

# J/ $\Psi$ radiative transitions to $\eta_c$

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based on

*Nora Brambilla, Yu Jia and Antonio Vairo*  
*Model-independent study of magnetic dipole transitions in quarkonium*  
*PRD 73 054005 (2006) [arXiv:hep-ph/0512369]*

*Nora Brambilla, Pablo **Roig** and Antonio Vairo*  
*(Work in progress)*

# Outline

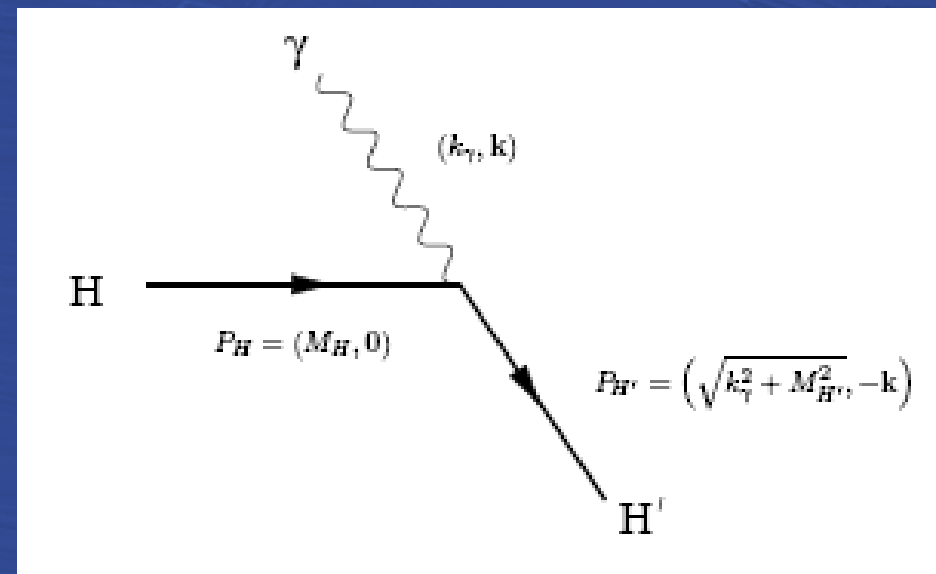
- Radiative transitions: Basics
- Experimental data on  $J/\Psi \rightarrow \gamma \eta_c$ 
  - EFT framework: pNRQCD
- Lineshape in  $J/\Psi \rightarrow \gamma \eta_c$  using pNRQCD
  - Conclusions and Outlook



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Two dominant single-photon transition processes:

- i) Electric dipole transitions (E1)
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In the non-relativistic limit

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- $n = n'$  Allowed transitions
- $n \neq n'$  Hindered transitions

# Experimental data on $J/\Psi \rightarrow \gamma \eta_c$

Only one direct experimental measurement existed for long time:

$$\Gamma(J/\Psi \rightarrow \gamma \eta_c) = (1.14 \pm 0.23) \text{ keV} \quad (\text{Crystal Ball '86})$$

There were also several measurements of the  $\text{BR}(J/\Psi \rightarrow \gamma \eta_c \rightarrow \gamma \phi \phi)$  and one independent measurement of  $\text{BR}(\eta_c \rightarrow \phi \phi)$  (Belle '03)

From them, one obtained  $\Gamma(J/\Psi \rightarrow \gamma \eta_c) = (2.9 \pm 1.5) \text{ keV}$

Recently, CLEO found  $\Gamma(J/\Psi \rightarrow \gamma \eta_c) = (1.85 \pm 0.08 \pm 0.28) \text{ keV}$  (CLEO '08)

The combination of these independent measurements leads to

$$\Gamma(J/\Psi \rightarrow \gamma \eta_c) = (1.44 \pm 0.18) \text{ keV} \text{ with a 13\% error}$$

$$m_{\eta_c} = 2977.3 \pm 1.3 \text{ MeV from } \Gamma(J/\Psi, \Psi(2S) \rightarrow \gamma \eta_c) \text{ vs.}$$

$$m_{\eta_c} = 2982.6 \pm 1.0 \text{ MeV from } \gamma\gamma \text{ or pp production}$$



# EFT framework: pNRQCD

- The LO NR expressions for the M1 transitions do not account well enough for the data.
- So one needs to supplement them with higher order corrections.
- EFT provide a systematic and controlled way of doing that.
- In the case of Quarkonia they are NRQCD (Caswell, Lepage '86) (Bodwin, Braaten and Lepage '95), pNRQCD (Pineda, Soto '97) (Brambilla, Pineda, Soto, Vairo '99) that exploit the hierarchy of scales the problem has.

