Shear Viscosities of Strongly Coupled Anisotropic Plasmas

Dominik Steineder

Institute for Theoretical Physics
Vienna University of Technology

work done in collaboration with Anton Rebhan, arXiv:1110.6825

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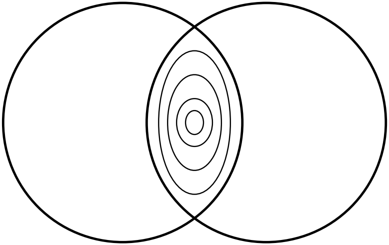
Elliptic flow and the quark-gluon plasma



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elliptic flow $\rightarrow n = 2$

ϕ_R ... orientation of reaction plane

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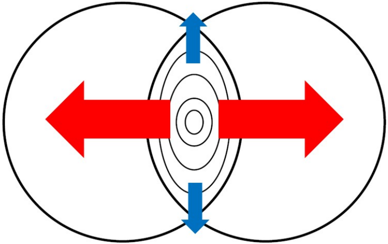

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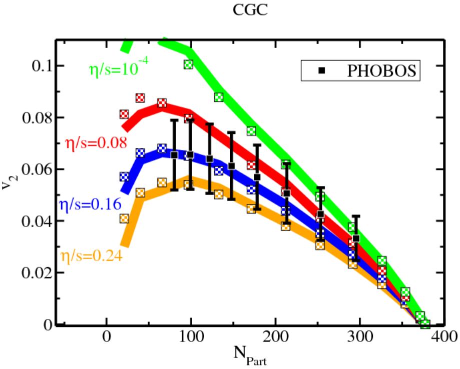

Luzum, Romatschke '08

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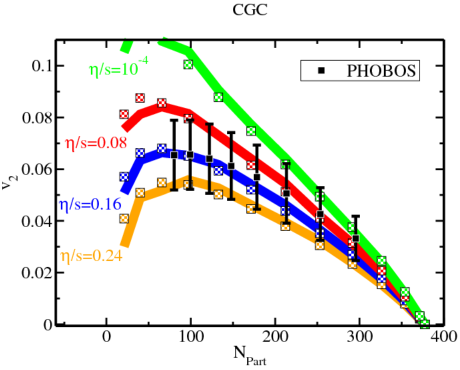

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"RHIC serves the perfect fluid" (2005)

large $v_2 \Rightarrow$ small η/s!

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$\eta/s \lesssim O(1) \Rightarrow$ **Strong coupling effect!**

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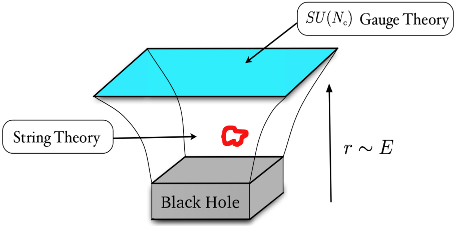
- **Gauge/gravity duality**
  - string theory inspired method to study large N gauge theories at strong coupling
  - not (yet ?) established for QCD
    $\Rightarrow$ Need to study "wrong" theory!



thermal state in boundary theory

$\Updownarrow$

black hole geometry

A brief history of η/s

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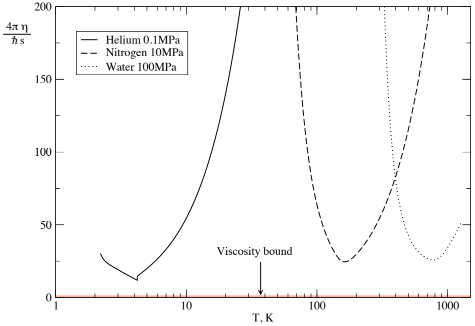
  [Kovtun, Son, Starinets '03; Buchel, Liu '03]
  $\Rightarrow$ conjectured **lower (quantum) bound for any fluid in nature**
  [Kovtun, Son, Starinets '04]
  with **856** citations



