\,\tilde{a}_m, \tilde{a}_n^\dagger\,] = \delta_{mn}$

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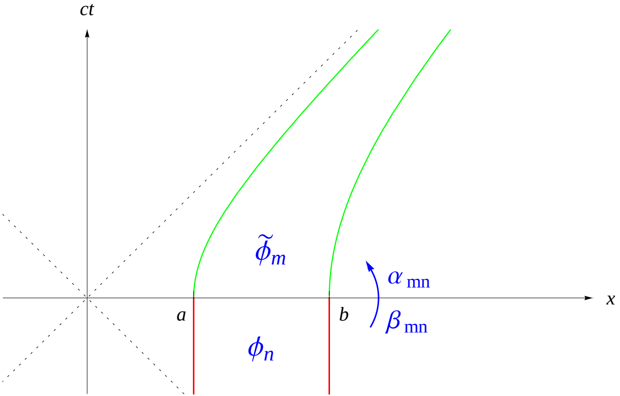

Solutions related by Bogoliubov transformation:

$$\phi_n = \sum_m (\alpha^*_{mn}\,\tilde{\phi}_m - \beta_{mn}\,\tilde{\phi}^*_m), \quad \alpha_{mn} = (\tilde{\phi}_m, \phi_n),\ \beta_{mn} = -(\tilde{\phi}_m, \phi^*_n)$$

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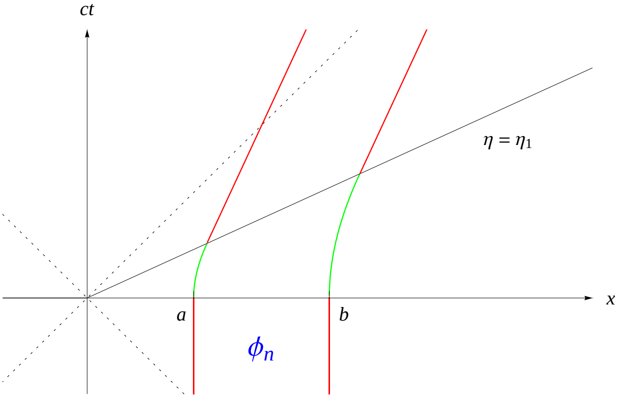

Finite duration of acceleration:

acceleration stops at fixed coordinate time $\eta = \eta_1$

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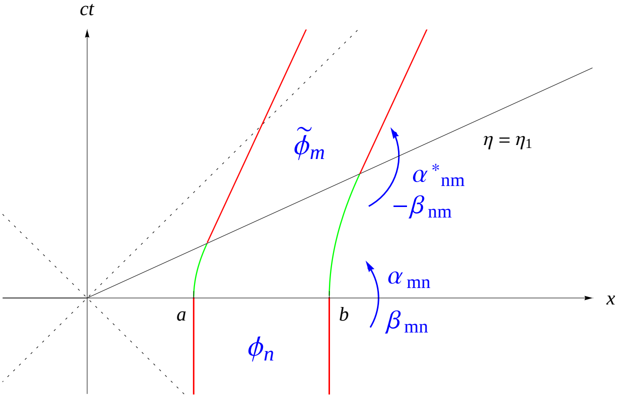

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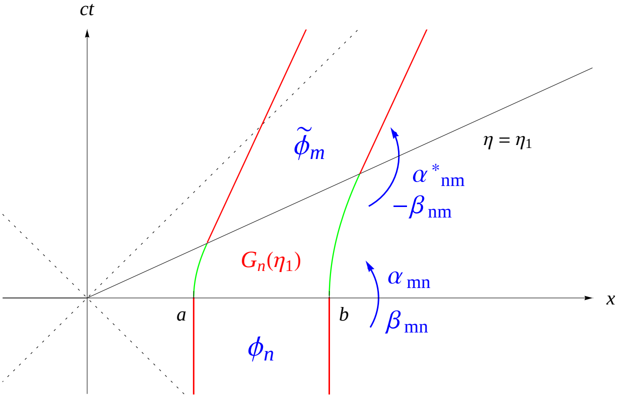

