Chiral Dynamics predictions for $\eta' \to \eta\pi\pi$

An EFT approach

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Preliminary results. Work in progress with R. Escribano and J.J Sanz Cillero

Outline

- Introduction & Motivation
- Large-N_c ChPT
- Large-N_c RChT
- Conclusion

Introduction & Motivation

η & η' Ideal for studying
- Symmetries
- Symmetry breaking in QCD

Quark masses

The Chiral Anomaly

E.M. Form Factors

Chiral invariant EFT

Introduction & Motivation

η & η' Ideal for studying
- Symmetries
- Symmetry breaking in QCD

$$\eta' \to \eta\pi\pi$$?

- Early calculation →few times less than exp. data
- Rescattering effects?
- Intermediate resonances effects? ($a_0 f_0$,σ)

Quark masses
The Chiral Anomaly
E.M. Form Factors
Chiral invariant EFT

Test EFT

$$\mathfrak{B}(\eta' \to \eta\pi^+\pi^-) = 44.6 \pm 1.4\%$$
$$\mathfrak{B}(\eta' \to \eta\pi^0\pi^0) = 20.7 \pm 1.2\%$$

PDG'09 (VES)
20.000 events

Chiral framework for η'

What EFT?

- η

Chiral framework for η'

What EFT?

- η

$m_\eta \simeq 550\mathrm{MeV}$

$\Downarrow$

Chiral Perturbation Theory (ChPT)

Gasser & Leutwyler '85

$p^2 \quad m_P^2$

π, K, η

8 Goldstone bosons

$SU(3)_R \times SU(3)_L \to SU(3)_V$

(SχSB)

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(SχSB)

However

• η'

$m_{\eta'} = 957.78 \pm 0.06\text{MeV}$

because

$U(1)_A$

~~ChPT~~

Chiral framework for η'

What EFT?

• η

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~~ChPT~~

't Hooft '74

But @ $N_c \to \infty$ $\cancel{U(1)_A}$

Chiral framework for η'

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~~ChPT~~

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But @ $N_c \to \infty$ $\cancel{U(1)_A}$

$U(3)_L \times U(3)_R$

Large-N_c ChPT

$p^2 \; m_P^2 \; 1/N_c$

π, K, η, η'

Kaiser & Leutwyler '94

Studying η'→ηππ

From Large-N_c
OZI
allowed-suppressed
components

$$\mathcal{M}_{\eta'\to\eta\pi\pi} = c_{qq}\,\mathcal{M}_{\eta_q\eta_q\pi\pi} + c_{sq}\,\mathcal{M}_{\eta_s\eta_q\pi\pi} + c_{ss}\,\mathcal{M}_{\eta_s\eta_s\pi\pi}$$

Studying η'→ηππ

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mixing

dynamics

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mixing

dynamics

$$c_{qq} = \frac{F^2}{3F_1^2F_8^2\cos^2(\theta_8-\theta_1)}\left[F_1^2\sin(2\theta_1) - F_8^2\sin(2\theta_8) + 2\sqrt{2}F_1F_8\cos(\theta_1+\theta_8)\right]$$

$$c_{sq} = \frac{F^2}{3F_1^2F_8^2\cos^2(\theta_8-\theta_1)}\left[\sqrt{2}F_1^2\sin(2\theta_1) + \sqrt{2}F_8^2\sin(2\theta_8) + F_1F_8\cos(\theta_1+\theta_8)\right]$$

$$c_{ss} = \frac{F^2}{3F_1^2F_8^2\cos^2(\theta_8-\theta_1)}\left[2F_1^2\sin(2\theta_1) - F_8^2\sin(2\theta_8) - 2\sqrt{2}F_1F_8\cos(\theta_1+\theta_8)\right]$$

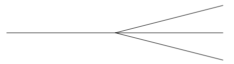
$$\begin{cases} F_1 = 1.1F_\pi \\ F_8 = 1.3F_\pi \\ F_\pi = 92.2\,\mathrm{MeV} \\ \theta_1 = -5^\circ \\ \theta_8 = -20^\circ \end{cases}$$

Defining the Amplitude

$$s = (p_{\pi^+} + p_{\pi^-})^2 = (p_{\eta'} - p_\eta)^2 \qquad t = (p_{\pi^+} + p_\eta)^2 = (p_{\eta'} - p_{\pi^+})^2$$

$$s + t + u = m_{\eta'}^2 + m_\eta^2 + 2m_\pi^2 \qquad u = (p_\eta + p_{\pi^-})^2 = (p_{\eta'} - p_{\pi^-})^2$$

Large-N_c ChPT



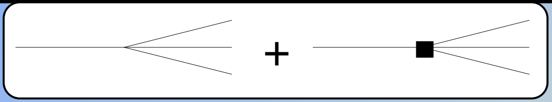
LO

$$\mathcal{M}_{\eta' \to \eta\pi^+\pi^-} = c_{qq} \times \frac{1}{F^2}\left[\frac{m_\pi^2}{2}\right]$$

