Complex stochastic dynamics in spin glasses and optimization problems

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Spin Glasses

$$\mathcal{H} = -\sum_{<ij>} J_{ij} s_i s_j$$

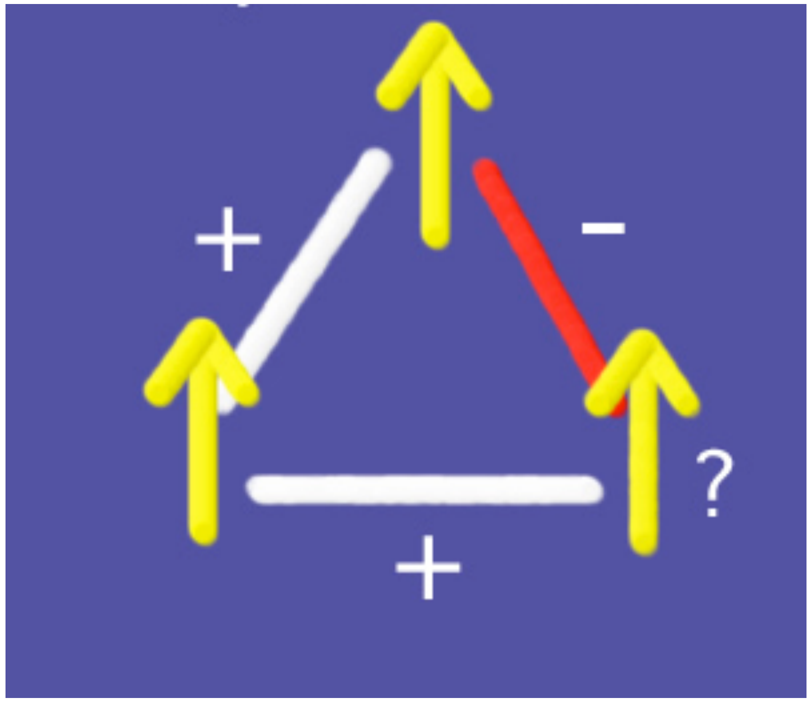
- $s_i = \pm 1$ classical (Ising) spins
  no quantum effects!
- J_{ij} quenched random variables
  e.g. $J_{ij} = \pm 1$ or $J_{ij} \sim N(0,1)$