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Scales: M

$$p \sim 1/r \sim Mv$$

$$E \sim Mv^2$$

$$\Lambda_{\text{QCD}}$$

$$k_\gamma$$



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pNRQCD already introduced by Antonio



# EFT framework: pNRQCD

$$\begin{aligned}
 \mathcal{L}_{\gamma \text{ pNRQCD}} = \int d^3r \text{Tr} \left\{ & V_A^{\text{em}} S^\dagger \mathbf{r} \cdot e e_Q \mathbf{E}^{\text{em}} S \right. \\
 & + \frac{1}{2m} V_S^{\frac{\sigma \cdot \mathbf{B}}{m}} \{S^\dagger, \boldsymbol{\sigma} \cdot e e_Q \mathbf{B}^{\text{em}}\} S \\
 & + \frac{1}{16m} V_S^{(r \cdot \nabla)^2 \frac{\sigma \cdot \mathbf{B}}{m}} \{S^\dagger, r^i r^j (\nabla^i \nabla^j \boldsymbol{\sigma} \cdot e e_Q \mathbf{B}^{\text{em}})\} S \\
 & + \frac{1}{2m} V_O^{\frac{\sigma \cdot \mathbf{B}}{m}} \{O^\dagger, \boldsymbol{\sigma} \cdot e e_Q \mathbf{B}^{\text{em}}\} O \\
 & + \frac{1}{4m^2} \frac{V_S^{\frac{\boldsymbol{\sigma} \cdot (\mathbf{r} \times \mathbf{r} \times \mathbf{B})}{m^2}}}{r} \{S^\dagger, \boldsymbol{\sigma} \cdot [\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times e e_Q \mathbf{B}^{\text{em}})]\} S \\
 & + \frac{1}{4m^2} \frac{V_S^{\frac{\sigma \cdot \mathbf{B}}{m^2}}}{r} \{S^\dagger, \boldsymbol{\sigma} \cdot e e_Q \mathbf{B}^{\text{em}}\} S \\
 & - \frac{1}{16m^2} V_S^{\frac{\boldsymbol{\sigma} \cdot \nabla \times \mathbf{E}}{m^2}} [S^\dagger, \boldsymbol{\sigma} \cdot [-i \nabla \times, e e_Q \mathbf{E}^{\text{em}}]] S \\
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 & + \frac{1}{4m^3} V_S^{\frac{\nabla_{\mathbf{r}}^2 \sigma \cdot \mathbf{B}}{m^3}} \{S^\dagger, \boldsymbol{\sigma} \cdot e e_Q \mathbf{B}^{\text{em}}\} \nabla_{\mathbf{r}}^2 S \\
 & \left. + \frac{1}{4m^3} V_S^{\frac{(\nabla_{\mathbf{r}} \cdot \boldsymbol{\sigma}) (\nabla_{\mathbf{r}} \cdot \mathbf{B})}{m^3}} \{S^\dagger, \boldsymbol{\sigma}^i e e_Q \mathbf{B}^{\text{em} j}\} \nabla_{\mathbf{r}}^i \nabla_{\mathbf{r}}^j S \right\}.
 \end{aligned}$$

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**E1** at  $O(1)$

**M1** at  $O(1)$

The matching procedure gives the coefficients  $V$  that appear at a given order in the  $v$  expansion

For the **M1** potential ( $\equiv V_1$ ):

$$V_1 = (\text{hard}) \times (\text{soft})$$

$$(\text{hard}) = c_F^{\text{em}} = 1 + 2\alpha_s(m_c)/3\pi + \dots$$

$$(\text{soft}) = 1$$

No large quarkonium anomalous magnetic moment

# EFT framework: pNRQCD

**M1** at  $O(v^2)$

$$\frac{1}{4m^2} \frac{V_S \frac{\sigma \cdot (r \times r \times B)}{m^2}}{r} \{S^\dagger, \sigma \cdot [\hat{r} \times (\hat{r} \times ee_Q B^{\text{em}})]\} S$$

$$\frac{1}{4m^2} \frac{V_S \frac{\sigma \cdot B}{m^2}}{r} \{S^\dagger, \sigma \cdot ee_Q B^{\text{em}}\} S$$