Bijnens '06

	$\eta' \to \eta\pi^+\pi^-$	$\eta' \to \eta\pi^0\pi^0$
Exp (PDG09)	$44.6 \pm 1.4\%$	$20.7 \pm 1.2\%$
ChPT@LO (Bijnens '06)	0.9%	0.5%

Large-N_c ChPT



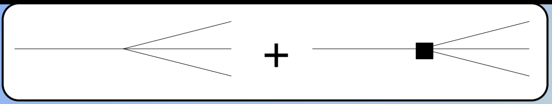
LO

NLO

$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = c_{qq}\times\frac{1}{F^2}\left[\frac{m_\pi^2}{2} - \frac{2L_5 m_\pi^2}{F^2}\left(m_{\eta'}^2+m_\eta^2+2m_\pi^2\right)+\right.$$

$$\left.+\ \frac{2(3L_2+L_3)}{F^2}\left(s^2+t^2+u^2-(m_{\eta'}^4+m_\eta^4+2m_\pi^4)\right)+\frac{24L_8 m_\pi^4}{F^2}+\frac{2\Lambda_2 m_\pi^2}{3}\right]+c_{sq}\times\frac{\sqrt{2}\Lambda_2 m_\pi^2}{3F^2}$$

Large-N_c ChPT



LO

NLO

$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = c_{qq}\times\frac{1}{F^2}\left[\frac{m_\pi^2}{2} - \frac{2L_5 m_\pi^2}{F^2}\left(m_{\eta'}^2 + m_\eta^2 + 2m_\pi^2\right) + \right.$$
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Suppressed

$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = c_{qq}\times\frac{1}{F^2}\left[\frac{m_\pi^2}{2} - \frac{2L_5 m_\pi^2}{F^2}\left(m_{\eta'}^2 + m_\eta^2 + 2m_\pi^2\right) + \right.$$
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Suppressed @ NLO

$$\mathcal{M}_{\eta_s\to\eta_s\pi^+\pi^-} = 0$$

Large-N_c ChPT

Results

$$3L_2 + L_3 = 1.1 \times 10^{-3}$$

Pich '08

$$L_5 = 2.1 \times 10^{-3}$$

$$L_8 = 0.8 \times 10^{-3}$$

Using $\Lambda_2 = 0.3$

$$BR_{\eta' \to \eta \pi^+ \pi^-} = 66.87\%$$

Large-N_c ChPT

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Using $\Lambda_2 = 0.3$

$$BR_{\eta' \to \eta \pi^+ \pi^-} = 66.87\%$$

$3L_2 + L_3 \longleftrightarrow$ dominance

+

$3L_2 + L_3 \lessgtr L_5 \longleftrightarrow$ interference

$$\left| \mathcal{M}_{\eta' \to \eta \pi \pi} \right|^2$$

Large-N_c ChPT

Results

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Pich '08

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$3L_2 + L_3 \longleftrightarrow$ dominance

+

$3L_2 + L_3 \otimes L_5 \longleftrightarrow$ interference

$$|\mathcal{M}_{\eta' \to \eta\pi\pi}|^2$$

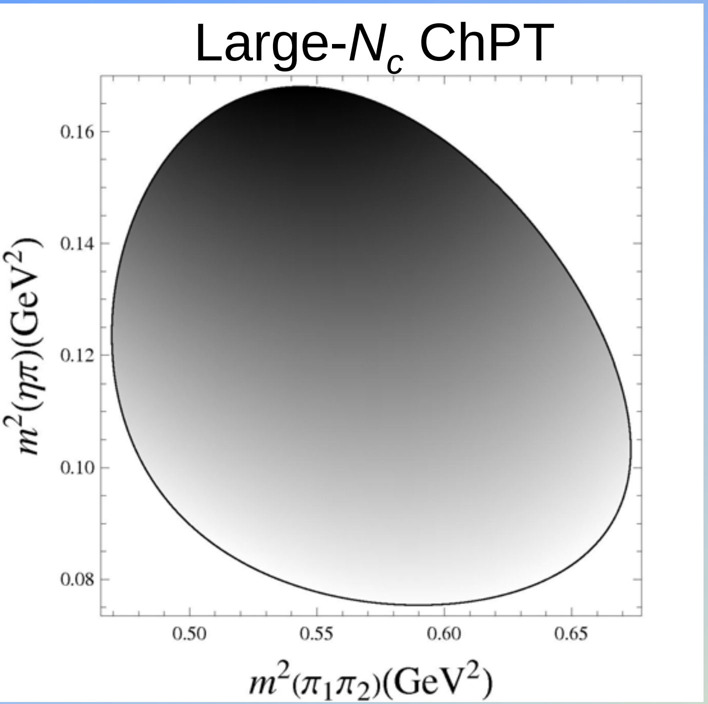
only $3L_2 + L_3 \longrightarrow BR_{\eta' \to \eta \pi^+ \pi^-} = 52.95\%$

w/o $3L_2 + L_3 \longrightarrow BR_{\eta' \to \eta \pi^+ \pi^-} = 1.20\%$

Isospin limit assumed

$$\mathfrak{B}(\eta' \to \eta \pi^+ \pi^-) = 2\, \mathfrak{B}(\eta' \to \eta \pi^0 \pi^0)$$

