Hadronic atoms

Akaki Rusetsky

Helmholtz-Institut für Strahlen- und Kernphysik

Abteilung Theorie, Universität Bonn, Germany

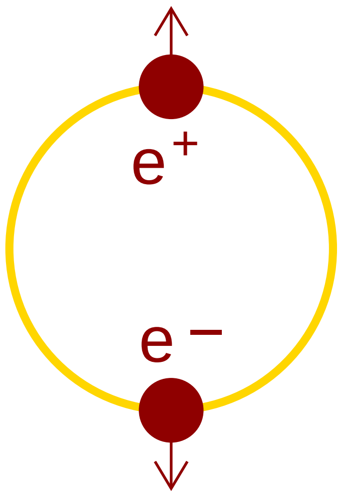
6th Vienna Central European Seminar

Vienna, November 27-29, 2009

Plan

- Introduction
- Physics background
- Systematic theory of hadronic atoms
- Isospin-breaking corrections
- Conclusions

Parapositronium



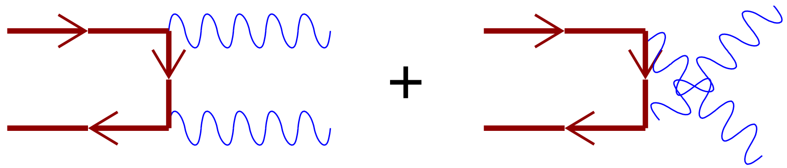
Reduced mass: $\mu = \frac{1}{2}\, m_e \simeq 0.26$ MeV

Binding energy: $E_B = -\frac{1}{2}\,\mu\alpha^2 + O(\alpha^4)$
$\simeq 6.8$ eV

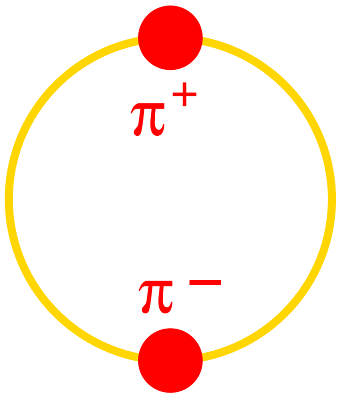
Bohr radius: $r_B = (\alpha\mu)^{-1} \simeq 10^5$ fm

Decay width: $\Gamma = \mu\alpha^5 + O(\alpha^6)$
$\simeq 5.3 \cdot 10^{-6}$ eV

Decays:



Pionium



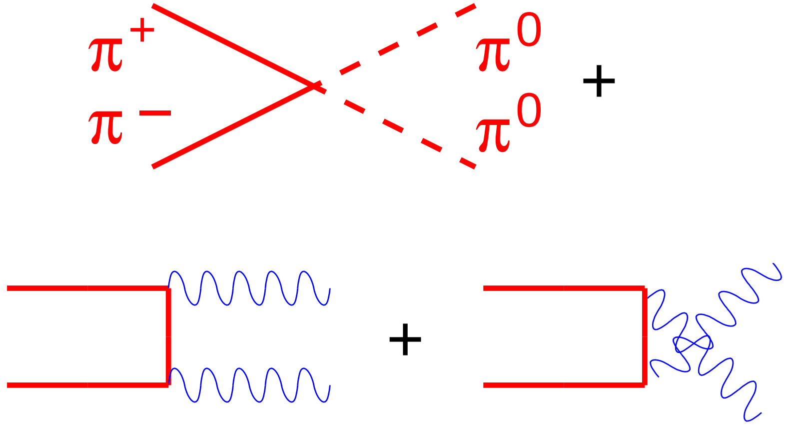
Reduced mass: $\mu = \frac{1}{2} M_\pi \simeq 70$ MeV $[0.27$ MeV$]$

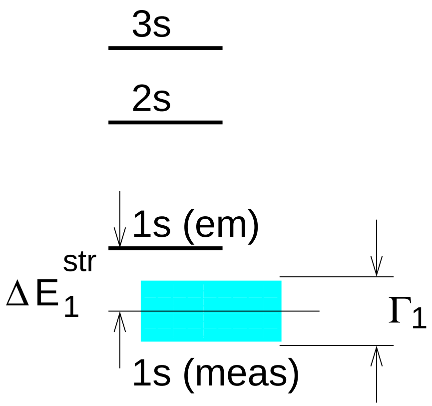
Binding energy: $E_B = -\frac{1}{2}\mu\alpha^2 + O(\alpha^4)$
$\simeq 1900$ eV $[6.8$ eV$]$

