# A new approach to effective field theories for few-nucleon physics

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Work in collaboration with Silas Beane (Univ. of New Hampshire) and David Kaplan (INT, Seattle); arXiv:0811.3938,...

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# Challenges in low energy nuclear physics

- Goal: Have quantitative control over interactions between a few (2-5) nucleons
  - NN and Nd scattering
  - Structure of nuclei / nuclear matter
  - Solar fusion, form factors, breakup of deuteron (radiative or neutrino), ...
- Methods: Potential models (traditional nuclear physics) vs. effective theories
  - Potential models: Fit nucleon-nucleon potential to data and apply it to *N*-body calculation
    - High precision, easy to implement — but no systematic expansion
  - EFT: Start from QCD, perform analysis of scale hierarchies in system and integrate out irrelevant dof's

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# Challenges is low energy nuclear physics

Potential models as summarized by Silas Beane:

NN phase shifts
↓

↓
NN potentials

To date, study of Nuclear Forces has relied on modeling that is disconnected from the Standard Model of particle interactions

# EFT for nuclear physics — why?

- Natural framework to bridge short and long distance physics and incorporate symmetries
- Clear hierarchy of operators: Expansion in $p/\Lambda$
  - In potential models no way to determine which observables calculable to desired accuracy
- EM / weak interactions, relativistic corrections, dynamical processes, etc. easy to incorporate
- **Numerical implementation** straightforward
  - Bridge between lattice QCD and nuclear structure
  - N body physics via lattice implementation of EFT extremely efficient (no QCD sign and signal/noise problems!)

# EFT for nuclear physics — how?

- Rules of EFT building
  - Hierarchy of energy scales: $E_{\text{interesting}} \ll m_{\text{irrelevant}}$
  - Identify symmetries and low-energy degrees of freedom
  - Devise consistent power counting scheme: Ordering of operators, estimation of errors
  - Integrate out irrelevant dof's
  - Fit parameters to data
- EFTs in nuclear physics
  - Parameters with scale: $m_N$, $m_\pi$, $a(^{2S+1}N_J)$, ...
  - Symmetries: Baryon number, Galilean, spin, isospin
  - Degrees of freedom: Nucleons, pion, ...
  - Data: Phase shifts in various channels from partial wave analysis of scattering results

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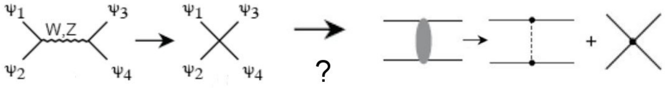
Rest of the talk: How to implement the above



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# Pionless theory — starting point

Consider NN scattering in ${}^1S_0$ channel: $S = 0$ and $L = 0$ for the two-particle system

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Assume $p \ll m_\pi$ and try to construct EFT expansion for scattering amplitude $\mathcal{A}$ with UV cutoff $\Lambda \sim m_\pi$...

...starting from the Lagrangian for non-relativistic nucleons

$$\mathcal{L} = N^\dagger \left(i\partial_t + \frac{\nabla^2}{2M}\right) N - C_0 \left(N^\dagger N\right)^2 - C_2 \left(N^\dagger \nabla^2 N\right)\left(N^\dagger N\right) + h.c. + \cdots$$

# Pionless theory — basic setup

1. Evaluate scattering amplitude diagrammatically to get

$$S = 1 + i\frac{Mp}{2\pi}\mathcal{A},$$
$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip}$$

2. Compare result to effective range expansion...

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}\Lambda^2\sum_{n=0}^{\infty} r_n\left(\frac{p^2}{\Lambda^2}\right)^{n+1}$$

   - $r_n \sim 1/\Lambda \sim$ range of potential, $a$ arbitrary

3. ...to write diagrammatic expansion of $\mathcal{A}$ in terms of the $r_n$

$$\mathcal{A} = -\frac{4\pi a}{M}\left[1 - iap + (ar_0/2 - a^2)p^2 + \mathcal{O}\left(p^3\right)\right]$$

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# Basic setup — pionless theory

- Problem: Scattering lengths anomalously large

$$a(^1S_0) = -23.7\,\text{fm} \sim (8\,\text{MeV})^{-1},$$
$$a(^3S_1) = 5.4\,\text{fm} \sim (35\,\text{MeV})^{-1}$$

$\therefore$ Nuclear EFT must be non-perturbative!!