Large-N_c ChPT



Dalitz plot

Using

$$x \equiv \frac{1}{\sqrt{3}}\frac{(T_1 - T_2)}{\langle T \rangle} \qquad T_1 = \frac{u-(m_{\eta'}-m_\pi)^2}{2m_{\eta'}} \quad T_2 = \frac{t-(m_{\eta'}-m_\pi)^2}{2m_{\eta'}} \quad T_3 = \frac{s-(m_{\eta'}-m_\eta)^2}{2m_{\eta'}}$$

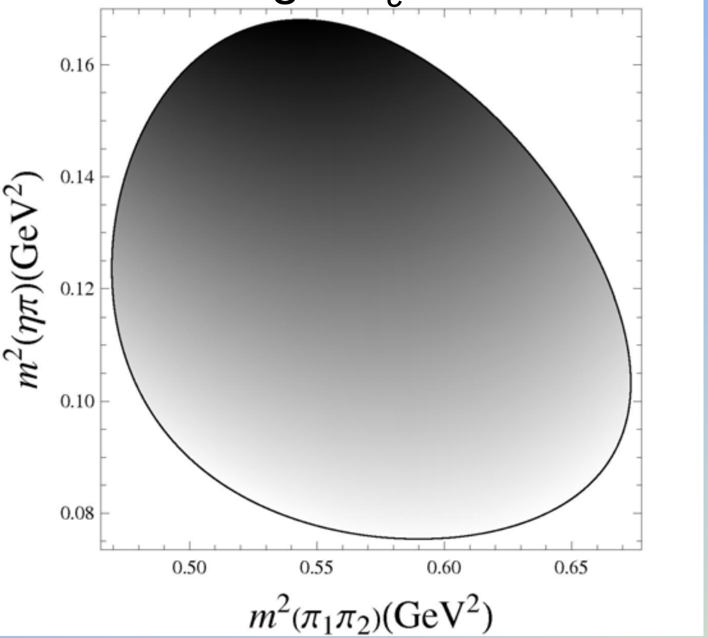
$$\langle T \rangle = \tfrac{1}{3}(T_1+T_2+T_3) = \tfrac{1}{3}(2m_\pi + m_\eta - m_{\eta'})$$

$$y \equiv \frac{1}{3}\left(2+\frac{m_\eta}{m_\pi}\right)\frac{T_3}{\langle T \rangle} - 1$$

$$\boxed{|\mathcal{M}|^2 = |N|^2(1 + ay + by^2 + \cancelto{0}{cx} + dx^2)}$$ PDG

c=0, if Charge Parity conservation holds

Large-N_c ChPT



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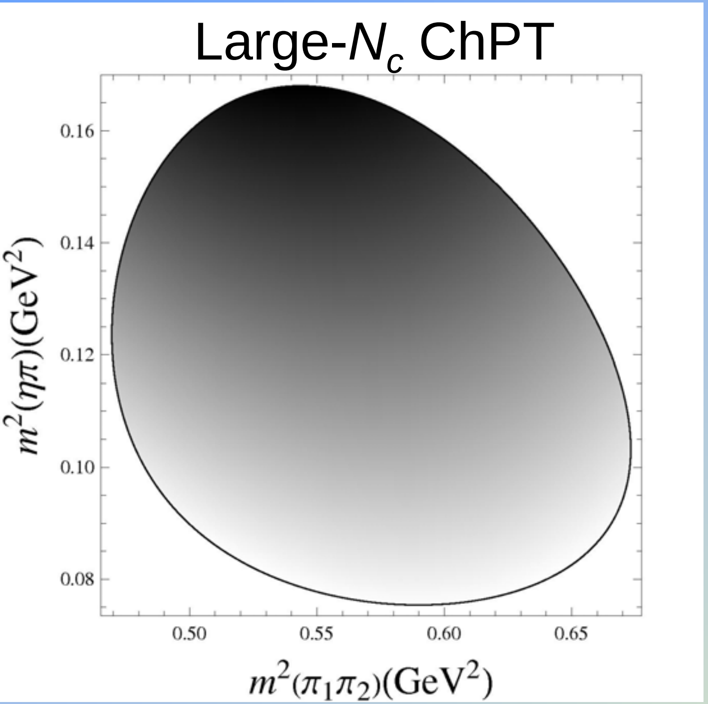
PDG

$$\boxed{a = -0.281} \qquad \boxed{d = -0.083}$$

Large-N_c ChPT



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$$\boxed{|\mathcal{M}|^2 = |N|^2(1 + ay + by^2 + dx^2)}$$