Bohr radius: $r_B = (\alpha\mu)^{-1}$
$\simeq 400$ fm $[10^5$ fm$]$

Decay width: $\Gamma \simeq 0.2$ eV
$[5.3 \cdot 10^{-6}$ eV$]$

Decays:





Observables of hadronic atoms

DGBT formulae

S. Deser, M.L. Goldberger, K. Baumann and W. Thirring, Phys. Rev. 96 (1954) 774.

Energy: $$E = 2M_\pi - \frac{\mu\alpha^2}{2} - \frac{4\pi}{3}\,|\Psi(0)|^2\,(2a_0 + a_2)/M_\pi^2 + \cdots$$

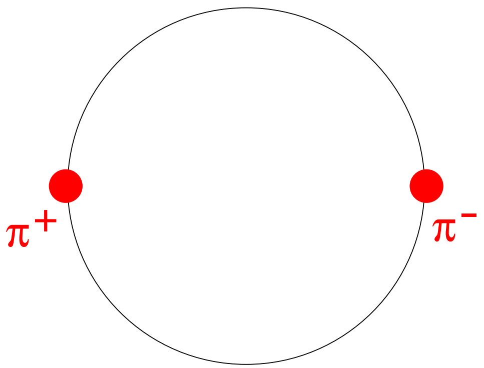
Width: $$\Gamma_{2\pi^0} = \frac{16\pi}{9}\,|\Psi(0)|^2\,p_1^*(a_0 - a_2)^2/M_\pi^3 + \cdots$$

$$|\Psi(0)|^2 = \frac{\alpha^3\mu^3}{\pi}\,, \qquad p_1^* = \left(M_\pi^2 - M_{\pi^0}^2 - \frac{1}{4}\,M_\pi^2\alpha^2\right)^{1/2}$$

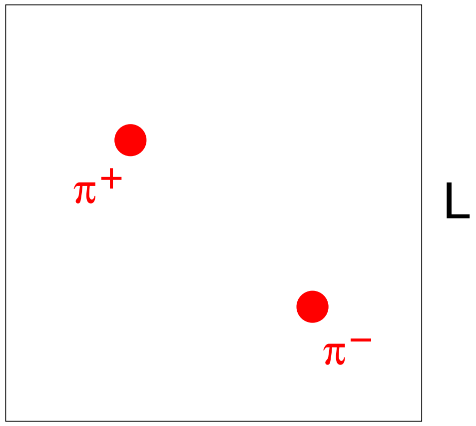
- ⇒ Valid in all orders in strong interactions
- ⇒ Since $R_{\text{str}}/r_B = O(\alpha) \ll 1$, short-range details of strong interactions do not matter. The final answer is written in terms of the scattering length
- ⇒ Can be used to extract the values of a_0, a_2 from the experiment on the pionium

