QCD like Theories at next-to-next-to leading Order

Jie Lu
in collaboration with Johan Bijnens

Department of Theoretical Physics
Lund University

6th Vienna Central European Seminar
EFFECTIVE FIELD THEORIES
Nov. 27 - 29, 2009

Outline

Reference: J. Bijnens and J. Lu, arXiv:0910.5424 [hep-ph]

Part I

Introduction

Technicolor Theories

Technicolor Theory: A candidate theory of beyond Standard Model.

- New strong interaction, new quark Q, new gauge boson at TeV scale.
- No fundamental Higgs Boson, no fine-tuning problem.
- Spontaneous ElectroWeak symmetry breaking in Technicolor Theory
  - Nonzero of quark condensate $\bar{Q}Q$.
  - $\bar{Q}Q \neq 0$ cause spontaneous breaking of chiral symmetry.
    Three different patterns in general:
    $SU(N_f) \times SU(N_f) \to SU(N_f)$ complex representation
    $SU(2N_f) \to SO(2N_f)$ real representation
    $SU(2N_f) \to Sp(2N_f)$ pseudo − real representation
    ▶ *M. E. Peskin, Nucl. Phys. B* **175** *(1980) 197.*
  - Some Goldstone Boson (technipion) become the longitudinal part of weak gauge Boson $\to$ EW symmetry breaking
- Many new resonance states: like techni-meson, techni-baryon, etc.
- Precision electroweak measurements are troublesome for classical Technicolor. But it can be avoided or amended in general, e.g., by the presence of Walking technicolor or new strong top quark dynamics . . . .

For a recent review: *Francesco Sannino arXiv:0911.0931*

Finite Baryon Density

Finite Baryon Density: study the color superconductivity, diquark Boson-Einstein condense, superfluid, etc.. at finite baryon density.

- Related phenomena: neutron stars, supernova explosion, relativistic heavy ion collisions....
- All of those need to understand diquark condensate mechanism.
- QCD lagrangian with finite baryon chemical potential term

$$\mathcal{L} = \bar{\psi}\gamma_\nu D_\nu \psi + m\bar{\psi}\psi - \mu\bar{\psi}\gamma_0\psi.$$

  But the lattice calculation suffers the problem that the determinant of the Euclidean QCD Dirac operator is not real.
- Other QCD-like theories without this problem could be helpful to understand the diquark condensate mechanism.
  QCD with adjoint quarks
  Two Color QCD
  ▶ *J. B. Kogut, M. A. Stephanov, D. Toublan, J. J. M. Verbaarschot and A. Zhitnitsky, Nucl. Phys. B* **582** *(2000) 477 [arXiv:hep-ph/0001171]*

QCD

QCD: $SU(3)_c$ gauge theory, quarks live in the fundamental representation.

- The Lagrangian with source terms

$$\begin{aligned}\mathcal{L} &= \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R + \bar{q}_L \gamma^\mu l_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R \\ &\quad -\bar{q}_L \mathcal{M}^\dagger q_R - \bar{q}_R \mathcal{M} q_L \\ D_\mu &= \partial_\mu q - iG_\mu q, \qquad i,j = 1,2,...,N_f\end{aligned}$$

- All the quarks have same mass in this Lagrangian.
- The flavor symmetry is $SU(N_f)_L \times SU(N_f)_R$.
- The vacuum condensate $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R \rangle + h.c.$, which breaks the $SU(N_f)_L \times SU(N_f)_R$ down to $SU(N_f)_V$.

QCD with adjoint quarks

- The Lagrangian with source terms

$$\begin{aligned} \mathcal{L} &= \mathrm{tr}_c\left(\overline{q}_{Li} i\gamma^\mu D_\mu q_{Li}\right) + \mathrm{tr}_c\left(\overline{q}_{Ri} i\gamma^\mu D_\mu q_{Ri}\right) + \mathrm{tr}_c\left(\overline{q}_{Li}\gamma^\mu l_{\mu ij} q_{Lj}\right) + \mathrm{tr}_c\left(\overline{q}_{Ri}\gamma^\mu r_{\mu ij} q_{Rj}\right) \\ &\quad - \mathrm{tr}_c\left(\overline{q}_{Ri}\mathcal{M}_{ij} q_{Lj}\right) - \mathrm{tr}_c\left(\overline{q}_{Li}\mathcal{M}^\dagger_{ij} q_{Rj}\right) \\ D_\mu q &= \partial_\mu q - iG_\mu q + iqG_\mu, \qquad i,j = 1,2,\ldots,N_f \end{aligned}$$