Spin Glasses

DISORDER

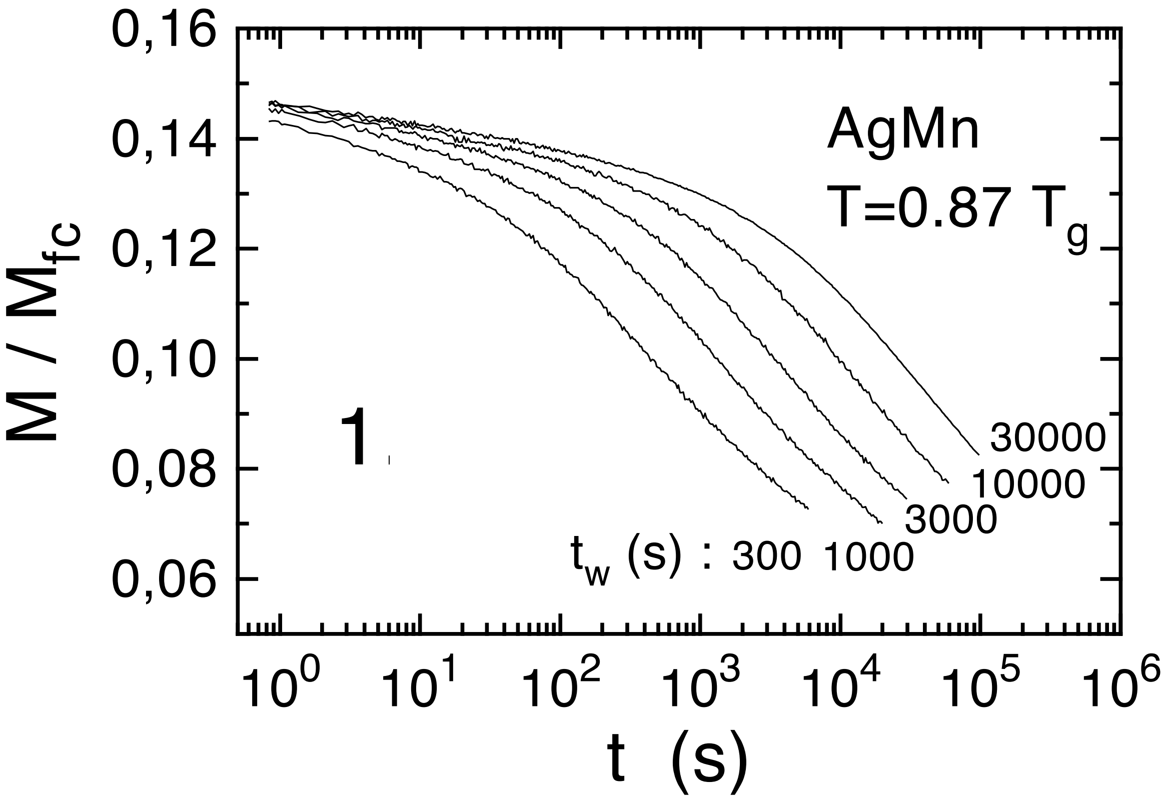
FRUSTRATION

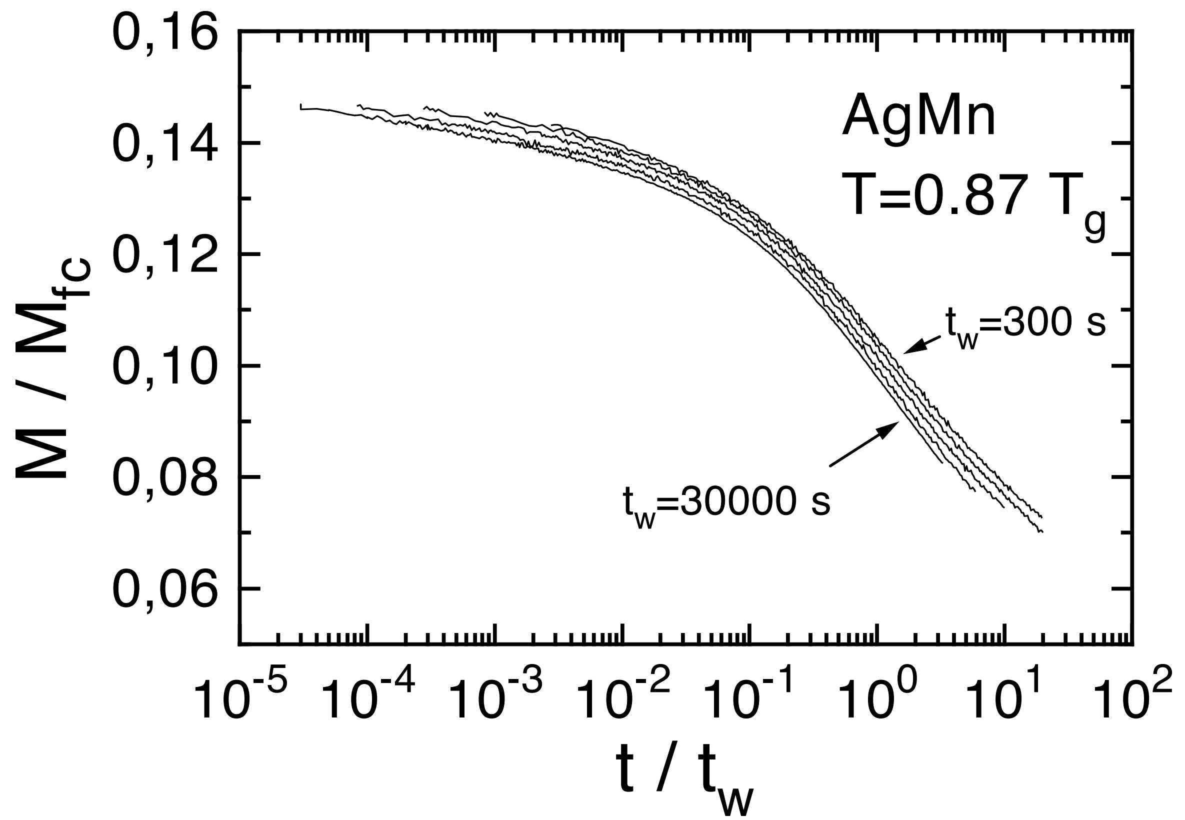
SLOW DYNAMICS



Experiments on SG

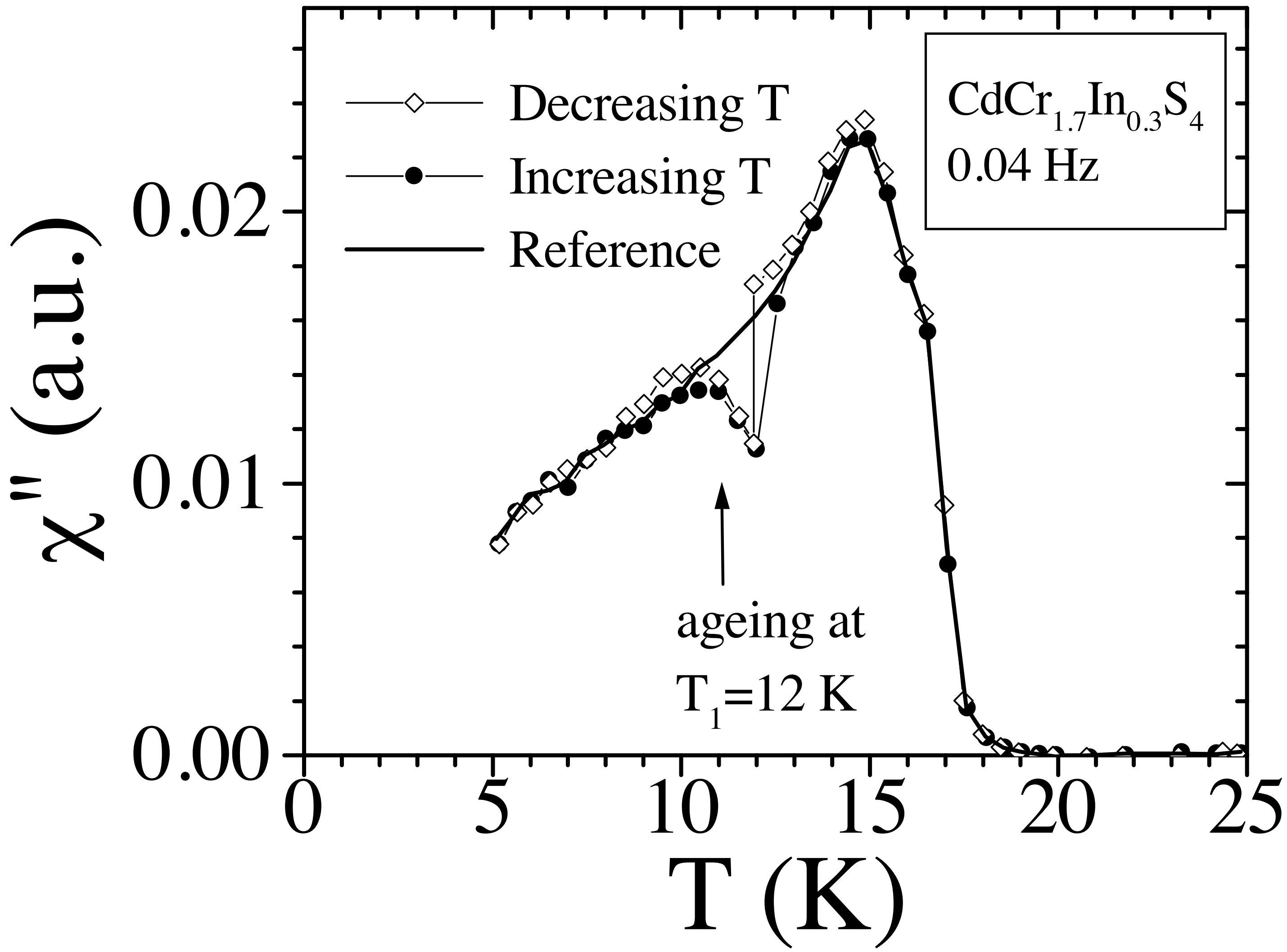
Below T_c -> always out of equilibrium





Experiments on SG

Rejuvenation and memory



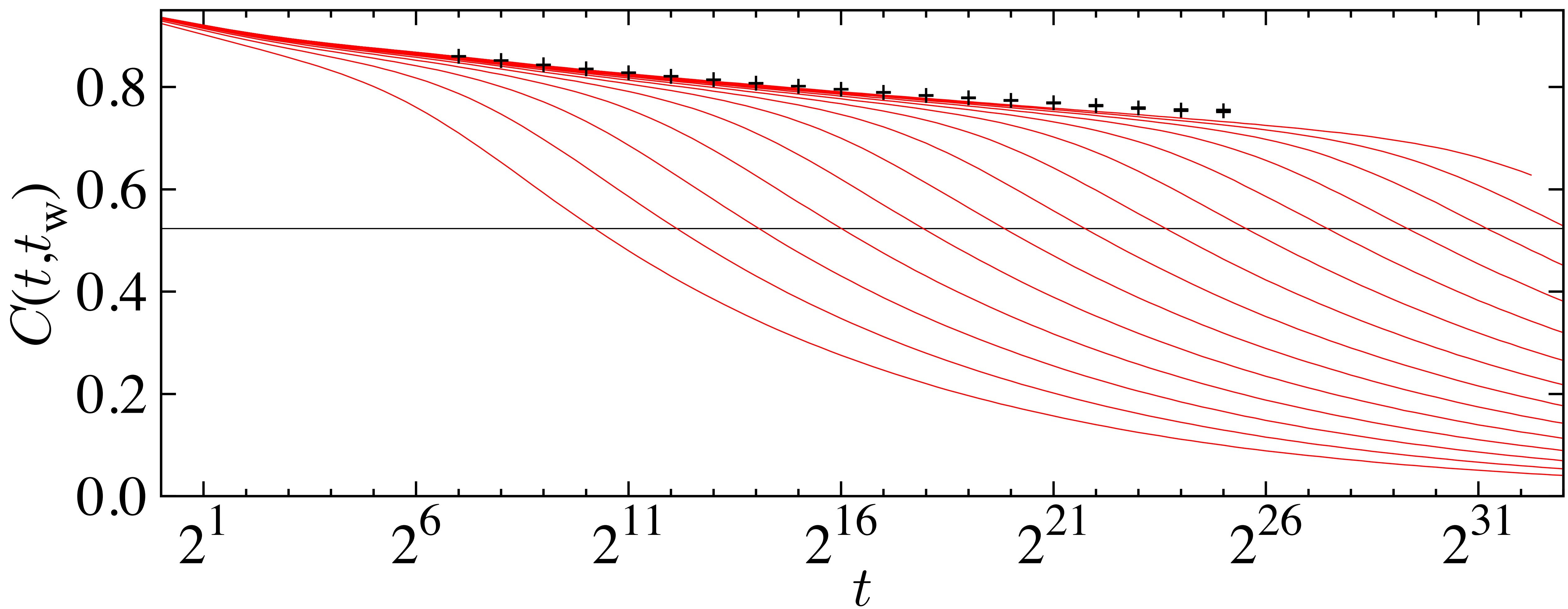
Slow growth of different kind of orders at different temperatures

Complex macroscopic dynamics but simple microscopic rules

- Energy relaxation
- Local in the configurational space
- Stochastic in nature

Monte Carlo, Glauber, Langevin, ...

Numerical simulations of SG



3D SG J=+/-1

T=0.64T$_c$

$$C(t, t_w) = \frac{1}{N} \sum_i s_i(t_w) s_i(t_w + t)$$

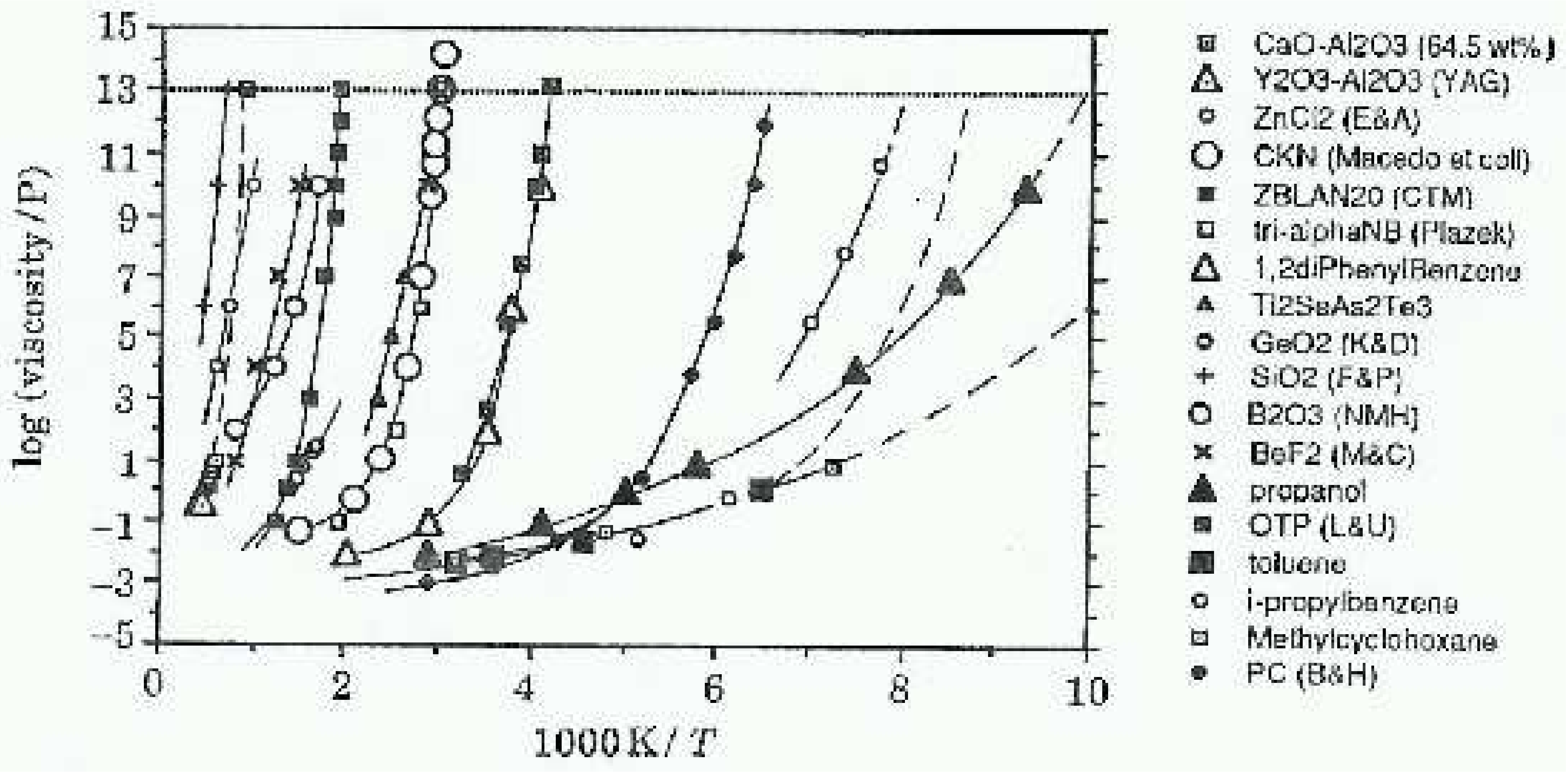
p-Spin Glass Model

$$\mathcal{H} = -\sum_{<ijk>} J_{ijk} \underbrace{S_i S_j S_k}_{\text{p-uples}}$$

p>2 is very different from p=2

Random First Order Transition (RFOT)

(Structural) Glasses



Impressive increase of relaxation timescales!

A Solvable pSG Model

- Fully connected topology
- Continuous unbounded variables $S_i \in \mathbb{R}$
- Spherical constraint $\sum_{i=1}^{N} S_i^2 = N$
- Langevin dynamics

$$\partial_t S_i(t) = -\frac{\partial \mathcal{H}}{\partial S_i} - \mu(t) S_i(t) + \eta_i(t)$$

A Solvable pSG Model

- Closed integral differential eqs. for

$$C(t,t') = \frac{1}{N}\sum_i \langle S_i(t) S_i(t')\rangle \quad R(t,t') = \frac{1}{N}\sum_i \frac{\partial\langle S_i(t)\rangle}{\partial H(t')}$$