PDG

$$\boxed{a = -0.281} \qquad \boxed{d = -0.083}$$

however $\boxed{b = -1.0 \cdot 10^{-3}}$

Same order
Subleading parameters

Large-N_c ChPT

Dalitz plot parameters

$$\cancel{|\mathcal{M}|^2 = |N|^2(1 + ay + by^2 + dx^2)}$$

VES ('07)

$$a_{\text{exp}} = -0.127(16)(8)$$
$$d_{\text{exp}} = -0.082(17)(8)$$
$$b_{\text{exp}} = -0.106(28)(14)$$

$$|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) + (by^2 + \kappa_{21} y x^2 + \kappa_{40} x^4)]$$

$\boxed{a = -0.281}$ $\boxed{d = -0.083}$ $\boxed{b = -1.0 \cdot 10^{-3}}$ $\boxed{\kappa_{21} = 11.6 \cdot 10^{-3}}$ $\boxed{\kappa_{40} = 1.7 \cdot 10^{-3}}$

Relations (m_p^2)

$$a/d = 3.4$$
$$\kappa_{40}/\kappa_{21} = 0.15$$
$$b/a > 0$$

$$x \equiv \frac{1}{\sqrt{3}} \frac{(T_1 - T_2)}{\langle T \rangle} \qquad T_1 = \frac{u-(m_{\eta'}-m_\pi)^2}{2m_{\eta'}} \quad T_2 = \frac{t-(m_{\eta'}-m_\pi)^2}{2m_{\eta'}} \quad T_3 = \frac{s-(m_{\eta'}-m_\eta)^2}{2m_{\eta'}}$$

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Large-N_c ChPT

Monte Carlo Simulation
Preliminary

VES, **PLB**651 ('07)
~20.000 events

with Large-N_c ChPT

$$|\mathcal{M}|^2 = |N|^2(1 + ay + by^2 + dx^2)$$

$\frac{k_i}{c_i}$: 1.05 ± 0.05 (a), -31 ± 28 (b), 1.32 ± 0.20 (d)

$$|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) +$$

$\frac{k_i}{c_i} \longrightarrow$ 1.19 ± 0.07 (a), 0.94 ± 0.60 (d)

$$+ (by^2 + \kappa_{21} yx^2 + \kappa_{40} x^4)]$$

-14 ± 22 (b), 13.9 ± 4.3 (κ_{21}), -31 ± 22 (κ_{40})

Large-N_c ChPT

Monte Carlo Simulation
Preliminary

VES, **PLB**651 ('07)
~20.000 events

Future plans (2010?):

- Crystal Ball at MAMI-C
- Crystal Barrel at ELSA
- KLOE2 at DAPHNE
- WASA at COSY

~10^6 events

with Large-N_c ChPT

$$|\mathcal{M}|^2 = |N|^2(1 + ay + by^2 + dx^2)$$

$\frac{k_i}{c_i}$ 1.05 ± 0.05 -31 ± 28 1.32 ± 0.20

$$|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) +$$

$\frac{k_i}{c_i} \longrightarrow$ 1.19 ± 0.07 0.94 ± 0.60

$$+ (by^2 + \kappa_{21} yx^2 + \kappa_{40} x^4)]$$

-14 ± 22 13.9 ± 4.3 -31 ± 22

$$|\mathcal{M}|^2 = |N|^2(1 + ay + by^2 + dx^2)$$

$\frac{k_i}{c_i}$ 0.98 ± 0.01 3.8 ± 4.3 0.97 ± 0.03

$$|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) +$$

$\frac{k_i}{c_i} \longrightarrow$ 0.97 ± 0.01 1.2 ± 0.1

$$+ (by^2 + \kappa_{21} yx^2 + \kappa_{40} x^4)]$$

2.9 ± 3.8 -0.83 ± 0.74 10.2 ± 3.7

Large-N_c ChPT

Partial Conclusion

- BR@NLO better BR@LO (68% vs 0.9%)
- BR still disagrees: 68% vs 44% (and 34% vs 21%)
- Dalitz parameters not correctly predicted:
    - $a=-0.28$ vs $a_{exp}=-0.12$
    - $d=-0.08$ vs $d_{exp}=-0.08$
    - $b=-1.0\ 10^{-3}$ vs $b_{exp}=-0.106$

VES'07

- However:
    - new parametrization including
        - $k_{21}x^2y$ & $k_{40}x^4$
    - and relations among the parameters
- Need to include NNLO:
    - local contribution $O(p^6)$
    - Final state interactions

What is more important?