- All the quarks have same mass in this Lagrangian.
- The lightest baryon is diquark(boson), with the same mass of pionic Goldstone Boson.
- Transfer the left hand quark to right hand anti-quark:

$$\tilde{q}_{Ri} \equiv C\overline{q}^T_{Li}, \qquad C = i\gamma^2\gamma^0$$

- The Lagrangian can be rewritten as

$$\begin{aligned} \mathcal{L} &= \mathrm{tr}_c\left(\overline{\hat{q}} i\gamma^\mu D_\mu \hat{q}\right) + \mathrm{tr}_c\left(\overline{\hat{q}}\gamma^\mu \hat{V}_\mu \hat{q}_j\right) - \frac{1}{2}\mathrm{tr}_c\left(\overline{\hat{q}} C\hat{\mathcal{M}}\overline{\hat{q}}^T\right) - \frac{1}{2}\mathrm{tr}_c\left(\hat{q}^T C\hat{\mathcal{M}}^\dagger \hat{q}\right) \\ \hat{q} &= \begin{pmatrix} q_R \\ \tilde{q}_R \end{pmatrix}, \qquad \hat{V}_\mu = \begin{pmatrix} r_\mu & 0 \\ 0 & -l^T_\mu \end{pmatrix}, \qquad \hat{\mathcal{M}} = \begin{pmatrix} 0 & \mathcal{M} \\ \mathcal{M}^T & 0 \end{pmatrix} \end{aligned}$$

- The flavor symmetry is enlarged to $SU(2N_f)$!
- The vacuum condensate $\langle \mathrm{tr}_c\left(\hat{q}^T CJ_S\hat{q}\right)\rangle + \mathrm{h.c.}$ breaks the $SU(2N_f)$ down to $SO(2N_f)$.

$$J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Two Color QCD

- The Lagrangian with source terms

$$\begin{aligned}\mathcal{L} &= \overline{q}_{Li} i\gamma^\mu D_\mu q_{Li} + \overline{q}_{Ri} i\gamma^\mu D_\mu q_{Ri} + \overline{q}_{Li}\gamma^\mu l_{\mu ij} q_{Lj} + \overline{q}_{Ri}\gamma^\mu r_{\mu ij} q_{Rj} \\ &\quad -\overline{q}_{Ri}\mathcal{M}_{ij} q_{Lj} - \overline{q}_{Li}\mathcal{M}^\dagger_{ij} q_{Rj} \\ D_\mu &= \partial_\mu q - iG_\mu q, \qquad i,j = 1,2,\ldots,N_f\end{aligned}$$

- All the quarks have same mass in this Lagrangian.
- The lightest baryon is diquark(boson), with the same mass of pionic Goldstone Boson.
- Transfer the left hand quark to right hand anti-quark:

$$\tilde{q}_{R\alpha i} = \epsilon_{\alpha\beta} C \overline{q}^T_{L\beta i}$$

- The Lagrangian can be rewritten as

$$\begin{aligned}\mathcal{L} &= \overline{\hat{q}} i\gamma^\mu D_\mu \hat{q} + \overline{\hat{q}}\gamma^\mu \hat{V}_\mu \hat{q}_{Lj} - \frac{1}{2}\overline{\hat{q}}_\alpha C\epsilon_{\alpha\beta}\hat{\mathcal{M}}\overline{\hat{q}}^T_\beta - \frac{1}{2}\hat{q}_\alpha\epsilon_{\alpha\beta}C\hat{\mathcal{M}}^\dagger\hat{q}_\beta \\ \hat{q} &= \begin{pmatrix} q_R \\ \tilde{q}_R \end{pmatrix}, \quad \hat{V}_\mu = \begin{pmatrix} r_\mu & 0 \\ 0 & -l^T_\mu \end{pmatrix}, \quad \hat{\mathcal{M}} = \begin{pmatrix} 0 & -\mathcal{M} \\ \mathcal{M}^T & 0 \end{pmatrix}\end{aligned}$$

- The flavor symmetry is enlarged to $SU(2N_f)$!
- The vacuum condensate $\langle \hat{q}_\alpha \epsilon_{\alpha\beta} C J_A \hat{q}_\beta \rangle + \mathrm{h.c.}$ breaks the $SU(2N_f)$ down to $Sp(2N_f)$.

$$J_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

Part II

Effective Theories

Broken Generator and Goldstone Bosons

The Goldstone Boson (π^a) manifold is parametrized in different way because different vacuum structure of the three theories.

- QCD: $SU(N_f) \times SU(N_f)/SU(N_f)$.
  - The number of broken generator: $N_g = N_f^2 - 1$
  - Goldstone Boson

$$U = u\mathbb{I}u, \qquad u = \exp\left(\frac{i}{\sqrt{2}F_0}\sum_{a=1}^{N_g}\pi^a T^a\right)$$

- QCD with Adjoint quarks: $SU(2N_f)/SO(2N_f)$.
  - The number of broken generator: $N_g = N_f(2N_f + 1) - 1$
  - Goldstone Boson

$$U = uJ_S u^T, \qquad u = \exp\left(\frac{i}{\sqrt{2}F_0}\sum_{a=1}^{N_g}\pi^a T^a\right)$$

- Two color QCD: $SU(2N_f)/Sp(2N_f)$.
  - The number of broken generator: $N_g = N_f(2N_f - 1) - 1$
  - Goldstone Boson

$$U = uJ_A u^T, \qquad u = \exp\left(\frac{i}{\sqrt{2}F_0}\sum_{a=1}^{N_g}\pi^a T^a\right)$$

Lagrangian at Leading order, Renormalization

- The Lagrangian is constructed by momentum expansion: p^2, p^4, p^6,....
- Assume all the mesons have same mass. At the leading order p^2, the three different theories have same form of Lagrangian:

$$\mathcal{L}_2 = \frac{F_0^2}{4}\langle D_\mu U^\dagger D^\mu U + \chi U^\dagger + U\chi^\dagger\rangle$$