Finite duration of acceleration:

modes pick up phases $G_n(\eta_1)$

Bogoliubov Transformations

(real, scalar) Bosonic quantum field ϕ with *discrete spectrum*:

$$\phi = \sum_n \Big(a_n \phi_n + a_n^\dagger \phi_n^* \Big)$$

Bogoliubov transformation: "In-Region" $\Leftrightarrow$ "Out-Region"

mode functions: $\phi_n = \sum_m \Big(\alpha_{mn}^* \, \tilde{\phi}_m - \beta_{mn} \, \tilde{\phi}_m^* \Big)$

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Studying Entanglement in Fock space

Transformation of Fock States

$|\,0\,\rangle \propto e^{W}\,|\,\tilde{0}\,\rangle$ with $W = \frac{1}{2}\sum_{p,q} V_{pq}\,\tilde{a}_p^\dagger\,\tilde{a}_q^\dagger$

act with creation operators $a_n^\dagger = \sum_m \left(\alpha_{mn}^*\,\tilde{a}_m^\dagger + \beta_{mn}\,\tilde{a}_m\right)$ on $|\,0\,\rangle$

Usual Procedure

- Select in-region state
- Transform to out-region
- Trace out inaccessible modes
- Study entanglement properties

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Bogoliubov transformations in phase space

Gaussian States - covariance matrix formalism

$$\sigma_{ij} = \langle \mathrm{X}_i \mathrm{X}_j + \mathrm{X}_j \mathrm{X}_i \rangle - 2 \langle \mathrm{X}_i \rangle \langle \mathrm{X}_j \rangle$$

quadratures: $\mathrm{X}_{(2n-1)} = \frac{1}{\sqrt{2}}(a_n + a_n^\dagger)$ and $\mathrm{X}_{(2n)} = \frac{-i}{\sqrt{2}}(a_n - a_n^\dagger)$

Bogoliubov transformation $\Rightarrow$ symplectic transformation S

$$\tilde{\sigma} = S\,\sigma\,S^T$$

$$S = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \cdots \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} & \cdots \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad \text{where} \quad \mathcal{M}_{mn} = \begin{pmatrix} \mathrm{Re}(\alpha_{mn} - \beta_{mn}) & \mathrm{Im}(\alpha_{mn} + \beta_{mn}) \\ -\mathrm{Im}(\alpha_{mn} - \beta_{mn}) & \mathrm{Re}(\alpha_{mn} + \beta_{mn}) \end{pmatrix}$$

N. Friis and I. Fuentes, e-print arXiv:1204.0617 [quant-ph] (2012) (published online in J. Mod. Opt.).

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Tracing & Entanglement

Transforming initial state

$$\sigma = \begin{pmatrix} C_{11} & C_{12} & C_{13} & \dots \\ C_{21} & C_{22} & C_{23} & \dots \\ C_{31} & C_{32} & C_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \Rightarrow \tilde{\sigma} = \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & \dots \\ \tilde{C}_{21} & \tilde{C}_{22} & \tilde{C}_{23} & \dots \\ \tilde{C}_{31} & \tilde{C}_{32} & \tilde{C}_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where $\tilde{C}_{mn} = \sum_{i,j} M_{mi} C_{ij} M_{nj}^{\dagger}$

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Compute entanglement measure of choice

e.g., (logarithmic) Negativity $\mathcal{N}$, Gaussian Contangle, Entropy of Entanglement, etc.