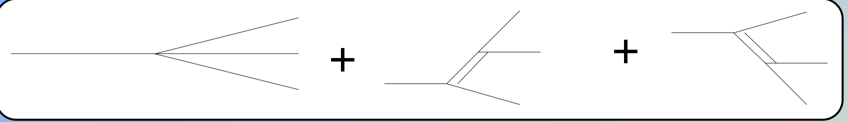
Large-N_c RChT

$$\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = \mathcal{L}^{\mathrm{GB}} + \mathcal{L}^{R_i} + \mathcal{L}^{R_iR_j} + \mathcal{L}^{R_iR_jR_k} + \ldots$$

$$\left.\begin{aligned} \mathcal{L}^{\mathrm{GB}} &= \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle \\ \mathcal{L}^{S}_{(2)} &= c_d\langle\, S u_\mu u^\mu \,\rangle + c_m\langle\, S\chi_+ \,\rangle \end{aligned}\right\} \text{Goldstone-Resonance interaction}$$

$\mathcal{L}^{S}_{(2)}$ includes vectors and scalars. We will see that in the particular channel $\eta' \to \eta\pi\pi$ only scalars contribute.

Large-N_c RChT



$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = c_{qq} \times \frac{1}{F_\pi^2}\Bigg[\frac{m_\pi^2}{2} + \frac{4c_d c_m}{F^2}\frac{m_\pi^4}{M_S^2}$$

$$+ \frac{1}{F^2}\frac{\left[c_d(t-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2\right]\left[c_d(t-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2\right]}{M_{a_0}^2 - t}$$

$$+ \frac{1}{F^2}\frac{\left[c_d(u-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2\right]\left[c_d(u-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2\right]}{M_{a_0}^2 - u}$$

$$+ \frac{1}{F^2}\frac{\left[c_d(s-m_\eta^2-m_{\eta'}^2)+2c_m^2 m_\pi^2\right]\left[c_d(s-2m_\pi^2)+2c_m m_\pi^2\right]}{M_{\sigma,f_0}^2 - s}\Bigg]$$

Large-N_c RChT

Chiral expansion at low energies

$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = c_{qq} \times \left\{ \frac{m_\pi^2}{2F^2} + \frac{12\,c_m^2}{F^4 M_S^2} m_\pi^4 - \frac{2\,c_d c_m}{F^4 M_S^2}\, m_\pi^2 (m_\eta^2 + m_{\eta'}^2 + 2m_\pi^2) \right.$$
$$\left. + \frac{c_d^2}{F^4 M_S^2} \left[s^2 + t^2 + u^2 - (m_\eta^4 + m_{\eta'}^4 + 2m_\pi^4) \right] \right\}$$

Large-N_c RChT

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$$3L_2 + L_3 = {c_d}^2/2M_S^2$$

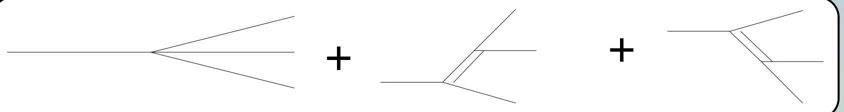
$$L_5 = c_d c_m/M_S^2$$

$$L_8 = {c_m}^2/2M_S^2$$

Subleading 1/Nc

$$\Lambda_1 = \Lambda_2 = 0$$

Large-N_c RChT



$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = c_{qq} \times \frac{1}{F_\pi^2}\Bigg[\frac{m_\pi^2}{2} + \frac{4c_d c_m}{F^2}\frac{m_\pi^4}{M_S^2}$$
$$+ \frac{1}{F^2}\frac{[c_d(t-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2][c_d(t-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2]}{M_{a_0}^2-t}$$
$$+ \frac{1}{F^2}\frac{[c_d(u-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2][c_d(u-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2]}{M_{a_0}^2-u}$$
$$+ \frac{1}{F^2}\frac{[c_d(s-m_\eta^2-m_{\eta'}^2)+2c_m^2 m_\pi^2][c_d(s-2m_\pi^2)+2c_m m_\pi^2]}{M_{\sigma,f_0}^2-s}\Bigg]$$

c_m Suppressed

c_d Dominant

Suppressed at this order

$$\mathcal{M}_{\eta_s\eta_q\pi\pi} = \mathcal{M}_{\eta_s\eta_s\pi\pi} = 0$$

Large-N_c RChT

Results

$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = c_{qq} \times \frac{1}{F_\pi^2}\Bigg[\frac{m_\pi^2}{2} + \frac{4c_d c_m}{F^2}\frac{m_\pi^4}{M_S^2}$$
$$+ \frac{1}{F^2}\frac{[c_d(t-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2]\,[c_d(t-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2]}{M_{a_0}^2-t}$$
$$+ \frac{1}{F^2}\frac{[c_d(u-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2]\,[c_d(u-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2]}{M_{a_0}^2-u}$$
$$+ \frac{1}{F^2}\frac{[c_d(s-m_\eta^2-m_{\eta'}^2)+2c_m^2 m_\pi^2]\,[c_d(s-2m_\pi^2)+2c_m m_\pi^2]}{M_{\sigma,f_0}^2-s}\Bigg]$$