To all orders: (hard) =  $2 c_F - c_S = 1$ ; (soft) =  $r^2 V_s' / 2$

(due to reparametrization/Poincaré invariance) (Brambilla, Gromes, Vairo '03)

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( $\equiv V_2$ )

$$\frac{1}{4m^2} \frac{V_S \frac{\sigma \cdot B}{m^2}}{r} \{S^\dagger, \sigma \cdot ee_Q B^{\text{em}}\} S$$

( $\equiv V_3$ )

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$$V_2 = r^2 V_s / 2$$

$$V_3 = 0 \text{ (No scalar interaction)}$$



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( $\equiv V_4$ )

(hard) = 1 (due to reparametrization invariance) (Manohar '97)

$$V_4 = 1 + O(\alpha_s \text{ soft contributions})$$

# Lineshape in $J/\Psi \rightarrow \gamma \eta_c$ using pNRQCD

$$\Gamma_{J/\Psi \rightarrow \eta_c \gamma} = \int \frac{d^3k}{(2\pi)^3} (2\pi) \delta(E_p^{J/\Psi} - k - E_k^{\eta_c}) |\langle \gamma(k) \eta_c | \mathcal{L}_\gamma | J/\Psi \rangle|^2$$

Up to  $O(v^2)$  this transition is completely accesible to perturbation theory

$$\begin{aligned} \Gamma_{J/\psi \rightarrow \eta_c \gamma} &= \frac{16}{3} \alpha e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2} \left[ 1 + C_F \frac{\alpha_s(M_{J/\psi}/2)}{\pi} + \frac{2}{3} \frac{\langle 1S | 3V_S^{(0)} - rV_S^{(0)'} | 1S \rangle}{M_{J/\psi}} \right] \\ &= \frac{16}{3} \alpha e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2} \left[ 1 + C_F \frac{\alpha_s(M_{J/\psi}/2)}{\pi} - \frac{2}{3} (C_F \alpha_s(p_{J/\psi}))^2 \right], \end{aligned}$$

The normalization scale for  $\alpha_s$  is the charm quark mass (in the contribution inherited from the quark magnetic moment) and the typical momentum transfer (for that one coming from the Coulomb potential).

$$\alpha_s(M_{J/\psi}/2) \approx 0.35$$

$$p_{J/\psi} \approx m C_F \alpha_s(p_{J/\psi}) / 2 \approx 0.8 \text{ GeV}$$

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$$\Gamma_{J/\psi \rightarrow \gamma \eta_c} = \frac{16}{3} \alpha e_c^2 \frac{k_\gamma^3}{M_{J/\Psi}^2} \left( 1 + \frac{4}{3} \frac{\alpha_s(M_{J/\Psi}/2)}{\pi} - \frac{2}{3} \frac{\langle 1|rV_s'|1 \rangle}{M_{J/\Psi}} + 2 \frac{\langle 1|V_s|1 \rangle}{M_{J/\Psi}} \right)$$

- If  $V_s = -\frac{4}{3} \frac{\alpha_s(\mu)}{r}$ :  $-\frac{2}{3} \frac{\langle 1|rV_s'|1 \rangle}{M_{J/\Psi}} + 2 \frac{\langle 1|V_s|1 \rangle}{M_{J/\Psi}} = -\frac{32}{27} \alpha_s(\mu)^2 < 0$
- If  $V_s = \sigma r$ :  $-\frac{2}{3} \frac{\langle 1|rV_s'|1 \rangle}{M_{J/\Psi}} + 2 \frac{\langle 1|V_s|1 \rangle}{M_{J/\Psi}} = \frac{4}{3} \frac{\sigma}{M_{J/\Psi}} \langle 1|r|1 \rangle > 0$

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$$\Gamma(J/\psi \rightarrow \eta_c \gamma) = (1.5 \pm 1.0) \text{ keV}$$

Experimentally  $\Gamma(J/\Psi \rightarrow \gamma \eta_c) = (1.44 \pm 0.18) \text{ keV}$