Relation to Lüscher's formula

Energy levels in a finite volume $\Longleftrightarrow$ scattering length



$r_B \gg R_{\text{str}}$



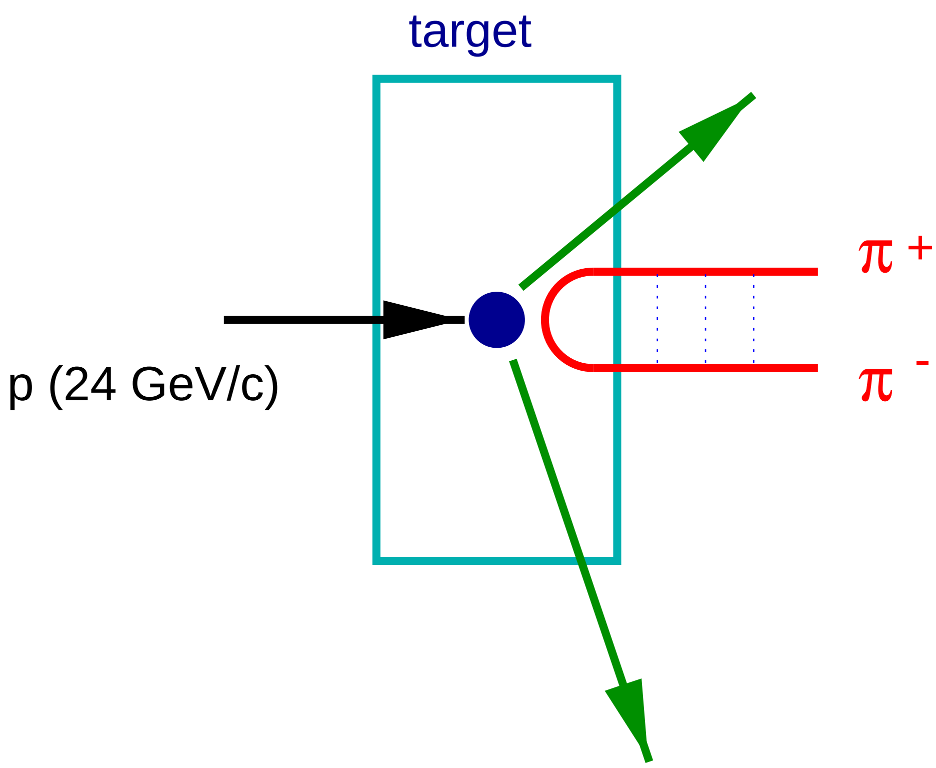
$L \gg R_{\text{str}}$

- $E_n = 2M_\pi - M_\pi \alpha^2 / 4n^2$
- $\langle \mathbf{p}^2 \rangle^{1/2} \sim \alpha M_\pi$
- Expansion in α

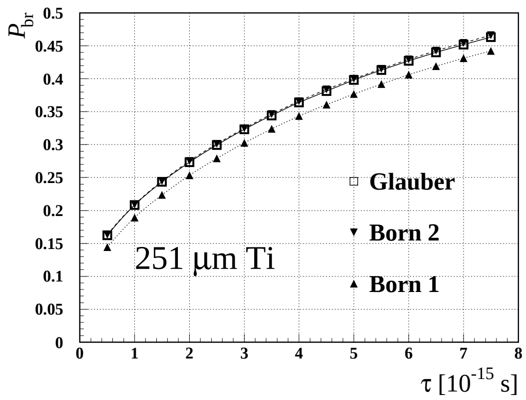
- Discrete levels in the box
- $\langle \mathbf{p}^2 \rangle^{1/2} \sim 1/L$
- Expansion in $1/L$

$$\Delta E \sim \frac{a}{L^3} \left\{ 1 + c_1 \frac{a}{L} + c_2 \frac{a^2}{L^2} \right\} + \cdots$$

DIRAC experiment at CERN



$$\Rightarrow \begin{cases} \pi^0\pi^0 & \text{(decay)} \\ \pi^+\pi^- & \text{(ionization)} \end{cases}$$



⇒ Separate "atomic pairs," which emerge in result of the ionization

⇒ Ionization probability on different targets ⇒ lifetime

C. Santamarina *et al*, J. Phys. B. At. Mol. Opt. Phys. **36** (2003) 4273

Pionium decays: physics background

- There exists a very precise prediction of a_0, a_2 within Chiral Perturbation Theory (ChPT) at two loops combined with dispersion relations
- The method assumes the standard scenario for the chiral symmetry breaking in QCD (developing a large quark condensate) [G. Colangelo, J. Gasser and H. Leutwyler, NPB 603 (2001) 125]

$$a_0 = 0.220 \pm 0.005\,, \quad a_2 = -0.444 \pm 0.0010$$

⇒ The DIRAC experiment tests the large/small condensate scenario in QCD with two flavors [J. Stern, arXiv:hep-ph/9510318]

$$M_\pi^2 = (m_u + m_d)B + \cdots; \quad B = -\langle \bar{u}u \rangle / F_\pi^2 \,|_{m_u, m_d \to 0}$$

⇒ Alternative methods to determine a_0, a_2:
cusps in $K \to 3\pi$ decays, K_{e4} decays (NA48/2 coll. at CERN)

DIRAC at CERN: $K\pi$ atom decays

- Deser-type formulae for the lifetime and the energy-level shifts

width	$\Rightarrow$	$a_{1/2} - a_{3/2}$
energy shift	$\Rightarrow$	$2a_{1/2} + a_{3/2}$

- Large/small condensate scenario in QCD with three flavors

$$B = -\langle \bar{u}u \rangle / F_\pi^2 \qquad \text{at } m_u, m_d \to 0,\ m_s \text{ fixed}$$

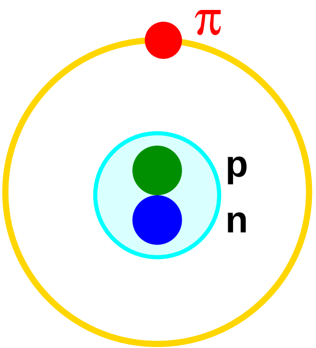
$$B_0 = -\langle \bar{u}u \rangle / F_\pi^2 \qquad \text{at } m_u, m_d, m_s \to 0$$

Flavor dependence: $B_0/B \simeq 1$ [large], $B_0/B << 1$ [small]
S.Descotes and J.Stern, PLB 488 (2000) 274; B.Moussallam, EPJC 14 (2000) 111