- The covariant derivative D_μ and mass parameter χ is different:

	QCD	Adjoint	2-colour
D_μ	$\partial_\mu U - ir_\mu U + iUl_\mu$	$\partial_\mu U - iV_\mu U - iU(J_S V_\mu^T J_S)$	$\partial_\mu U - iV_\mu U - iU(J_A V_\mu^T J_A)$
χ	m_0^2	$m_0^2 J_S$	$m_0^2 J_A$

- Renormalization, The cancelation of divergence.
  - The one loop divergence of $\mathcal{L}_2$ is absorbed by the bare coefficient of $\mathcal{L}_4$: L_i
  - The two loop divergence of $\mathcal{L}_2$ and one loop divergence of $\mathcal{L}_4$ are absorbed by the bare coefficient of $\mathcal{L}_6$: K_i.

Lagrangian at Next-to-Leading order

- The p^4 Lagrangian of the three case has the same form:

$$
\begin{aligned}
\mathcal{L}_4 \quad = \quad & L_0 \mathrm{Tr}\left[D_\mu U (D_\nu U)^\dagger D^\mu U (D^\nu U)^\dagger\right] \\
& + L_1 \left\{\mathrm{Tr}[D_\mu U (D^\mu U)^\dagger]\right\}^2 + L_2 \mathrm{Tr}\left[D_\mu U (D_\nu U)^\dagger\right] \mathrm{Tr}\left[D^\mu U (D^\nu U)^\dagger\right] \\
& + L_3 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger D_\nu U (D^\nu U)^\dagger\right] + L_4 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger\right] \mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right) \\
& + L_5 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger (\chi U^\dagger + U \chi^\dagger)\right] + L_6 \left[\mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)\right]^2 \\
& + L_7 \left[\mathrm{Tr}\left(\chi U^\dagger - U \chi^\dagger\right)\right]^2 + L_8 \mathrm{Tr}\left(U \chi^\dagger U \chi^\dagger + \chi U^\dagger \chi U^\dagger\right) \\
& - i L_9 \mathrm{Tr}\left[f^R_{\mu\nu} D^\mu U (D^\nu U)^\dagger + f^L_{\mu\nu} (D^\mu U)^\dagger D^\nu U\right] + L_{10} \mathrm{Tr}\left(U f^L_{\mu\nu} U^\dagger f_R^{\mu\nu}\right) \\
& + H_1 \mathrm{Tr}\left(f^R_{\mu\nu} f_R^{\mu\nu} + f^L_{\mu\nu} f_L^{\mu\nu}\right) + H_2 \mathrm{Tr}\left(\chi \chi^\dagger\right).
\end{aligned}
$$

- The modified minimal subtraction scheme ($\overline{\mathrm{MS}}$) of p^4 Lagrangian:

$$
\begin{aligned}
L_i &= (c\mu)^{d-4}\left[L_i^r(\mu) + \Gamma_i \Lambda\right] \\
\ln c &= -\frac{1}{2}[\ln 4\pi + \Gamma'(1) + 1], \qquad \Lambda = \frac{1}{16\pi^2(d-4)}
\end{aligned}
$$

Lagrangian at Next-to-Leading order

The coefficients Γ_i for the three cases that are needed to absorb the divergences at NLO. The last two lines correspond to the terms with H_1 and H_2

i	QCD $SU(N)_L \times SU(N_f)_R/SU(N_f)$	Adjoint $SU(2N_f)/SO(2N_f)$	2-colour $SU(2N_f)/Sp(2N_f)$
0	$N_f/48$	$(N_f+4)/48$	$(N_f-4)/48$
1	$1/16$	$1/32$	$1/32$
2	$1/8$	$1/16$	$1/16$
3	$N_f/24$	$(N_f-2)/24$	$(N_f+2)/24$
4	$1/8$	$1/16$	$1/16$
5	$N_f/8$	$N_f/8$	$N_f/8$
6	$(N_f^2+2)/(16N_f^2)$	$(N_f^2+1)/(32N_f^2)$	$(N_f^2+1)/(32N_f^2)$
7	0	0	0
8	$(N_f^2-4)/(16N_f)$	$(N_f^2+N_f-2)/(16N_f)$	$(N_f^2-N_f-2)/(16N_f)$
9	$N_f/12$	$(N_f+1)/2$	$(N_f-1)/2$
10	$-N_f/12$	$-(N_f+1)/2$	$-(N_f-1)/2$
1'	$-N_f/24$	$-(N_f+1)/4$	$-(N_f+1)/4$
2'	$(N_f^2-4)/(8N_f)$	$(N_f^2+N_f-2)/(8N_f)$	$(N_f^2-N_f-2)/(8N_f)$

- QCD: *J. Gasser and H. Leutwyler, Annals Phys.* **158** *(1984) 142.*
- 2 color: small disagreement with *K. Splittorff, et al, Nucl. Phys. B* **620** *(2002) 290 [arXiv:hep-ph/0108040].*
- Adjoint: new.

