Complex behaviour in chaotically quantized field theories

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Background

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There are about 20 free parameters in the standard model (SM)

- coupling constants
- masses
- mixing angles

Not fixed by any first principle in the SM. Need something in addition!

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Fundamental constants are different in various parts of the universe (string landscape) and we just happen to live in a part where they are such that life could evolve and we are there to observe things...

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Do you really believe that...?

Background

In string theory/M-theory fundamental parameters correspond to so-called moduli fields to approach minima of their potentials—but nobody knows what these potentials are and how they are generated. One needs an additional (so far unknown sector) that fixes the fundamental parameters.

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Background

In this talk we construct such an additional sector, starting from a generalization of the stochastic quantization method: so-called chaotic quantization. Main idea: Gaussian white noise (in fictitious time) of Parisi-Wi approach of stochastic quantization replaced by suitable deterministic chaotic dynamics, a coupled noise network on a microscopic scale This leads to complexity at a fundamental level: nontrivial correlations in quantization noise network will distinguish fundamental parameters

Stochastic quantization

To 2nd quantize classical field eq. $\frac{\delta S}{\delta \varphi} = 0$ introduce fictitious time variable s and consider Langevin equation

$$\frac{\partial}{\partial s}\varphi(x,s) = \frac{\delta S}{\delta \varphi}(x,s) + L(x,s)$$

$L(x,s)$: spatio-temporal Gaussian white noise, i.e.

$\langle L(x,s) \rangle = 0$

$\langle L(x,s)L(x',s') = 2\delta(x-x')\delta(s-s')$

Quantum mechanical expectations can be calculated as stochastic expectations with respect to this Langevin eq.

PH Damgaard, H. Hueffel, Stochastic quantization, Word Scientific (1988))

Chaotic quantization

- Basic idea is to generate ‘noise’ by smooth deterministic chaotic map T
- If T is strongly mixing then a functional Central Limit Theorem for weakly dependent events guaranties convergence or rescaled sum of iterates to Wiener process
- In other words, uncoupled deterministic chaotic maps looked at ‘from far away’ generate Gaussian white noise in a suitable scaling limit

  C. Beck, G. Roepstorff, Physica A 145, 1 (1987)
- application to stochastic quantization C. Beck, Nonlinearity 8, 423 (1995) Particular coupled chaotic noise model: ‘chaotic string’

What is a chaotic string?

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- . . . 1 fictitious time parameter n

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described by iteration

$$\Phi^i_{n+1} = (1-a)T_N(\Phi^i_n) + s\frac{a}{2}\left(T^b_N(\Phi^{i-1}_n) + T^b_N(\Phi^{i+1}_n)\right)$$

with $T^0_N(\Phi) = \Phi$, $T^1_N(\Phi) = T_N(\Phi)$, where $T_N(\Phi)$ is N-th order Chebychev polynomial, $s = \pm 1$.

Chebyshev polynomials

Defined by $T_N(\Phi) = \cos(N \arccos \Phi)$

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$$T_1(\Phi) = \Phi$$

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$$T_2(\Phi) = 2\Phi^2 - 1$$

Chebyshev polynomials

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$$T_3(\Phi) = 4\Phi^3 - 3\Phi \qquad \ldots$$

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- . . . semi-conjugated to a Bernoulli shift with N symbols (shift of information)

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- ... ergodic, strongly mixing, distinguished by minimum skeleton of higher-order correlations (C. Beck, Nonlinearity 4, 1131 (1991))

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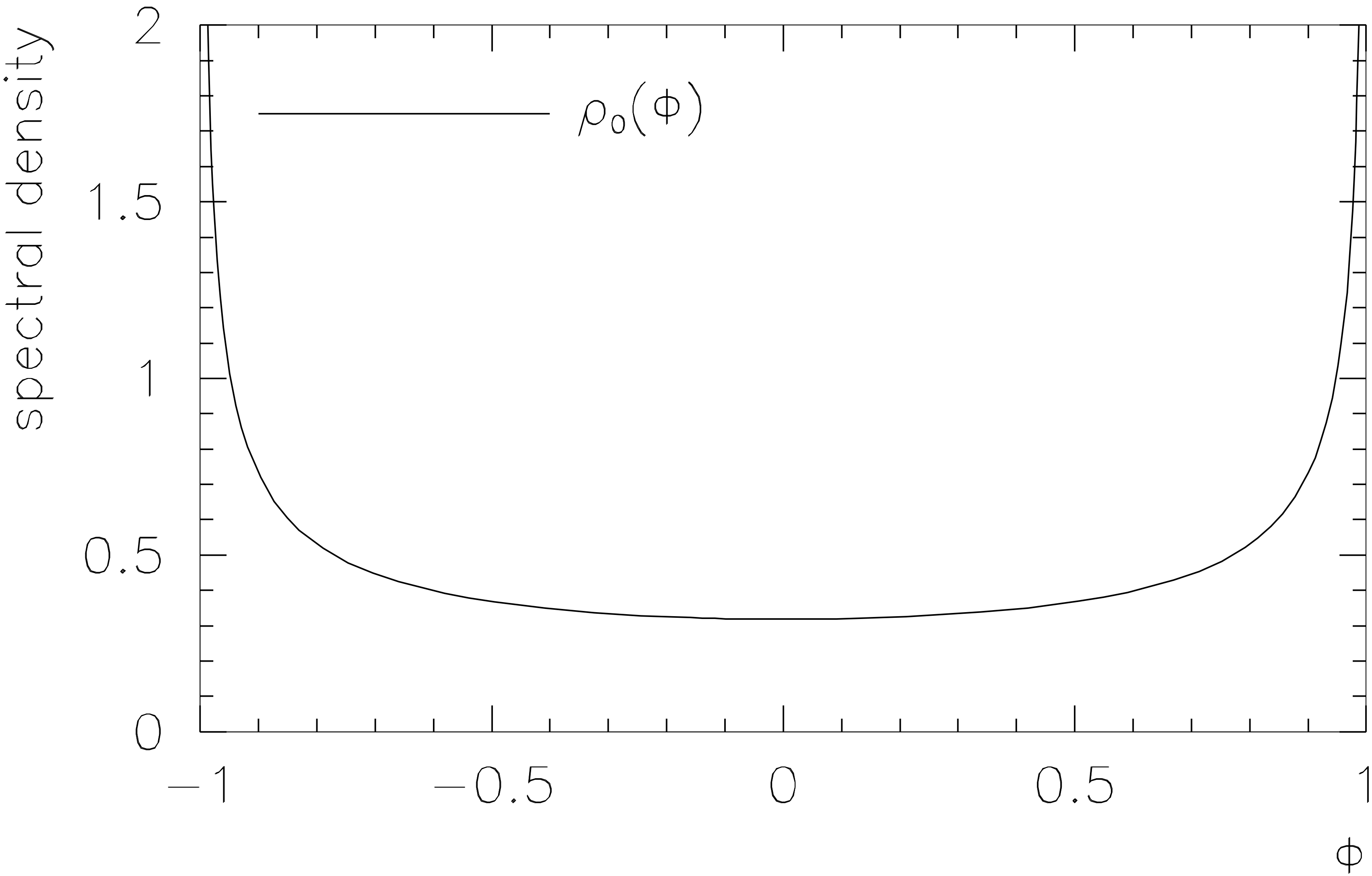
invariant density $\rho_0(\Phi) = \frac{1}{\pi\sqrt{1-\Phi^2}}$

Invariant density



Generalized statistical mechanics

- Invariant density is q-Gaussians with $q=3$ (or $q=-1$ is escort formalism is used) C. Beck, F. Schloegl, Thermodynamics of Chaotic Systems, Cambridge University Press (1993)
- $E=\frac{1}{2}mv^2$, $v=i\Phi$, $m=\beta_0^{-1}$ kind of $q=3$ Tsallis statistics with complex momenta
- You can obtain such a deformed statistical mechanics from a 'superstatistics' where the inverse temperature β is distributed according to a χ^2-distribution $f(\beta)$ of degree 1: $\int d\beta f(\beta)e^{-\beta E}=\rho_0(\Phi)$
- Formally this system maximizes more general entropies relevant for superstatistical systems (R. Hanel, S. Thurner, M. Gell-Mann, PNAS (2010))

Symmetry and classification

Laplacian coupling of the chaotic string

$$\Phi_{n+1}^{i} = (1-a)T_N(\Phi_n^i) + s\frac{a}{2}\left(T_N^b(\Phi_n^{i-1}) + T_N^b(\Phi_n^{i+1})\right)$$

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  diffusive forward coupling ($2A, 3A, \ldots$)

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- NB string ($b=0$, $s=+1$, $T_N^0(\Phi) = \Phi$)
  diffusive backward coupling ($2B, 3B, \ldots$)