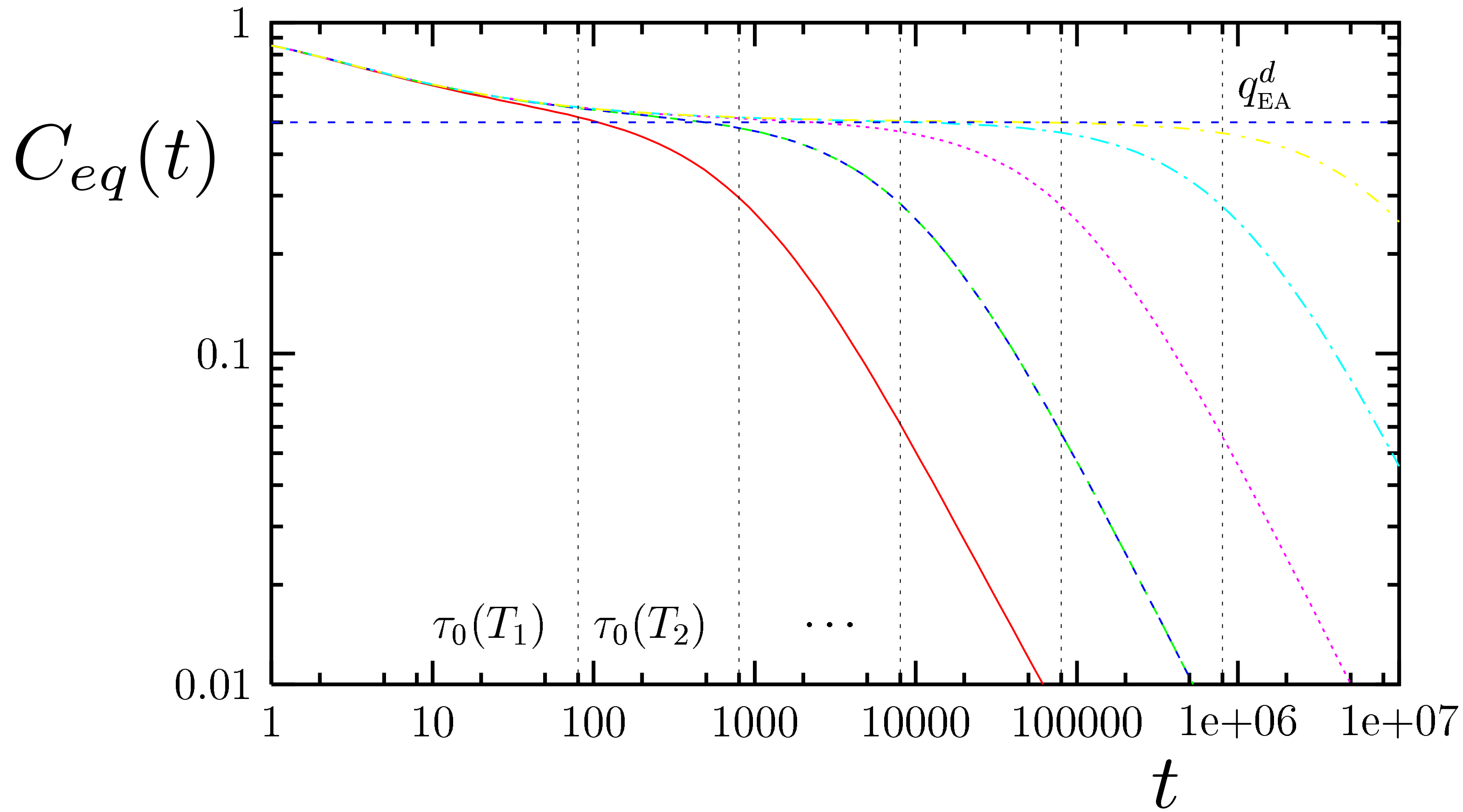
- For $T > T_c$ reaches equilibrium

$$C(t,t') = C_{eq}(t-t')$$

- For $T < T_c$ out of equilibrium (aging)

$$C(t,t') = \widetilde{C}(h(t)/h(t')) \sim \tilde{C}(t/t')$$

pSG Dynamics for T->T_c^+

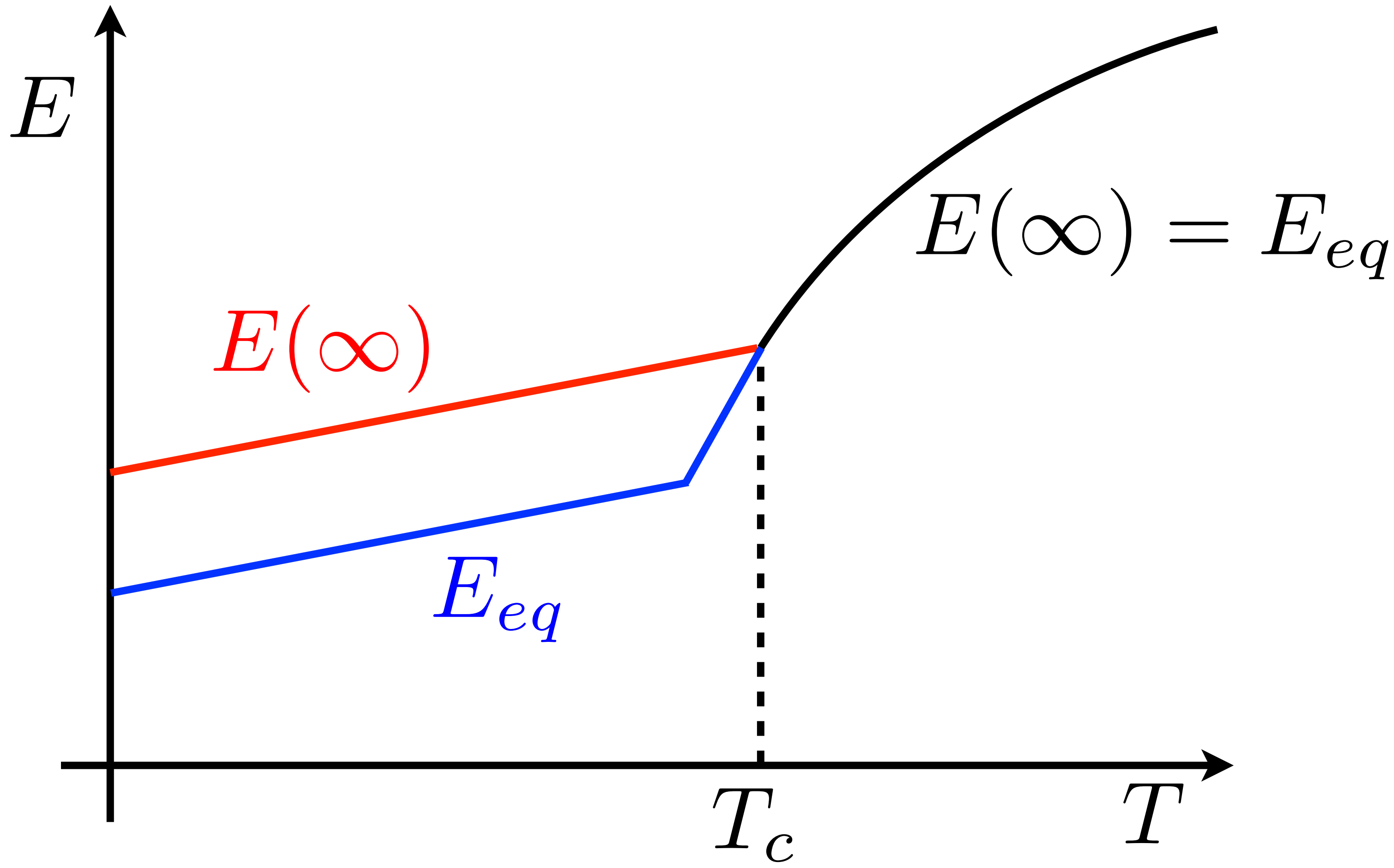


$$\tau(T) \propto (T - T_c)^{-\gamma}$$

pSG Dynamics for T<Tc

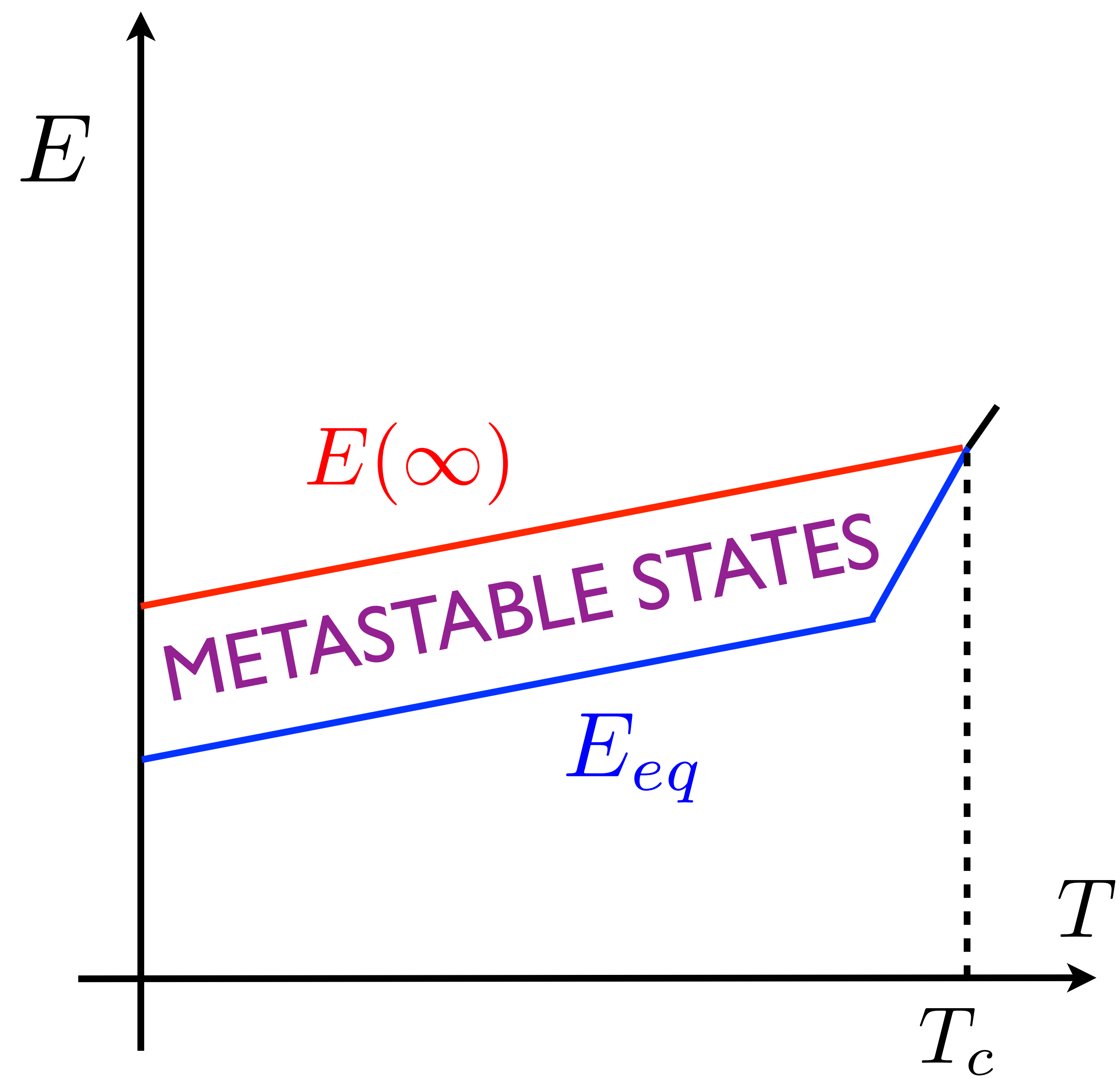
Does not relax to equilibrium states!!

$$E(\infty) \equiv \lim_{t\to\infty} E(t) \neq E_{eq}$$



both for quenches & coolings

Why dynamics get stuck?



