Newtonian vs. Relativistic Cosmology

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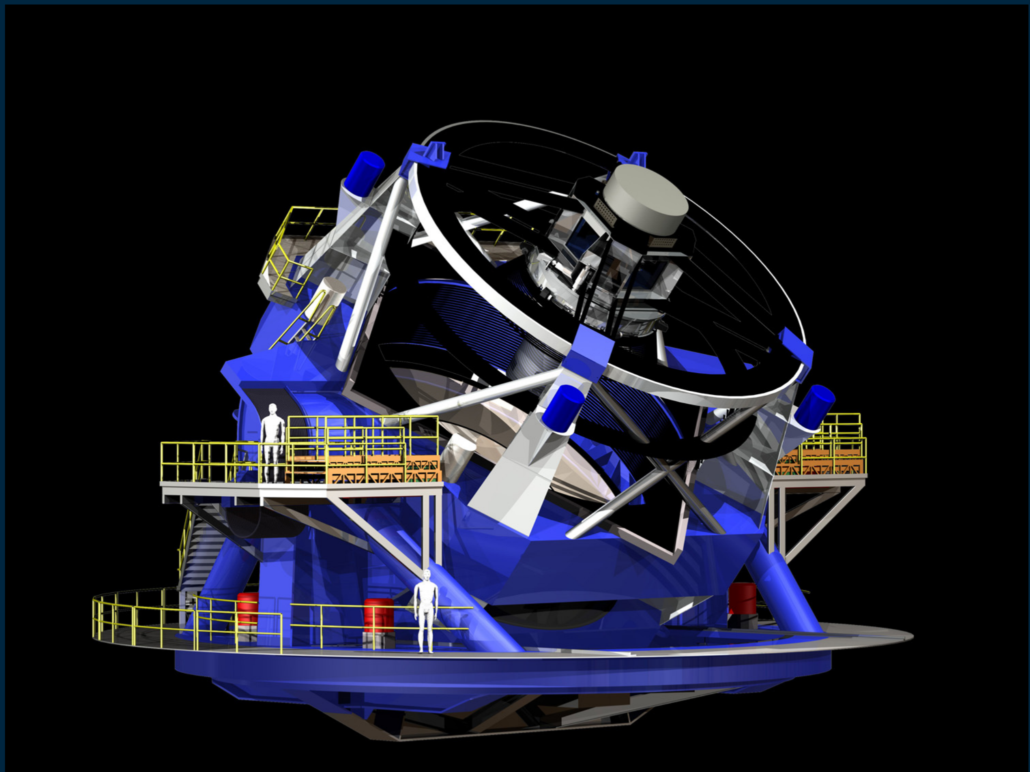
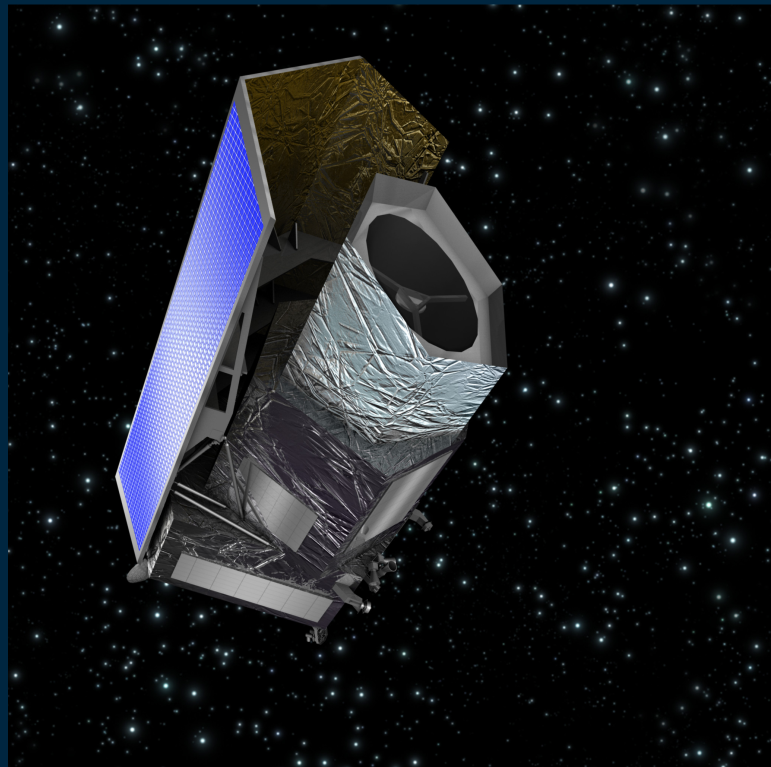
Motivation: new surveys and simulations
"new" space-time foliation
accuracy of Newtonian cosmology at large scales