Since the dominance

$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} \sim c_d^2$$

$$\downarrow$$

$$|\mathcal{M}_{\eta'\to\eta\pi^+\pi^-}|^2 \sim c_d^4$$

Large-N_c RChT

Results

$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = c_{qq} \times \frac{1}{F_\pi^2}\left[\frac{m_\pi^2}{2} + \frac{4c_d c_m}{F^2}\frac{m_\pi^4}{M_S^2}\right.$$
$$+\frac{1}{F^2}\frac{\left[c_d(t-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2\right]\left[c_d(t-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2\right]}{M_{a_0}^2-t}$$
$$+\frac{1}{F^2}\frac{\left[c_d(u-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2\right]\left[c_d(u-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2\right]}{M_{a_0}^2-u}$$
$$\left.+\frac{1}{F^2}\frac{\left[c_d(s-m_\eta^2-m_{\eta'}^2)+2c_m^2 m_\pi^2\right]\left[c_d(s-2m_\pi^2)+2c_m m_\pi^2\right]}{M_{\sigma,f_0}^2-s}\right]$$

Since the dominance

$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} \sim c_d^2$$

$$|\mathcal{M}_{\eta'\to\eta\pi^+\pi^-}|^2 \sim c_d^4$$

$$c_m = F^2/4c_d$$

Jamin, Oller, Pich ‘02

$$M_S = 980\text{MeV}$$

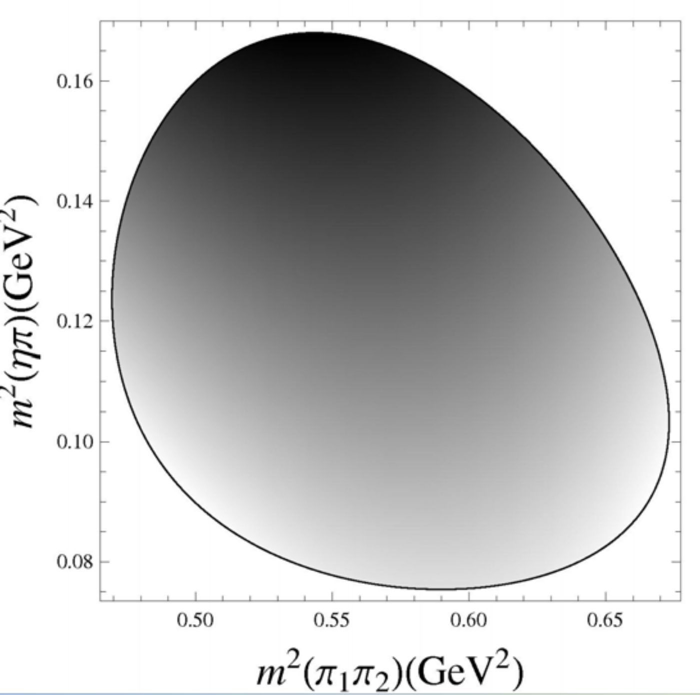
$$\text{Br}(\eta'\to\eta\pi^+\pi^-) = (44.6\pm1.4)\%$$

VES ‘07

$$\boxed{c_d = (28.9\pm0.2)\ \text{MeV}}$$

Large-N_c RChT

Dalitz plot: new parametrization



$$\boxed{c_d = (28.9 \pm 0.2)\ \mathrm{MeV}}$$

+

$$\boxed{|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) + (by^2 + \kappa_{21} y x^2 + \kappa_{40} x^4)]}$$

$$\boxed{a = -0.1166(6)} \qquad \boxed{d = -0.0539(4)}$$

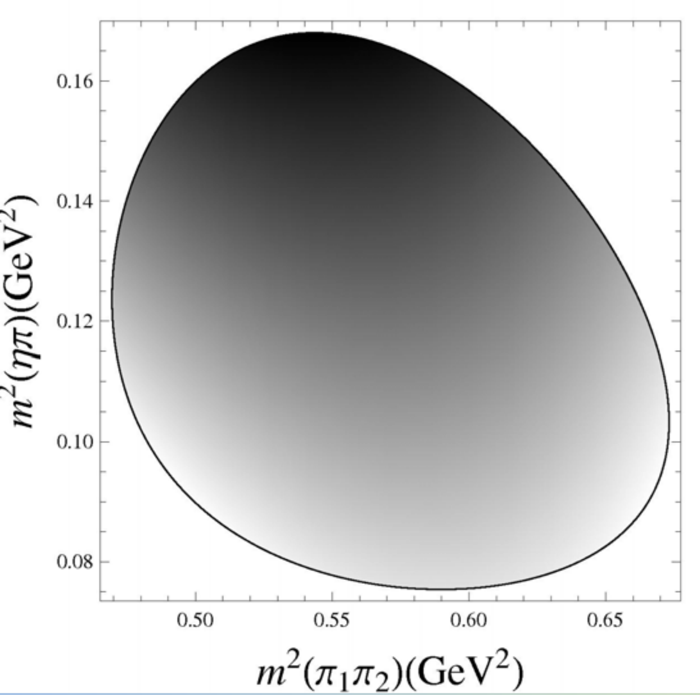
$$\boxed{a_{exp} = -0.127(16)(8)} \qquad \boxed{d_{\mathrm{exp}} = -0.082(17)(8)}$$