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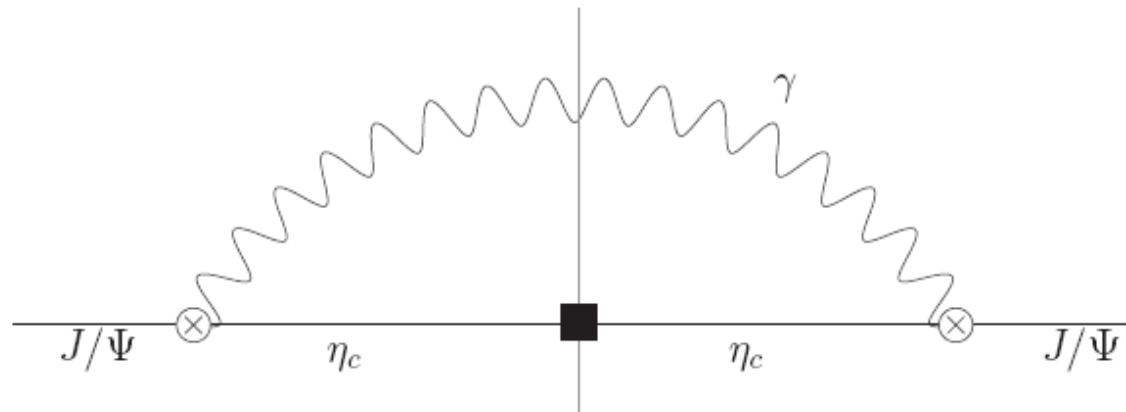
However, the theoretical (EFT) description of the decay  $J/\Psi \rightarrow \gamma \eta_c$  is not a priori what should be directly compared to experimental data

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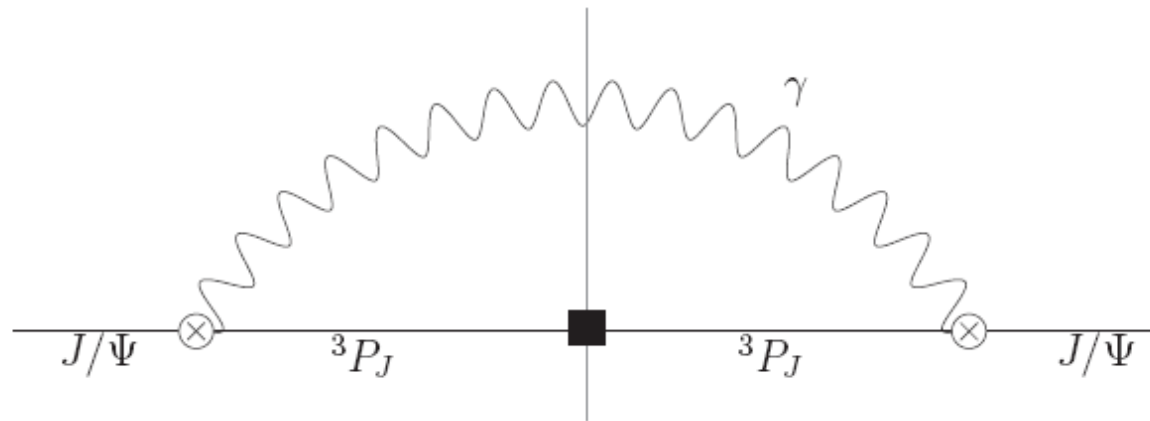