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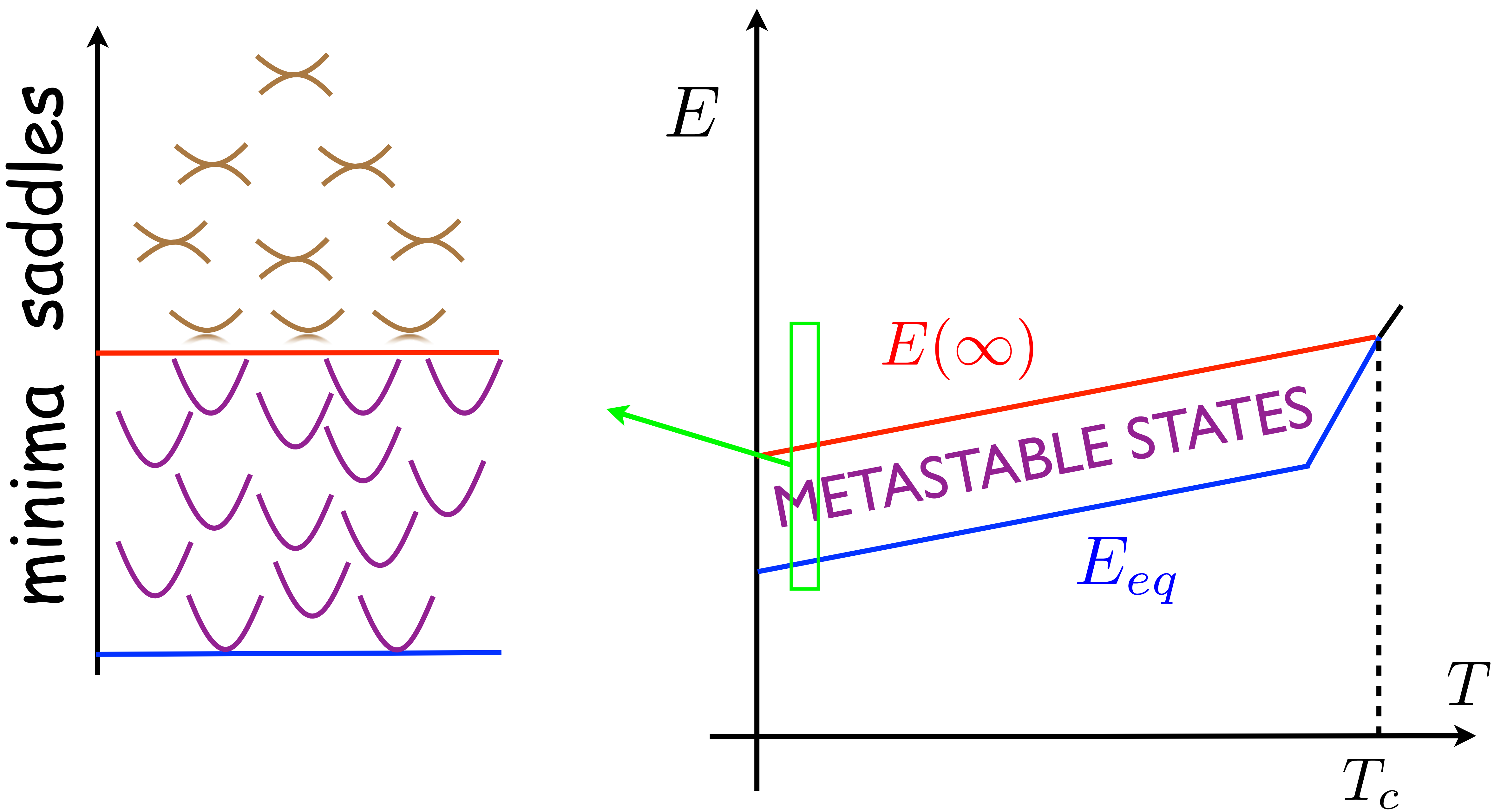


Static-Dynamics Connection ?

- It is too difficult to solve in general the dynamics (e.g. for Ising spins)
- Is long time dynamics determined by few thermodynamical (i.e. static) properties of the energy potential?
- E.g. does energy relaxation stop at the highest metastable states?

How to count metastable states

Compute the replicated free-energy $\Phi(m,T)$
by the replica or the cavity method

$$e^{-\beta m \Phi(m,T) N} \equiv \sum_\alpha Z_\alpha^m = \int e^{-\beta m f N + N\Sigma_f(f,T)} \mathrm{d}f$$

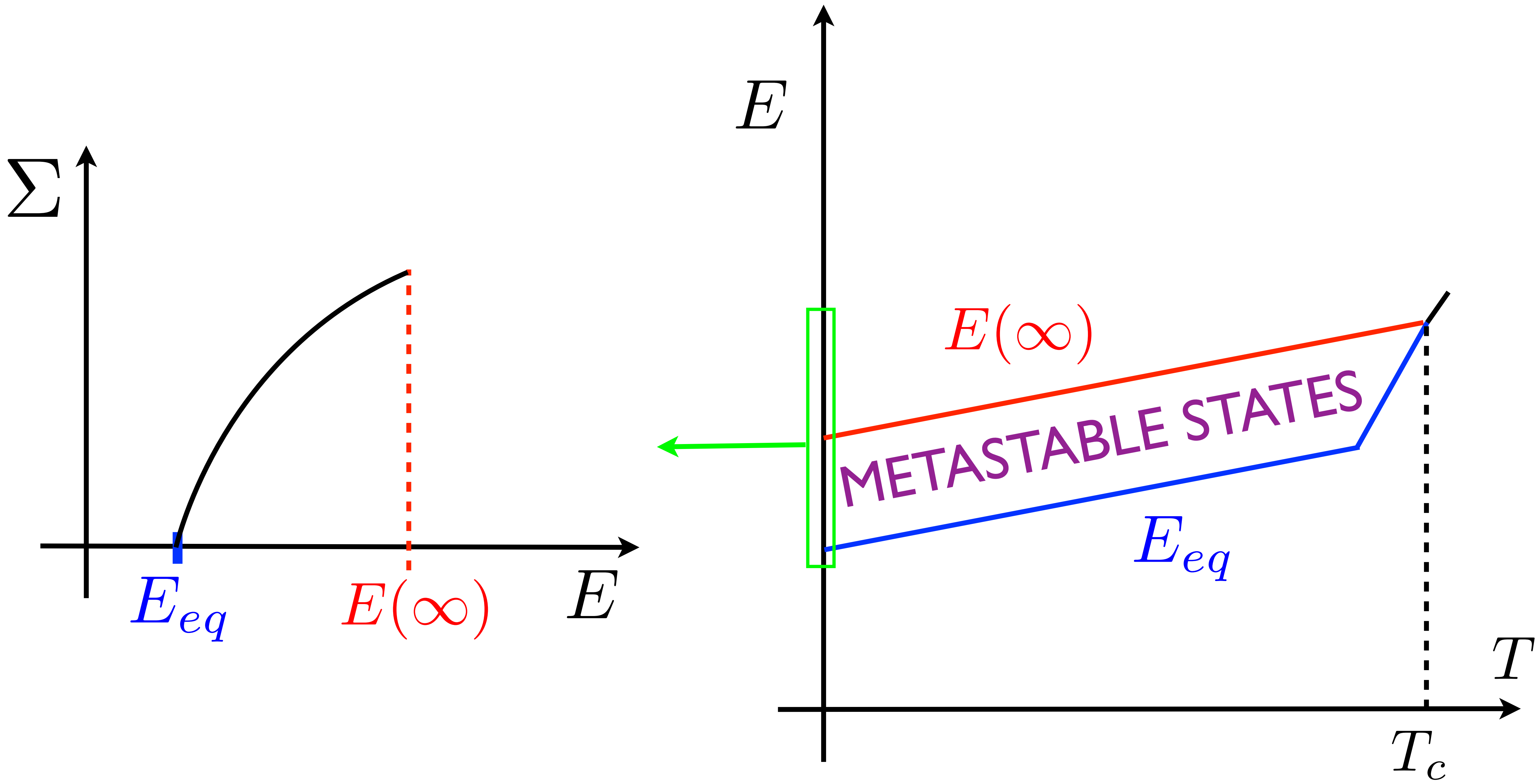
get the complexity or configurational entropy

$\Sigma_f(f,T)$=ln(# states with free-energy f)

by Legendre transforming

$$\Sigma_f(f,T) = \beta m f - \beta m \Phi(m,T)|_{\beta f = \partial_m (m\Phi)}$$

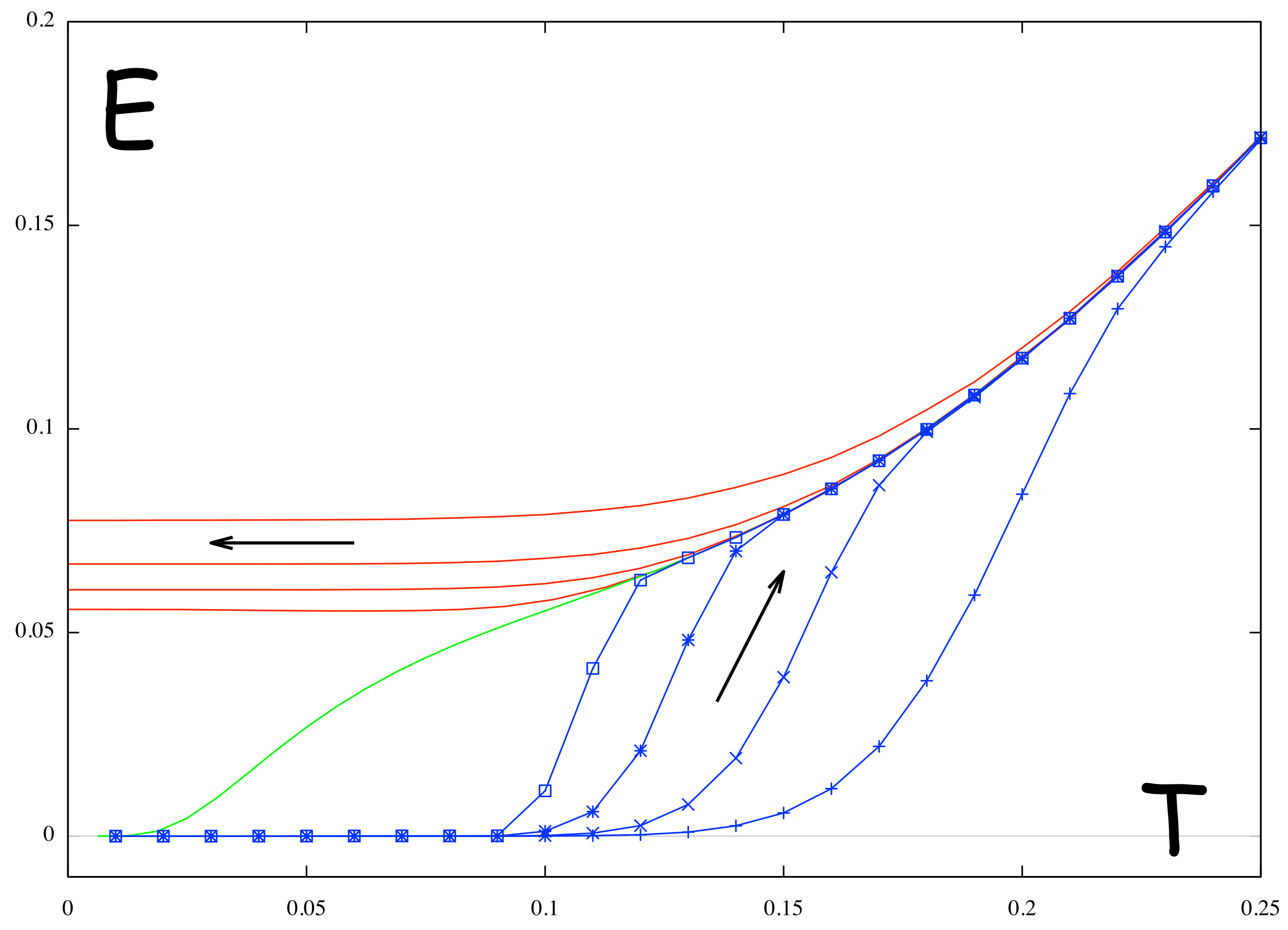
Why dynamics get stuck?



