The Quantum Geometric Limit

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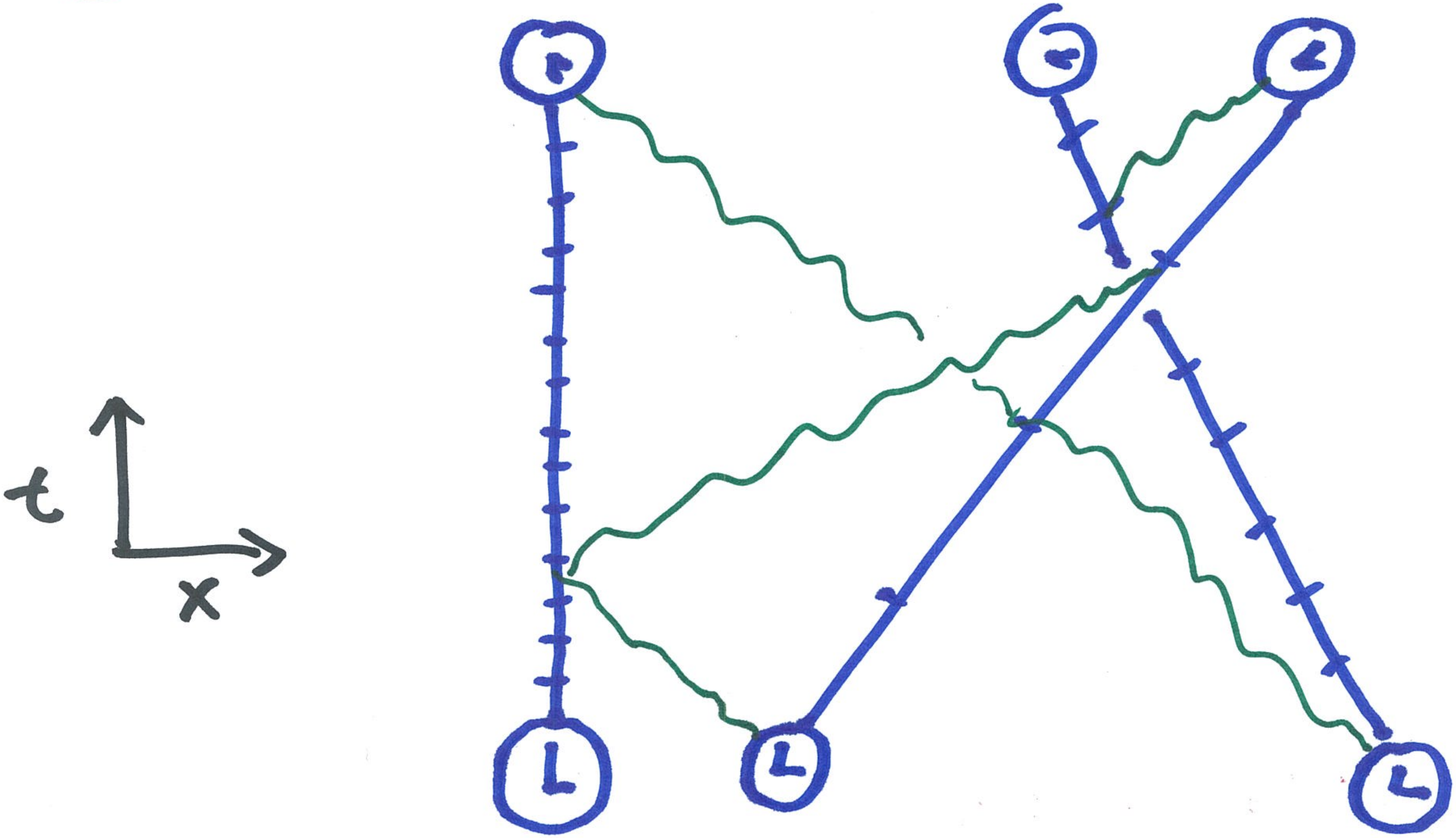
GPS



Clocks in space

Quantum limits to GPS

- How many ticks of clocks + clicks of detectors can occur within a volume of radius r over time t?