Flender & Schwarz, PRD 86 (2012) 063527; arXiv:1207.2035

Vienna, 2012

New generation of optical surveys



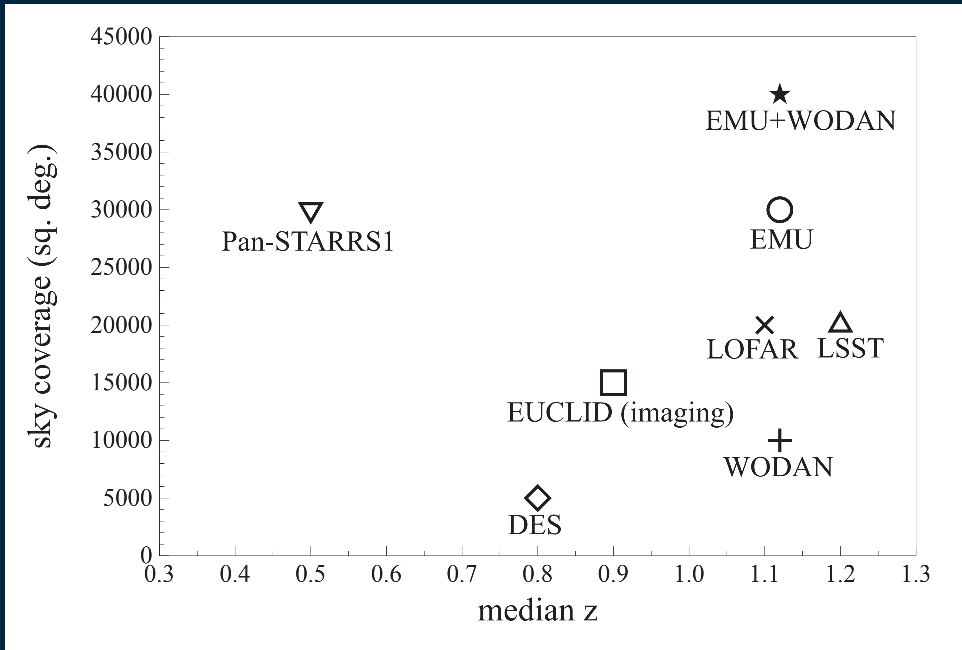


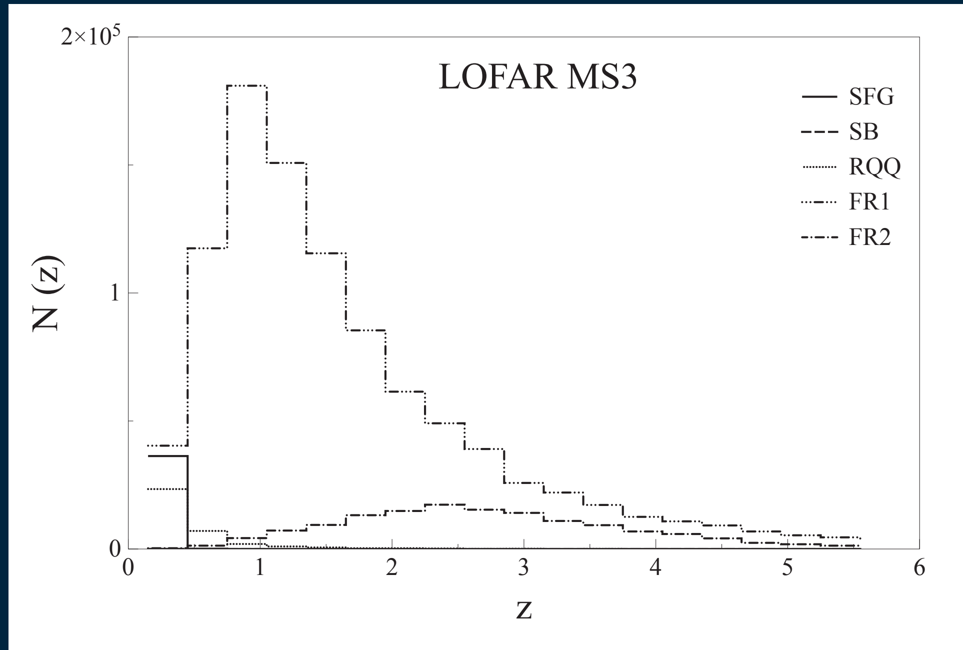
DES, Pan-STARRS, ..., LSST, Euclid

New generation of radio surveys

LOFAR, ASKAP, MeerKat, Apertif, ... SKA