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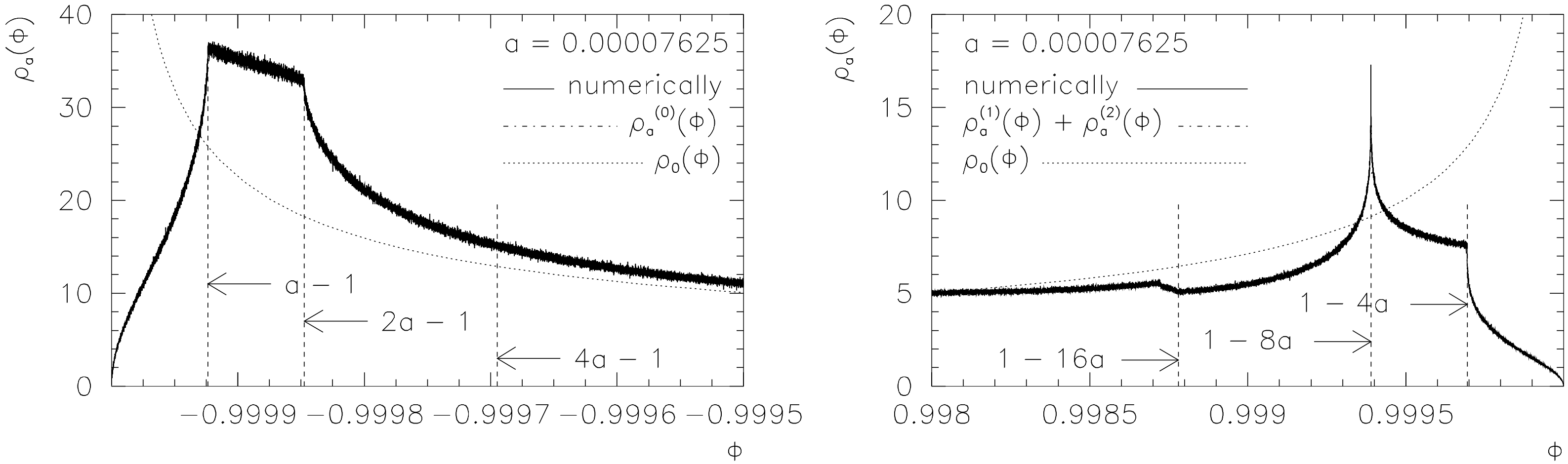
- NB string ($b=0$, $s=+1$, $T^0_N(\Phi) = \Phi$)
  diffusive backward coupling ($2B$, $3B$, $\ldots$)
- NA^-/NB^- string ($s=-1$, only for even N)
  antidiffusive coupling ($2A^-$, $2B^-$, $\ldots$)

The coupling a

$a > 0$: deviation from $\rho_0(\Phi)$

The coupling a

$a > 0$: deviation from $\rho_0(\Phi) \quad \Rightarrow \quad \rho_a(\Phi)$



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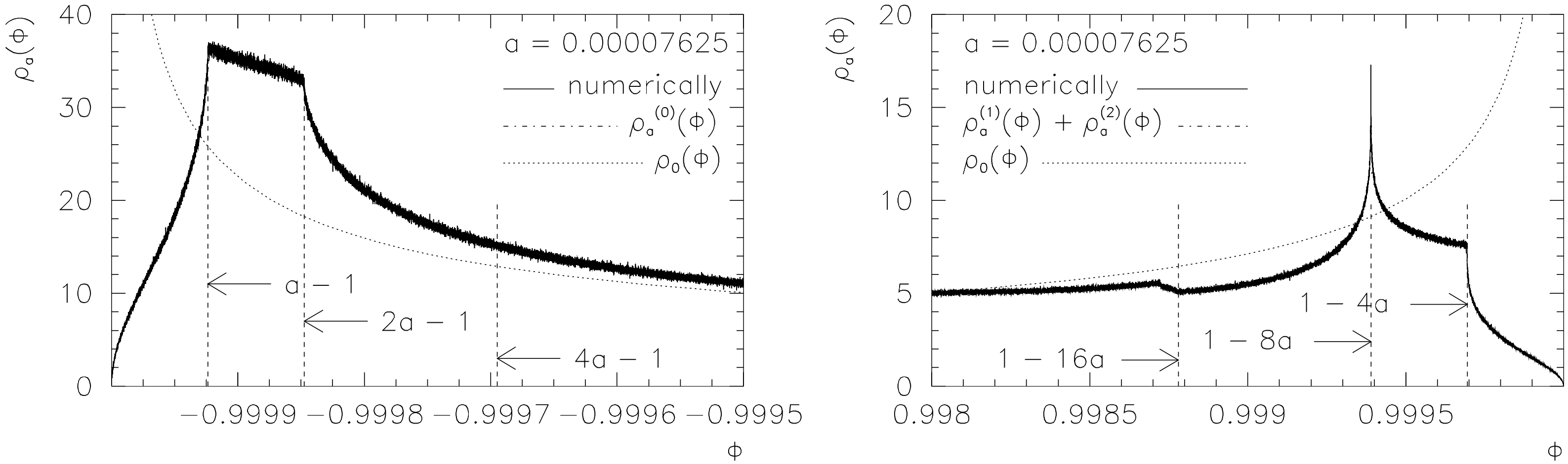

$\Rightarrow$ scaling behavior of observables $\langle f(\Phi)\rangle$

S. Groote, C. Beck, Phys. Rev. **E74** (2006) 046216; Dyn. Sys. **22(2)** (2007) 219

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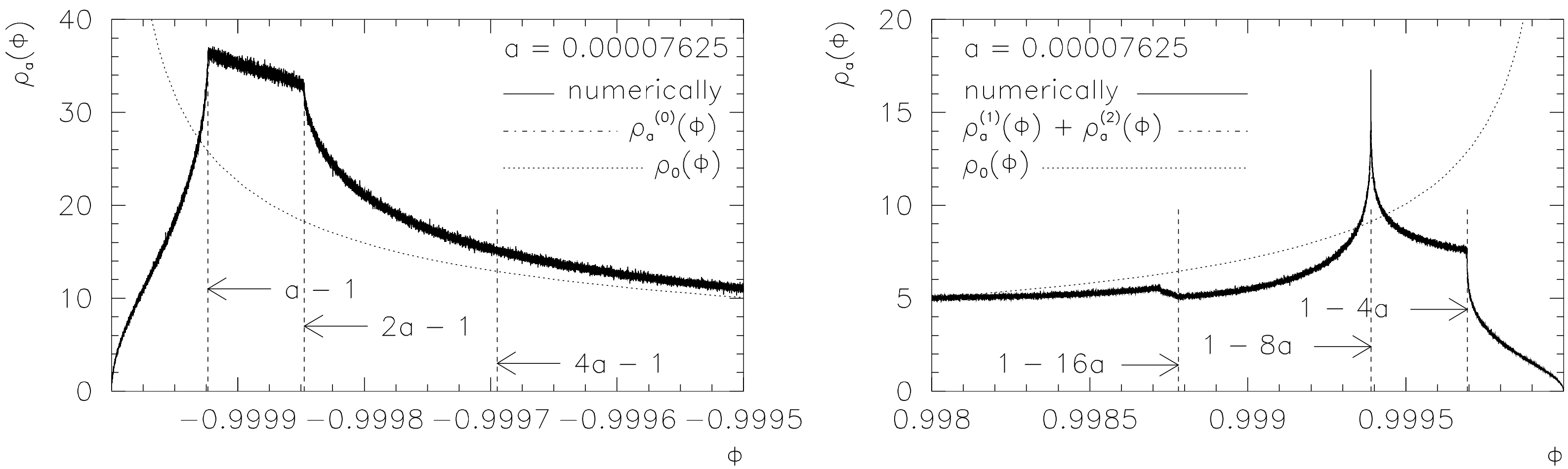

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What are suitable observables for this chaotic noise network?