- Convergence of chiral expansion in $SU(3) \times SU(3)$ ChPT
  - V. Bernard, N. Kaiser and U.-G. Meißner, NPB 357 (1991) 129
  - J. Bijnens, P. Dhonte and P. Talavera, JHEP 0405 (2004) 036
  - J. Schweizer, PLB 625 (2005) 217
  - Roy-Steiner equations: P. Büttiker *et al*, EPJC 33 (2004) 409

πH and πd at PSI

	ΔE_{1s}	Γ	
pionic hydrogen:	7.120 ± 0.010 MeV	0.823 ± 0.019 MeV	(2006)
pionic deuterium:	-2.323 ± 0.031 MeV	1.171 ± 0.049 MeV	(2009)



$$\Rightarrow \quad a_{\pi^- d} = \underbrace{(a_{\pi^- p} + a_{\pi^- n})}_{\text{small}} + \underbrace{(\text{double scattering}) + \cdots}_{\text{corrections}}$$

- Output: precise values of the scattering lengths a_{0+}^{+} and a_{0+}^{-}

$\Rightarrow$ Precise value of the πNN coupling constant (GMO sum rule)

$\Rightarrow$ πN σ-term (explicit chiral symmetry breaking)

$$\sigma_{\pi N} = (m_u + m_d)\langle N|\bar{u}u + \bar{d}d|N\rangle / m_p$$

$\Rightarrow$ Strangeness content of the nucleon

$$y = 2\langle N|\bar{s}s|N\rangle / \langle N|\bar{u}u + \bar{d}d|N\rangle$$

KH and Kd: SIDDHARTA coll. at LNF-INFN

Determination of the S-wave $\bar{K}N$ scattering lengths a_0 and a_1:

- a_0, a_1 are *complex:* inelastic thresholds (e.g., $\pi\Sigma$) lie below
  - $\Rightarrow$ Need *both* kaonic hydrogen and kaonic deuterium data are necessary to extract a_0 and a_1 separately
- a_0, a_1 are *large:* $\Lambda(1405)$ lies in the vicinity of threshold
  - $\Rightarrow$ Multiple-scattering series for the kaonic deuterium are non-perturbative
- Confronting the theory with the experiment:
  - $\Rightarrow$ Compare with the theoretical calculations carried out in *coupled-channel unitarized ChPT*
  - $\Rightarrow$ Preliminary DEAR/SIDDHARTA result: not compatible with the scattering data
  - $\Rightarrow$ Useful input for the theory of antikaon interaction with nuclear medium

Extraction of the scattering lengths

$$E = 2M_\pi - \frac{\mu\alpha^2}{2} - \frac{1}{6}\,\alpha^3 M_\pi\,(2a_0 + a_2) + O(\alpha^4, \alpha^3(m_d - m_u)^2)$$

$$\Gamma = \frac{2}{9}\,\alpha^3 p_1^*\,(a_0 - a_2)^2 + O(\alpha^{9/2}, \alpha^{7/2}(m_d - m_u)^2)$$

- a_0, a_2 are defined in pure QCD with $\alpha = 0$ and $m_u = m_d$, hadronic atoms exist in the real world $\alpha \neq 0$ and $m_u \neq m_d$. How are the parameters of these two theories related?

In QFT, this relation is ambiguous!

- How does one evaluate isospin-breaking corrections in a systematic manner?
- In case of pionic/kaonic deuterium: three-body dynamics

. . . the biggest theoretical challenge at present

Theory of hadronic atoms: essentials

Characteristic momenta in the atom:

$$\langle p \rangle \sim r_B^{-1} \sim \alpha\mu \simeq 0.5 \text{ MeV} \ll M_\pi$$

$\hookrightarrow$ The non-relativistic expansion in $\langle p \rangle / M_\pi$ translates into the expansion in the fine structure constant α

$\hookrightarrow$ NR EFT: no massive particle creation/annihilation

$\hookrightarrow$ All dynamics from higher energy scales is hidden in the couplings of the NR effective Lagrangian

Scale hierarchy

NR EFT			ChPT	QCD+QED
strong width	binding energy	mass splittings	chiral	hard
$\alpha^3 p_1^\star M_\pi^4/\Lambda^4$	$\alpha^2 M_\pi$	$\alpha\Lambda,\ (m_d - m_u)^2$	M_π	$\Lambda \sim 1$ GeV

