Vienna Central European Seminar
on Particle Physics and Quantum Field Theory

First steps toward an asymptotically safe model of electroweak interactions

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Higgs boson exclusion limits

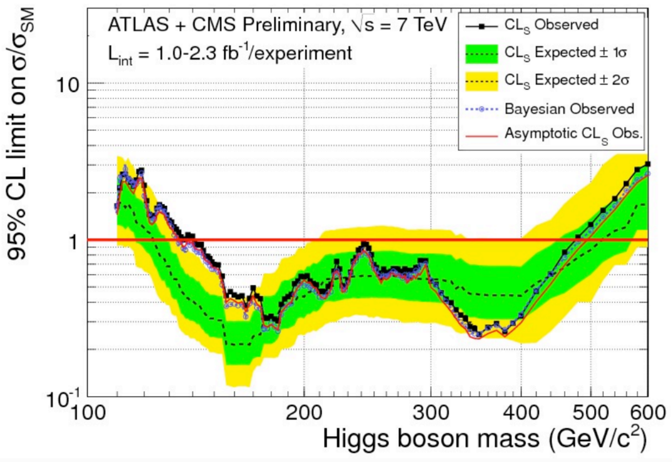


Figure: Higgs boson excluded in the mass range 141-476 GeV/c^2.

Motivations

The minimal realization of the SM would include only fundamental particles already discovered (no Higgs boson). The breaking of the EW symmetry would be implemented by coupling the gauge bosons and fermions to the corresponding Nambu-Goldstone bosons described by a Nonlinear σ Model. Two strong arguments against this picture, suggesting the existence of a fundamental Higgs:

- Violation of unitarity;
- EW precision measurements;

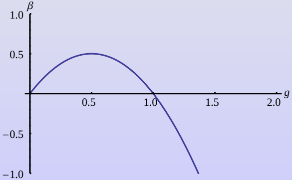
Both these arguments may be avoided, in principle, if the EW symmetry breaking sector is described by a Nonlinear σ Model which is Asymptotically Safe.

Asymptotic Safety

A quantum field theory is said to be Asymptotically Safe (AS) if it has a fixed point with a finite number of UV-attractive (relevant) directions.

Examples of AS theories:

- QCD is asymptotically free, admits a Gaussian UV fixed point [Wilczek, Gross and Politzer].
- Nonlinear sigma model in d=2+ϵ is asymptotically safe [A.M. Polyakov, PLB 57, 79 (1975), W.A.Bardeen et al., PRD 14, 985(1976)]



The nonlinear σ model (NLσM)

The NLσM action describes the physics of the Goldstone bosons (Gb) of a spontaneously broken symmetry $(G \to H)$:

$$\mathcal{S} = \frac{1}{2f^2}\int d^dx\, h_{\alpha\beta}\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta$$

- f is the Gb coupling with mass dimension $(2-d)/2$.
- the flow of $\frac{1}{f^2}h_{\alpha\beta}$ is governed by the Ricci tensor:

$$\frac{d}{dt}\left(\frac{1}{f^2}h_{\alpha\beta}\right) = 2c_d k^{d-2} R_{\alpha\beta}\,.$$

- one-loop UV non-trivial fixed point $(d > 2)$:

$$\tilde{f}^2{}_* = \frac{d-2}{2}\frac{D}{c_d\, R}. \qquad (\tilde{f}^2 = k^{d-2}f^2)$$

(R Ricci scalar) [A. Codello and R. Percacci, PLB **672** (2009) 280]

$SU(2) \times U(1)$ gauged NLσM

The spontaneous breaking of $SU(2) \times U(1) \to U(1)$ is implemented by coupling the gauge bosons to the NLσM Nambu-Goldstone bosons:

$$\mathcal{S} = \frac{1}{2f^2} \int d^4x \, h_{\alpha\beta} D_\mu \phi^\alpha D^\mu \phi^\beta + \frac{1}{4g^2} \int d^4x \, W^i_{\mu\nu} W_i^{\mu\nu} + \frac{1}{4g'^2} \int d^4x \, B_{\mu\nu} B^{\mu\nu} \qquad \text{(Euclidean action)}$$

- $1/f^2 = v^2/4$ (v is the EW VEV), g and g' are the gauge couplings.
- the gauge covariant derivative is

$$D_\mu \phi^\alpha = \partial_\mu \phi^\alpha + W^i_\mu R^\alpha_i - B_\mu L^\alpha_3 \qquad \alpha \, , i = 1, 2, 3 \, .$$

- R/L are right/left invariant Killing vectors.