Ongoing and near future surveys are large and deep





Raccanelli et al. 2012

Numerical simulations

analysis of deep and wide galaxy surveys requires simulated data

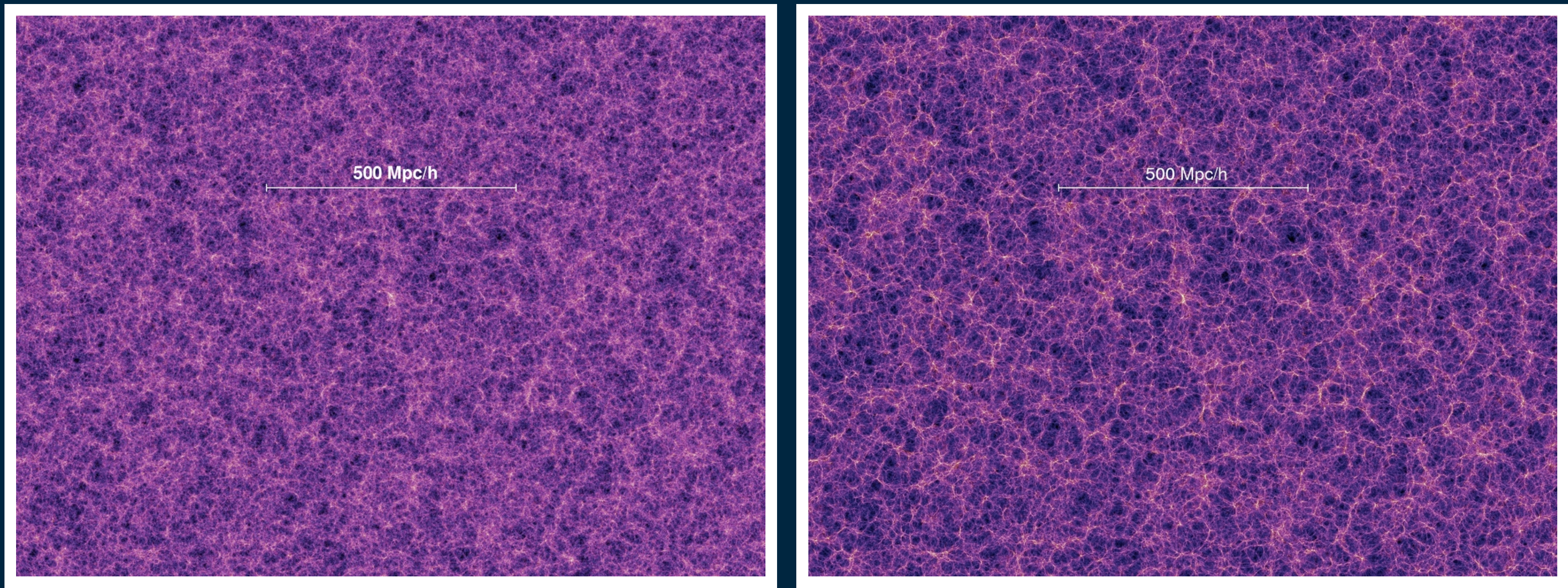
there are NO relativistic cosmological simulations

simulations are based on Newtonian cosmology

(both N-body and hydrodynamics)