Two quantum limits:

$$\Delta E \Delta t \geq \frac{\pi \hbar}{2}$$ Heisenberg

$$E \Delta t \geq \frac{\pi \hbar}{2}$$ Margolus-Levitin

where $\Delta E = (\mathrm{tr}_\rho H^2 - (\mathrm{tr}_\rho H)^2)^{1/2}$ is the spread in energy, and

$E = \mathrm{tr}_\rho H - E_0$ is the average energy above ground state

Here, Δt is the time it takes for a system to go from one state to an orthogonal state.

Δt = time for clock to tick
for detector to click
for bit to flip.

Maximum energy within a volume of radius r:

$$R_s = \frac{2Gm}{c^2} \le r$$

or, since $E = mc^2$

$$\frac{2GE}{c^4} \le r$$

Op = accumulation of phase of $\pi/2$ relative to ground state.

$$\#\text{ops} = \frac{2Et}{\pi\hbar}, \qquad \frac{2GE}{c^4} \leq r$$

$\Rightarrow$

$$\boxed{\#\text{ops} \leq \frac{c^4}{\pi\hbar G}\, rt = \frac{rt}{\pi l_p t_p}}$$

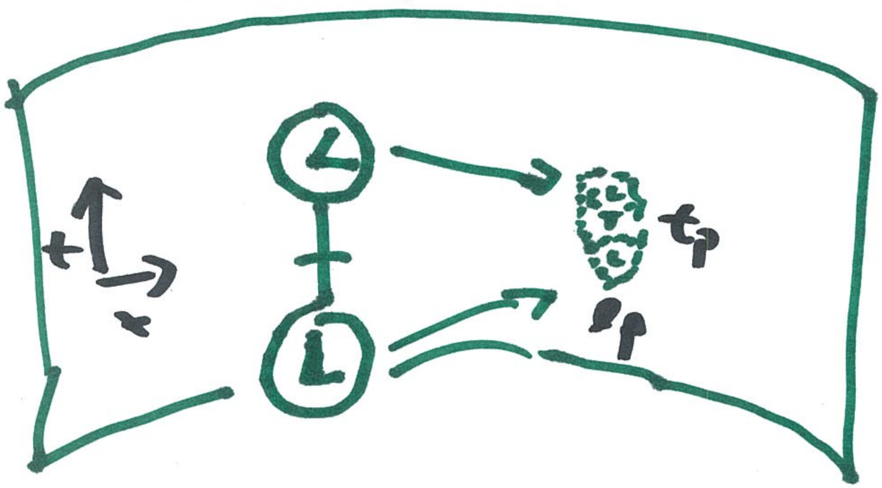
l_p = Planck ~~length~~ length

t_p = Planck time

The quantum geometric limit

The quantum geometric limit:

- easy to derive
- no quantum gravity necessary
- associates each op with a 1+1 dimensional Planck-scale area on some world sheet



- Not $\frac{t}{t_p}\left(\frac{r}{\ell_p}\right)^3$!
- Not $\frac{t}{t_p}\left(\frac{r}{\ell_p}\right)^2$!

Compare with holography:

each qubit in 3-volume associated with area $\sim l_p^2$ on 2-d surface.

Quantum geometric limit:

each δp in 3+1 volume associated with area $\sim l_p t_p$ on 1+1-d surface.