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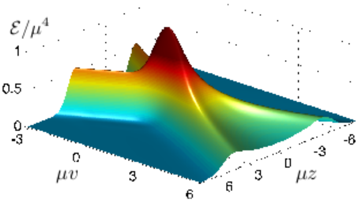
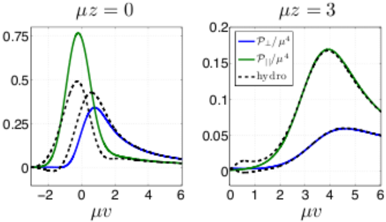
- **and the story continues ...**

Anisotropy and heavy ion collisions

Shock waves in AdS_5 [Chesler, Yaffe '10]





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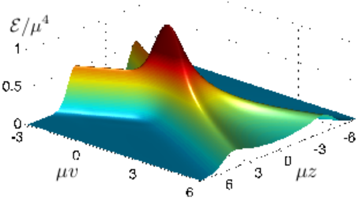

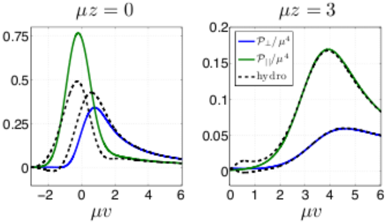

start with something simpler: **stationary anisotropic plasma**

Anisotropic axion-dilaton gravity [Mateos, Trancanelli '11]

Boundary

$$S = S_{\mathcal{N}=4} + \frac{1}{8\pi^2}\int \theta(z)\mathrm{Tr}\, F \wedge F$$

with $\theta(z) = 2\pi a z$

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$s = \frac{(\epsilon + P_\perp)}{T}$	$s = \frac{A_h}{4GV_3}$

Calculating η/s with gauge/gravity duality

Kubo formula

$$\eta_{ijkl} = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,G^R_{ij,kl}(\omega,0)$$

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perturb metric by $\psi_a = h^i_j$ and expand action to second order in ψ_a
$\Rightarrow$ effective action for massless scalar ψ_a

$$G^R_a(q) = -\lim_{u\to 0}\frac{\Pi_a(u,q)}{\psi_a(u,q)} \qquad \text{with } \Pi_a = \frac{\partial\mathcal{L}^{(2)}}{\partial(\partial_u\psi_a)} \propto \partial_u\psi_a$$

retarded correlator $\leftrightarrow$ infalling boundary conditions at horizon

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Either solve numerically or simplify further ...

Calculating η/s with gauge/gravity duality

Membrane paradigm [Iqbal, Liu '08]

generic transport coefficient of boundary theory

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shear viscosity

$$\eta_a = \frac{\Pi_a(u_h, q)}{i\omega\psi_a(u_h, q)} \qquad \text{with } \Pi_a(u_h, q) \propto i\omega\psi_a$$

and check whether $\partial_u \eta_a = 0$.

η/s for anisotropic plasmas [Rebhan, DS '11]

In anisotropic plasma we find **2 shear viscosities**:

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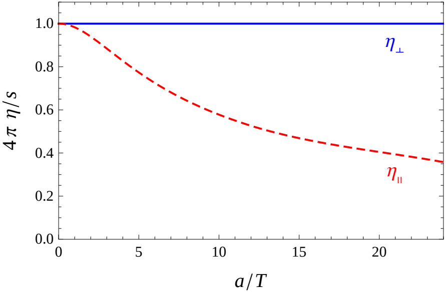

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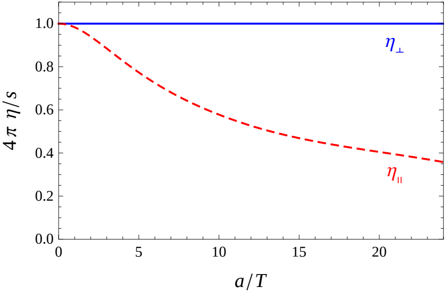
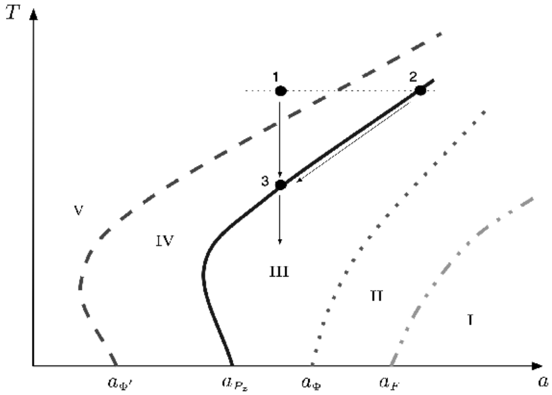


