Genuine multipartite entanglement in complex systems

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Outline

Introduction and motivation

Multipartite entanglement is a key resource in many fields

Importance of multipartite entanglement

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- Plays a fundamental role in quantum phase transitions [d]
- It is required to understand ionization procedures [e]
- Possibly responsible for transport efficiency in biological systems [f]

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[a] *e.g.* D. Markham, B. C. Sanders, Physical Review A **78**, 042309 (2008)
[b] *e.g.* R. Raussendorf and H.-J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001)
[c] *e.g.* D. Bruss and C. Macchiavello, arXiv:1007.4179
[d] *e.g.* S. Sachdev, *Quantum Phase Transitions* , (1999)
[e] *e.g.* D. Akoury et al., Science **9** Vol. 318. no. 5852, p. 949 -952, (2007)
[f] *e.g.* M. Sarovar et al., Nature Physics, **6**, 462 (2010)

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- **However**

[a] *e.g.* R. A. Bertlmann, W. Grimus and B.C. Hiesmayr, Phys. Lett. A **289**, 21-26 (2001)

[b] *e.g.* H. Bartosik, J. Klepp, C. Schmitzer, S. Sponar, A. Cabello, H. Rauch, Y. Hasegawa, Phys. Rev. Lett. **103**, 040403 (2009)

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- **We need criteria applicable to arbitrary dimensional systems involving many particles**
- **Those criteria should be locally implementable and feasible**
- **The more complex a system, the more important noise resistance becomes**

pure states

Consider pure states $|\psi\rangle\langle\psi|$ in $\mathcal{H} = \underbrace{\mathbb{C}^{d_1}}_{1} \otimes \underbrace{\mathbb{C}^{d_2}}_{2} \otimes (\cdots) \otimes \underbrace{\mathbb{C}^{d_n}}_{n}$

The state is called k-separable with respect to a given k-partition $\{\alpha_1|\alpha_2|\cdots|\alpha_k\}$ if it can be written as a product

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- if $k = n$ the state is *fully separable*
- if $k = 1$ the state is *genuinely multipartite entangled*

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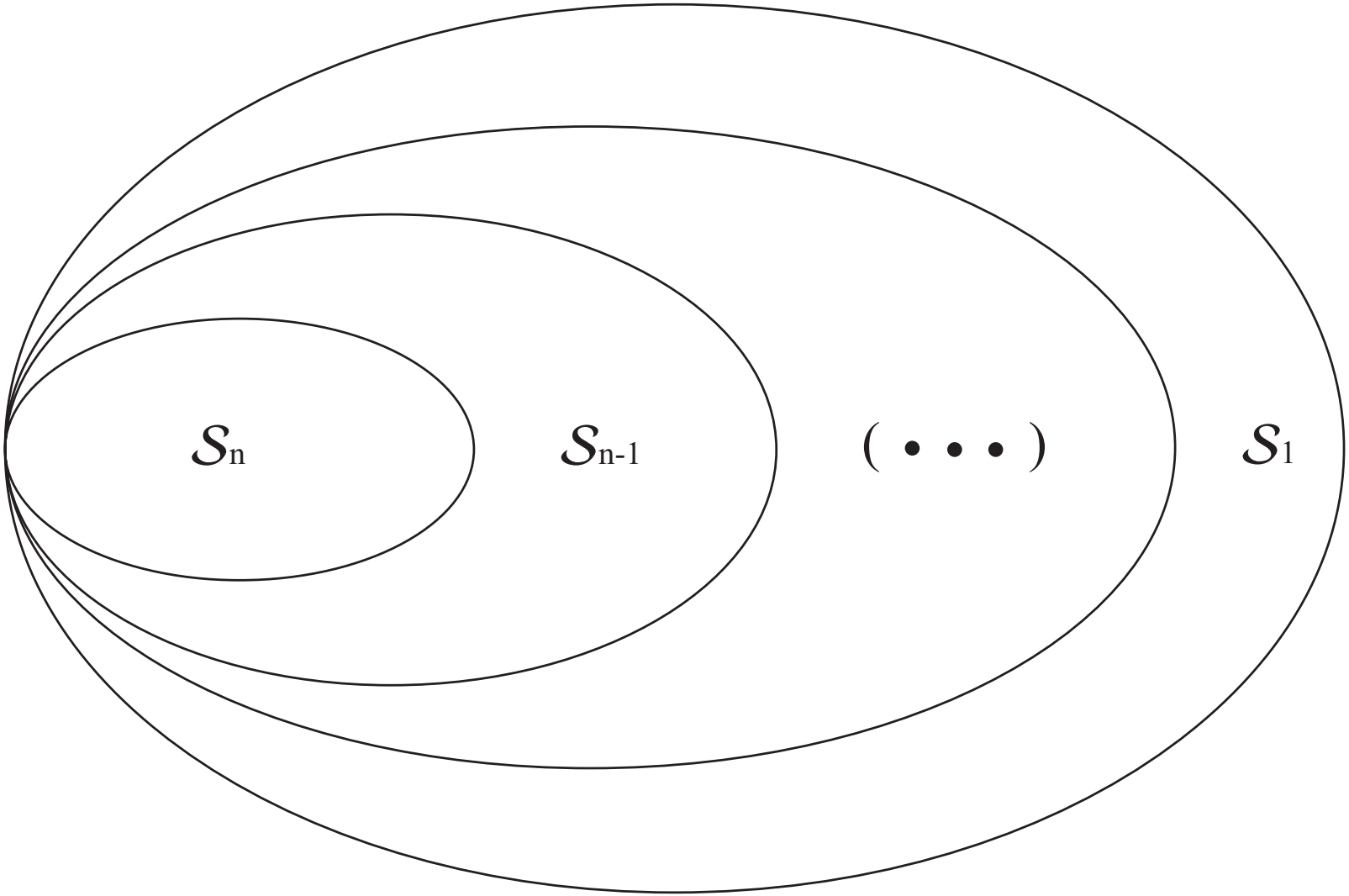
The convex structure of separable states