VES'07

Good prediction within the errors

Large-N_c RChT

Dalitz plot: new parametrization



$$\boxed{c_d = (28.9 \pm 0.2)\ \mathrm{MeV}}$$

+

$$\boxed{|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) + (by^2 + \kappa_{21} y x^2 + \kappa_{40} x^4)]}$$

$$\boxed{b = 0.666(5) \cdot 10^{-3}}$$

$$\boxed{b_{exp} = -0.106(28)(14)}$$

VES‘07

$$\kappa_{21} = -5,71(3) \cdot 10^{-3}$$
$$\kappa_{40} = -1,207(4) \cdot 10^{-3}$$

Need for new fit

Large-N_c RChT

Monte Carlo Simulation
Preliminary

VES, **PLB**651 ('07)
~20.000 events

with Large-N_c RChT

$$|\mathcal{M}|^2 = |N|^2(1 + ay + by^2 + dx^2)$$

$\frac{k_i}{c_i}$ $\quad 0.71 \pm 0.17 \quad -107 \pm 53 \quad 1.47 \pm 0.43$

$$|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) +$$

$\frac{k_i}{c_i} \longrightarrow \quad 0.72 \pm 0.26 \quad 0.2 \pm 1.7$

$$+ (by^2 + \kappa_{21} y x^2 + \kappa_{40} x^4)]$$

$-110 \pm 58 \quad -1 \pm 15 \quad 45 \pm 55$

Large-N_c RChT

Monte Carlo Simulation
Preliminary

VES, **PLB**651 ('07)
~20.000 events

Future plans (2010?):

- Crystal Ball at MAMI-C
- Crystal Barrel at ELSA
- KLOE2 at DAPHNE
- WASA at COSY

~10^6 events

with Large-N_c RChT

$$|\mathcal{M}|^2 = |N|^2(1 + ay + by^2 + dx^2)$$

$\frac{k_i}{c_i}$: a: 0.71 ± 0.17, b: -107 ± 53, d: 1.47 ± 0.43

$$|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) + + (by^2 + \kappa_{21} yx^2 + \kappa_{40} x^4)]$$

$\frac{k_i}{c_i}$: a: 0.72 ± 0.26, d: 0.2 ± 1.7, b: -110 ± 58, κ_{21}: -1 ± 15, κ_{40}: 45 ± 55

$$|\mathcal{M}|^2 = |N|^2(1 + ay + by^2 + dx^2)$$

$\frac{k_i}{c_i}$: a: 1.01 ± 0.01, b: -1.6 ± 4.3, d: 1.08 ± 0.04

$$|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) + + (by^2 + \kappa_{21} yx^2 + \kappa_{40} x^4)]$$

$\frac{k_i}{c_i}$: a: 1.00 ± 0.02, d: 1.02 ± 0.12, b: 0.3 ± 4.4, κ_{21}: 1.7 ± 1.2, κ_{40}: 0.4 ± 4.0

Large-N_c RChT

Summary of predictions

$$c_d = (28.9 \pm 0.2)\ \text{MeV}$$

+

$$|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) + (by^2 + \kappa_{21} y x^2 + \kappa_{40} x^4)]$$

$$a = -0.1166(6)$$
$$d = -0.0539(4)$$
$$b = 0.666(5) \cdot 10^{-3}$$
$$\kappa_{21} = -5,71(3) \cdot 10^{-3}$$
$$\kappa_{40} = -1,207(4) \cdot 10^{-3}$$

$$a_{\text{exp}} = -0.127(16)(8)$$
$$d_{\text{exp}} = -0.082(17)(8)$$
$$b_{\text{exp}} = -0.106(28)(14)$$