...and beyond mean field ?

Metastable states have infinite lifetime in MF
...but even in finite dimensions they can be huge!

1D model with local 4-spin interactions (no disorder at all)



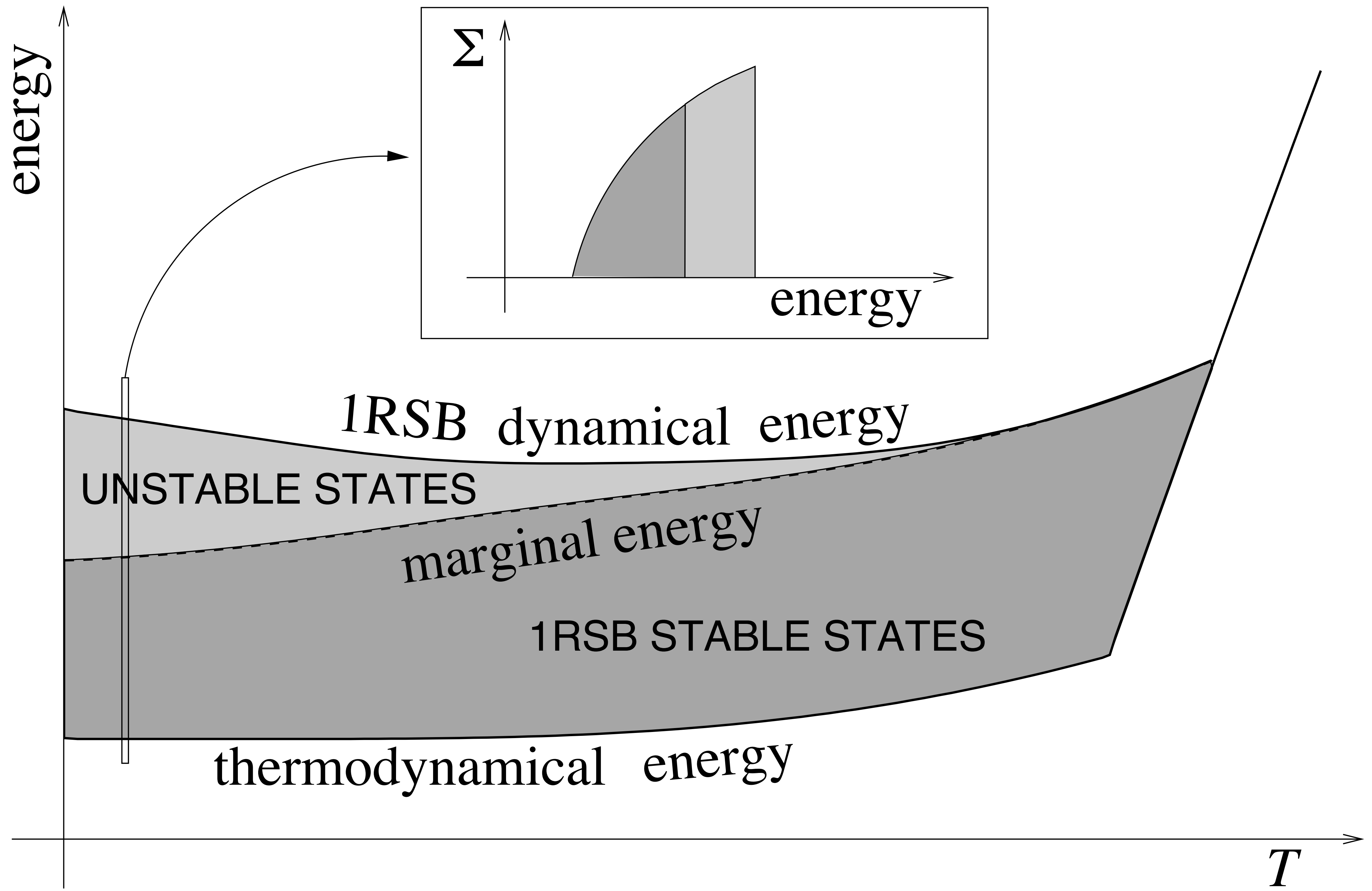
Ising SG are more complex

$$\mathcal{H} = - \sum_{<ijk>} J_{ijk} S_i S_j S_k \qquad \boxed{S_i = \pm 1}$$

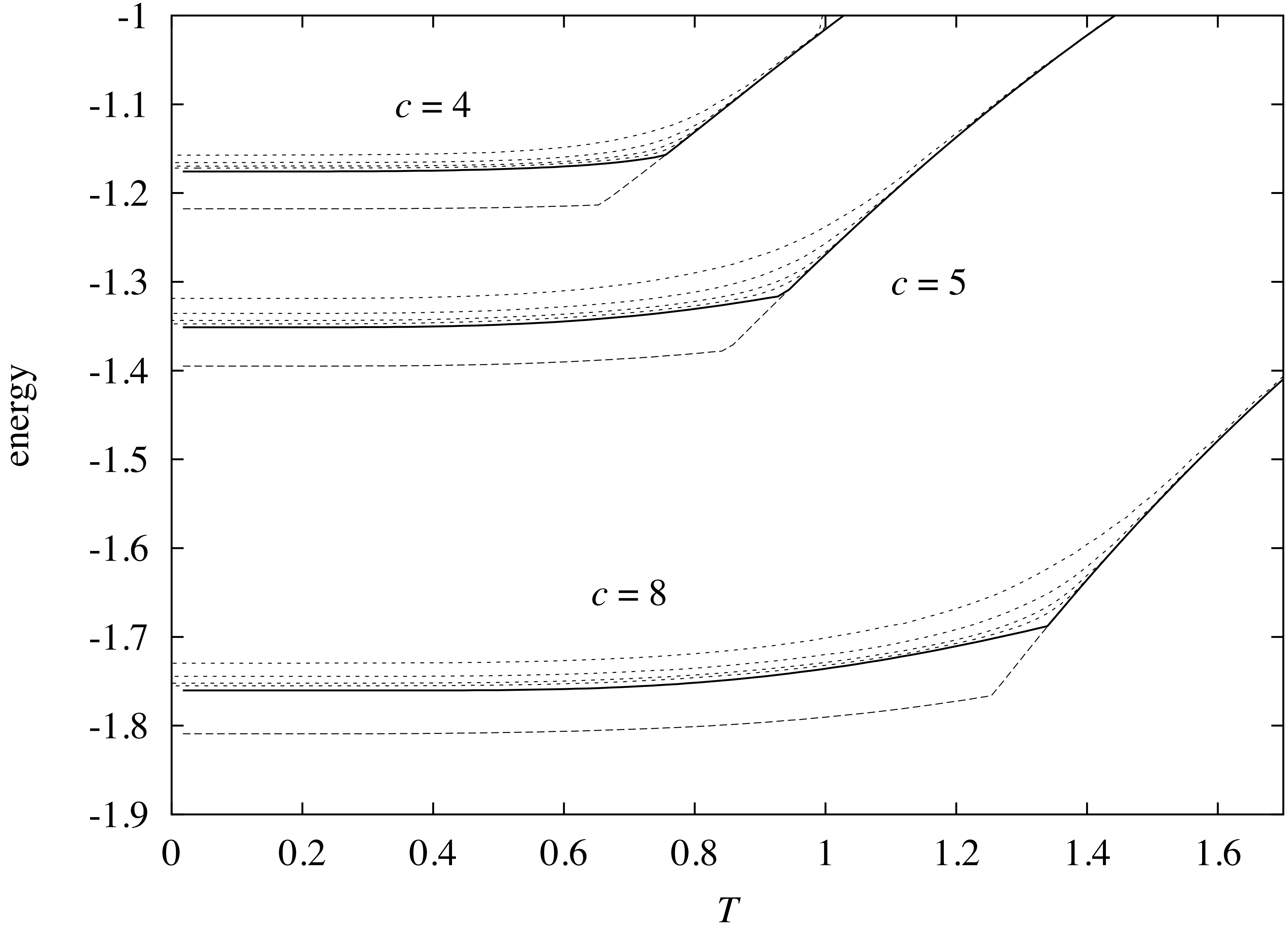
J=+/-1   p=3

interaction network -> random c-regular graph:
<ijk> randomly chosen such that each spin participate exactly to c interactions

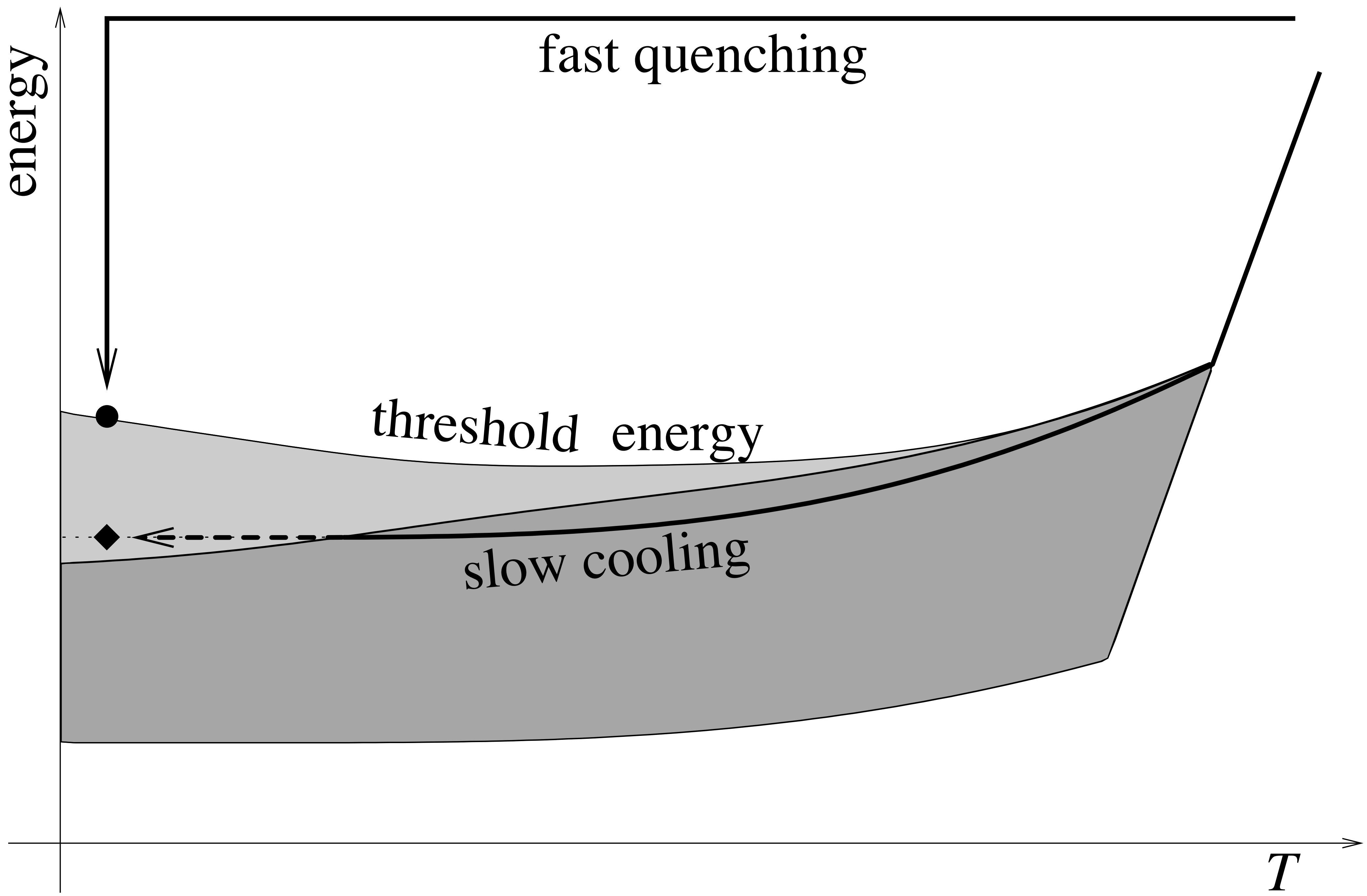
More Complex Complexity >:-!