Beta functions

One-loop RGEs (optimized cutoff):

$$\frac{d\tilde{f}^2}{dt} = 2\tilde{f}^2 - \frac{1}{2}\frac{\tilde{f}^2}{(4\pi)^2}\left(\tilde{f}^2 + 6g^2 + 3g'^2\right)$$

$$\frac{dg^2}{dt} = -\frac{g^4}{(4\pi)^2}\frac{29}{2}$$

$$\frac{dg'^2}{dt} = \frac{1}{6}\frac{g'^4}{(4\pi)^2}$$

For constant gauge couplings ($g_* = 0.65$, $g'_* = 0.35$) we have

$$\tilde{f}_* = \sqrt{64\pi^2 - 6g_*^2 - 3g_*'^2} \simeq 25.06$$

Approximate solution:

$$f^2(k) = \frac{\tilde{f}_*^2 f_0^2}{\tilde{f}_*^2 + (k^2 - k_0^2) f_0^2}.$$

Scattering amplitude and perturbative unitarity

Elastic amplitude of pions scattering $\pi^i\pi^j \to \pi^k\pi^l$:

$A(ij \to kl) = A(s,t,u)\delta^{ij}\delta^{kl} + A(t,s,u)\delta^{ik}\delta^{jl} + A(u,s,t)\delta^{il}\delta^{jk}$

Leading order amplitude $A(s,t,u) = sf^2/4$.

Isospin amplitudes:

$$A_0 = 3A(s,t,u) + A(t,s,u) + A(u,s,t)$$
$$A_1 = A(t,s,u) - A(u,s,t)$$
$$A_2 = A(t,s,u) + A(u,s,t)$$

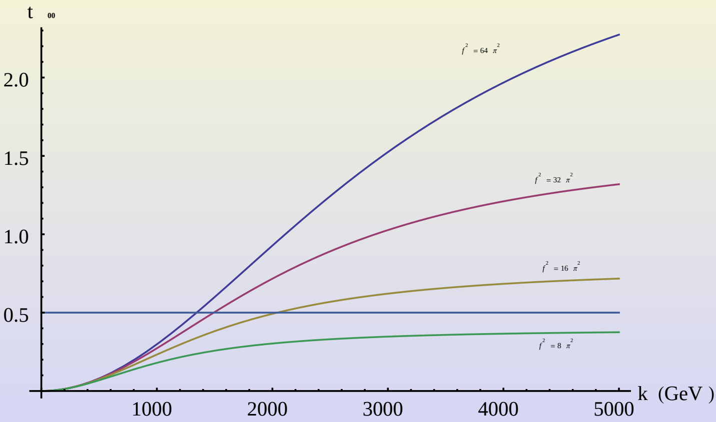
Partial wave decomposition of A_I $(I = 0,1,2)$:

$$t_{IJ} = \frac{1}{64\pi}\int_{-1}^{1} \mathrm{dcos}\theta \; P_J(\cos\theta)\, A_I$$

The unitarity bound on the first partial wave is:

$$\mathrm{Re}(t_{00}) = \frac{sf^2}{64\pi} < \frac{1}{2}\,.$$

RG improved unitarity bound



The value of the fixed point depends on the regularization scheme and on the inclusion of higher derivative operators [R. Percacci and O. Zanusso, Phys. Rev. D **81** (2010) 065012].