VES'07

$$a/d = 2.2$$
$$\kappa_{40}/\kappa_{21} = 0.21$$
$$b/a < 0$$

Large-N_c RChT

Summary of predictions

$$c_d = (28.9 \pm 0.2)\ \mathrm{MeV}$$ + $$|\mathcal{M}|^2 = |N|^2[1 + (ay + dx^2) + (by^2 + \kappa_{21} yx^2 + \kappa_{40} x^4)]$$

$$\begin{aligned}\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} &= c_{qq} \times \frac{1}{F_\pi^2}\left[\frac{m_\pi^2}{2} + \frac{4c_d c_m}{F^2}\frac{m_\pi^4}{M_S^2} \right.\\ &+ \frac{1}{F^2}\frac{\left[c_d(t - m_\eta^2 - m_\pi^2) + 2c_m^2 m_\pi^2\right]\left[c_d(t - m_{\eta'}^2 - m_\pi^2) + 2c_m m_\pi^2\right]}{M_{a_0}^2 - t}\\ &+ \frac{1}{F^2}\frac{\left[c_d(u - m_\eta^2 - m_\pi^2) + 2c_m^2 m_\pi^2\right]\left[c_d(u - m_{\eta'}^2 - m_\pi^2) + 2c_m m_\pi^2\right]}{M_{a_0}^2 - u}\\ &\left. + \frac{1}{F^2}\frac{\left[c_d(s - m_\eta^2 - m_{\eta'}^2) + 2c_m^2 m_\pi^2\right]\left[c_d(s - 2m_\pi^2) + 2c_m m_\pi^2\right]}{M_{\sigma,f_0}^2 - s}\right]\end{aligned}$$

a_0

$$|\mathcal{M}_{\eta'\to\eta\pi^+\pi^-}|^2$$

a_0

$+$

$a_0 \leftrightarrow \sigma, f_0$

$$a = -0.1166(6)$$
$$d = -0.0539(4)$$
$$b = 0.666(5)\cdot 10^{-3}$$
$$\kappa_{21} = -5.71(3)\cdot 10^{-3}$$
$$\kappa_{40} = -1.207(4)\cdot 10^{-3}$$

$$a_{\exp} = -0.127(16)(8)$$
$$d_{\exp} = -0.082(17)(8)$$
$$b_{\exp} = -0.106(28)(14)$$

VES'07

$$a/d = 2.2$$
$$\kappa_{40}/\kappa_{21} = 0.21$$
$$b/a < 0$$

Large-N_c RChT

Corrections

$$\mathcal{M}_{\eta'\to\eta\pi^+\pi^-} = c_{qq} \times \frac{1}{F_\pi^2}\left[\frac{m_\pi^2}{2} + \frac{4c_d c_m}{F^2}\frac{m_\pi^4}{M_S^2} + \frac{1}{F^2}\frac{[c_d(t-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2]\,[c_d(t-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2]}{M_{a_0}^2-t} + \frac{1}{F^2}\frac{[c_d(u-m_\eta^2-m_\pi^2)+2c_m^2 m_\pi^2]\,[c_d(u-m_{\eta'}^2-m_\pi^2)+2c_m m_\pi^2]}{M_{a_0}^2-u} + \frac{1}{F^2}[c_d(s-m_\eta^2-m_{\eta'}^2)+2c_m^2 m_\pi^2]\,[c_d(s-2m_\pi^2)+2c_m m_\pi^2] \times \left\{ \frac{\cos^2\phi_S}{M_\sigma^2-s} + \frac{\sin^2\phi_S}{M_{f_0}^2-s} \right\} \right]$$

Φ_S=-8º

$$\left(M_\sigma^2 - s - c_\sigma s \overline{B}_0(s,m_\pi^2,m_\pi^2)\right)^{-1}$$

$$\overline{B}_0(s,m_\pi^2,m_\pi^2) = \frac{1}{16\pi^2}\left(2-\rho(s)\ln\frac{\rho(s)+1}{\rho(s)-1}\right)$$

$$(M_\sigma^{poles} - i\Gamma_\sigma^{poles}/2)^2 = (0.445 - i0.555/2)^2$$

$$c_d = (28.9 \pm 0.2)\ \text{MeV}$$

$$c_d = (29 \pm 2)\text{MeV}$$

$$a = -0.1166(6)$$
$$d = -0.0539(4)$$
$$b = 0.666(5)\cdot 10^{-3}$$
$$\kappa_{21} = -5.71(3)\cdot 10^{-3}$$
$$\kappa_{40} = -1.207(4)\cdot 10^{-3}$$

$$a = -0.1154$$
$$d = -0.0676$$
$$b = -0.0160$$
$$\kappa_{21} = -7.40\cdot 10^{-3}$$
$$\kappa_{40} = -1.278\cdot 10^{-3}$$

$$a_{\exp} = -0.127(16)(8)$$
$$d_{\exp} = -0.082(17)(8)$$
$$b_{\exp} = -0.106(28)(14)$$

Conclusions

- Large-N_c ChPT:
  - BR@NLO→correct order of magnitude
    - Need NNLO contributions
  - New parametrization for the Dalitz plot
- Large-N_c RChT:
  - Prediction for c_d, c_m
  - Better BR
  - Good Dalitz parameters predictions
  - Indication of small ππ rescattering