- Resolution: Must sum powers of $(ap)$ to all orders to extend validity of EFT to $\Lambda \sim m_\pi$

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# Two approaches to pionless EFT

Weinberg's proposal

1. Compute NN potential in a derivative expansion
2. Solve Schrödinger eq. numerically
3. Match $C_{2n}$'s to phase shift data

KSW (Kaplan, Savage, Wise) power counting

1. Introduce physical subtraction scheme
2. Sum $C_0$ to all orders
3. Expand amplitude in $p/\Lambda$ and fit $a$ and $r_n$

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$\approx$ Expand theory around $a = \infty$

# KSW power counting

- Want to expand $\mathcal{A}$ in powers of $p$ while retaining $(ap)$ to all orders:

$$\mathcal{A} = -\frac{4\pi}{M}\frac{1}{(1/a+ip)}\left[1+\frac{r_0/2}{(1/a+ip)}p^2+\frac{(r_0/2)^2}{(1/a+ip)^2}p^4+\frac{(r_1/2\Lambda^2)}{(1/a+ip)}p^4+\ldots\right]$$

- How? Sum $C_0$ to all orders and expand in $C_{2n}$, $n \geq 1$!

$$\mathcal{L} \quad = \quad \cdots - C_0\left(N^\dagger N\right)^2 - C_2\left(N^\dagger \nabla^2 N\right)\left(N^\dagger N\right) + h.c. + \cdots$$

- The bubble in PDS (power divergence subtraction) scheme

$$I_0 \quad = \quad \left(\frac{\mu}{4}\right)^{4-d}\int\frac{d^{d-1}q}{(2\pi)^{d-1}}\frac{1}{E-q^2/M+i\epsilon} \rightarrow -\frac{M}{4\pi}(\mu+ip)$$

  - PDS = Subtract also power law divergences
  - Leads to $C_0 = 4\pi/(M(-\mu+1/a))$, $\mu \sim p$

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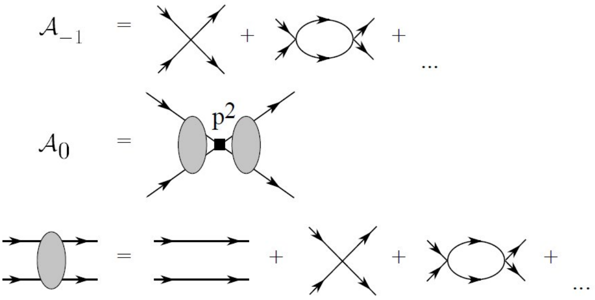
# KSW power counting

Expansion of the amplitude:



# KSW power counting

- Want to expand $\mathcal{A}$ in powers of $p$ while retaining $(ap)$ to all orders:

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- How? Sum $C_0$ to all orders and expand in $C_{2n}$, $n \geq 1$!

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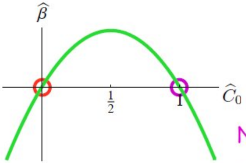
  - PDS = Subtract also power law divergences
  - Leads to $C_0 = 4\pi/(M(-\mu+1/a)),\quad \mu\sim p$

# KSW power counting

- Running of dimensionless coupling $\hat{C}_0 \equiv -\frac{M\mu}{4\pi} C_0 = \mu/(\mu + 1/a)$:

$$\mu \frac{\partial \hat{C}_0}{\partial \mu} = \hat{C}_0 \left(1 - \hat{C}_0\right)$$




Trivial IR fixed point
...like Fermi theory!
...**not** like nuclear physics!

Nontrivial UV fixed point
...very much like nuclear physics! Large a!

- KSW expansion = Expansion around non-trivial UV fixed point $\hat{C}_0 = 1$!

# Limitations of pionless EFT

Pionless EFT a'la KSW a success...

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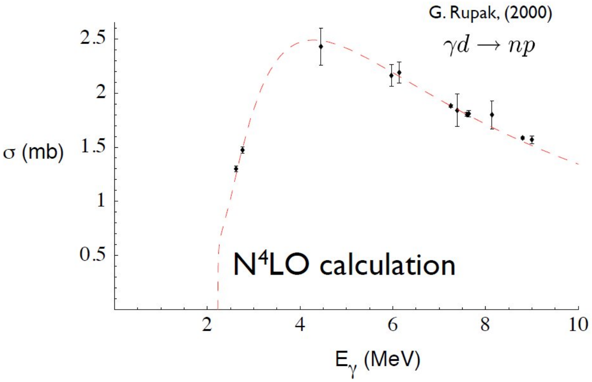
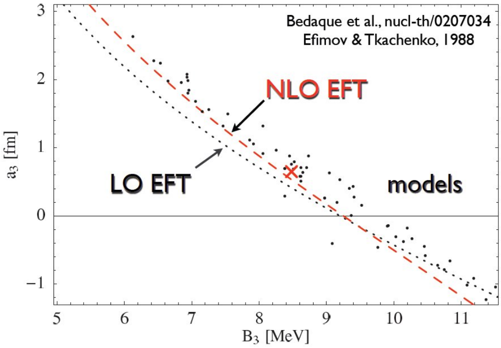


# Limitations of pionless EFT

Pionless EFT a'la KSW a success...