surveys and simulations start to reach Hubble volume

Newtonian simulations



$z = 5.7$ and $z = 0$

Springel et al. 2005

Newtonian cosmology

consider dust ($p = 0$) and a cosmological constant Λ

$(\eta, \mathbf{x})$ = (conformal time, comoving distance)

physical distance: $\mathbf{r} \equiv a(\eta)\mathbf{x}$; a scale factor
matter density: $\rho \equiv \bar{\rho}(1+\delta)$, peculiar velocity: $\mathbf{v} \equiv \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\eta}$
Newtonian potential: $\bar{\Phi} + \Phi$

$$\mathcal{H} \equiv \frac{a'}{a}, \quad \bar{\rho}' + 3\mathcal{H}\bar{\rho} = 0, \quad \mathcal{H}'\mathbf{x} = -\nabla\bar{\Phi}, \quad \Delta\bar{\Phi} = (4\pi G\bar{\rho} - \Lambda)\, a^2$$

isotropic and homogeneous background is equivalent to Friedmann-Lemaître model

$$\delta' + \nabla\cdot[(1+\delta)\mathbf{v}] = 0, \quad \mathbf{v}' + \mathcal{H}\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla\Phi, \quad \Delta\Phi = 4\pi G a^2 \bar{\rho}\delta$$

scalar and vector contributions: $\mathbf{v} = \nabla v + \nabla \times \mathbf{w}$
below: focus on irrotational dust ($\mathbf{w} = 0$) and thus scalar sector

Confusing literature

Narlikar 1963: dust shear-free NC can expand and rotate
Ellis 1967, 2011: Dust Shear-Free Theorem in RC
$\{\dot{u}^a = 0, \sigma_{ab} = 0\} \Rightarrow \theta\,\omega = 0$
$\Rightarrow$ the limit to Newtonian cosmology is singular

Hwang & Noh 2006, 2012:
correspondance of NC and RC up to 2nd order perturbations
for gauge-invariant quantities (scalar sector)

Chisari & Zaldarriaga 2011:
dictionary, shift initial conditions, use ray tracing
Green & Wald 2011, 2012: another dictionary, extra equations

Relativistic cosmology

$$R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G T_{ab}, \qquad T^{ab}{}_{;b} = 0$$

fluctuations around a (spatially flat) Friedmann-Lemaître model

$$\mathrm{d}^2 s = a^2(\eta) \left[-(1+2\phi)\mathrm{d}\eta^2 + 2B_{,i}\mathrm{d}\eta\mathrm{d}x^i + ((1+2\psi)\delta_{ij} + h_{ij})\mathrm{d}x^i\mathrm{d}x^j \right]$$

What does $\eta =$ const mean physically? $\Rightarrow$ different slicing conditions

for each slicing there exists an adapted coordinate system
infinitesimal transformations: $\tilde{\eta} = \eta + \xi^0, \tilde{\mathbf{x}} = \mathbf{x} + \nabla\xi$