Violation of the viscosity bound!

The issue of instabilities [Mateos, Trancanelli '11; Rebhan, DS '11]



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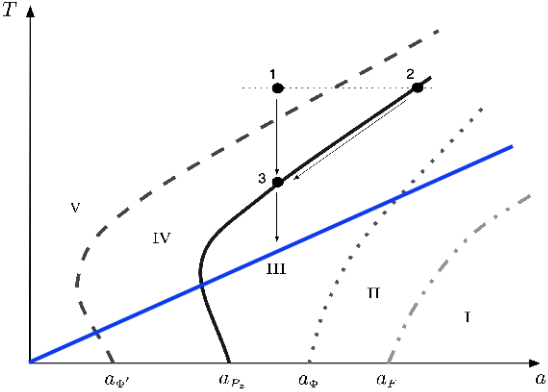

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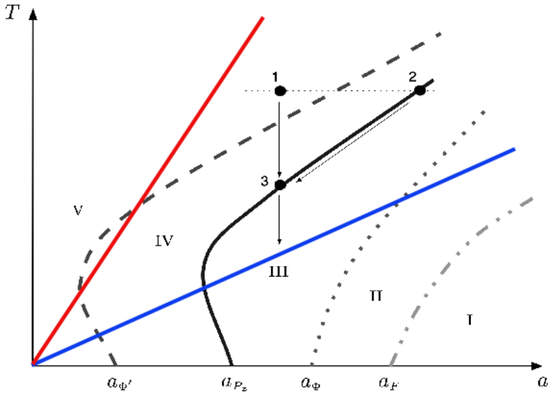

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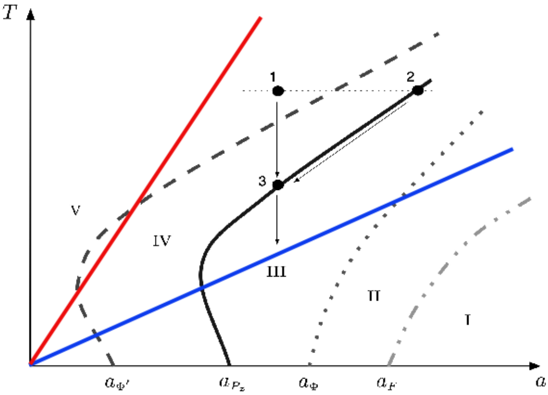

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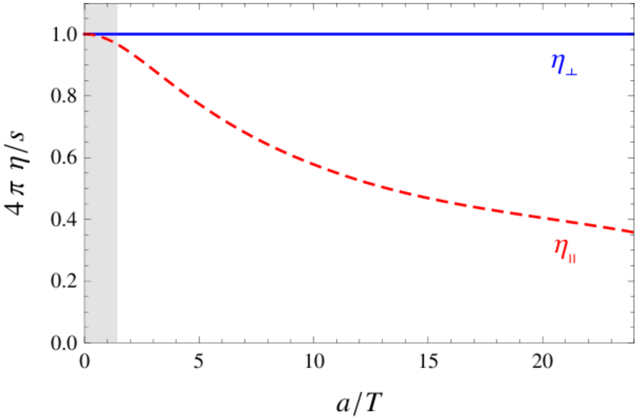

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I would like to talk about ...

- full study of all hydrodynamic modes (in progress)
- implications of the different shear viscosities for heavy ion collisions