S and T parameters

We add to the effective lagrangian two higher order operators $\mathcal{L}^{(0)}$ and $\mathcal{L}^{(1)}$:

$$\mathcal{L} = \frac{1}{2f^2} h_{\alpha\beta} D_\mu \varphi^\alpha D^\mu \varphi^\beta + \frac{1}{4g^2} W^i_{\mu\nu} W_i^{\mu\nu} + \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} - \frac{a_0}{f^2} D_\mu \varphi^\alpha D^\mu \varphi^\beta L^3_\alpha L^3_\beta - \frac{a_1}{2} B^{\mu\nu} W^i_{\mu\nu} R_{i\alpha} L^\alpha_3$$

They give contribution to Peskin and Takeuchi (S and T) oblique parameters:

$$\begin{aligned} S &= -16\pi a_1(m_Z) + \frac{1}{6\pi}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right], \\ T &= \frac{2}{\alpha_{em}} a_0(m_Z) - \frac{3}{8\pi \cos^2\theta_W}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right]. \end{aligned}$$

Beta functions

One-loop RGEs (optimized cutoff):

$$\frac{d\tilde{f}^2}{dt} = 2\tilde{f}^2 - \frac{1}{2}\frac{\tilde{f}^2}{(4\pi)^2}\left(\tilde{f}^2(1+2a_0)+6g^2+3g'^2\right),$$

$$\frac{da_0}{dt} = \frac{1}{2}\frac{1}{(4\pi)^2}\left(\tilde{f}^2 a_0(1-2a_0)+\frac{3}{2}g'^2\right),$$

$$\frac{da_1}{dt} = \frac{1}{(4\pi)^2}\left(\tilde{f}^2 a_1+\frac{1}{6}\right).$$

Two nontrivial fixed points:

FPI: $\tilde{f}_* = 25.1$ $\quad a_{0*} = -0.000292$ $\quad a_{1*} = -0.000265$
(1 relevant and 2 irrelevant directions)

FPII: $\tilde{f}_* = 17.7$ $\quad a_{0*} = 0.501$ $\quad a_{1*} = -0.000530$
(2 relevant and 1 irrelevant directions)

Comparison with experimental bounds

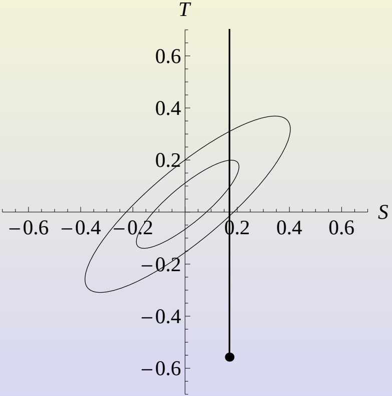


Figure: The half-line (FPII endpoints) and the dot (FPI endpoint) show the values permitted by asymptotic safety. The ellipses show the 1 and 2 σ experimental bounds with m_H=117GeV.

Fermions and Goldstone bosons

$SU(N)_L \times SU(N)_R$ invariant nonlinear sigma model lagrangian coupled to fermions:

$$\mathcal{L} = -\frac{1}{f^2}\mathrm{Tr}\left(U^\dagger \partial_\mu U U^\dagger \partial^\mu U\right) + \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R$$
$$- \frac{2h}{f}\left(\bar{\psi}_L^{ia} U^{ij} \psi_R^{ja} + \mathrm{h.c.}\right). \qquad (1/f = v/2)$$

$U = e^{if\pi^a T_a}$ is $SU(N)$ valued scalar field, π^a Goldstone bosons. $\psi^{ia}_{L/R}$ in the fundamental of $SU(N)_{L/R}$ and $SU(N_c)$

Degenerate multiplet of fermions with mass

$$m = 2\frac{h}{f} = hv\,,$$

h is the Yukawa coupling.

Beta functions

One-loop RG equations for $\tilde{f}$ and h (sharp cutoff):

$$\begin{aligned} \frac{d\tilde{f}}{dt} &= \tilde{f} - \frac{N}{64\pi^2}\tilde{f}^3 + \frac{N_c}{4\pi^2}h^2\tilde{f} \,, \\ \frac{dh}{dt} &= \frac{1}{16\pi^2}\left(4N_c - 2\frac{N^2-1}{N}\right)h^3 + \frac{1}{64\pi^2}\frac{N^2-2}{N}h\tilde{f}^2 \,. \end{aligned}$$

Fixed Points:

FPI $(h_* = 0, \tilde{f}_* = 0) \Rightarrow$ trivial

FPII $(h_* = 0, \tilde{f}_* = 8\pi/\sqrt{N}) \Rightarrow h = 0$ at all scales

FPIII $(h_* \neq 0, \tilde{f}_* \neq 0) \Rightarrow N > 2N_c$ (not true for the most phenomenologically important case $N = 2$, $N_c = 3$)

Four-fermion interactions

Fix $N = 2$, we add to the lagrangian a complete set of $SU(2)_L \times SU(2)_R$ four fermion operators:

$$\begin{aligned}\mathcal{L}_{\psi^4} &= \lambda_1 \left(\bar{\psi}_L^{ia}\psi_R^{ja}\bar{\psi}_R^{jb}\psi_L^{ib}\right) + \lambda_2 \left(\bar{\psi}_L^{ia}\psi_R^{jb}\bar{\psi}_R^{jb}\psi_L^{ia}\right) \\ &+ \lambda_3 \left(\bar{\psi}_L^{ia}\gamma_\mu\psi_L^{ia}\bar{\psi}_L^{jb}\gamma^\mu\psi_L^{jb} + \bar{\psi}_R^{ia}\gamma_\mu\psi_R^{ia}\bar{\psi}_R^{jb}\gamma^\mu\psi_R^{jb}\right) \\ &+ \lambda_4 \left(\bar{\psi}_L^{ia}\gamma_\mu\psi_L^{ib}\bar{\psi}_L^{jb}\gamma^\mu\psi_L^{ja} + \bar{\psi}_R^{ia}\gamma_\mu\psi_R^{ib}\bar{\psi}_R^{jb}\gamma^\mu\psi_R^{ja}\right) .\end{aligned}$$

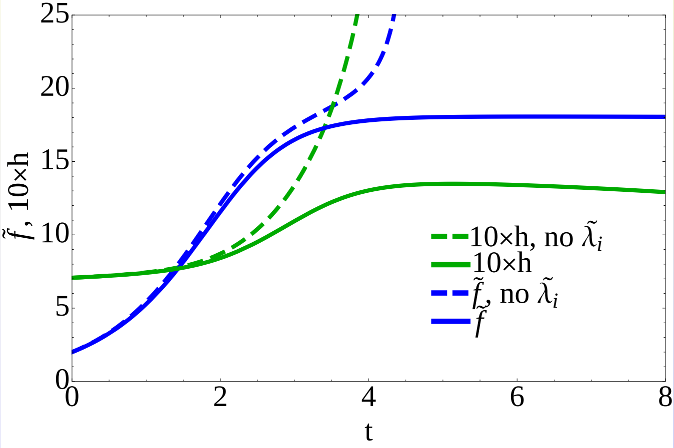
The RGE of h is modified by these interactions:

$$\frac{dh}{dt} = \frac{1}{16\pi^2}\left(4N_c - 3 + \cdots\right)h^3 + \frac{1}{64\pi^2}\left[\tilde{f}^2 - 16(N_c\tilde{\lambda}_1 + \tilde{\lambda}_2)\right]h$$

RGE for $\tilde{\lambda}$ (sharp cutoff):

$$\frac{d\tilde{\lambda}_1}{dt} = 2\tilde{\lambda}_1 - \frac{1}{4\pi^2}\left[N_c\tilde{\lambda}_1^2 + \frac{3}{2}\tilde{\lambda}_1\tilde{\lambda}_2 - 2\tilde{\lambda}_1\tilde{\lambda}_3 - 4\tilde{\lambda}_1\tilde{\lambda}_4\right]$$

Numerical solutions



Running of $\tilde{f}$ and h for $N = 2$ and $N_c = 3$. Initial conditions for the third quark family: $h_0 = m_t/v$ and $\tilde{f}_0 = 2$..

Summary and Conclusions

- There seem to exist fixed point for the gauged NLσM in which only the leading two-derivative operator is considered.
- The position of the fixed point depends on the scheme of regularization and inclusion of higher derivative operators may also move the fixed point.
- In the case of the electroweak chiral lagrangian our approach is to assume the existence of the AS picture and study some phenomenological consequences.
- AS seems to be compatible with electroweak precision measurement. Estimations of S and T in agreement with experimental data.
- In the case of $SU(2) \times U(1)$, the model is no more AS when the NLσM is coupled to fermion. AS can be restored introducing effective four-fermion interactions.