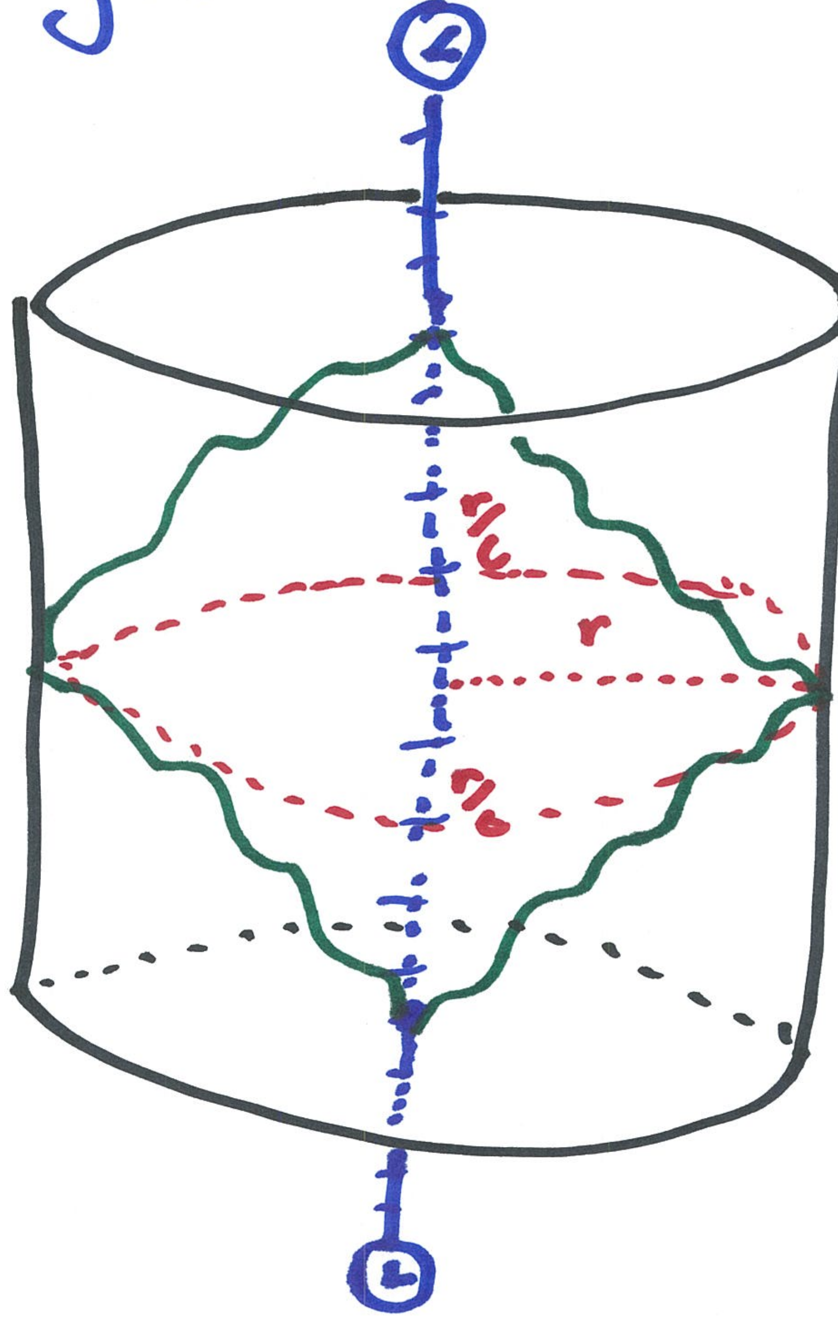
Jacobson 1995:

Holography + Unruh radiation

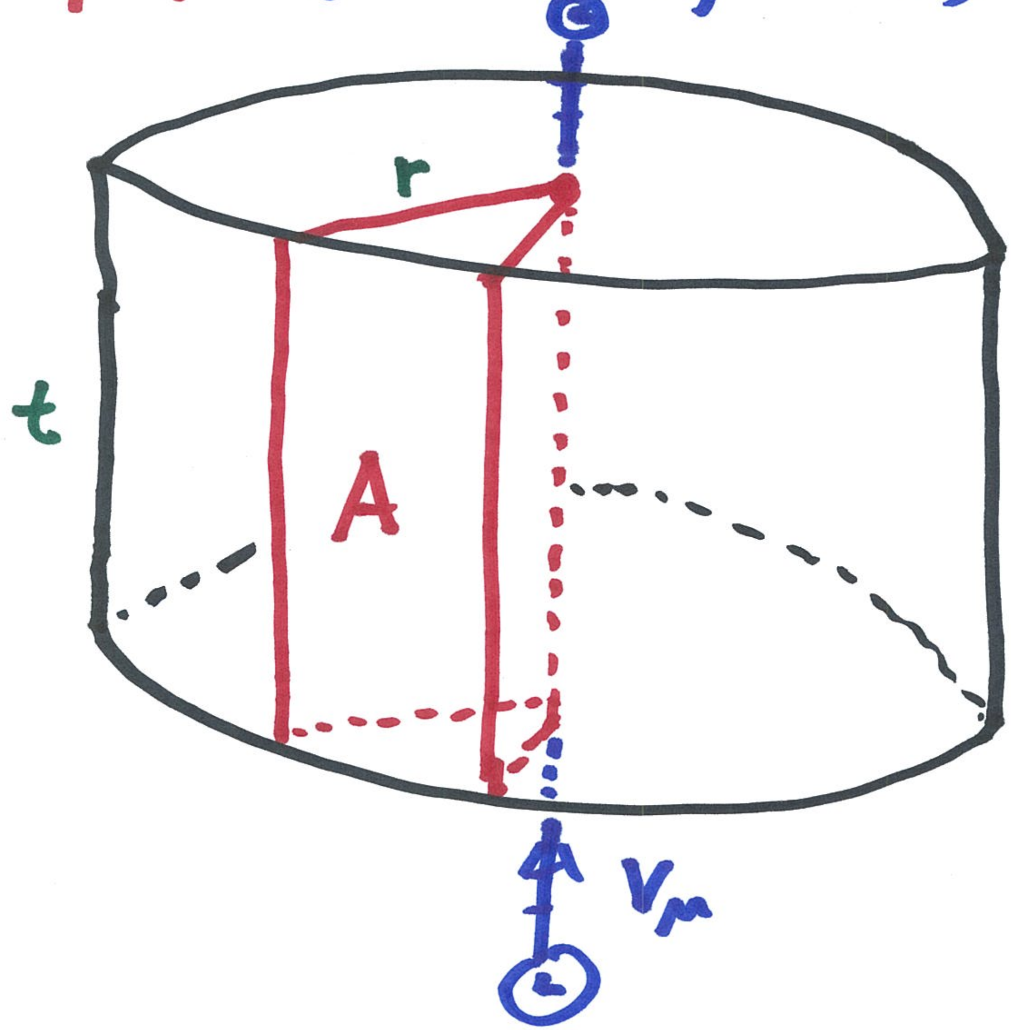
$\Rightarrow$ Einstein's equations

I'll now show that the quantum geometric limit $\Rightarrow$ Einstein's equations.