Hadronic atoms in NR EFT: latest work

P. Labelle and K. Buckley, arXiv:hep-ph/9804201

D. Eiras and J. Soto, PRD 61 (2000) 114027

B.R. Holstein, PRD 60 (1999) 114030

X. Kong and F. Ravndal, PRD 59 (1999) 014031; PRD 61 (2000) 077506

A. Gall, J. Gasser, V.E. Lyubovitskij and A. Rusetsky, PLB 462 (1999) 335

J. Gasser, V.E. Lyubovitskij and A. Rusetsky, PLB 471 (1999) 244

V.E. Lyubovitskij and A. Rusetsky, PLB 494 (2000) 9

J. Gasser, V.E. Lyubovitskij, A. Rusetsky and A. Gall, PRD 64 (2001) 016008

J. Gasser, M.A. Ivanov, E. Lipartia, M. Mojžiš and A. Rusetsky, EPJC 26 (2002) 13

J. Schweizer, PLB 587 (2004) 33; EPJC 36 (2004) 483

P. Zemp, *Pionic Hydrogen in QCD+QED: Decay width at NNLO*, PhD thesis, Univ. Bern, 2004.

U.-G. Meißner, U. Raha and A. Rusetsky, EPJC 35 (2004) 349; EPJC 41 (2005) 213; PLB 639 (2006) 478; EPJC 47 (2006) 473

. . .

Reviews:

J. Gasser, V. E. Lyubovitskij and A. Rusetsky, Phys. Rept. 456 (2008) 167

J. Gasser, V. E. Lyubovitskij and A. Rusetsky, Ann. Rev. Part. Nucl. Sci. (in print)

Strong non-relativistic Lagrangian and Feynman rules

A. Gall, J. Gasser, V.E. Lyubovitskij and AR,

PLB 462 (1999) 335; PLB 471 (1999) 244; PRD 64 (2001) 016008

$$\mathcal{L}_0 = \sum_{i=\pm,0} \Phi_i^\dagger \left(i\partial_t - M_{\pi^i} + \frac{\triangle}{2M_{\pi^i}} + \frac{\triangle^2}{8M_{\pi^i}^3} + \cdots \right) \Phi_i$$

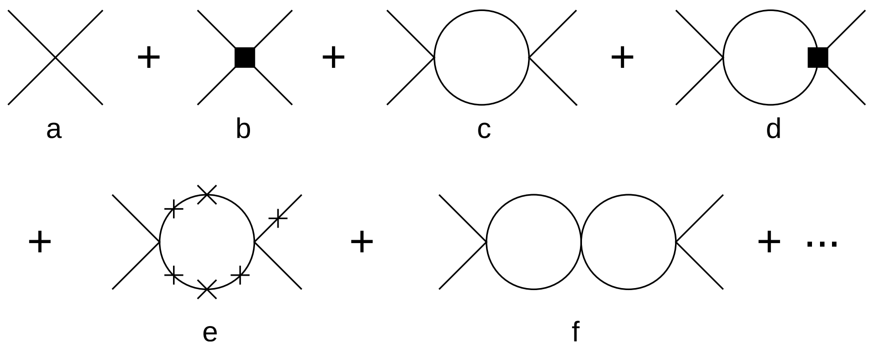
$$i\langle 0|T\Phi_\pm(x)\Phi_\pm^\dagger(0)|0\rangle = \int \frac{d^4p}{(2\pi)^4} \frac{\mathrm{e}^{-ipx}}{M_\pi + \mathbf{p}^2/2M_\pi - p^0 - i0}$$

4-pion interactions:

$$\mathcal{L}_I = c_1\, \Phi_+^\dagger \Phi_-^\dagger \Phi_+ \Phi_- + c_2\, (\Phi_+^\dagger \Phi_-^\dagger \Phi_0 \Phi_0 + \text{h.c.}) + c_3\, \Phi_0^\dagger \Phi_0^\dagger \Phi_0 \phi_0$$
$$+ \quad \text{derivative terms}$$

- Power counting: $\mathbf{p}/M_\pi \quad \Leftrightarrow \quad \nabla/M_\pi$
- Particle number is conserved by construction

Loops



$$\text{Loop} = \frac{iM_\pi}{4\pi} \underbrace{\sqrt{M_\pi(E - 2M_\pi)}}_{=|\mathbf{p}|}$$

$$T_{\text{NR}} = c_1 + c_1'\mathbf{p}^2 + c_1^2 \frac{iM_\pi|\mathbf{p}|}{4\pi} + \cdots = P_0(\mathbf{p}^2) + i|\mathbf{p}|P_1(\mathbf{p}^2)$$ [S-wave]

c_1 ⇒ scattering length

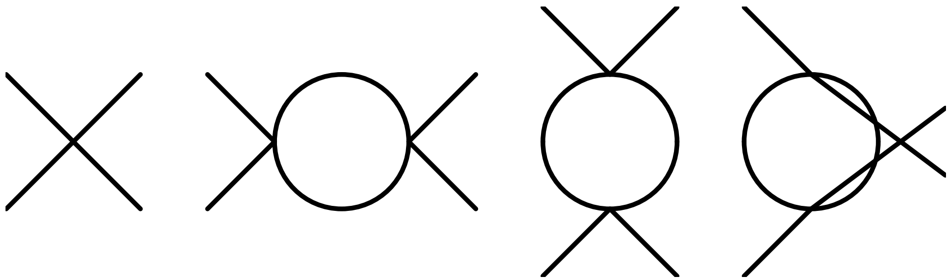
c_1' ⇒ scattering length, effective radius