Self energy

Dynamics of $\Phi_{n+1} = \pm T_N(\Phi_n)$ described by "potentials" ($V(\Phi) := \Phi^2 - \langle \Phi^2 \rangle$)

$$
\begin{aligned}
V_\pm^{(2)}(\Phi) &= \pm\left(-\tfrac{2}{3}\Phi^3 + \Phi\right) + \tfrac{1}{2}V(\Phi) \\
V_\pm^{(3)}(\Phi) &= \pm\left(-\Phi^4 + \tfrac{3}{2}\Phi^2\right) + \tfrac{1}{2}V(\Phi) \\
V_\pm^{(4)}(\Phi) &= \pm\left(-\tfrac{8}{5}\Phi^5 + \tfrac{8}{3}\Phi^3 - \Phi\right) + \tfrac{1}{2}V(\Phi) \qquad \ldots
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Self energy can also be regarded as a generalized thermodynamic potential

Interaction energy

Dynamics of

$$\Phi_{n+1}^{i} = \Phi_n'^{i} + \frac{a}{2}\left(\pm\Phi_n'^{i-1} - 2\Phi_n'^{i} \pm \Phi_n'^{i+1}\right)$$

($\Phi_n'^{i} = \pm T_N(\Phi_n^i)$) described by “potentials”

$$W_{\pm}(\Phi_n^i, \Phi_n^{i-1}) = \frac{1}{4}\left((\Phi_n^i \pm \Phi_n^{i-1})^2 - \langle(\Phi_n^i)^2\rangle - \langle(\Phi_n^{i-1})^2\rangle\right)$$

Interaction energy

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Observable: interaction energy $\langle W_\pm(\Phi^i_n, \Phi^{i-1}_n)\rangle$

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Observable: interaction energy $\langle W_\pm(\Phi_n^i, \Phi_n^{i-1})\rangle$

$W(a) \sim \langle \Phi_n^i \Phi_n^{i+1} \rangle$

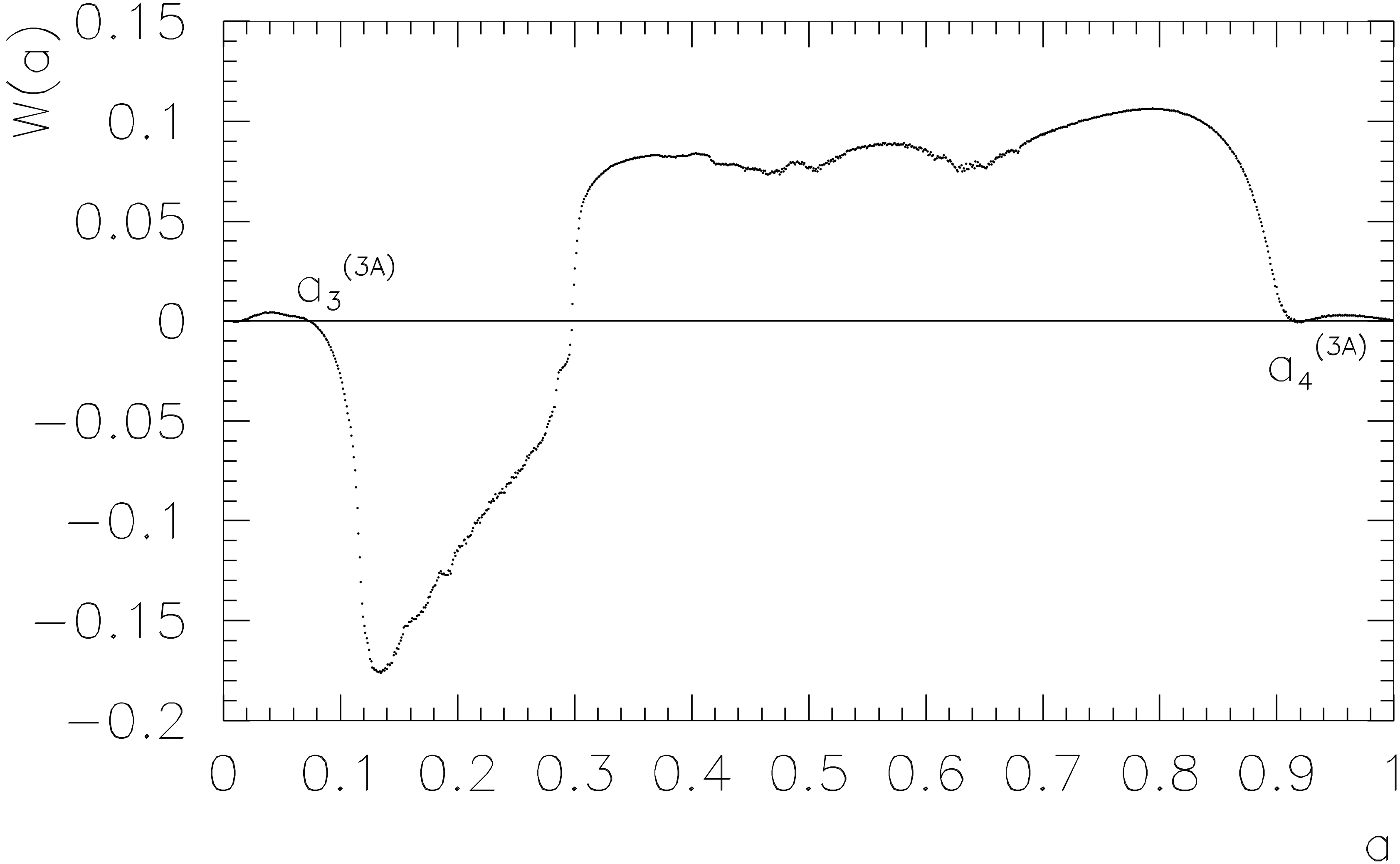
Stable zeros

- Principal idea: A priori any coupling a possible.
- But distinguished states given by those a where $W(a) = 0$ (coupled chaotic noise network as random as possible!)
- Need also consider stability: if

$$\frac{\partial}{\partial s} a \sim W(a)$$

then only zeros with negative slope are stable (moduli field dynamics)

Numerical results for $3A$ interaction energy



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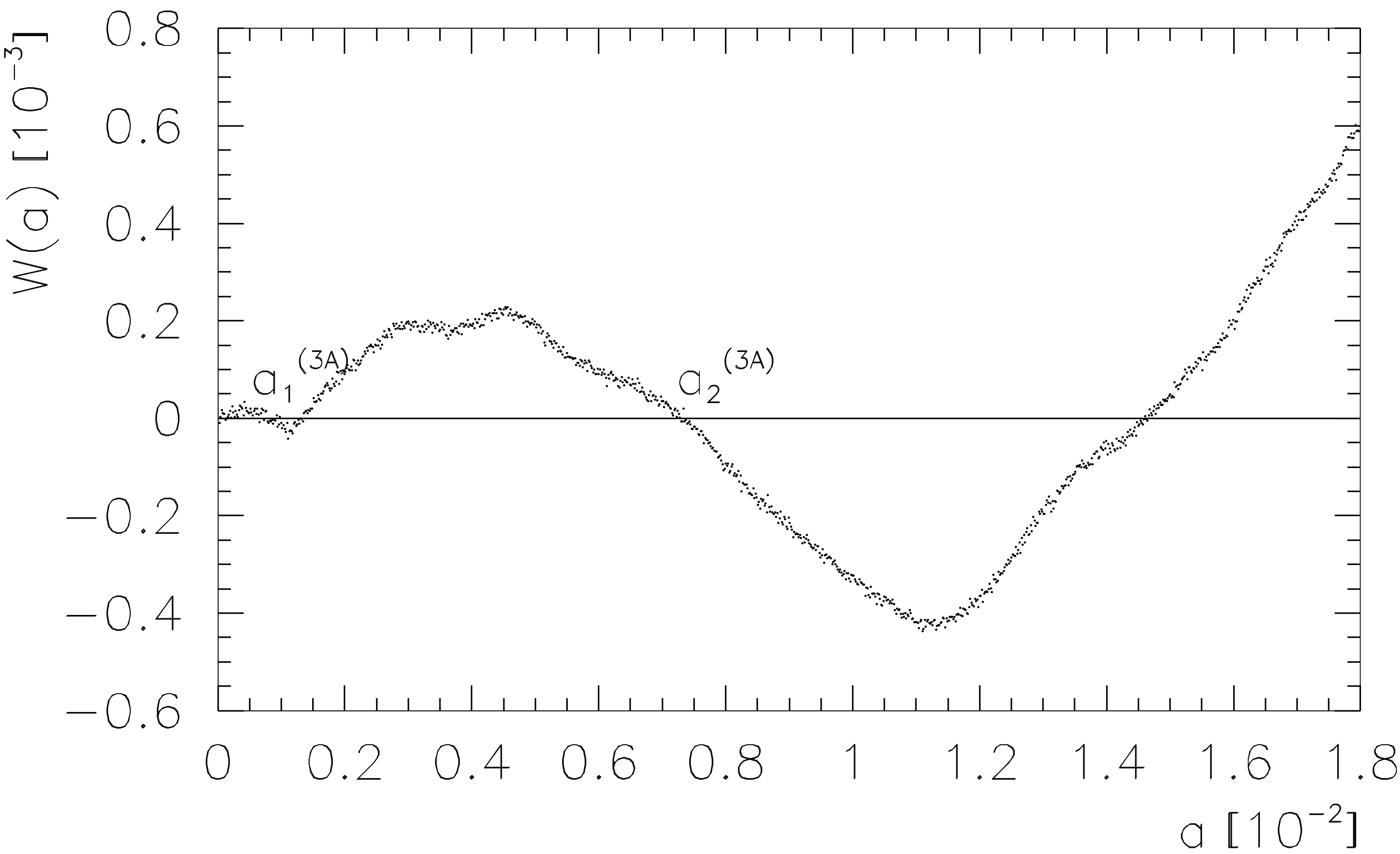