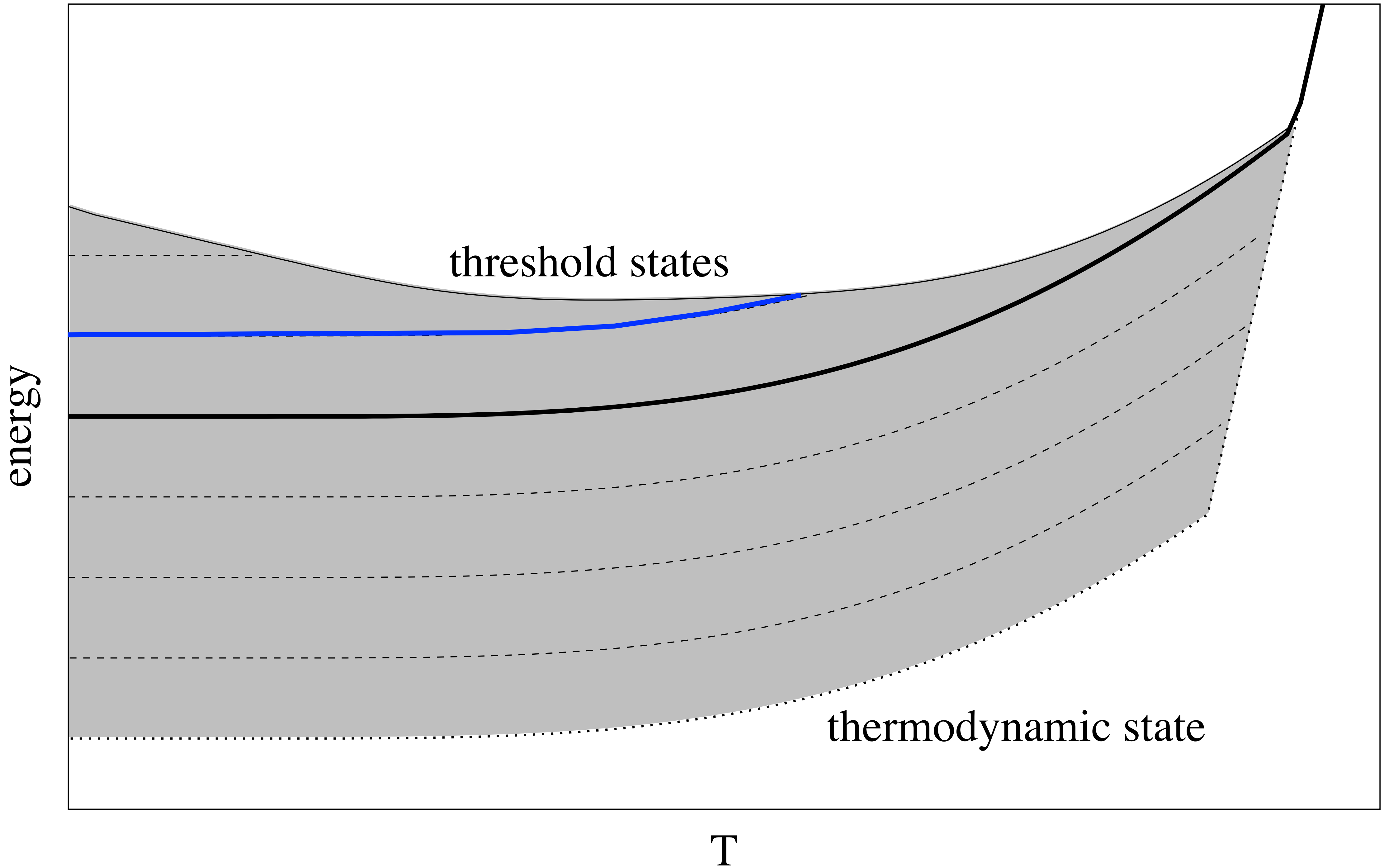
Coolings get stuck, as usual...



Coolings vs. Quenches



Coolings follow the states...



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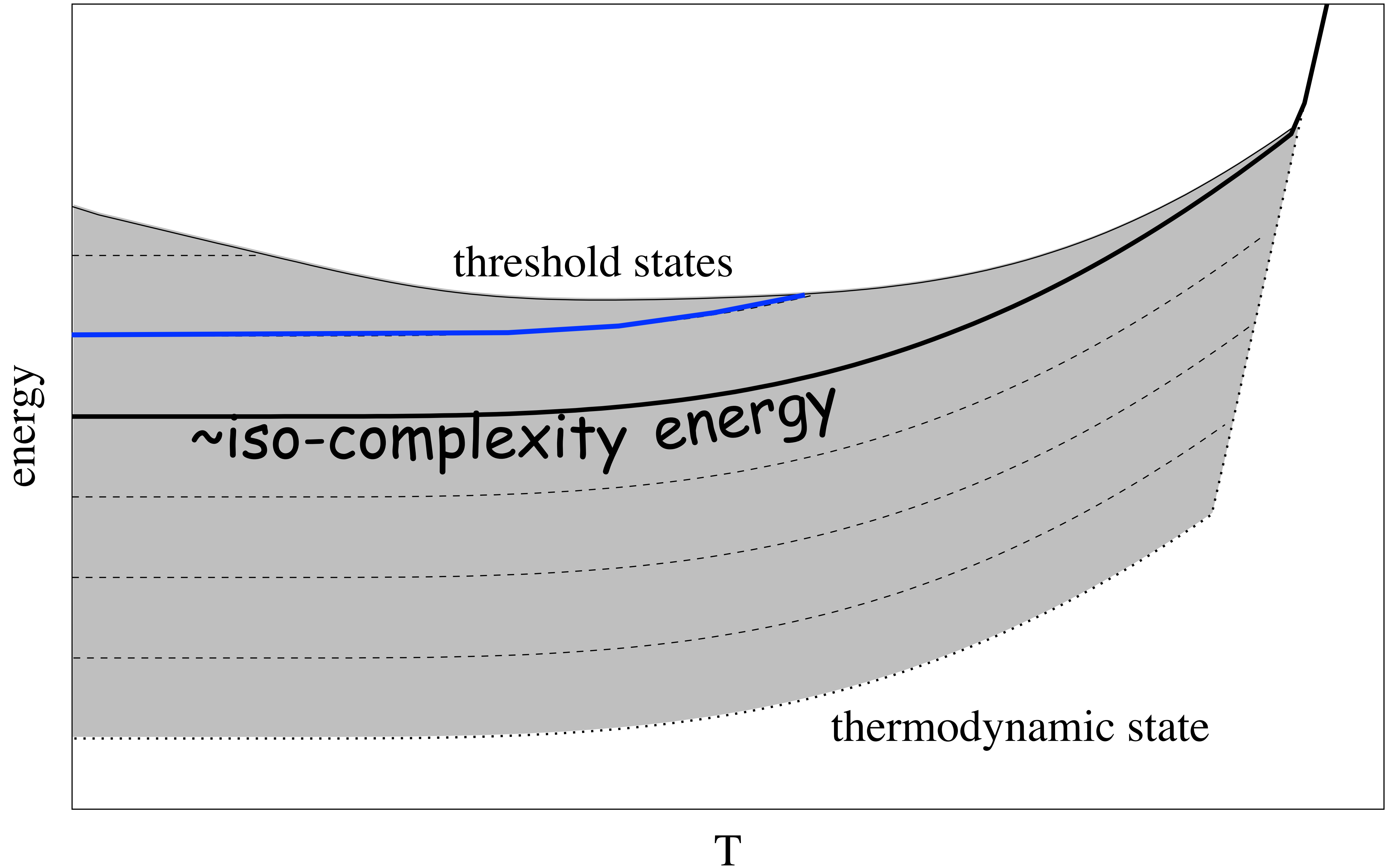


Another Static-Dynamics Link

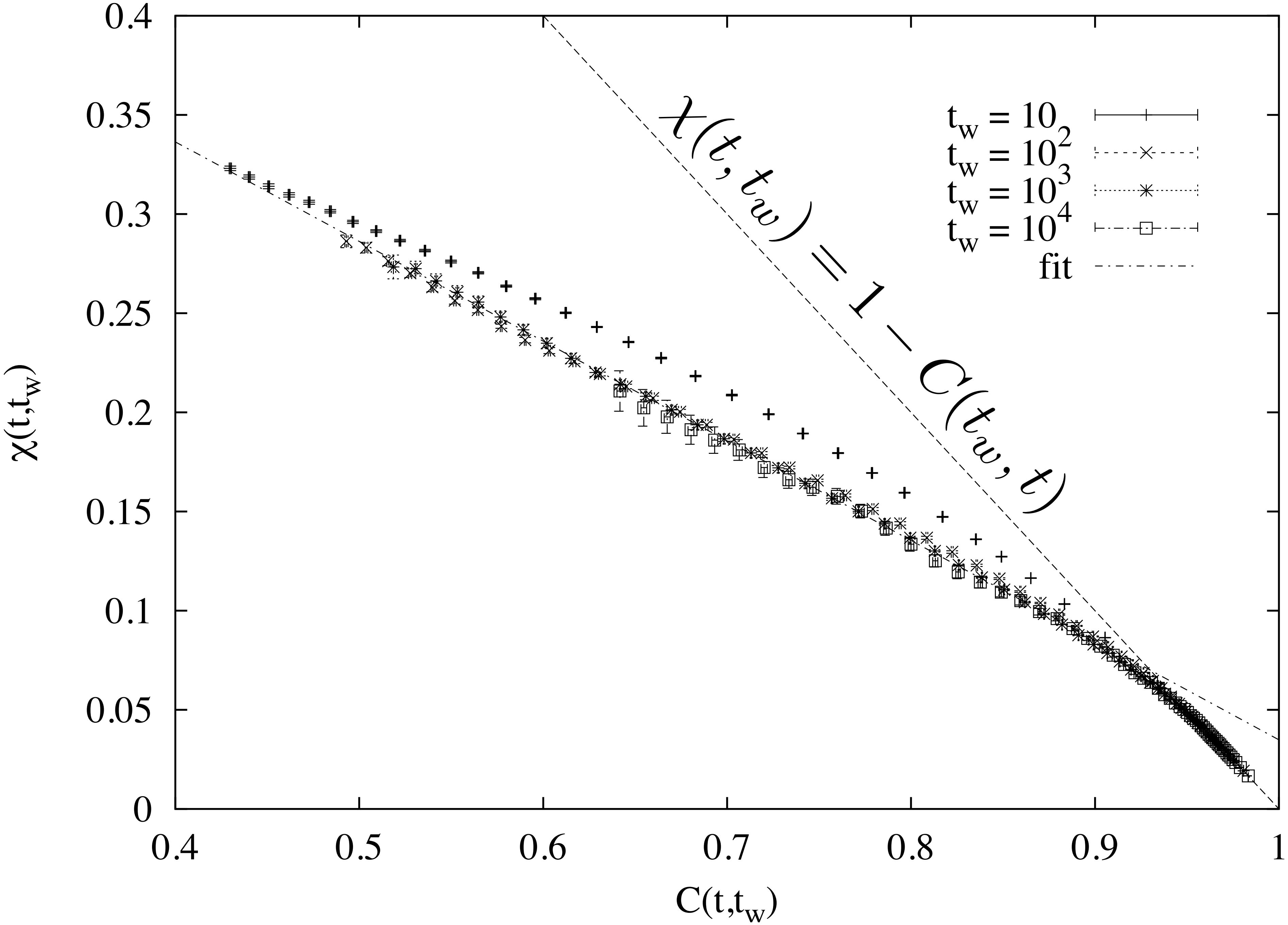
Measure correlation and integrated response