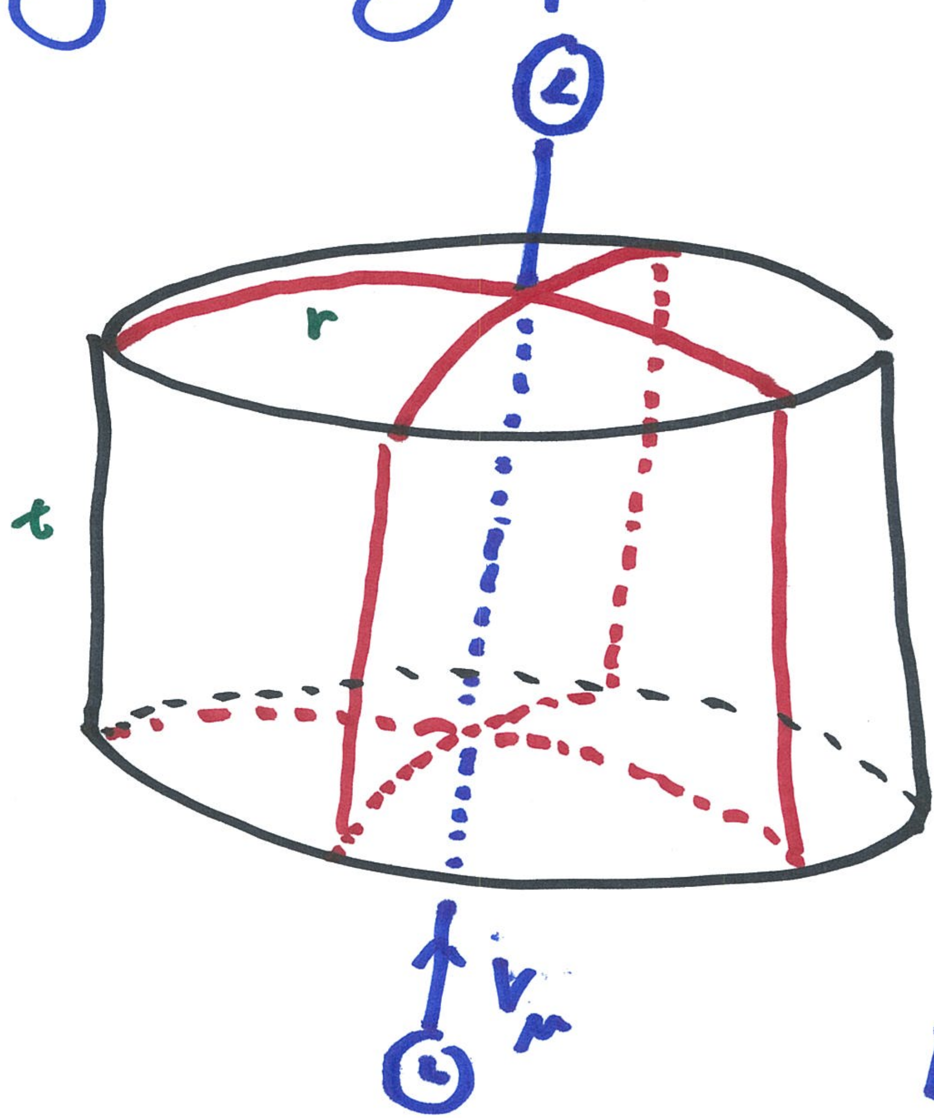
Covariant cylinder:



Suppose each op removes an area $\propto l_p t_p$ from surface of covariant cylinder



Thereby causing spacetime to curve:



$$K = \frac{3A}{\pi r^2 t}$$

So

$$\alpha\, l_p t_p \#\mathrm{ops} = \frac{\pi}{3}\left(K_{12}+K_{23}+K_{13}\right) r^3 t$$

$$= \frac{\pi}{3}\left(R^{\mu\nu} - \frac{g^{\mu\nu}}{2} R\right) V_\mu V_\nu\, r^3 t$$

But

$$\#\mathrm{ops} = \frac{2}{\pi\hbar}\, \underbrace{T^{\mu\nu} V_\mu V_\nu}_{\text{energy/unit volume}} \cdot \underbrace{\frac{4\pi r^3}{3}}_{\text{volume}} \cdot t$$

So

$$\left(\frac{8\pi}{\hbar}\right)\alpha\, l_p t_p\, T^{\mu\nu} V_\mu V_\nu = \left(R^{\mu\nu} - \frac{g^{\mu\nu}}{2} R\right) V_\mu V_\nu$$

$\alpha = \pi^2 \Rightarrow$

V_μ arbitrary $\Rightarrow$

$$8\pi G T^{\mu\nu} = R^{\mu\nu} - \frac{g^{\mu\nu}}{2} R$$

Einstein's equations

Each op removes area $\pi^2 l_p t_p$

$\Rightarrow$ Einstein's equations.

Summary:

Physics of computation + black hole limit

$\Rightarrow$ Quantum geometric limit: $\# \leq \dfrac{rt}{\pi l_p t_p}$

In turn, quantum geometric limit

+ area removal

$\Rightarrow$ Einstein's equations