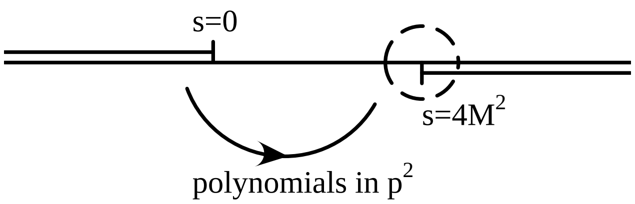
$\ldots$

↪ NR EFT = effective-range expansion

↪ Compare with ChPT: expansion in powers of the quark mass!

Relativistic QFT to one loop





$$T_\mathsf{R}(s,\cos\theta) = \lambda_r - \frac{\lambda_r^2}{16\pi^2}\left(\bar J(s) + \bar J(t) + \bar J(u)\right)$$

$$\begin{aligned}
\bar J(s) &= \int_0^1 d\tau \ln\frac{M^2 - s\tau(1-\tau)}{M^2} = -i\pi\sigma - \sigma\ln\frac{1-\sigma}{1+\sigma} - 2\,, \quad \sigma^2 = 1 - \frac{4M^2}{s} \\
T^l_\mathsf{R}(s) &= \frac{1}{64\pi}\int_{-1}^1 d\cos\theta P_l(\cos\theta) T_\mathsf{R}(s,\cos\theta)
\end{aligned}$$

Same analytic structure as in the NR amplitude for $|s - 4M^2| \le 4M^2$

$$T^0_\mathsf{R}(s) = \left[\lambda_r + \frac{\lambda_r^2}{16\pi^2}\left(2 - \frac{5\mathbf{p}^2}{3M^2}\right)\right] + \frac{i\lambda_r^2|\mathbf{p}|}{16\pi M}\left(1 - \frac{\mathbf{p}^2}{2M^2}\right) + \cdots$$

Matching condition

$$T_{\mathsf{R}}(p_1,p_2;p_3,p_4)=(2w_a(\mathbf{p}_1))^{1/2}\cdots(2w_d(\mathbf{p}_4))^{1/2}T_{\mathsf{NR}}(p_1,p_2;p_3,p_4)$$

- Fixes the couplings $c_1,c_2,c_3,\cdots$ in the non-relativistic Lagrangian (polynomial part)

In the ϕ^4 theory: $$c_1=\frac{1}{(2M)^2}\left(\lambda_r+\frac{\lambda_r^2}{8\pi^2}+O(\lambda_r^3)\right)$$

- Non-analytic part $\sim|\mathbf{p}|$ is reproduced by loops
- Matching to ChPT: the couplings $c_1,c_2,c_3,\cdots$ are given as an expansion in the quark mass

Electromagnetic interactions: Lagrangian

- Guiding principles: C, P, T, gauge invariance, rotational invariance
- Write down all possible terms allowed by symmetries at a given order

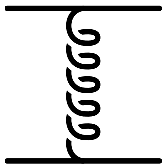
$$
\begin{aligned}
\mathcal{L} &= \sum_{\pm} \Phi_{\pm}^{\dagger}\Big(iD_t - M_\pi + \frac{\mathbf{D}^2}{2M_\pi} + \frac{\mathbf{D}^4}{8M_\pi^3} + \cdots \mp \frac{e\kappa_1(\mathbf{DE}-\mathbf{ED})}{6M_\pi^2}\Big)\Phi_{\pm} \\
&+ \frac{1}{2}(\mathbf{E}^2-\mathbf{B}^2) + c_1(\Phi_+^{\dagger}\Phi_-^{\dagger})(\Phi_+\Phi_-) + \cdots \\
&+ \text{space derivatives}+\text{nonminimal terms with } \mathbf{E},\ \mathbf{B}+\text{neutral pions}+\cdots
\end{aligned}
$$

$$
D_t\Phi_{\pm} = \partial_t\Phi_{\pm} \mp ieA_0\Phi_{\pm}\,; \qquad \mathbf{D}\Phi_{\pm} = \nabla\Phi_{\pm} \pm ie\mathbf{A}\Phi_{\pm}
$$