$3A$ *interaction energy*

$$
\begin{aligned}
a_1^{(3A)} &= 0.0008164(8) \\
a_2^{(3A)} &= 0.0073038(17)
\end{aligned}
$$

Numerical observation:

$$a_2^{(3A)} \approx \alpha_{\mathrm{el}} \approx \frac{1}{137}, \qquad a_1^{(3A)} \approx \frac{1}{9}\alpha_{\mathrm{el}}$$

It turns out that the zeros are equal to the electromagnetic coupling for a particle of charge $\pm e$ ($a_2^{(3A)}$, e.g. electron), or of charge $\pm e/3$ ($a_1^{(3A)}$, e.g. down-quark)

Running coupling

Due to the renormalization group equation the coupling runs with the energy. Using the 1-st order QED formula

$$\begin{aligned}\alpha_{\rm el}(E) &= \alpha_{\rm el}(0)\left\{1+\frac{2\alpha_{\rm el}(0)}{\pi}\sum_i Q_i^2 \times \right.\\ &\left. \times \int_0^1 d(1-x)\ln\left(1+\frac{E^2}{m_i^2}x(1-x)\right)dx\right\}\end{aligned}$$

where the sum runs over all charged elementary particle flavors with charges Q_i and masses m_i, the best fit is obtained for

$$\alpha_{\rm el}(3m_e) = 0.007303, \qquad \frac{1}{9}\alpha_{\rm el}(3m_d) = 0.0008165$$

Some comment on masses

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- we use “free” masses, i.e.
  $\overline{\mathrm{MS}}$ mass $\bar{m}_q(E)$ at fixed $E = 1$ GeV for light flavors
  $\overline{\mathrm{MS}}$ mass $\bar{m}_q(\bar{m}_q)$ for heavy flavors

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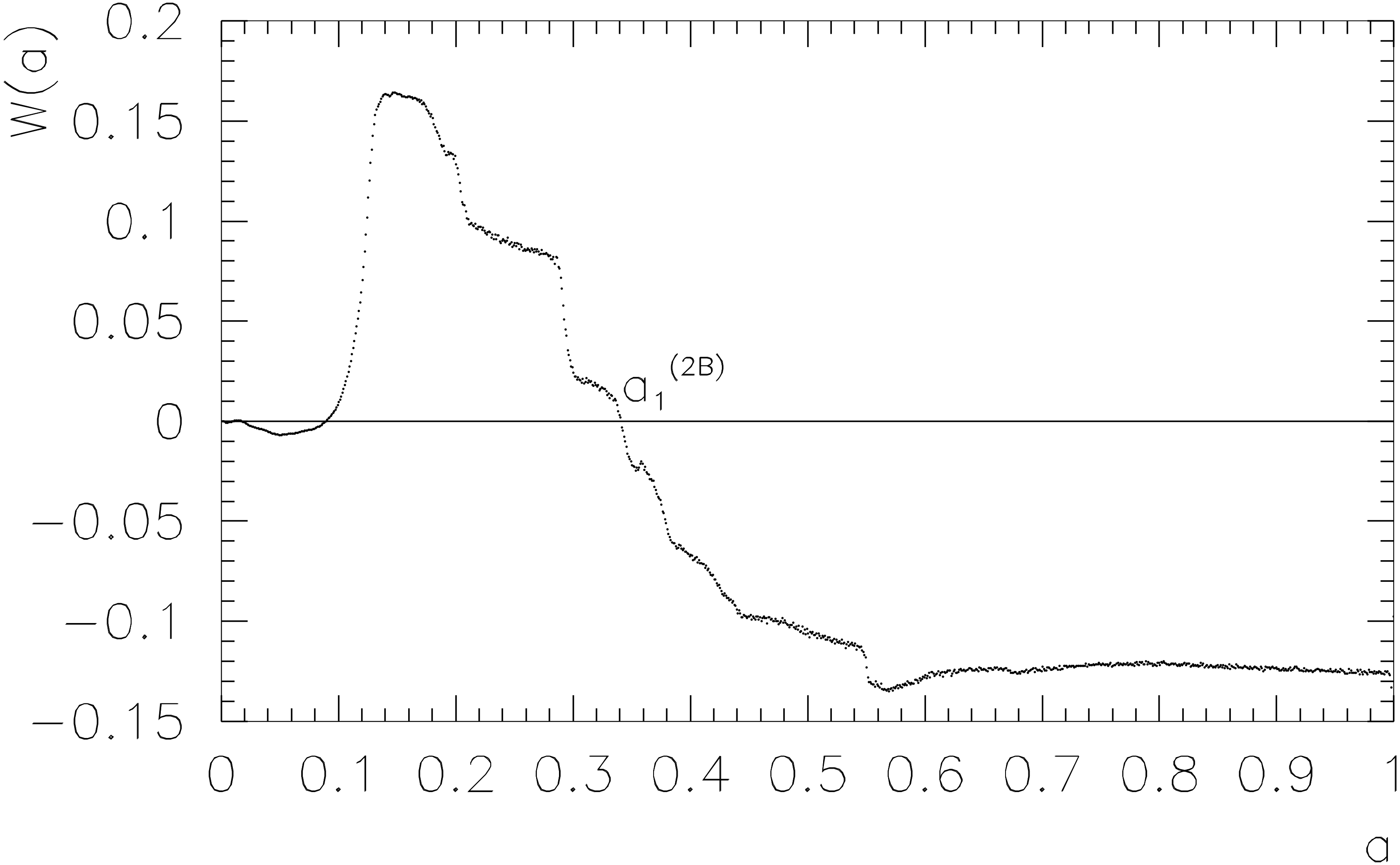
Vice versa, we can use the zeros to determine masses:

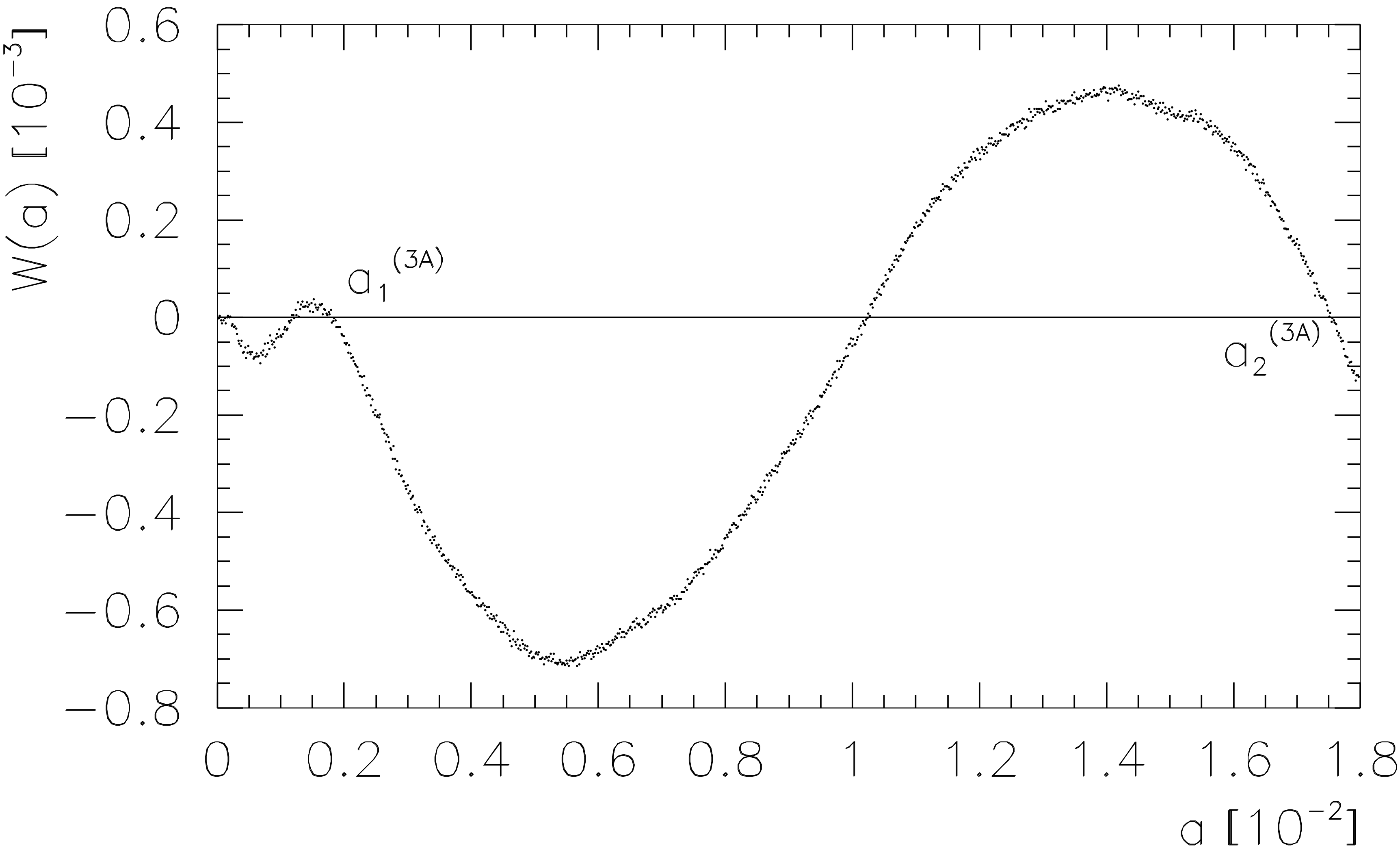
$$m_d^{\mathrm{est}} = (8.7 \pm 2.1)\,\mathrm{MeV}$$

coincides with $\bar{m}_d(1\,\mathrm{GeV})$ (PDG)