Well studied space-time foliations

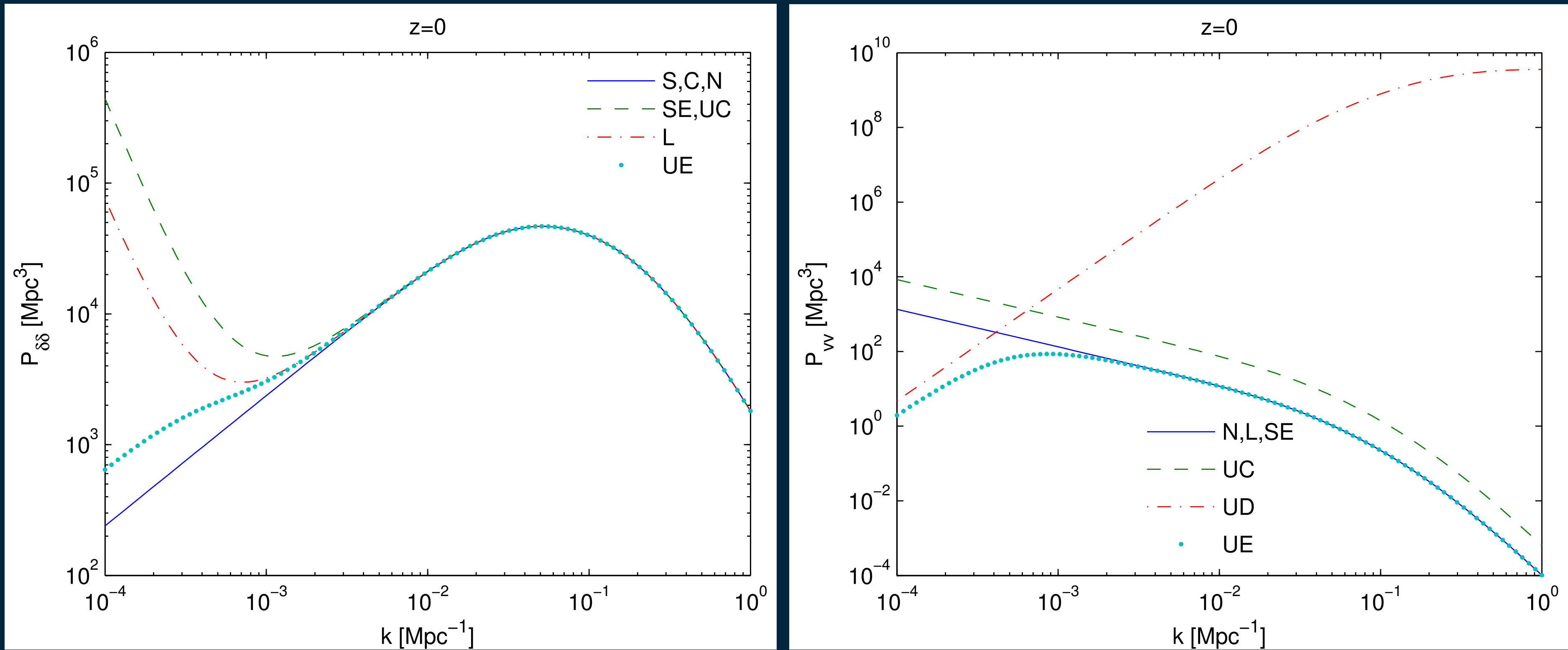
uniform density (UD): $\delta = B = 0$
comoving (C): $B = v = 0$

synchronous (S): $\phi = B = 0$, for dust synchronous & comoving $v = 0$

uniform curvature (UC): $\psi = B = 0$
uniform expansion (UE): $\theta = B = 0$
longitudinal (L) = vanishing shear: $B = h = 0$
spatially Euclidian (SE): $\psi = h = 0$

for each slicing "gauge invariant" combinations can be defined,
e.g. $\Phi_{\mathrm{gi}} \equiv \phi + [(B - h')a]'/a$

Power-spectra of well studied space-time foliations



no agreement with Newtonian Cosmology in δ AND $\mathbf{v}$ at all scales

Flender & Schwarz 2012

The Newtonian matter slicing

Flender & Schwarz 2012

start from longitudinal gauge ($v_{\mathsf{L}} = v_{\mathsf{N}}$) and pick $\xi^0 = 2\Phi_{\mathsf{gi}}/3\mathcal{H} \Rightarrow$

$$\delta_{\mathsf{NM}} \equiv \delta_{\mathsf{L}} + 3\mathcal{H}\xi^0 = \delta_{\mathsf{N}}, \quad v_{\mathsf{NM}} = v_{\mathsf{N}}, \quad \phi_{\mathsf{NM}} = 0$$

δ and $\mathbf{v}$ agree with NC at all times and scales due to choice of slicing