BACKUP

S and T ellipses

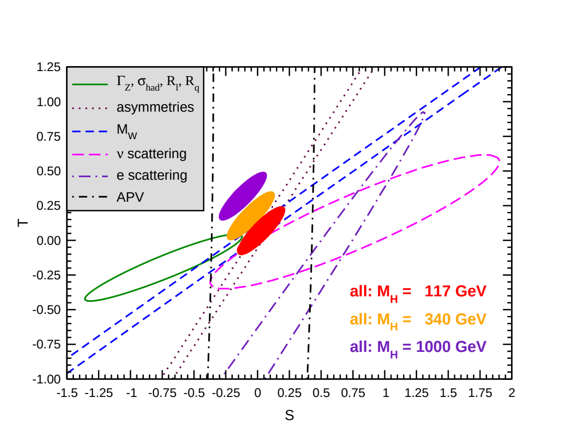


Figure: One sigma S and T ellipses, PDG, J. Phys. G, 37, 075021 (2010).

Complete set of beta functions

$$
\begin{aligned}
\frac{d\tilde{f}}{dt} &= \tilde{f} - \frac{1}{32\pi^2}\tilde{f}^3 + \frac{N_c}{4\pi^2}h^2\tilde{f} \\
\frac{dh}{dt} &= \frac{1}{16\pi^2}\left[4N_c - 3 + \frac{16}{\tilde{f}^2}(N_c\tilde{\lambda}_1 + \tilde{\lambda}_2)\right] h^3 \\
&+ \frac{1}{64\pi^2}\left[\tilde{f}^2 - 16(N_c\tilde{\lambda}_1 + \tilde{\lambda}_2)\right] h \\
\frac{d\tilde{\lambda}_1}{dt} &= 2\tilde{\lambda}_1 - \frac{1}{4\pi^2}\left[N_c\tilde{\lambda}_1^2 + \frac{3}{2}\tilde{\lambda}_1\tilde{\lambda}_2 - 2\tilde{\lambda}_1\tilde{\lambda}_3 - 4\tilde{\lambda}_1\tilde{\lambda}_4\right] \\
\frac{d\tilde{\lambda}_2}{dt} &= 2\tilde{\lambda}_2 + \frac{1}{4\pi^2}\left[\frac{1}{4}\tilde{\lambda}_1^2 + 4\tilde{\lambda}_1\tilde{\lambda}_3 + 2\tilde{\lambda}_1\tilde{\lambda}_4 - \frac{3}{4}\tilde{\lambda}_2^2 + 2(2N_c - 1)\tilde{\lambda}_2\tilde{\lambda}_3\right. \\
\frac{d\tilde{\lambda}_3}{dt} &= 2\tilde{\lambda}_3 + \frac{1}{4\pi^2}\left[\frac{1}{4}\tilde{\lambda}_1\tilde{\lambda}_2 + \frac{N_c}{8}\tilde{\lambda}_2^2 + (2N_c - 1)\tilde{\lambda}_3^2 + 2(N_c + 2)\tilde{\lambda}_3\tilde{\lambda}_4\right. \\
\frac{d\tilde{\lambda}_4}{dt} &= 2\tilde{\lambda}_4 + \frac{1}{4\pi^2}\left[\frac{1}{8}\tilde{\lambda}_1^2 - 4\tilde{\lambda}_3\tilde{\lambda}_4 + (N_c + 2)\tilde{\lambda}_4^2\right]
\end{aligned}
$$

Fixed points table ($\tilde{f}_* = 17.78$, $h_* = 0$)