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**M1**



**E1**



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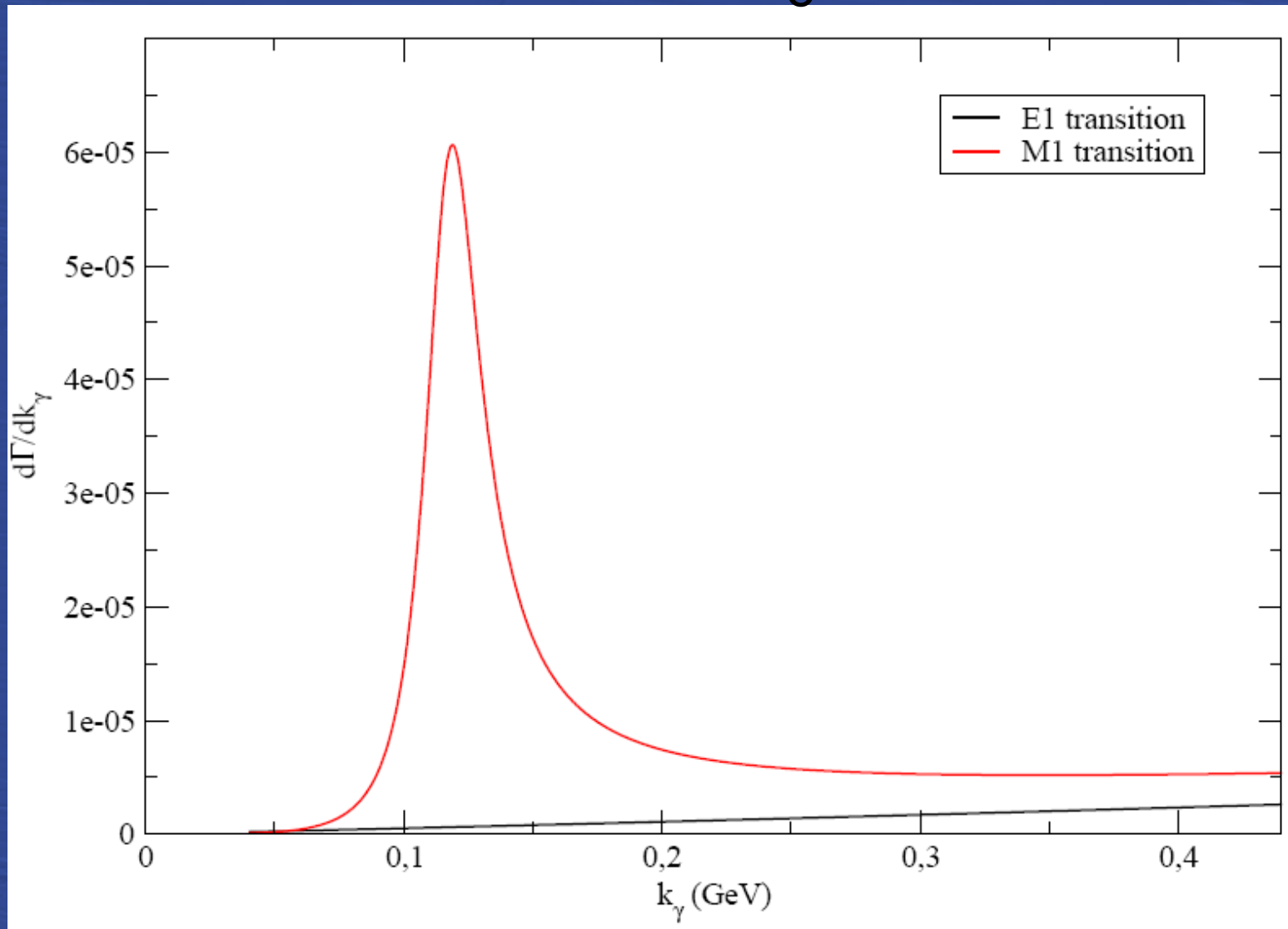
$$\frac{d\Gamma_{J/\Psi \rightarrow \eta_c \gamma}^{\text{mag}}}{dk_\gamma} = \frac{4 c_F^{em 2} \alpha e_Q^2 k_\gamma^3}{3 m^2 \pi} \frac{\frac{\Gamma_{\eta_c}}{2}}{(E_{J/\Psi} - k_\gamma - E_{\eta_c})^2 + \frac{\Gamma_{\eta_c}^2}{4}}$$

$$\frac{d\Gamma_{J/\Psi \rightarrow \eta_c \gamma}^{\text{el}}}{dk_\gamma} = \frac{8 \alpha e_Q^2}{9 \pi} k_\gamma \left| \phi_{J/\Psi}(0) \right|^2 \frac{\left| a_e(k_\gamma) \right|^2}{m^4} [C_A \Im m f_1 ({}^3P_0) + 5 C_A \Im m f_1 ({}^3P_2)]$$

The function  $a_e(k_\gamma)$  has been discussed in (Manohar, Ruiz-Femenía '03, Ruiz-Femenía '07, '09) (Voloshin '03)

We have checked the results in these papers for the orthopositronium decay spectrum in (p)NRQED.