Lagrangian at Next-to-Next-to-Leading order

- The p^6 lagrangian of $SU(N_f) \times SU(N_f)$ case is known.

$$r_i = (c\mu)^{2(d-4)} \left[r_i^r - \Gamma_i^{(2)} \Lambda^2 - \left(\frac{1}{16\pi^2} \Gamma_i^{(1)} + \Gamma_i^{(L)} \right) \Lambda \right]$$

  - *J. Bijnens, G. Colangelo and G. Ecker, JHEP* **9902** *(1999) 020*
- The complete form of the p^6 lagrangian of "Adjoint QCD" and "2-color QCD" is not know yet. Whether they have the same form of QCD is also not known.
- The divergence structure form at p^6 for "Adjoint QCD" and "2-color QCD" is the same except the coefficients $\Gamma_i^{(L)}$, $\Gamma_i^{(1)}$ and $\Gamma_i^{(2)}$, they are unknown coefficients. The nonlocal divergence should be canceled in the calculation.
- The related NNLO coupling constants r_i can be combined as a single unknown parameters for each calculated quantity.

Part III

The Calculation

Mass Equation

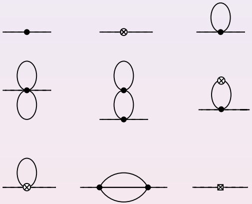
- Assume all the Mesons have the same mass
- The full propagator of Meson field

$$\begin{aligned} i\Delta(p) &= \frac{i}{p^2 - M_0^2 + i0^+} + \frac{i}{p^2 - M_0^2 + i0^+}[-i\Sigma(p^2)]\frac{i}{p^2 - M_0^2 + i0^+} + \cdots \\ &= \frac{i}{p^2 - M_0^2 - \Sigma(p^2) + i0^+}. \end{aligned}$$

- The physical mass as defined in the pole of the full propagator:

$$\begin{aligned} m^2 &= m_0^2 + \Sigma(m^2) \\ \Sigma(p^2) &= \Sigma^{(2)}(p^2) + \Sigma^{(4)}(p^2) + \Sigma^{(6)}(p^2) + \ldots \qquad \text{(The self energy)} \end{aligned}$$

The feynman diagrams and correction of mass



The set of diagrams contributing to the 1PI quantities.

$\bullet$: a p^2 vertex,
$\times$: a p^4 vertex,
$\boxtimes$ a p^6 vertex.

The mass of meson m_π^2 up to order of p^6.

$$
\begin{aligned}
M^2_{\text{phys}} &= M^2_{\text{LO}} + M^2_{\text{NLO}} + M^2_{\text{NNLO}} \\
M^2_{\text{NLO}} &= \Sigma^{(4)}(m_0^2) = M^2\left[a_M \frac{\overline{A}(M^2)}{F^2} + b_M \frac{M^2}{F^2}\right] \\
M^2_{\text{NNLO}} &= \Sigma^{(6)}(m_0^2) + \Sigma^{(4)}(m_0^2)\Sigma^{(4)'}(m_0^2) \\
&= M^2\left[\left(c_M \frac{\overline{A}(M^2)^2}{F^4} + \frac{M^2\overline{A}(M^2)}{F^4}\left(d_M + \frac{e_M}{16\pi^2}\right) + \frac{M^4}{F^4}\left(f_M + \frac{g_M}{16\pi^2} + \frac{h_M}{(16\pi^2)^2}\right)\right]
\end{aligned}
$$