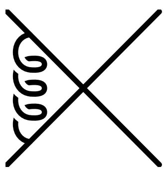
$$
\mathbf{E} = -\nabla A_0 - i\partial_t\mathbf{A}\,; \qquad \mathbf{B} = \text{rot}\,\mathbf{A}
$$

Matching in the presence of photons at $O(\alpha)$

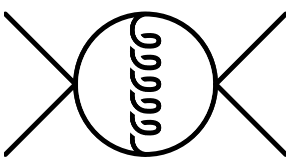
- Attach 1 photon in all possible ways to the “strong” diagrams



$$= \frac{4\pi\alpha}{|\mathbf{p}-\mathbf{q}|^2}$$



$$= \frac{\pi\alpha M_\pi}{4|\mathbf{p}|} + \underbrace{i\alpha\theta_C}_{\text{Coulomb phase}} + \cdots$$



$$= -\frac{\alpha M_\pi^2}{8\pi}\left(\text{UV div} + 2\ln\frac{2|\mathbf{p}|}{\mu} - 1 - i\pi\right) + \cdots$$

$\pi^+\pi^- \to \pi^+\pi^-$ elastic scattering amplitude

$$T_{\text{NR}} = T_{1\gamma} + \exp(2i\alpha\theta_C)\left\{\frac{B_1}{|\mathbf{p}|} + B_2 \ln\frac{|\mathbf{p}|}{\mu} + \mathcal{A}_{\text{NR}}\right\} + O(|\mathbf{p}|)$$

Matching: $\mathcal{A}_{\text{R}} = (2M_\pi)^2\,\mathcal{A}_{\text{NR}}$

Bound states

- Bound states in the NR EFT are described by the Schrödinger equation
- Predominately Coulomb bound states:

$$(H_0 + H_C)|\Psi_{nl}\rangle = E_n|\Psi_{nl}\rangle\,, \qquad H_C = -\frac{4\pi\alpha}{|\mathbf{p}-\mathbf{q}|^2}$$

$$E_n = 2M_\pi - \frac{M_\pi\alpha^2}{4n^2}\,, \quad n = 1, 2, \cdots$$

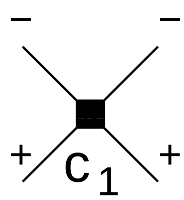
- The strong interactions are included perturbatively. The shift of the pole position in the scattering matrix is calculated by using the Feshbach formalism: Master equation

$$z - E_n = \langle\Psi_{nl}|\bar{\tau}_{nl}(E_n)|\Psi_{nl}\rangle + O(\alpha^5)$$

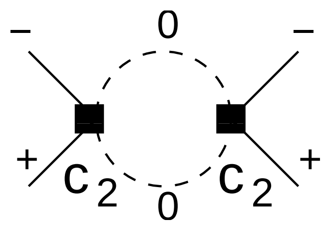
$$\bar{\tau}_{nl}(E_n) = H_I + \sum_{\{ml\}\neq\{nl\}} H_I \frac{|\Psi_{ml}\rangle\langle\Psi_{ml}|}{E_n - E_m} H_I + \cdots$$

The pionium decay at lowest order

$\bar{\tau} =$



$+$



$= -c_1 - 2c_2^2\, J_0(z)$

$$\Delta E_1 = \text{Re}\, z = -|\Psi_{10}(\mathbf{r}=0)|^2\, c_1 = \frac{\alpha^3 M_\pi^3}{8\pi}\, c_1$$

$$-\frac{\Gamma_1}{2} = \text{Im}\, z = -|\Psi_{10}(\mathbf{r}=0)|^2\, c_2^2\, \frac{M_\pi}{2\pi}\, \sqrt{2M_\pi(M_\pi - M_{\pi^0})}$$

$$= \frac{\alpha^3 M_\pi^4}{16\pi^2}\, c_2^2 \sqrt{2M_\pi(M_\pi - M_{\pi^0})}$$

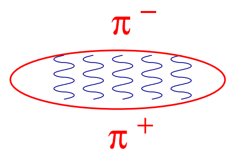
The pole automatically emerges on the second Riemann sheet