$$C(t_w, t), \quad \chi(t, t_w) = T \int_{t_w}^{t} R(t, s) ds$$

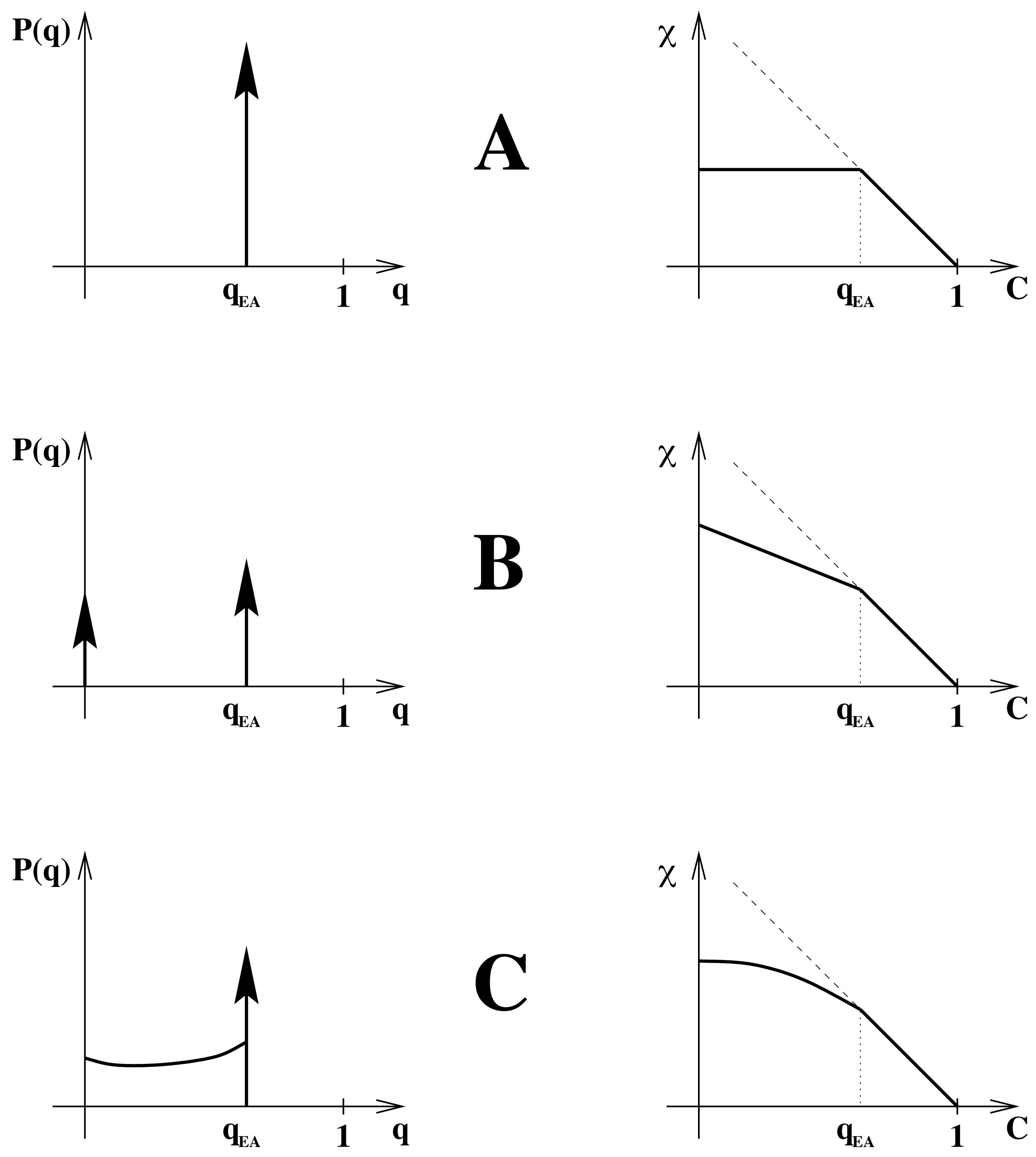
At equilibrium Fluctuation Dissipation Th.

$$\chi(t, t_w) = 1 - C(t_w, t)$$

Fluctuation Dissipation Ratio



SG order parameter vs. FDR



Combinatorial Optimization

Constraint Satisfaction Problems (CSP)

Find an assignment to N binary variables such as to satisfy M constraints.

(each constraint involves few variables)

$$\alpha = \frac{M}{N}$$

constraints per variable ratio

SAT/UNSAT threshold
$$\begin{cases} \alpha < \alpha_s & \text{SAT} \\ \alpha > \alpha_s & \text{UNSAT} \end{cases}$$