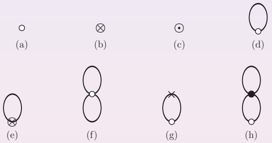
Results

	QCD
a_M	$-\frac{1}{N_F}$
b_M	$8N_F\,(2L_6^r - L_4^r) + 8\,(2L_8^r - L_5^r)$
c_M	$-\frac{1}{2} + \frac{9}{2N_F^2} + \frac{3}{8}N_F^2$
d_M	$8L_0^r(-\frac{3}{N_F} + N_F) + 8L_1^r(-1 + 2N_F^2) + 4L_2^r(4 + N_F^2) + L_3^r(-\frac{24}{N_F} + 20N_F)$ $+L_4^r(40 - 16N_F^2) + L_5^r(\frac{40}{N_F} - 16N_F) + L_6^r(-16 + 16N_F^2) + L_8^r(-\frac{80}{N_F} + 32N_F)$
e_M	$-\frac{5}{3} + \frac{4}{N_F^2} + \frac{19}{16}N_F^2$
f_M	$-32K_{17}^r - 16K_{19}^r - 16K_{23}^r + 48K_{25}^r + 32K_{39}^r$ $+N_F\,(-32K_{18}^r - 16K_{20}^r - 16K_{21}^r + 48K_{26}^r + 32K_{40}^r)$ $+N_F^2\,(-16K_{22}^r + 48K_{27}^r) + 64(N_F L_4^r + L_5^r)(N_F L_4^r + L_5^r - 2N_F L_6^r - 2L_8^r)$
g_M	$-\frac{4}{N_F}(L_0^r + L_3^r) + 4L_1^r + 2N_F(2L_0^r + L_3^r) + 2N_F^2 L_2^r$ $-8[L_4^r - 2L_6^r + \frac{1}{N_F}(L_5^r - 2L_8^r)]$
h_M	$-\frac{1}{4} + \frac{3}{4}\frac{1}{N_F^2} + \frac{169}{384}N_F^2$
	Adjoint
a_M	$\frac{1}{2} - \frac{1}{2N_F}$
b_M	$16N_F(2L_6^r - L_4^r) + 8(2L_8^r - L_5^r)$
c_M	$\frac{3}{8}\left(1 + \frac{3}{N_F^2} - \frac{4}{N_F} + N_F + N_F^2\right)$
d_M	$L_0^r(12 - 12\frac{1}{N_F} + 8N_F) + 8L_1^r(-1 + 2N_F + 4N_F^2)$ $+4L_2^r(4 + N_F + 2N_F^2) + L_3^r(12 - \frac{12}{N_F} + 20N_F)$ $+L_4^r(40 - 40N_F - 32N_F^2) + L_5^r(-20 + \frac{20}{N_F} - 16N_F)$ $+16L_6^r(-1 + 3N_F + 2N_F^2) + L_8^r(40 - \frac{40}{N_F} + 32N_F)$
e_M	$-\frac{2}{3} + \frac{1}{N_F^2} - \frac{3}{4}\frac{1}{N_F} + \frac{77}{48}N_F + \frac{19}{16}N_F^2$
f_M	$r_{MA}^r + 64(2N_F L_4^r + L_5^r)(2N_F L_4^r + L_5^r - 4N_F L_6^r - 2L_8^r)$
g_M	$2L_0^r(1 - \frac{1}{N_F} + 2N_F) + 4L_1^r + 2N_F L_2^r(1 + 2N_F) + 2L_3^r(1 - \frac{1}{N_F} + N_F)$ $-8(1 - N_F)(L_4^r - 2L_6^r) + 4(1 - \frac{1}{N_F})(L_5^r - 2L_8^r)$
h_M	$-\frac{1}{16} + \frac{3}{16}\frac{1}{N_F^2} - \frac{3}{16}\frac{1}{N_F} + \frac{193}{384}N_F + \frac{169}{384}N_F^2$
	2-colour
a_M	$-\frac{1}{2} - \frac{1}{2N_F}$
b_M	$16N_F(2L_6^r - L_4^r) + 8(2L_8^r - L_5^r)$
c_M	$\frac{3}{8}\left(1 + \frac{3}{N_F^2} + \frac{4}{N_F} - N_F + N_F^2\right)$
d_M	$L_0^r(-12 - 12\frac{1}{N_F} + 8N_F) + 8L_1^r(-1 - 2N_F + 4N_F^2)$ $+4L_2^r(4 - N_F + 2N_F^2) + L_3^r(-12 - \frac{12}{N_F} + 20N_F)$ $+L_4^r(40 + 40N_F - 32N_F^2) + L_5^r(20 + \frac{20}{N_F} - 16N_F)$ $+16L_6^r(-1 - 3N_F + 2N_F^2) + L_8^r(-40 - \frac{40}{N_F} + 32N_F)$
e_M	$-\frac{2}{3} + \frac{1}{N_F^2} + \frac{3}{4}\frac{1}{N_F} - \frac{77}{48}N_F + \frac{19}{16}N_F^2$
f_M	$r_{MT}^r + 64(2N_F L_4^r + L_5^r)(2N_F L_4^r + L_5^r - 4N_F L_6^r - 2L_8^r)$
g_M	$-2L_0^r(1 + \frac{1}{N_F} - 2N_F) + 4L_1^r - 2N_F L_2^r(1 - 2N_F) - 2L_3^r(1 + \frac{1}{N_F} - N_F)$ $-8(1 + N_F)(L_4^r - 2L_6^r) - 4(1 + \frac{1}{N_F})(L_5^r - 2L_8^r)$
h_M	$-\frac{1}{16} + \frac{3}{16}\frac{1}{N_F^2} + \frac{3}{16}\frac{1}{N_F} - \frac{193}{384}N_F + \frac{169}{384}N_F^2$

17

Table 3: The coefficients $a_M, \ldots, g_M$ appearing in the expansion of the mass.

Vacuum Condensate



The set of diagrams contributing to the 1PI quantities.

$\circ$: a p^2 insertion of $\overline{q}q$,
$\otimes$: a p^4 insertion of $\overline{q}q$,
$\odot$: a p^6 insertion of $\overline{q}q$,
$\bullet$: a p^2 vertex,
$\times$: a p^4 vertex.

The vacuum condensate $\overline{q}q$ up to order of p^6.

$$\langle\overline{q}q\rangle = \langle\overline{q}q\rangle_{\mathrm{LO}} + \langle\overline{q}q\rangle_{\mathrm{NLO}} + \langle\overline{q}q\rangle_{\mathrm{NNLO}}$$

$$\langle\overline{q}q\rangle_{\mathrm{LO}} \equiv \sum_{i=1,N_F} \langle\overline{q}_{Ri}q_{Li} + \overline{q}_{Li}q_{Ri}\rangle_{\mathrm{LO}} = -N_F B_0 F^2$$

$$\langle\overline{q}q\rangle_{\mathrm{NLO}} = \langle\overline{q}q\rangle_{\mathrm{LO}}\left(a_V\frac{\overline{A}(M^2)}{F^2} + b_V\frac{M^2}{F^2}\right)$$

$$\langle\overline{q}q\rangle_{\mathrm{NNLO}} = \langle\overline{q}q\rangle_{\mathrm{LO}}\left[c_V\frac{\overline{A}(M^2)^2}{F^4} + \frac{M^2\overline{A}(M^2)}{F^4}\left(d_V + \frac{e_V}{16\pi^2}\right) + \frac{M^4}{F^4}\left(f_V + \frac{g_V}{16\pi^2}\right)\right]$$