Separability criteria

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where $\mathcal{P}_{\alpha_i}$ is the cyclic permutation operator acting on $\mathcal{H}_{\alpha_i}^{\otimes 2}$, *i.e.*

$$\mathcal{P}_{\alpha_i}\,|\varphi_1\rangle \otimes |\varphi_2\rangle = |\varphi_2\rangle \otimes |\varphi_1\rangle \ ,$$

$\mathcal{P}_n$ is the cyclic permutation operator acting on $\mathcal{H}^{\otimes 2}$ and $|\Phi\rangle$ is an arbitrary fully separable state, e.g. $|\Phi\rangle = |000111\rangle$

Inequality II

As the LHS of inequality I is convex we can derive a criterion fulfilled by all k-separable states

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[1] Any violation of this inequality implies *genuine* k-nonseparability

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This framework can be employed in order to

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- Study the effect of Lorentz transformations on multipartite entanglement classification [c]
- Provide security proofs in multipartite quantum cryptography [d]

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[a] *in* M. Huber, H. Schimpf, A. Gabriel, Ch. Spengler, D. Bruß, B.C. Hiesmayr arXiv:1011.4087

[b] *in* M. Huber, P. Erker, H. Schimpf, A. Gabriel, B.C. Hiesmayr, arXiv:1011.4579

[c] *in* M. Huber, N.Friis, A. Gabriel, Ch. Spengler, B.C. Hiesmayr arXiv:1011.3374

[d] *in* S. Schauer, M. Huber, B.C. Hiesmayr arXiv:1009.4796

Observables

As the previous inequalities decompose into products of density matrix elements no full state tomography is required.

Properties

- full state tomography requires all $\prod_{i=1}^{n} d_i^2$ d.m.e. to be measured, i.e. scales with $\mathcal{O}(d^{2n})$

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- The derived inequalities even scale polynomially

Noise resistance

Consider the generalized d-dimensional n-partite GHZ state $|\psi_{dn}\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle^{\otimes n}$ with additional isotropic (white) noise: $\rho = p|\psi_{dn}\rangle\langle\psi_{dn}| + (1-p)\frac{1}{d^n}\mathbb{1}$

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With inequality II we can show analytically that these states are genuinely multipartite entangled for $p > \frac{3}{d^{n-1}+3}$

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- Entanglement quantification?
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The End

References:

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Thank you!