# Lineshape in $J/\Psi \rightarrow \gamma \eta_c$ using pNRQCD





# Lineshape in $J/\Psi \rightarrow \gamma \eta_c$ using pNRQCD

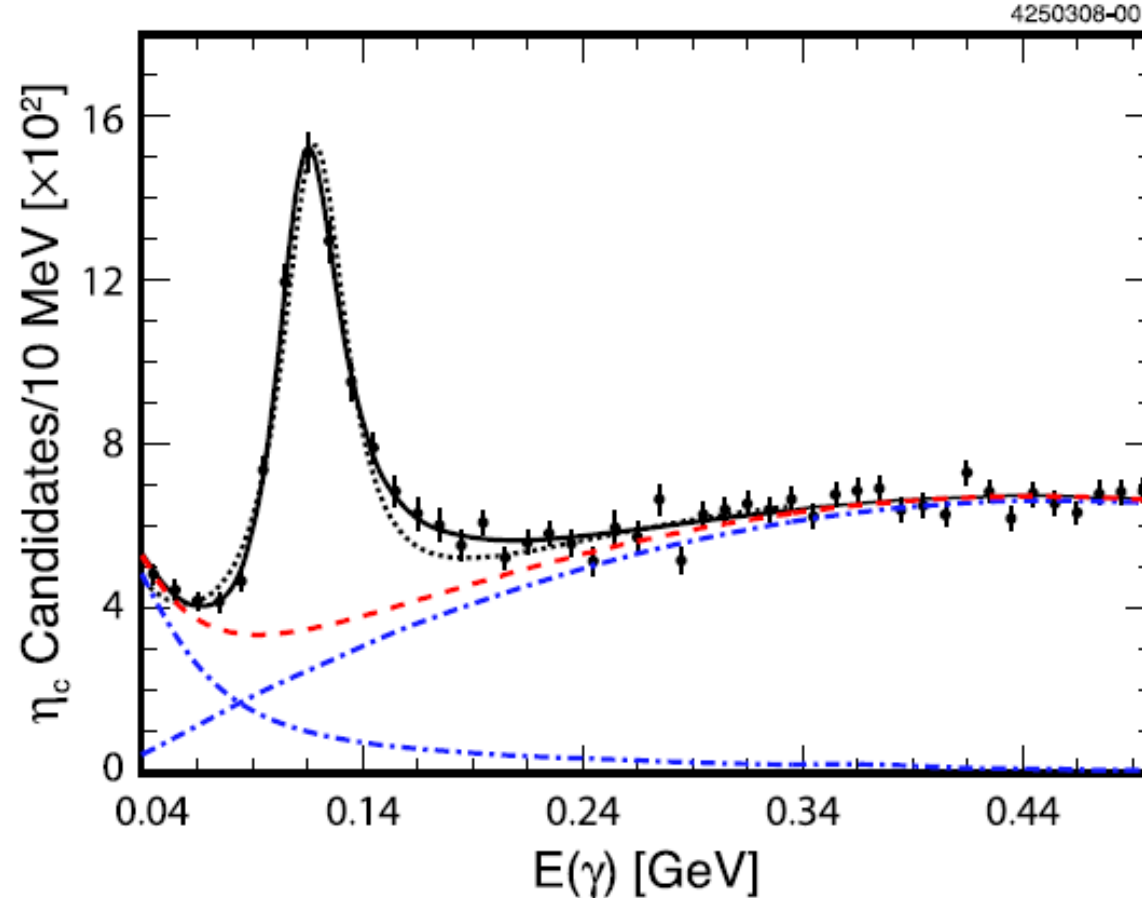


FIG. 1: Fits to the photon spectrum in exclusive  $J/\psi \rightarrow \gamma \eta_c$  decays using relativistic Breit-Wigner (dotted) and modified (solid) signal line shapes convolved with a 4.8 MeV wide resolution function. Total background is given by the dashed line. The dot-dashed curves indicate two major background components described in the text.

# Conclusions and Outlook

- Radiative decays of quarkonia are/will be subject of research in CLEO-c, BaBar, Belle, BES-III.
- One can take advantage of the hierarchy of scales the problem has and develop an EFT approach (pNRQCD) able to study them systematically.
- Within pNRQCD at  $O(v^2)$  one obtains  $\Gamma(\mathbf{J}/\Psi \rightarrow \gamma \eta_c)$  in agreement with experiment.
- A description of the lineshape of this process is currently in progress:  $\Gamma(\mathbf{J}/\Psi \rightarrow \gamma \eta_c)$  and  $m_{\eta_c}$ .
- We find that the M1 contribution overcomes completely that of E1.
- We still have to get the background subtracted data to compare our predictions to it.
- We are also considering the importance of fragmentation contributions in the low-E region.