$3B$ *interaction energy*



$3B$ *interaction energy*



$3B$ *interaction energy*

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\begin{aligned}
a_1^{(3B)} &= 0.0018012(4) \\
a_2^{(3B)} &= 0.017550(1)
\end{aligned}
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For symmetry reasons

- ... would also expect to see ν, u
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Indeed, for the B string one observes

$$
\begin{aligned}
a_1^{(3B)} &= \alpha_{\mathrm{el}}(3m_{d_L}) + \alpha_{\mathrm{weak}}^{u_R}(3m_{u_R}) \\
a_2^{(3B)} &= \alpha_{\mathrm{el}}(3m_{e_R}) + \alpha_{\mathrm{weak}}^{\nu_L}(3m_{\nu_{eL}})
\end{aligned}
$$

Electroweak coupling

We use

$$\alpha_{\text{weak}}^{f} = \alpha_{\text{el}} \frac{(T_f^3 - Q_f \sin^2 \theta_W)^2}{\sin^2 \theta_W \cos^2 \theta_W}$$

where T_f^3 is the weak isospin of the fermion and Q_f its charge (in units of e), and the fact that left handed and right handed particles cannot interact.

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where T_f^3 is the weak isospin of the fermion and Q_f its charge (in units of e), and the fact that left handed and right handed particles cannot interact. Result:

$$\begin{aligned} \alpha_{\text{el}}(3m_{d_L}) + \alpha_{\text{weak}}^{u_R}(3m_{u_R}) &= 0.001801 \\ \alpha_{\text{el}}(3m_{e_R}) + \alpha_{\text{weak}}^{\nu_L}(3m_{\nu_{eL}}) &= 0.01755 \end{aligned}$$

which is a 4-digit coincidence with the zeros of 3B string.

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\end{aligned}$$

which is a 4-digit coincidence with the zeros of 3B string.
mass estimate:

$$m_u = (7.1 \pm 3.0)\ \text{MeV}$$

Electroweak parameters

Vice versa, knowing $\alpha_{\rm el}(E=3m_e)$, we determine

$$\alpha_{\rm el}(0) = \alpha_{\rm el}(E)\left\{1 - \frac{2\alpha_{\rm el}(E)}{\pi}\sum_i Q_i^2 \times \right.$$
$$\left. \times \int_0^1 d(1-x)\ln\left(1+\frac{E^2}{m_i^2}x(1-x)\right)dx\right\}$$

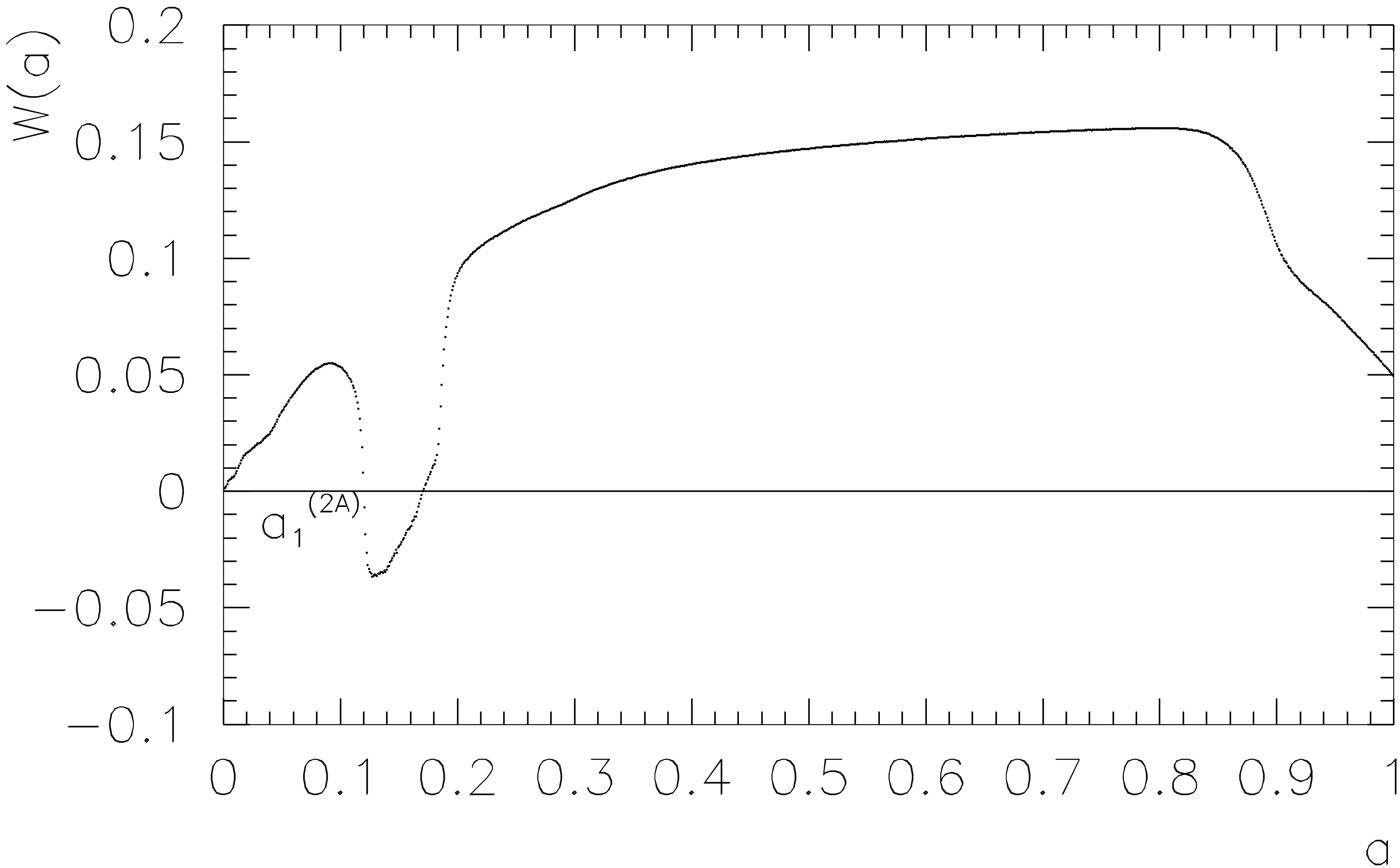
as $\alpha_{\rm el}(0) = 1/137.03(3)$, to be compared with $1/137.036$.

In the same manner,

$$\sin^2\theta_W = \frac{1}{2}\left(1-\sqrt{1-\frac{\alpha_{\rm el}(0)}{a_2^{(3B)}-a_2^{(3A)}}} = 0.23177(7)\right)$$

perfectly agrees with measurements in (PDG)