Results

	QCD
a_M	$-\frac{1}{N_F}$
b_M	$8N_F\,(2L_6^r - L_4^r) + 8\,(2L_8^r - L_5^r)$
c_M	$-\frac{1}{2} + \frac{9}{2N_F^2} + \frac{3}{8}N_F^2$
d_M	$8L_0^r(-\frac{3}{N_F} + N_F) + 8L_1^r(-1 + 2N_F^2) + 4L_2^r(4 + N_F^2) + L_3^r(-\frac{24}{N_F} + 20N_F)$
	$+L_4^r(40 - 16N_F^2) + L_5^r(\frac{40}{N_F} - 16N_F) + L_6^r(-16 + 16N_F^2) + L_8^r(-\frac{80}{N_F} + 32N_F)$
e_M	$-\frac{5}{3} + \frac{4}{N_F^2} + \frac{19}{16}N_F^2$
f_M	$-32K_{17}^r - 16K_{19}^r - 16K_{23}^r + 48K_{25}^r + 32K_{39}^r$
	$+N_F\,(-32K_{18}^r - 16K_{20}^r - 16K_{21}^r + 48K_{26}^r + 32K_{40}^r)$
	$+N_F^2\,(-16K_{22}^r + 48K_{27}^r) + 64(N_FL_4^r + L_5^r)(N_FL_4^r + L_5^r - 2N_FL_6^r - 2L_8^r)$
g_M	$-\frac{4}{N_F}(L_0^r + L_3^r) + 4L_1^r + 2N_F(2L_0^r + L_3^r) + 2N_F^2L_2^r$
	$-8[L_4^r - 2L_6^r + \frac{1}{N_F}(L_5^r - 2L_8^r)]$
h_M	$-\frac{1}{4} + \frac{3}{4}\frac{1}{N_F^2} + \frac{169}{384}N_F^2$
	Adjoint
a_M	$\frac{1}{2} - \frac{1}{2N_F}$
b_M	$16N_F(2L_6^r - L_4^r) + 8(2L_8^r - L_5^r)$
c_M	$\frac{3}{8}\left(1 + \frac{3}{N_F^2} - \frac{4}{N_F} + N_F + N_F^2\right)$
d_M	$L_0^r(12 - 12\frac{1}{N_F} + 8N_F) + 8L_1^r(-1 + 2N_F + 4N_F^2)$
	$+4L_2^r(4 + N_F + 2N_F^2) + L_3^r(12 - \frac{12}{N_F} + 20N_F)$
	$+L_4^r(40 - 40N_F - 32N_F^2) + L_5^r(-20 + \frac{20}{N_F} - 16N_F)$
	$+16L_6^r(-1 + 3N_F + 2N_F^2) + L_8^r(40 - \frac{40}{N_F} + 32N_F)$
e_M	$-\frac{2}{3} + \frac{1}{N_F^2} - \frac{3}{4}\frac{1}{N_F} + \frac{77}{48}N_F + \frac{19}{16}N_F^2$
f_M	$r_{MA}^r + 64(2N_FL_4^r + L_5^r)(2N_FL_4^r + L_5^r - 4N_FL_6^r - 2L_8^r)$
g_M	$2L_0^r(1 - \frac{1}{N_F} + 2N_F) + 4L_1^r + 2N_FL_2^r(1 + 2N_F) + 2L_3^r(1 - \frac{1}{N_F} + N_F)$
	$-8(1 - N_F)(L_4^r - 2L_6^r) + 4(1 - \frac{1}{N_F})(L_5^r - 2L_8^r)$
h_M	$-\frac{1}{16} + \frac{3}{16}\frac{1}{N_F^2} - \frac{3}{16}\frac{1}{N_F} + \frac{193}{384}N_F + \frac{169}{384}N_F^2$
	2-colour
a_M	$-\frac{1}{2} - \frac{1}{2N_F}$
b_M	$16N_F(2L_6^r - L_4^r) + 8(2L_8^r - L_5^r)$
c_M	$\frac{3}{8}\left(1 + \frac{3}{N_F^2} + \frac{4}{N_F} - N_F + N_F^2\right)$
d_M	$L_0^r(-12 - 12\frac{1}{N_F} + 8N_F) + 8L_1^r(-1 - 2N_F + 4N_F^2)$
	$+4L_2^r(4 - N_F + 2N_F^2) + L_3^r(-12 - \frac{12}{N_F} + 20N_F)$
	$+L_4^r(40 + 40N_F - 32N_F^2) + L_5^r(20 + \frac{20}{N_F} - 16N_F)$
	$+16L_6^r(-1 - 3N_F + 2N_F^2) + L_8^r(-40 - \frac{40}{N_F} + 32N_F)$
e_M	$-\frac{2}{3} + \frac{1}{N_F^2} + \frac{3}{4}\frac{1}{N_F} - \frac{77}{48}N_F + \frac{19}{16}N_F^2$
f_M	$r_{MT}^r + 64(2N_FL_4^r + L_5^r)(2N_FL_4^r + L_5^r - 4N_FL_6^r - 2L_8^r)$
g_M	$-2L_0^r(1 + \frac{1}{N_F} - 2N_F) + 4L_1^r - 2N_FL_2^r(1 - 2N_F) - 2L_3^r(1 + \frac{1}{N_F} - N_F)$
	$-8(1 + N_F)(L_4^r - 2L_6^r) - 4(1 + \frac{1}{N_F})(L_5^r - 2L_8^r)$
h_M	$-\frac{1}{16} + \frac{3}{16}\frac{1}{N_F^2} + \frac{3}{16}\frac{1}{N_F} - \frac{193}{384}N_F + \frac{169}{384}N_F^2$