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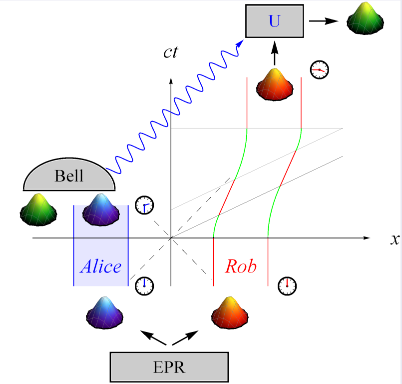
Results - Degradation of Entanglement

"in-region": 2-mode squeezed state of *Alice's* mode k and *Rob's* mode k'



"out-region": Trace out all other modes: entanglement degradation

N. Friis, A. R. Lee, K. Truong, C. Sabín, E. Solano, G. Johansson and I. Fuentes in preparation (2012).

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Bogoliubov coefficients: perturbative in $h := 2\frac{b-a}{b+a}$

$$\alpha_{mn} = \alpha_{mn}^{(0)} + \alpha_{mn}^{(1)}\, h + O(h^2), \quad \beta_{mn} = \beta_{mn}^{(1)}\, h + O(h^2)$$

Teleportation Fidelity: $\mathcal{F} = \mathcal{F}^{(0)} - \mathcal{F}^{(2)}\, h^2 + O(h^4)$

where

$$\mathcal{F}^{(0)} = \left(1 + \text{Cosh}(2r) - \text{Cos}(\phi)\,\text{Sinh}(2r)\right)^{-1},$$

$$\mathcal{F}^{(2)} = \left(\mathcal{F}^{(0)}\right)^2 \left(1 + e^{-2r}\right)\left(f_{k'}^{\beta} + f_{k'}^{\alpha}\,\text{Tanh}(2r)\right)$$

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"in-region": 2-mode squeezed state of *Alice's* mode k and *Rob's* mode k'

Bogoliubov coefficients: perturbative in $h := 2\frac{b-a}{b+a}$

$$\alpha_{mn} = \alpha_{mn}^{(0)} + \alpha_{mn}^{(1)}\, h + O(h^2), \quad \beta_{mn} = \beta_{mn}^{(1)}\, h + O(h^2)$$

Teleportation Fidelity: $\tilde{\mathcal{F}} = \tilde{\mathcal{F}}^{(0)} - \tilde{\mathcal{F}}^{(2)}\, h^2 + \mathcal{O}(h^4)$

where

$$\tilde{\mathcal{F}}^{(0)} = \left(1 + \text{Cosh}(2r) - \text{Cos}(\phi)\,\text{Sinh}(2r)\right)^{-1},$$

$$\tilde{\mathcal{F}}^{(2)} = \left(\tilde{\mathcal{F}}^{(0)}\right)^2 \left(1 + e^{-2r}\right)\left(f_k^{\beta} + f_k^{\alpha}\,\text{Tanh}(2r)\right)$$

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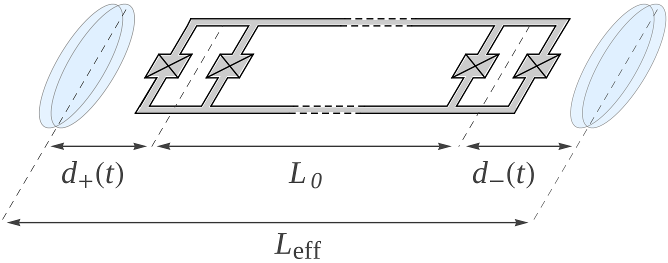
Experimental Proposal

$1 - dim$ transmission line for microwave radiation



Superconducting circuits (SQUIDS) simulate boundary conditions

$\Rightarrow$ cavity of effective length $L_{\text{eff}} = L_0 + d_+(t) + d_-(t)$

predicted degradation of optimal teleportation fidelity: $\approx 4\%$

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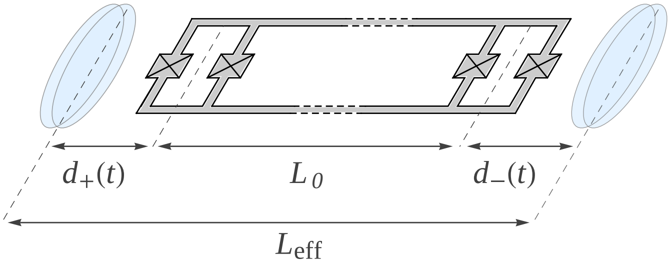

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Thank you for your attention.