$2A$ interaction energy



$2A$ *interaction energy*

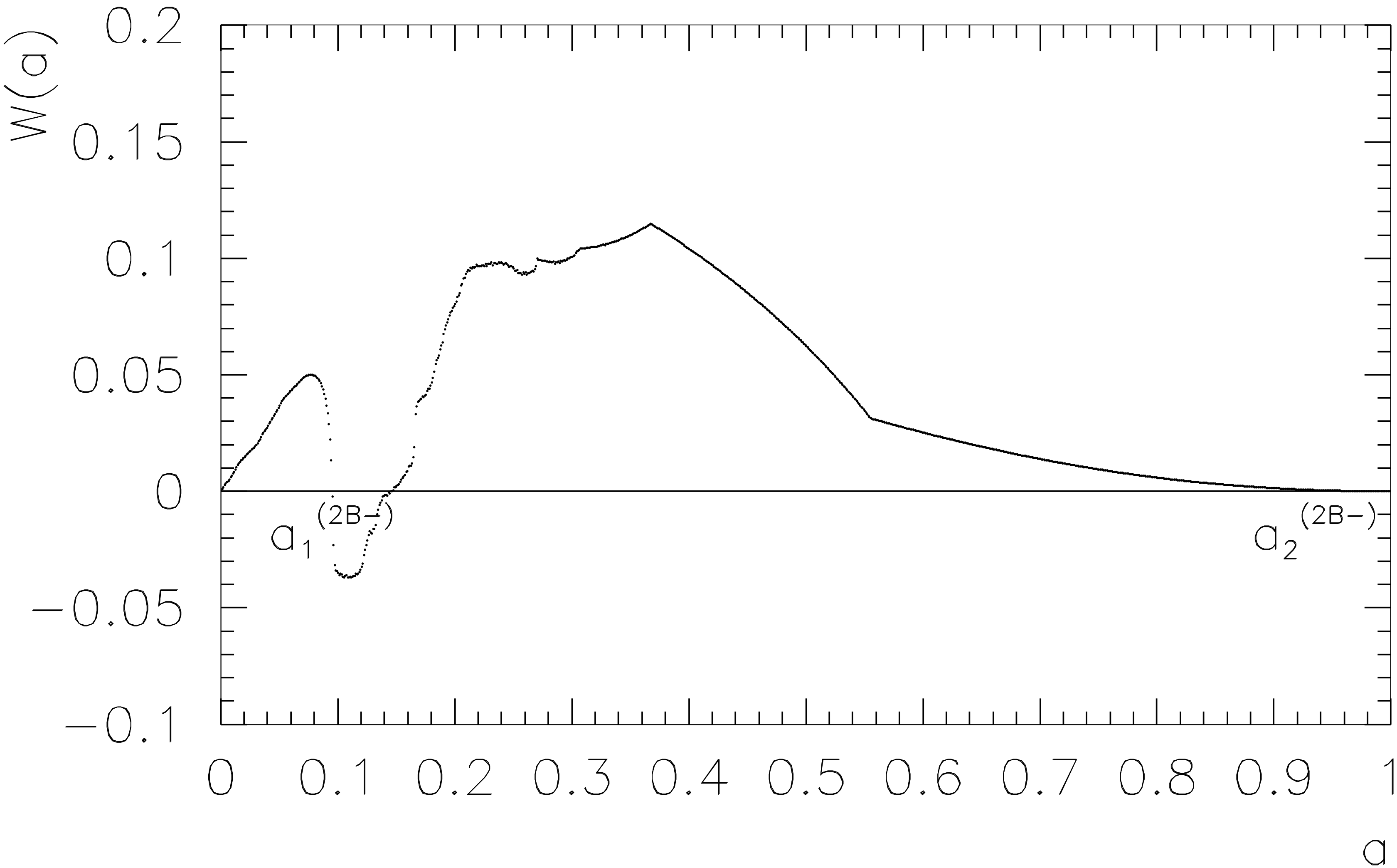
The only zero

$$a_1^{(2A)} = 0.120093(3)$$

numerically appears to coincide with the strong coupling constant α_s at the W boson mass scale, again in excellent agreement with measurements.

Observe symmetry: $N = 3$ strings fix electroweak couplings at lightest fermion mass scales, $N = 2$ strings strong couplings at lightest bosonic mass scales. B-coupling form describes spinless states, A-coupling states with spin.

$2B^-$ interaction energy



$2B^-$ interaction energy

- The first stable zero of the $2B^-$ dynamics provides a prediction for the Higgs mass: $m_H = (154.4 \pm 2)$ GeV
- $a_1^{(2B^-)} = 0.095370(1)$ interpreted as $a_1^{(2B^-)} = \alpha_s(E)$ with $E = m_H + 2m_t$
- Discrete symmetry transformations for chaotic strings studied in detail in M. Schaefer, C. Beck, Dyn. Syst. 25, 253 (2010)

Relevant energy scales

- Lots of other coincidences with standard model couplings for local minima of self energy
- relevant energy scales E always given by $E = \frac{N}{2}(m_B + 2m_f)$
- Factor $N/2$ can be understood from superstatistical generalized statistical mechanics approach (formally χ^2-distributed inverse local temperature and complex momenta)

Feynman webs

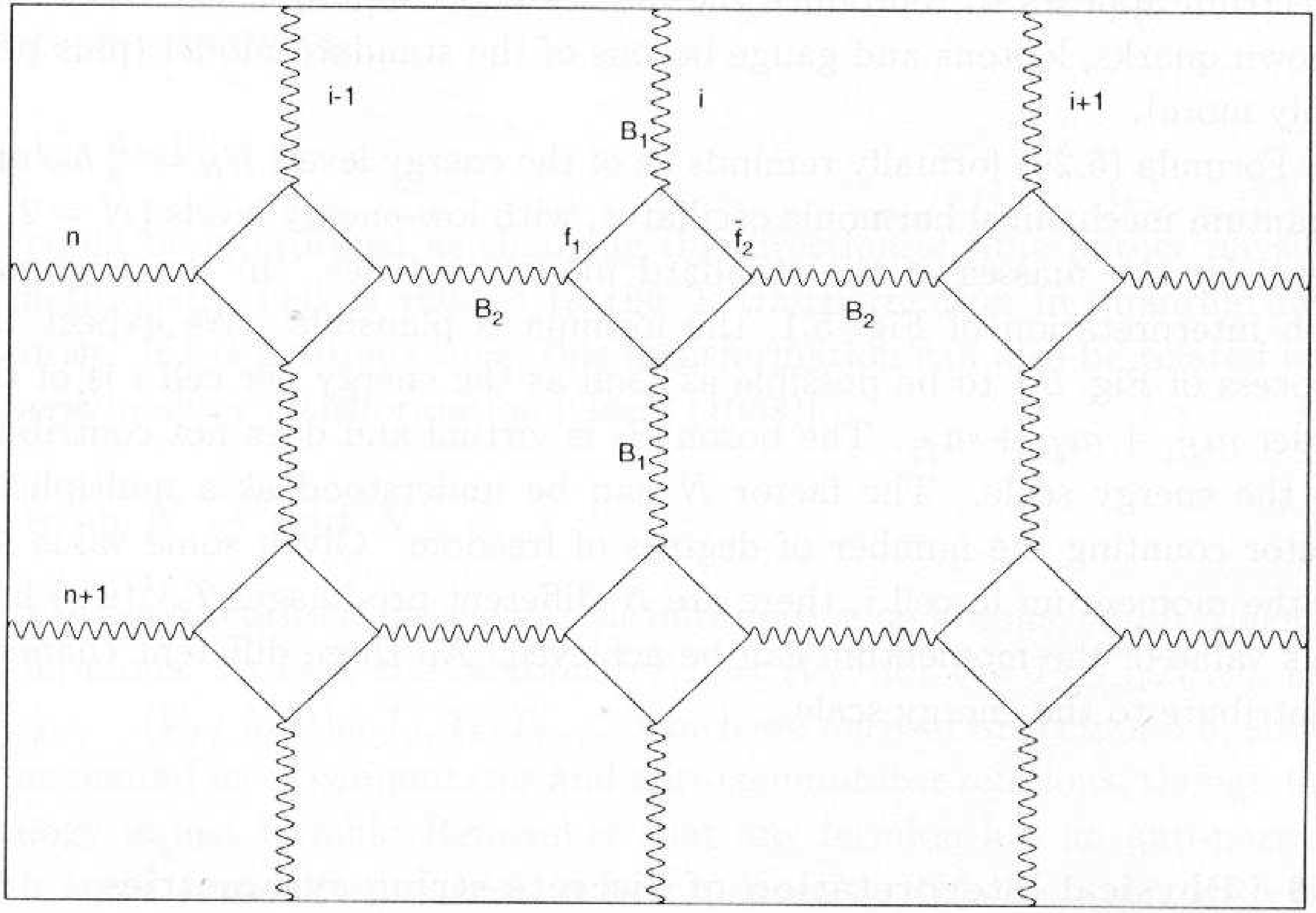
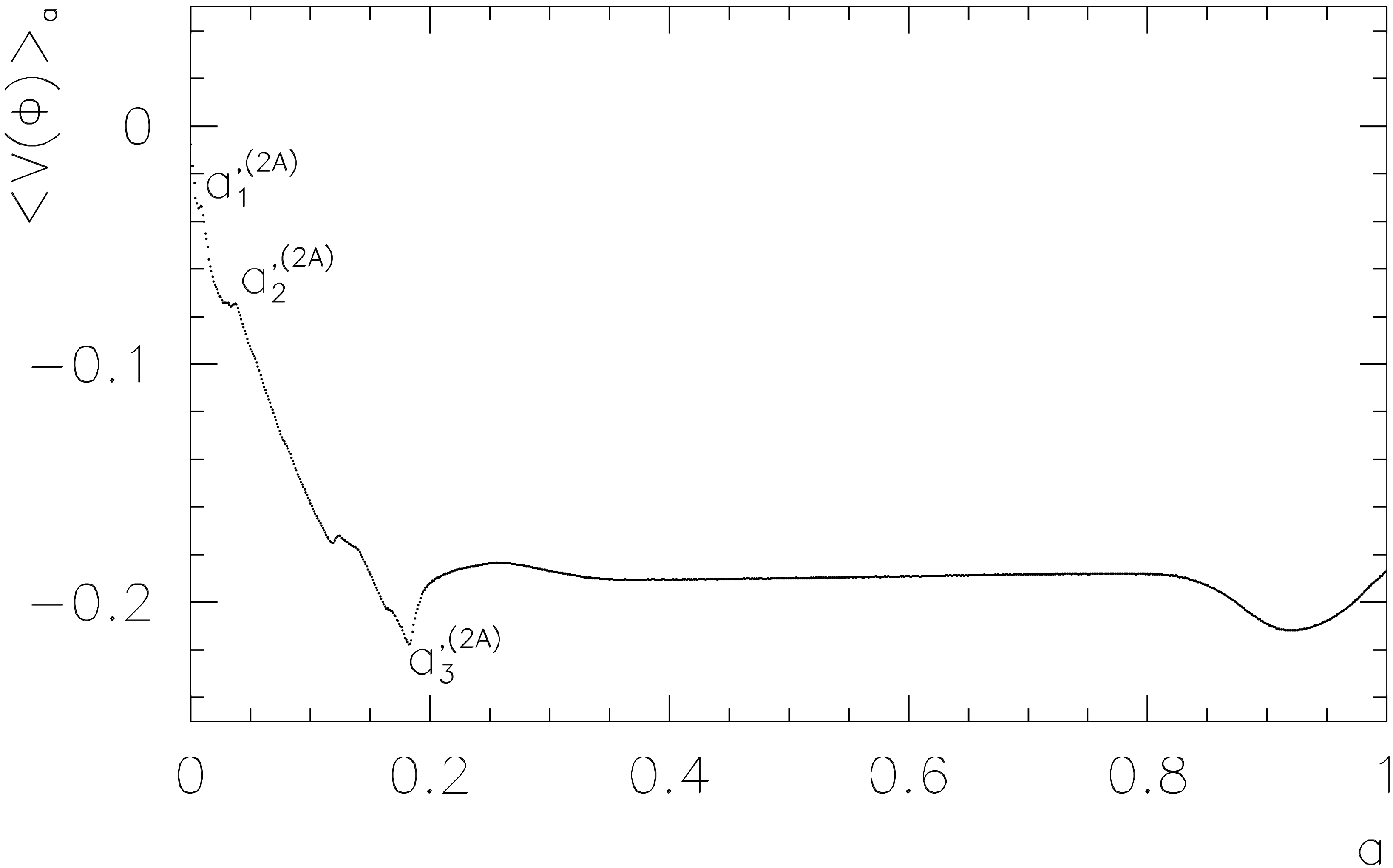