17

Table 3: The coefficients $a_M, \ldots, g_M$ appearing in the expansion of the mass.

Definition of Decay Constant and Wave Function Renormalization

- Decay constant F_π:

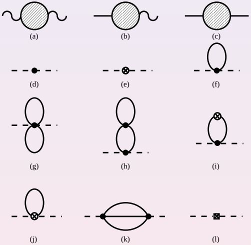
$$\begin{aligned} \langle 0|A^a_\mu|\pi(p)\rangle &= i\sqrt{2}F_\pi p_\mu \\ F_\pi &= F_0 + \Delta^{(4)}F_\pi + \Delta^{(6)}F_\pi \end{aligned}$$

- Field Renormalization

$$i\Delta(p) \approx \frac{iZ_\Phi}{p^2 - m^2}, \qquad Z_\Phi = \frac{1}{1-\Sigma'(m^2)}$$

The renormalized fields: $\Phi_R = (\sqrt{Z_\Phi})^{-1} \cdot \Phi_0$

The feynman diagrams and correction of



The set of diagrams contributing to the 1PI quantities.

$\odot$: vertex of order p^2.
$\otimes$: vertex of order p^4.
$\boxtimes$: vertex of order p^6.

The decay constant F_π up to order of p^6

$$F_{\text{phys}} = F_{\text{LO}} + F_{\text{NLO}} + F_{\text{NNLO}}$$

$$F_{\text{NLO}} = F\left(a_F\frac{\overline{A}(M^2)}{F^2} + b_F\frac{M^2}{F^2}\right)$$

$$F_{\text{NNLO}} = F\left[c_F\frac{\overline{A}(M^2)^2}{F^4} + \frac{M^2\overline{A}(M^2)}{F^4}\left(d_F + \frac{e_F}{16\pi^2}\right) + \frac{M^4}{F^4}\left(f_F + \frac{g_F}{16\pi^2} + \frac{h_F}{(16\pi^2)^2}\right)\right]$$