Random 3-xorsat

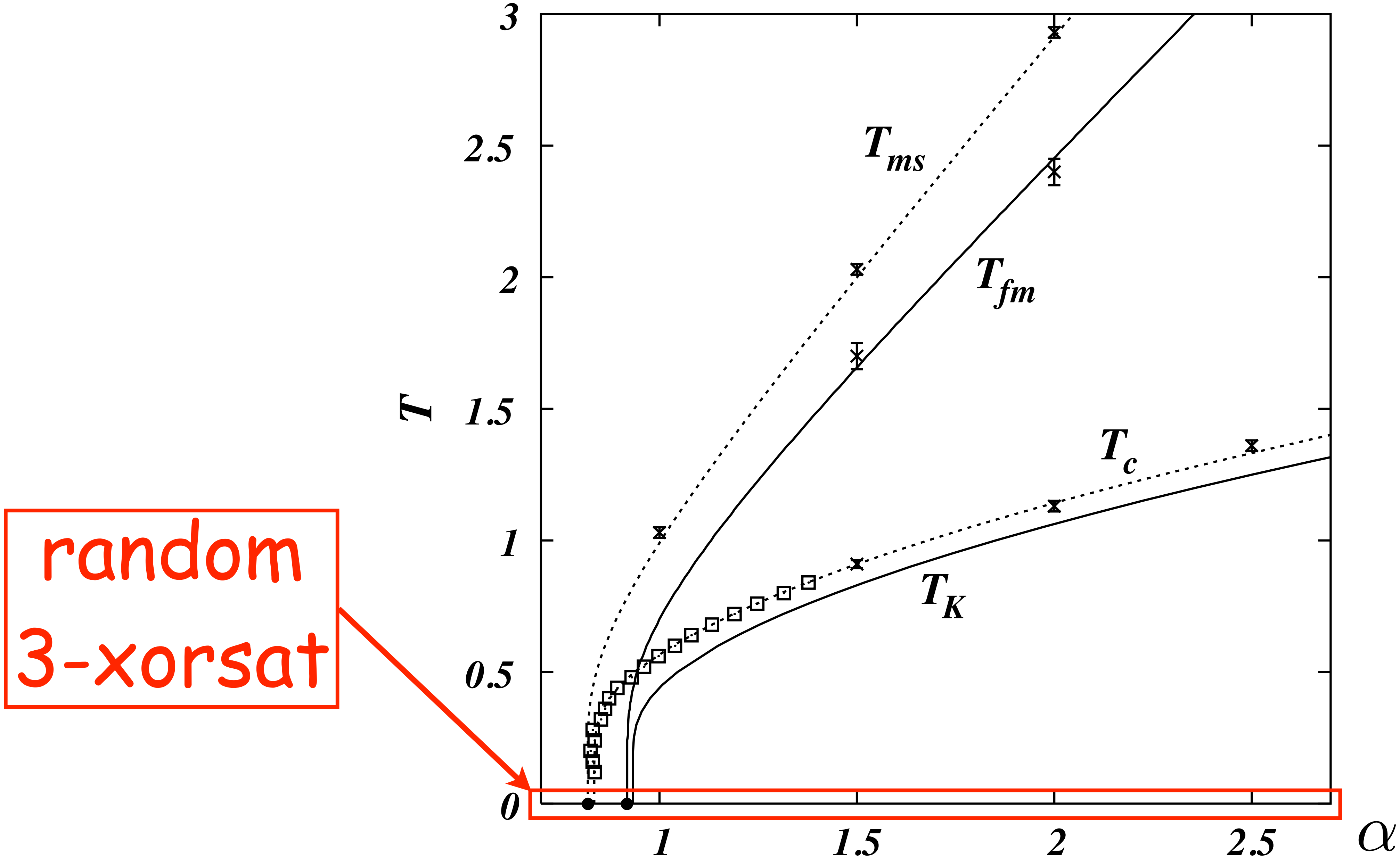
Satisfy αN equations like

$$S_i S_j S_k = J_{ijk}$$

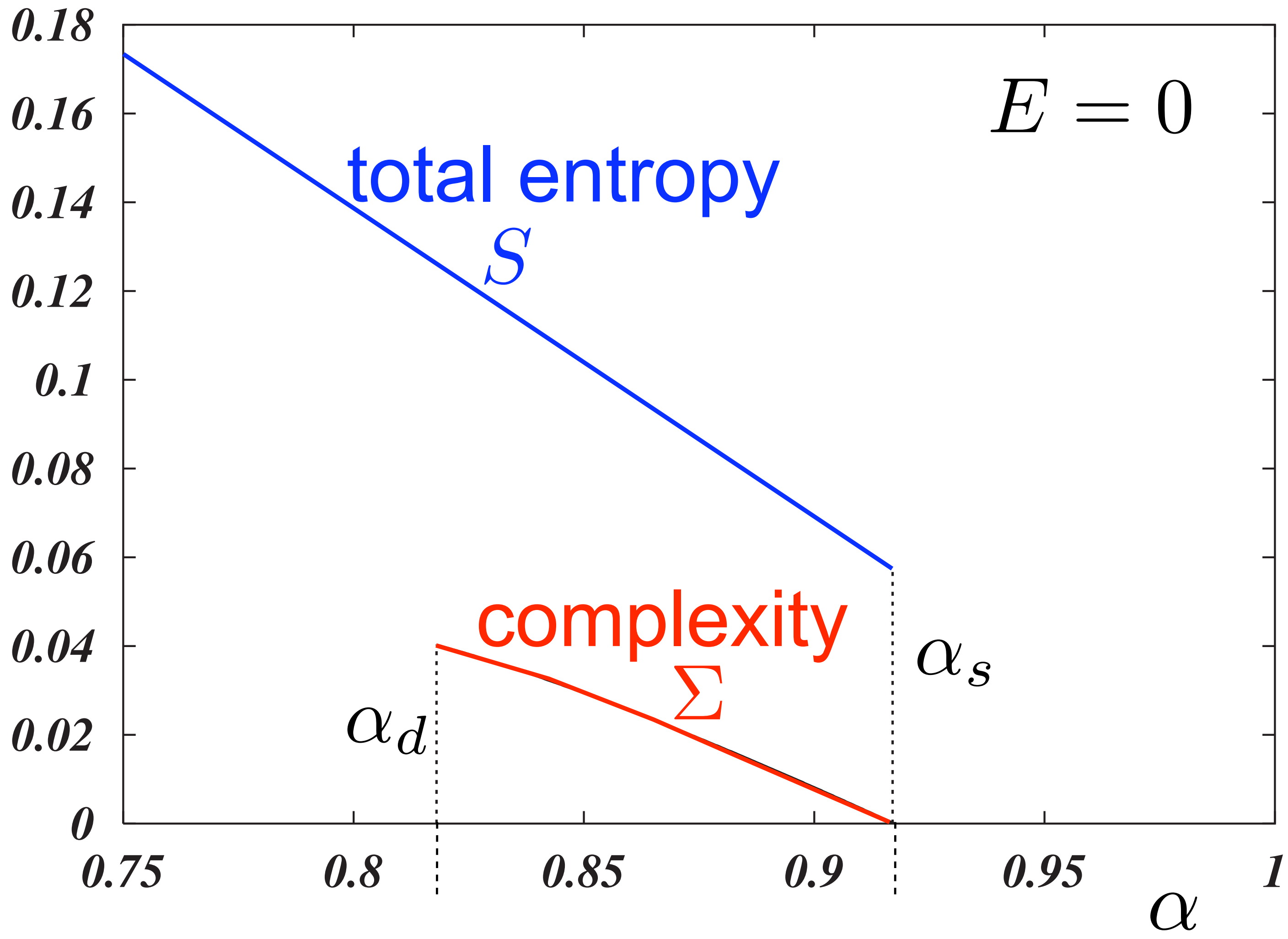
where the triplets <ijk> are randomly chosen uniformly among $\binom{N}{3}$

It corresponds to computing the ground state of a 3SG model

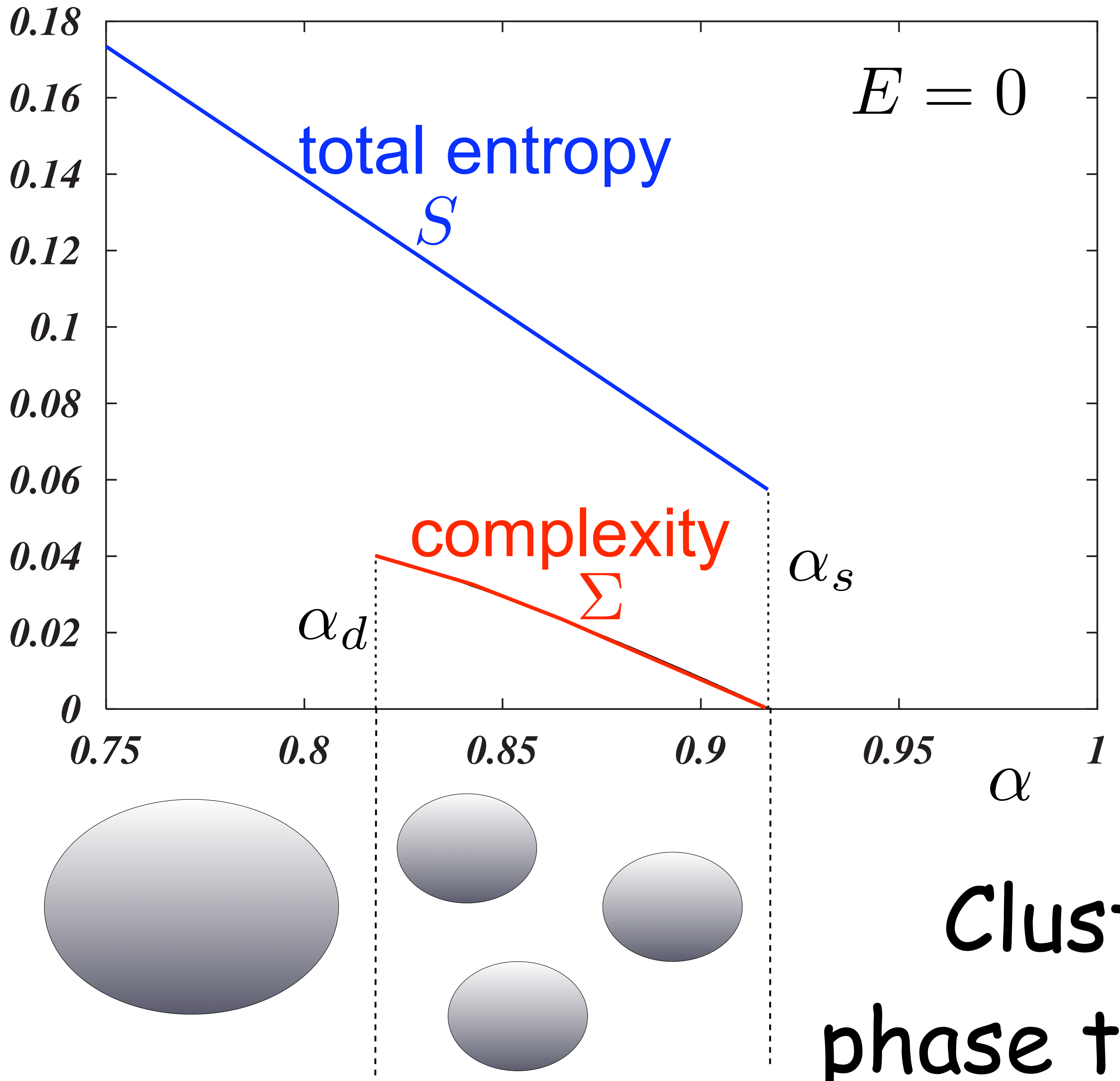
3-spin glass on a random graph



Random 3-xorsat



Random 3-xorsat



Clustering phase transition

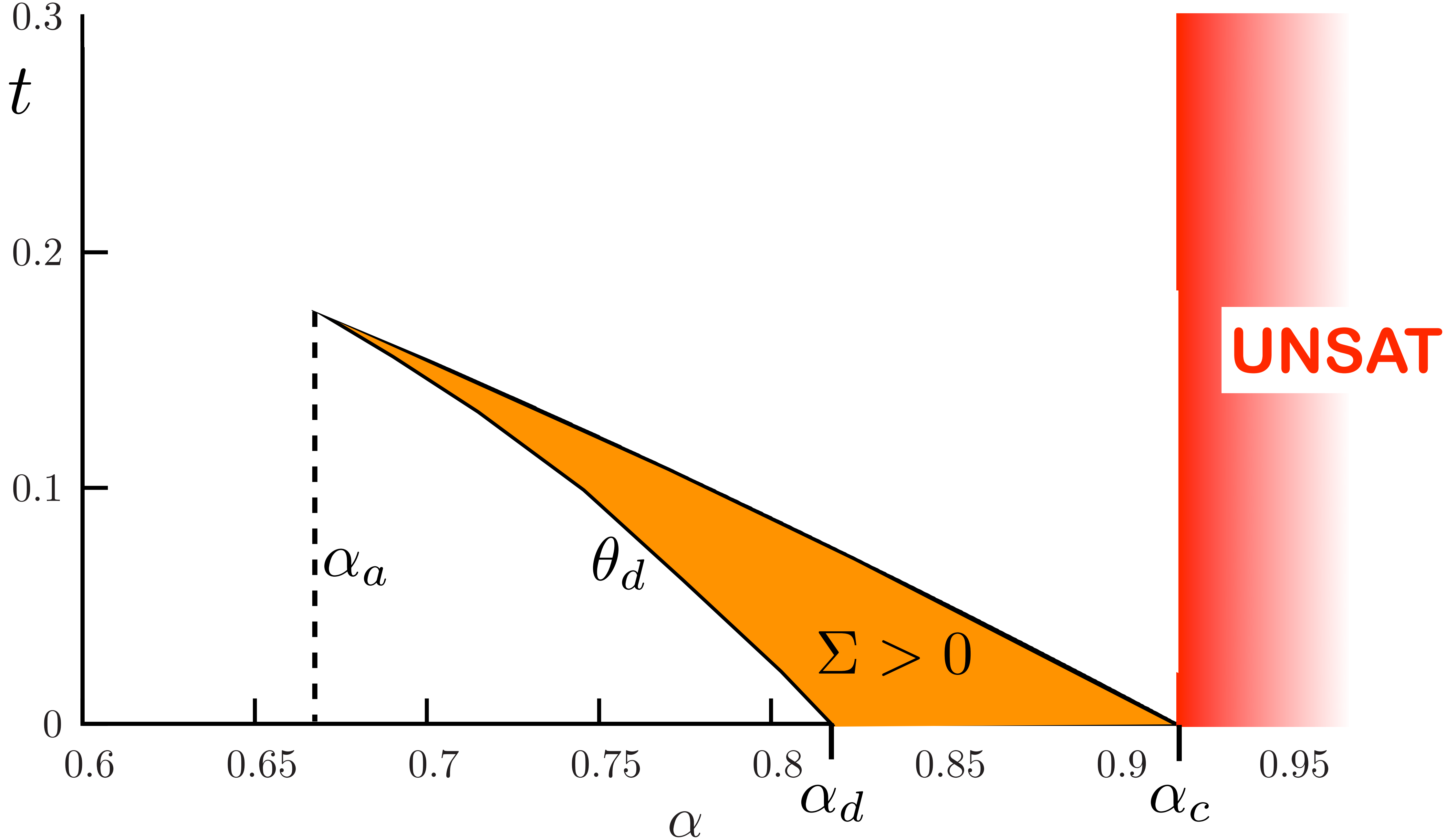
Dynamics: searching solutions

- Monte Carlo is very inefficient for T -> 0
- Sequential construction algorithm:

`while`(there are unassigned variables)

  - compute marginals
  - choose randomly an unassigned variable
  - fix it according to the marginal

Mapping the dynamics to a static problem...



Conclusions

- Spin glasses are prototype for complex systems
- The link between statics and dynamics has provided very useful
- ...but many aspects of this connection are still unclear and need to be improved.

Bibliography

- Too long to fit in a single slide ;-)
- Depends on which specific issue you are interested in
- Ask me via e-mail
  `Federico.Ricci@roma1.infn.it`

Phase transitions in Random SAT