Fig. 5.1 Feynman web interpretation of the coupled map dynamics.

$2A$ *self energy*



$2A$ *self energy*

$$a_3'^{(2A)} = 0.1848(1) \;\leftrightarrow\; \alpha_s(2m_b) = 0.1848$$

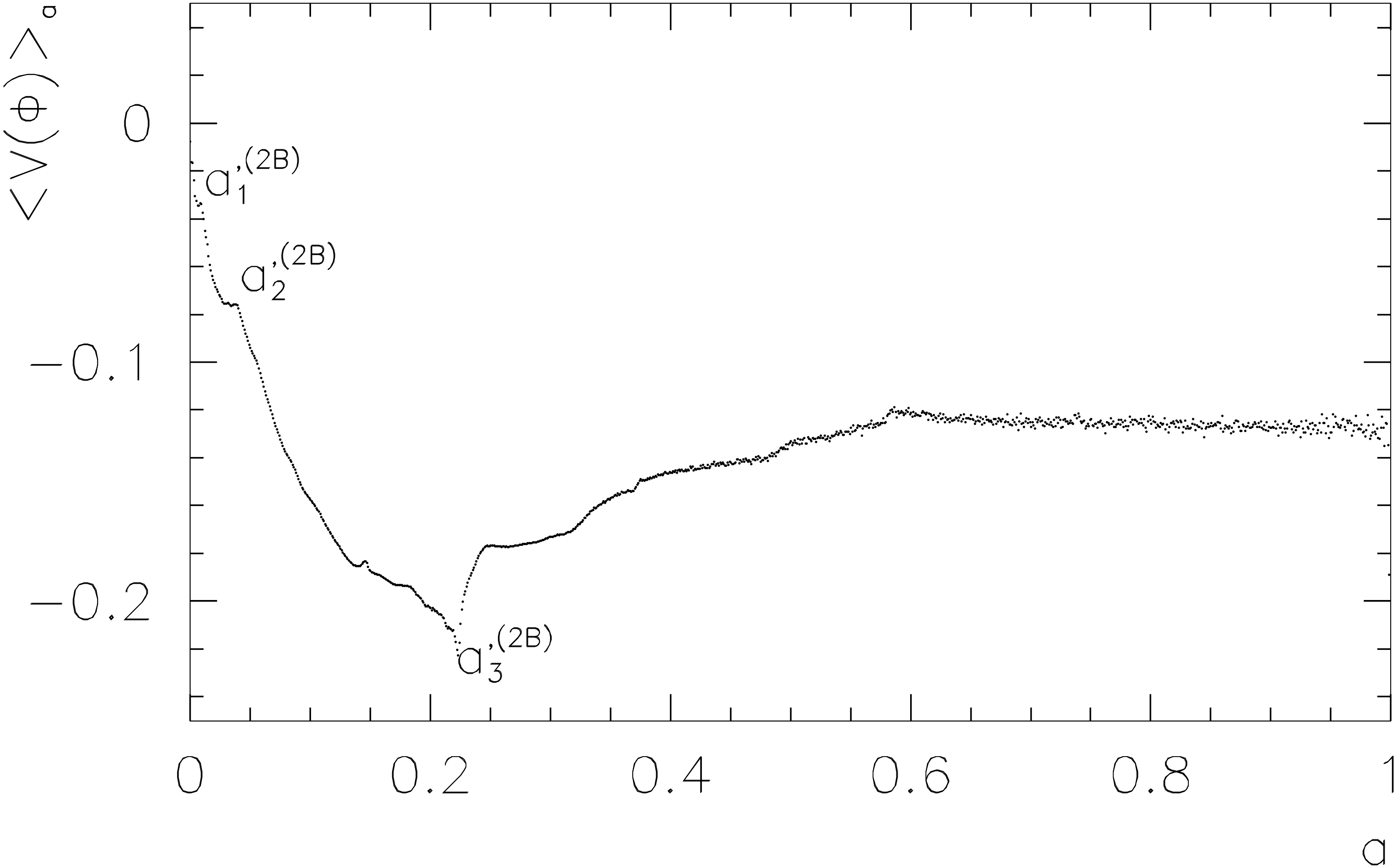
obtained for $m_b = 4.23$ GeV

$$a_2'^{(2A)} = 0.03369(2) \;\leftrightarrow\; \alpha_2(m_Z + 2m_b) = 0.03369(1)$$

with $\alpha_2(E)$ the $SU(2)$ part of the electroweak coupling

$$a_1'^{(2A)} = 0.00755(3) \;\leftrightarrow\; \alpha_{\mathrm{el}}(2m_\tau)$$

$2B$ self energy



More details

A detailed description of all numerical coincidences observed (both for interaction energy and self energy), can be found in

- C. Beck, Spatio-temporal Chaos and Vacuum Fluctuations of Quantized Fields, World Scientific (2002)
- These results have also been published in
  C. Beck, Chaotic strings and standard model parameters, Physica D 171, 72 (2002)
- Connection with dark energy:
  C. Beck, Phys. Rev. D 69, 123515 (2004)

Grand unification

Energy dependence of the three SM couplings given by the second-order formula

$$\frac{1}{\alpha_i(E')} = \beta_0 \ln\left(\frac{E'}{E}\right) + \frac{1}{\alpha_i(E)} + \frac{\beta_1}{\beta_0} \ln\left(\frac{1/\alpha_i(E') + \beta_1/\beta_0}{1/\alpha_i(E) + \beta_1/\beta_0}\right)$$

$$\beta_0 = \frac{-1}{2\pi}\left(b_i + \sum_{j\neq i} \frac{b_{ij}}{4\pi}\alpha_j(E)\right), \qquad \beta_1 = \frac{-b_{ii}}{8\pi^2}$$

$$(b_i) = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_F \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_H \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

$$(b_{ij}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & -102 \end{pmatrix} + N_F \begin{pmatrix} 19/15 & 3/5 & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & 76/3 \end{pmatrix} + N_H \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 13/6 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

U. Amaldi, W. de Boer, H. Fürstenau, Phys. Lett. **B260** (1991) 447

Grand unification

Three Standard Model (SM) couplings

$$\begin{aligned} \alpha_1 &= \frac{g_1^2}{4\pi}, \quad g_1 = \frac{e}{\cos\theta_W} && U(1) \\ \alpha_2 &= \frac{g_2^2}{4\pi}, \quad g_2 = \frac{e}{\sin\theta_W} && SU(2) \\ \alpha_3 &= \frac{g_s^2}{4\pi} = \alpha_s && SU(3) \end{aligned}$$

N_F: number of active families

N_H: number of active Higgs bosons (SM: 0 or 1)

For a quick and systematical overview see e.g.