	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	ϵ_h
fp0	0	0	0	0	0.5
fp1a	0	-28.71	-7.18	0	1.22
fp1b	0	0	7.85	-9.51	0.5
fp1c	0	25.61	-4.27	0	-0.15
fp1d	25.80	-1.77	0.19	-1.15	-1.42
fp2a	13.41	20.10	-3.80	-0.24	-1.03
fp2b	20.86	-3.56	7.04	-8.94	-1.00
fp2c	0	-36.55	2.34	-13.92	1.43
fp2d	0	0	-15.79	0	0.5
fp2e	37.17	-37.36	-8.43	-1.65	-1.38
fp2f	-2.92	32.59	4.67	-12.04	-0.10
fp3a	$0.$	31.67	4.67	-12.06	-0.30
fp3b	19.95	-8.59	-15.27	-0.36	-0.80
fp3c	31.22	-44.52	0.73	-13.38	-0.74
fp3d	-4.87	1.54	-5.42	-20.10	0.83

Experimental constraints

Current bounds on contact interactions, only one operator is considered [E. Eichten, K. D. Lane, M. E. Peskin, PRL **50** (1983) 811], $\psi = (u\ d)$:

$$\mathcal{L}_{qqqq} = \frac{4\pi A}{2\Lambda^2}\,\bar{\psi}_L^{ia}\gamma_\mu\psi_L^{ia}\bar{\psi}_L^{jb}\gamma^\mu\psi_L^{jb} \qquad (A = \pm 1)\,.$$

Lower bound of the contact interaction scale Λ:

$$\lambda(k) = \frac{2\pi}{\Lambda^2}\,.$$

Current published bound [ATLAS Collaboration, New J. Phys. 13 (2011) 053044]:

$$\Lambda > 9.5\text{TeV} \qquad \text{with } 36\,\text{pb}^{-1}.$$

Future expected bound [ATLAS and CMS, arXiv:0709.2518 hep-ph]:

$$\Lambda > 30\text{TeV} \qquad \text{with } 100\,\text{fb}^{-1}.$$

Comparison with the experimental limit

$\tilde{\lambda}_i(k) < 2\pi k^2/\Lambda_{bound}$

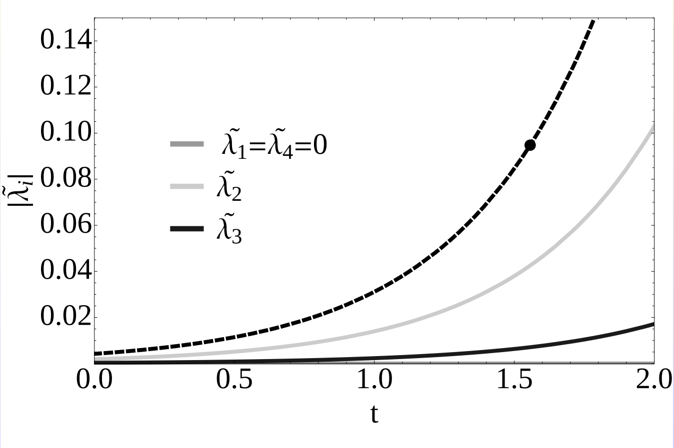


Figure: RG evolution of $\tilde{\lambda}_i$ towards the IR from the FP fp1c.

Two operators analysis (preliminary)[w. U. De Sanctis]

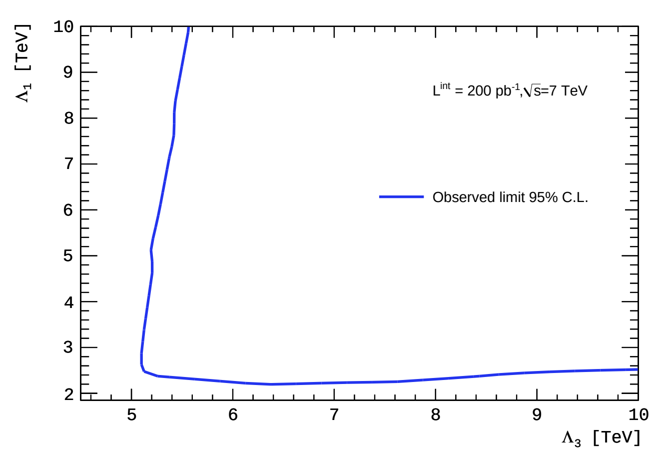


Figure: 95% CL limit on Λ_1 and Λ_3