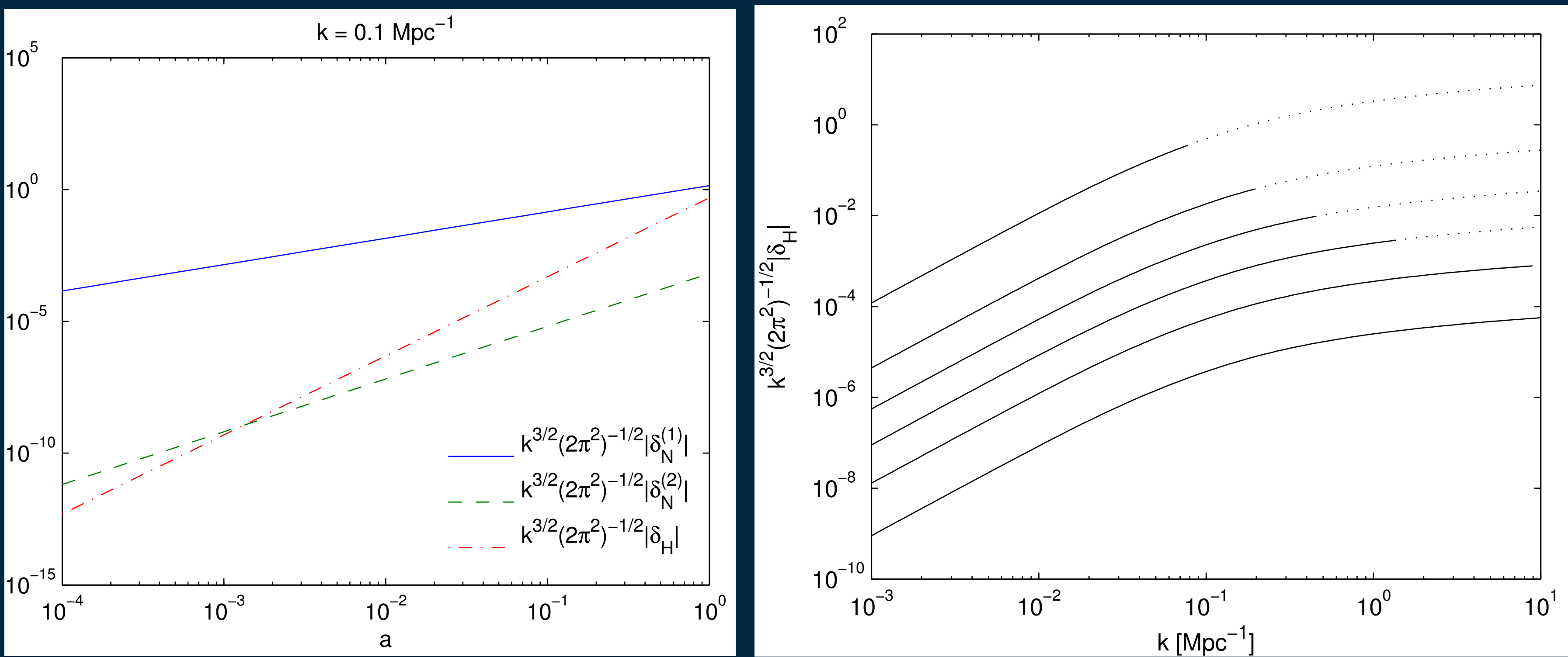
a dictionary: $\{\delta, v, \Phi\}_{\mathsf{N}} \leftrightarrow \{\delta_{\mathsf{NM}}, v_{\mathsf{NM}}, \Phi_{\mathsf{gi}}\}$ Haugg, Hofmann & Kopp 2012

${}^3R_{\mathsf{NM}} = \frac{20}{3}\Delta_{\mathbf{r}}\Phi_{\mathsf{gi}}$ spatial curvature; $-\nabla_{\mathbf{r}}\Phi_{\mathsf{gi}} = \dot{\mathbf{v}}_{\mathsf{NM}} + H\mathbf{v}_{\mathsf{NM}}$ peculiar acceleration

no spatial curvature, fluctuation of expansion, geometric shear in NC

to test NC against RC consider fluctuation of expansion rate:
$\delta_H \equiv \frac{\theta_{\mathsf{NM}}}{3H} = \frac{1}{3H^2}\Delta_{\mathsf{r}}\Phi_{\mathsf{gi}}$

Time evolution of relativistic and non-linear terms



significant GR effects at $k < 0.1$/Mpc, proper distances are modified

Flender & Schwarz 2012

Conclusions

◇ need three observables to investigate NC/RC correspondance

◇ a one-to-one correspondence does not hold (even at linear level)

◇ Newtonian cosmology can be mapped to relativistic cosmology for irrotational dust (at linear order)

◇ initial conditions must be specified on a well defined slicing

e.g. HZ spectrum in S,C slicing is not the same as HZ in NM slicing

◇ Newtonian simulations are correct for δ, v, Φ, but not for geometry

e.g. physical distance, redshift space-distortion, etc.