Results

	QCD
a_M	$-\frac{1}{N_F}$
b_M	$8N_F\,(2L_6^r - L_4^r) + 8\,(2L_8^r - L_5^r)$
c_M	$-\frac{1}{2} + \frac{9}{2N_F^2} + \frac{3}{8}N_F^2$
d_M	$8L_0^r(-\frac{3}{N_F} + N_F) + 8L_1^r(-1 + 2N_F^2) + 4L_2^r(4 + N_F^2) + L_3^r(-\frac{24}{N_F} + 20N_F)$
	$+L_4^r(40 - 16N_F^2) + L_5^r(\frac{40}{N_F} - 16N_F) + L_6^r(-16 + 16N_F^2) + L_8^r(-\frac{80}{N_F} + 32N_F)$
e_M	$-\frac{5}{3} + \frac{4}{N_F^2} + \frac{19}{16}N_F^2$
f_M	$-32K_{17}^r - 16K_{19}^r - 16K_{23}^r + 48K_{25}^r + 32K_{39}^r$
	$+N_F\,(-32K_{18}^r - 16K_{20}^r - 16K_{21}^r + 48K_{26}^r + 32K_{40}^r)$
	$+N_F^2\,(-16K_{22}^r + 48K_{27}^r) + 64(N_F L_4^r + L_5^r)(N_F L_4^r + L_5^r - 2N_F L_6^r - 2L_8^r)$
g_M	$-\frac{4}{N_F}(L_0^r + L_3^r) + 4L_1^r + 2N_F(2L_0^r + L_3^r) + 2N_F^2 L_2^r$
	$-8[L_4^r - 2L_6^r + \frac{1}{N_F}(L_5^r - 2L_8^r)]$
h_M	$-\frac{1}{4} + \frac{3}{4}\frac{1}{N_F^2} + \frac{169}{384}N_F^2$
	Adjoint
a_M	$\frac{1}{2} - \frac{1}{2N_F}$
b_M	$16N_F(2L_6^r - L_4^r) + 8(2L_8^r - L_5^r)$
c_M	$\frac{3}{8}\left(1 + \frac{3}{N_F^2} - \frac{4}{N_F} + N_F + N_F^2\right)$
d_M	$L_0^r(12 - 12\frac{1}{N_F} + 8N_F) + 8L_1^r(-1 + 2N_F + 4N_F^2)$
	$+4L_2^r(4 + N_F + 2N_F^2) + L_3^r(12 - \frac{12}{N_F} + 20N_F)$
	$+L_4^r(40 - 40N_F - 32N_F^2) + L_5^r(-20 + \frac{20}{N_F} - 16N_F)$
	$+16L_6^r(-1 + 3N_F + 2N_F^2) + L_8^r(40 - \frac{40}{N_F} + 32N_F)$
e_M	$-\frac{2}{3} + \frac{1}{N_F^2} - \frac{3}{4}\frac{1}{N_F} + \frac{77}{48}N_F + \frac{19}{16}N_F^2$
f_M	$r_{MA}^r + 64(2N_F L_4^r + L_5^r)(2N_F L_4^r + L_5^r - 4N_F L_6^r - 2L_8^r)$
g_M	$2L_0^r(1 - \frac{1}{N_F} + 2N_F) + 4L_1^r + 2N_F L_2^r(1 + 2N_F) + 2L_3^r(1 - \frac{1}{N_F} + N_F)$
	$-8(1 - N_F)(L_4^r - 2L_6^r) + 4(1 - \frac{1}{N_F})(L_5^r - 2L_8^r)$
h_M	$-\frac{1}{16} + \frac{3}{16}\frac{1}{N_F^2} - \frac{3}{16}\frac{1}{N_F} + \frac{193}{384}N_F + \frac{169}{384}N_F^2$
	2-colour
a_M	$-\frac{1}{2} - \frac{1}{2N_F}$
b_M	$16N_F(2L_6^r - L_4^r) + 8(2L_8^r - L_5^r)$
c_M	$\frac{3}{8}\left(1 + \frac{3}{N_F^2} + \frac{4}{N_F} - N_F + N_F^2\right)$
d_M	$L_0^r(-12 - 12\frac{1}{N_F} + 8N_F) + 8L_1^r(-1 - 2N_F + 4N_F^2)$
	$+4L_2^r(4 - N_F + 2N_F^2) + L_3^r(-12 - \frac{12}{N_F} + 20N_F)$
	$+L_4^r(40 + 40N_F - 32N_F^2) + L_5^r(20 + \frac{20}{N_F} - 16N_F)$
	$+16L_6^r(-1 - 3N_F + 2N_F^2) + L_8^r(-40 - \frac{40}{N_F} + 32N_F)$
e_M	$-\frac{2}{3} + \frac{1}{N_F^2} + \frac{3}{4}\frac{1}{N_F} - \frac{77}{48}N_F + \frac{19}{16}N_F^2$
f_M	$r_{MT}^r + 64(2N_F L_4^r + L_5^r)(2N_F L_4^r + L_5^r - 4N_F L_6^r - 2L_8^r)$
g_M	$-2L_0^r(1 + \frac{1}{N_F} - 2N_F) + 4L_1^r - 2N_F L_2^r(1 - 2N_F) - 2L_3^r(1 + \frac{1}{N_F} - N_F)$
	$-8(1 + N_F)(L_4^r - 2L_6^r) - 4(1 + \frac{1}{N_F})(L_5^r - 2L_8^r)$
h_M	$-\frac{1}{16} + \frac{3}{16}\frac{1}{N_F^2} + \frac{3}{16}\frac{1}{N_F} - \frac{193}{384}N_F + \frac{169}{384}N_F^2$

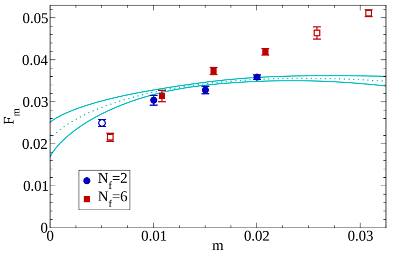
17

Table 3: The coefficients $a_M, \ldots, g_M$ appearing in the expansion of the mass.

A Lattice Calculation

- Toward TeV Conformality, Thomas Appelquist et al. arXiv:0910.2224

QCD with flavor number varies from $N_f = 2 \longrightarrow 6$, only used NLO χPT results.



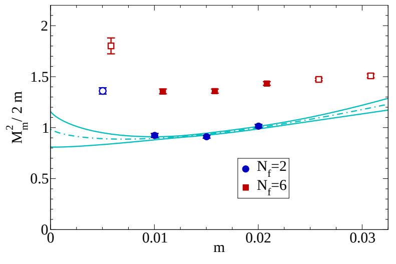


Figure: The Goldstone-boson decay constant F_m and the slope of the pseudoscalar mass squared $M_m^2/2m$ in lattice units, as a function of fermion mass. The fit for $N_f = 2$ is from a joint fit to M_m^2, F_m and $\langle\overline{\psi}\psi\rangle_m$ using the (solid) points at $m_f = 0.01 - 0.02$, constrained to match NLO χPT.

Part IV

Summary

Summary

- We have found the Effective Field Theory of QCD, Adjoint QCD and 2-color QCD can be written in an extremely similar way.
- We have calculated the one loop divergence coefficient Γ_i for Adjoint QCD and 2-color QCD.
- We have calculated the m_π, F_π and $\langle\bar{q}q\rangle$ in the $SU(N_f)$ Chiral Perturbation Theory and other two QCD-like Theories for equal mass case up to order of p^6.
- All the three theories maybe useful for the study of Technicolor Theory and Finite Baryon Density, especially for people who are working in Lattice.