Gordon Kane, "Modern Elementary Particle Physics", Addison-Wesley 1993

Grand unification

Observed local minima of the self energy of chaotic strings do not provide evidence for supersymmetric particles. They do provide, however, evidence for a grand unification energy scale near 10^{16} GeV (based on non-supersymmetric energy dependence of the running coupling constants).

Grand unification scenario

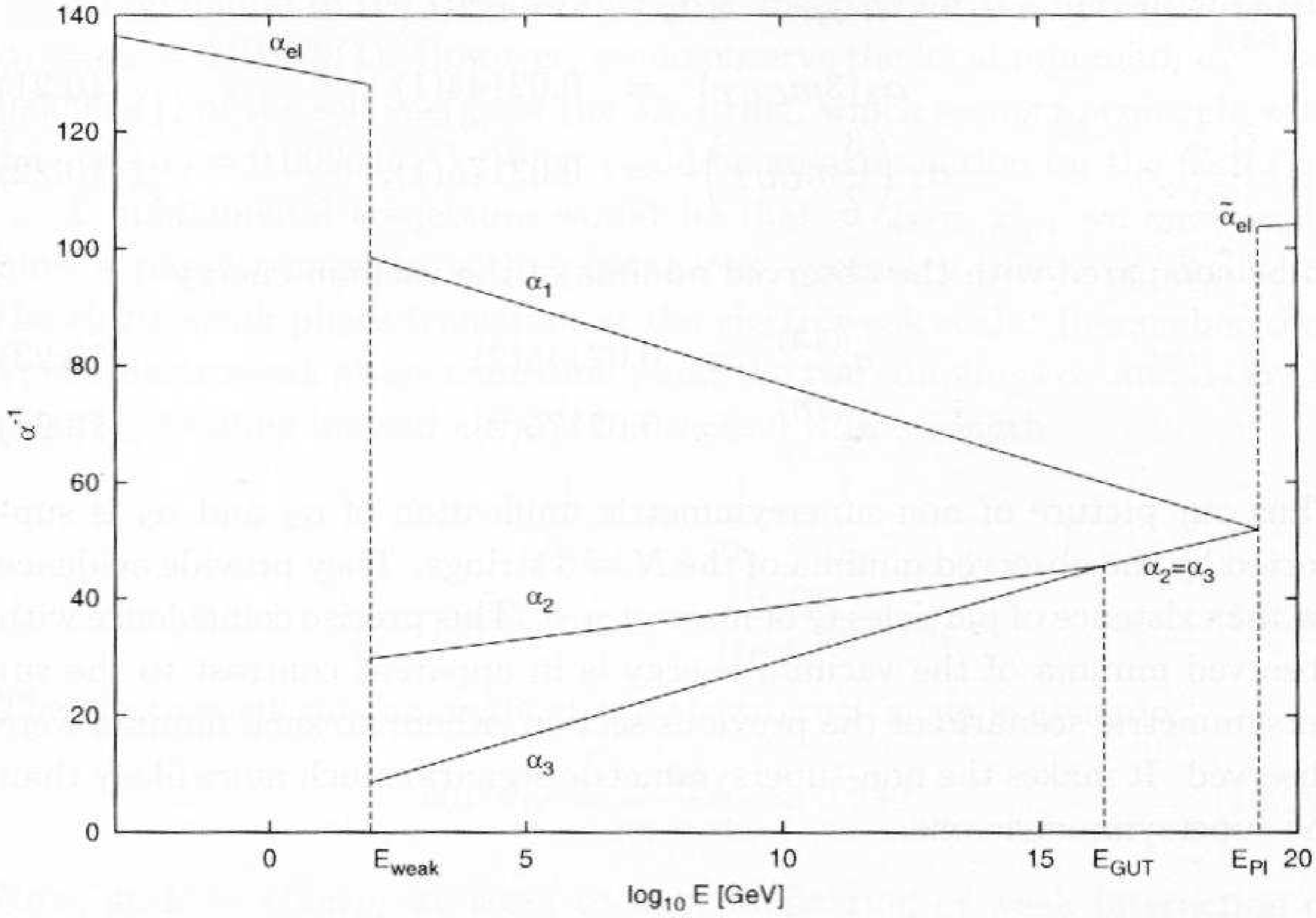


Fig. 10.1 Grand unification scenario based on non-supersymmetric beta functions.

Dark energy

Parisi-Wu approach for stochastic quantization applied to self-interacting scalar field ϕ of the Robertson-Walker metric

$$\frac{\partial}{\partial s}\phi = \ddot{\phi} + 3H\dot{\phi} + V'(\phi) + L(s,t)$$

$L(s,t)$: Gaussian white noise
H: Hubble constant
V: potential under consideration

Discretization $s = n\tau$, $t = i\delta$, $\phi_n^i = p_{\max}\Phi_n^i$ under the assumption that $a := 2\tau/\delta^2$ remains finite leads to

$$\Phi_{n+1}^i = (1-a)T(\Phi_n^i) + \frac{3}{2}a\delta H(\Phi_n^i - \Phi_n^{i-1}) + \frac{a}{2}(\Phi_n^{i-1} + \Phi_n^{i+1}) + \tau \times \text{noise}$$

Dark energy

Connection between $V'(\phi)$ and $T(\Phi)$:

$$V'(\phi) = \frac{1-a}{\tau} \{-\phi + p_{\max} T(\Phi)\}$$

or integrated over ϕ

$$V(\phi) = \frac{1-a}{\tau} p_{\max}^2 \left\{ -\frac{1}{2}\Phi^2 + \int T(\Phi) d\Phi \right\} + C$$

For $T(\Phi) = T_{-3}(\Phi) = -4\Phi^3 + 3\Phi$ and a suitable choice for C we obtain

$$\langle V_-^{(3)}(\phi) \rangle = \frac{3p_{\max}^2}{8\tau}$$

where $\langle A \rangle = \int A(\Phi)\rho_0(\Phi) d\Phi$.

Dark energy

The chaotic $N = 3$ field with $a = 0$ generates dark energy given by $\langle V_-^{(3)}(\phi)\rangle = \frac{3p_{\max}^2}{8\tau}$
Current observational estimate of dark energy density is $\rho_\phi = (2.9 \pm 0.3) \times 10^{-47}\,\text{GeV}^4$.
All fields with coupling $a > 0$ have an equation of state $w = p/\rho > -1$ and their energy density rapidly decays during the expansion of the universe as $\rho \sim R^{-3(1+w)}$
(R: scale factor of RW metric)

C. Beck, Phys. Rev. **D69** (2004) 123515

So what is really going on?

- Initially everything is a Bernoulli shift of information ($\Phi_{n+1} = T_N(\Phi_n)$) in fictitious time n

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- Coupling parameters move to zeros of interaction energy, local minima of self energy, and are fixed for the future universe
- Then physical space-time is created. Fundamental constants of SM fixed by chaotic strings (which generate moduli potentials)

Brief history of the universe

- Chaotic string dynamics can now be re-interpreted as a (singular) scalar field in a Robertson-Walker metric, eq. of state $w = -1$ for $a = 0$ only.

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Brief history of the universe

- Chaotic string dynamics can now be re-interpreted as a (singular) scalar field in a Robertson-Walker metric, eq. of state $w = -1$ for $a = 0$ only.
- All chaotic scalar fields with couplings $a \neq 0$ thin out immediately, only $a = 0$ fields remain with constant density and generate dark energy (cosmological constant).
- Chaotic uncoupled fields may serve as perfect noise source for Parisi-Wu approach of quantization, there is equivalence with stochastic quantization.

Conclusion 1

- Chaotic quantization approach gives physical meaning and complex structure to the noise of the Parisi-Wu approach.
- In the chaotic noise space things are discrete, nonlinear, chaotic, coupled and complex. This is complexity science at a fundamental level.
- The most relevant concept in the chaotic noise space is information (and a shift of symbol sequences). Generalized statistical mechanics methods come in as useful tools.

Conclusion 2

- Anthropic principle is wrong —there is good reason why SM parameters have the values they actually have.
- SM parameters are chosen in an optimum way —they minimize the interaction energy/self energy of chaotic strings. For zero correlation function they look as random as possible.
- Fictitious time has physical meaning. Before creation of space-time the universe may start out of a Bernoulli shift of information in fictitious time, later this becomes stochastic quantization noise.