Ground-state width at NLO

Counting of the isospin-breaking corrections: $\alpha \sim (m_d - m_u)^2 \sim \delta$

$$c_i = \bar{c}_i + \alpha c_i^{(1)} + (m_d - m_u)^2 c_i^{(2)} + O(\delta^2)$$

$$\Gamma_{2\pi 0} = -\frac{\alpha^3 M_\pi^3}{4\pi} X \left\{ 1 - 2c_1 \cdot \right.$$



$$\left. \right\} + O(\delta^5)$$

$$X = -\frac{M_{\pi^0}}{2\pi}\rho^{1/2}\left(1 + \frac{5\rho}{8M_{\pi^0}^2}\right) c_2^2 \left(1 - \rho \frac{M_{\pi^0}^2}{4\pi^2} c_3^2\right) + O(\delta^{5/2})$$

$$\rho = 2M_{\pi^0}(M_\pi - M_{\pi^0} - M_\pi \alpha^2/8)$$

- Perform matching of c_1, c_2, c_3 to the relativistic threshold $\pi\pi$ amplitudes, calculated at a pertinent order in δ
- Substitute in the expression of the decay width

Final result for the width at NLO in isospin breaking

$$\boxed{\Gamma_{2\pi^0} = \tfrac{2}{9}\,\alpha^3 p_1^* \mathcal{A}^2 (1+K) + O(\delta^{11/2})}$$

$$p_1^* = (\Delta M_\pi^2 - M_\pi^2\alpha^2/4)^{1/2}$$: phase space

$$\mathcal{A} = a_0 - a_2 + O(\delta)$$: threshold amplitude

$$K = \frac{\Delta M_\pi^2}{9M_\pi^2}(a_0+2a_2)^2 - \frac{2\alpha}{3}(\ln\alpha - 1)(2a_0+a_2) + O(\delta)$$

: bound-state correction factor

$$\Delta M_\pi^2 = M_\pi^2 - M_{\pi^0}^2$$

- No reference to NF EFT after matching
- $\mathcal{A}$ is the <u>relativistic</u> threshold amplitude for $\pi^+\pi^- \to \pi^0\pi^0$

Isospin breaking corrections: the definition

Quantum mechanics:

$$i\frac{\partial\Psi}{\partial t} = (\hat{H}_0 + \hat{V})\Psi\,, \qquad \hat{V} = \hat{V}_{\mathsf{str}} - \underbrace{\frac{\alpha}{r}}_{\text{switch off}}$$

The procedure is ambiguous in Quantum Field Theory, due to the presence of UV divergences

$$\text{QCD + QED} \quad : \quad \mu\frac{dg}{d\mu} = \beta_g(g,e)\,, \qquad \mu\frac{de}{d\mu} = \beta_e(g,e)$$

$$\text{pure QCD} \quad : \quad \mu\frac{d\bar{g}}{d\mu} = \beta_g(\bar{g},0)$$

What is the relation of $g(\mu)$ to $\bar{g}(\mu)$ defined at $\alpha = 0$ and $m_d = m_u$?
In QCD, the answer is convention dependent

J. Gasser, AR and I. Scimemi, EPJC 32 (2003) 97

Isospin-breaking corrections in ChPT

Definition of the isospin-symmetric world:

$M_\pi = M_{\pi^+} = 139.57$ MeV $\longrightarrow$ fixes quark mass $\hat{m}$

$F_\pi = 92.4$ MeV $\longrightarrow$ fixes Λ_{QCD}

Isospin-breaking corrections to the $\pi^+\pi^- \to \pi^0\pi^0$ amplitude:

$$\mathcal{A} = \underbrace{a_0 - a_2}_{\text{symmetric}} + \epsilon + O(\delta^2)$$

$$O(p^2) \quad : \quad \mathcal{A} = \frac{3}{32\pi F^2}\,(4M_\pi^2 - M_{\pi^0}^2) = a_0 - a_2 + \frac{3\Delta M_\pi^2}{32\pi F^2}$$

$$O(p^4) \quad : \quad \Gamma_{2\pi^0} = \frac{2}{9}\,\alpha^3 p_1^* (a_0 - a_2)^2 (1+\delta)\,, \qquad \delta = (5.8 \pm 1.2) \times 10^{-2}$$

$$\tau = \Gamma_{2\pi^0}^{-1} = (2.9 \pm 0.1) \times 10^{-15}\ \text{s} \qquad \text{(standard scenario)}$$

Conclusions

- Experiments on hadronic atoms enable one to extract the values of hadronic scattering lengths in QCD
- In order to ensure that the accuracy of the theoretical description of the atoms matches with the theoretical precision, the isospin-breaking corrections to the leading-order DGBT formula should be evaluated
  - $\Rightarrow$ Definition of the isospin-symmetry limit in QCD is convention-dependent
- Due to a huge difference between the atomic and strong energy scales, the NR EFT provides a systematic framework to investigate this sort of the bound states
  - $\Rightarrow$ This approach is universal and applies to all known hadronic atoms $\pi^+\pi^-$, πK, πH, πd, KH, Kd, $\ldots$, despite their physically very different nature