*Phillips line: $^3H$ binding energy - Nd scattering length correlation*



# Limitations of pionless EFT

...but clearly we want to go beyond that:

- To describe scattering for $p \gtrsim 100$ MeV
- To describe nuclei heavier than $^3$H and $^3$He

$\therefore$ Try including pions into EFT

# Pionful EFTs

Again two approaches... Weinberg and KSW

# Pionful EFTs

Weinberg's proposal:

1. Perform $\chi$ expansion of NN potential $V$
2. Solve Schrödinger equation for nucleons with $V$
3. Obtain phase shift from the above

KSW approach:

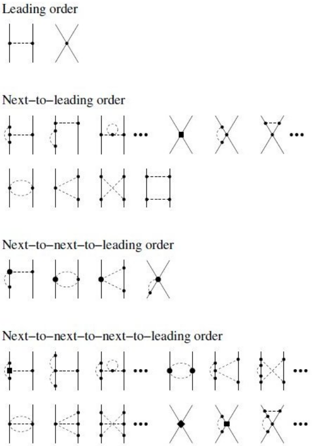
1. Start from pionless KSW counting — expand around non-trivial fixed point
2. Include one pion exchange *perturbatively* at the same order as $C_2$

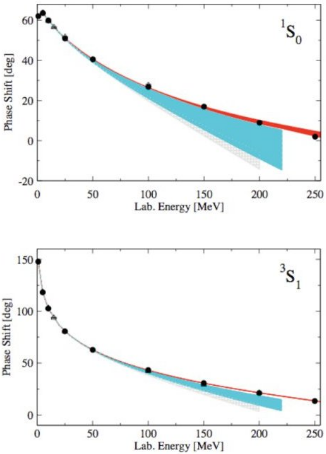
# Pionful EFT a'la Weinberg

Virtues:

- Pion exchange correctly incorporated in EFT
- Systematically improvable scheme
- Extendable to high orders and accuracies: Now at NNNLO

# Pionful EFT a'la Weinberg



# Pionful EFT a'la Weinberg

Virtues:

- Pion exchange correctly incorporated in EFT
- Systematically improvable scheme
- Extendable to high orders and accuracies: Now at NNNLO

Vices:

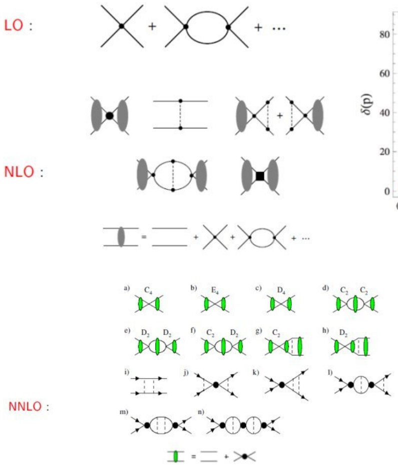
- Inconsistent power counting: Higher order counter terms needed
  - Non-renormalizable EFT: Cannot remove cutoff!
- Tensor potential singular
- Typically close to 10 free parameters per channel
- Numerical calculations show no advantage to potential models

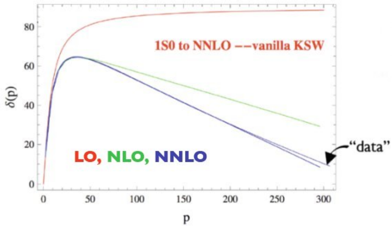
# Pionful EFT a'la KSW

Virtues:

- Consistent power counting: Renormalizable scheme
- Very few fit parameters
- Also systematically improvable: Calculations up to NNLO

# Pionful EFT a'la KSW




- Renormalized
- 2 parameter fit
- Analytic

# Pionful EFT a'la KSW

Virtues:

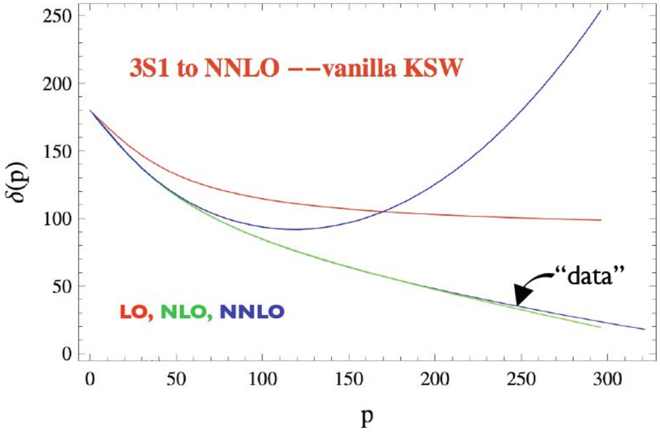
- Consistent power counting: Renormalizable scheme
- Very few fit parameters
- Also systematically improvable: Calculations up to NNLO

Vices:

- Does not converge for $^3S_1$ and $^3D_1$!
  - Reason: $1/r^3$ pion exchange potential in tensor channel
  - Singular short distance physics screws up expansion

$$
\begin{aligned}
V_C(r) &= -\frac{\alpha_\pi}{r}\, m_\pi^2 \mathrm{e}^{-m_\pi r}, \\
V_T(r) &= -\frac{\alpha_\pi}{r}\, m_\pi^2 \mathrm{e}^{-m_\pi r}\left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right)
\end{aligned}
$$

# Pionful EFT a'la KSW



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# Introducing PV fields (Beane, Kaplan, AV, 0812:3938)

New idea: Modify pion propagator by adding new fields that

- Regulate singular short distance potential...
- ...but leave correct long distance behavior unaffected

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- Regulate singular short distance potential...
- ...but leave correct long distance behavior unaffected

$$G_\pi(q,m) = i\frac{g_A^2}{4f_\pi^2}\frac{(\mathbf{q}\cdot\boldsymbol{\sigma}_1)(\mathbf{q}\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{\mathbf{q}^2+m^2}$$ 

pion, I=J=1, mass $m$

$$G_{(1,0)}(q,\lambda) = -i\frac{g_A^2}{4f_\pi^2}\frac{\lambda^2}{\mathbf{q}^2+\lambda^2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$$ 

I=1, J=0, mass $\lambda$

1/r³ cancels for S=1

S=0 unchanged

$$G_\pi(q,m_\pi) - G_\pi(q,\lambda) + G_{(1,0)}(q,\lambda)$$

# Introducing PV fields (Beane, Kaplan, AV, 0812:3938)

New idea: Modify pion propagator by adding new fields that

- Regulate singular short distance potential...
- ...but leave correct long distance behavior unaffected

Interpretation of $\lambda$:

- Extra mass scale counted as $\lambda \sim m_\pi$
  - But $\lambda \geq 2\Lambda_{\text{NN}}$!
- Regain KSW expansion as $\lambda \to \infty$
- Roughly analogous to renormalization scale in pQCD

Now perform some straightforward algebra...

$$
\begin{aligned}
F_1(x_1,x_2) &= \frac{1}{8}\,\mathrm{Tr}\left[P_\tau \tau^i \tau^j \left(P_\tau\right)^* \left(\tau^j\right)^T \left(\tau^i\right)^T\right] \\
&\times \int_k\int_l\int_r \frac{\mathrm{Tr}\left[P_\sigma\, \bar\sigma\cdot\mathbf{k}\,\bar\sigma\cdot\mathbf{l}\,(P_\sigma)^*\;\bar\sigma^T\cdot\mathbf{l}\,\bar\sigma^T\cdot\mathbf{k}\right]}{((\mathbf{k}+\mathbf{r})^2-p^2)((\mathbf{k}+\mathbf{l}+\mathbf{r})^2+x_1^2)(r^2+x_2^2)(k^2+m_1^2)(l^2+m_2^2)} \\
&= -\frac{(d-2)^2}{2}\int_k\int_l\int_r \frac{1}{((\mathbf{k}+\mathbf{r})^2-p^2)((\mathbf{k}+\mathbf{l}+\mathbf{r})^2+x_1^2)(r^2+x_2^2)} \\
&\times \left\{1-\frac{m_1^2}{k^2+m_1^2}-\frac{m_2^2}{l^2+m_2^2}+\frac{m_1^2m_2^2}{(k^2+m_1^2)(l^2+m_2^2)}\right\}, \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
F_2(x_1,x_2) &= \frac{m_2^2}{8}\,\mathrm{Tr}\left[P_\tau \tau^i \tau^j \left(P_\tau\right)^* \left(\tau^j\right)^T \left(\tau^i\right)^T\right] \\
&\times \int_k\int_l\int_r \frac{\mathrm{Tr}\left[P_\sigma\, \bar\sigma\cdot\mathbf{l}\,(P_\sigma)^*\,\bar\sigma^T\cdot\mathbf{l}\right]}{((\mathbf{k}+\mathbf{r})^2-p^2))((\mathbf{k}+\mathbf{l}+\mathbf{r})^2+x_1^2)(r^2+x_2^2)(k^2+m_2^2)(l^2+m_1^2)} \\
&= \frac{(d-2)^2 m_2^2}{2}\int_k\int_l\int_r \frac{1}{((\mathbf{k}+\mathbf{r})^2-p^2)((\mathbf{k}+\mathbf{l}+\mathbf{r})^2+x_1^2)(r^2+x_2^2)(k^2+m_2^2)} \\
&\times \left\{1-\frac{m_1^2}{l^2+m_1^2}\right\}, \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{F_2}(x_1,x_2) &= \frac{m_2^2}{8}\,\mathrm{Tr}\left[P_\tau \tau^i \tau^j \left(P_\tau\right)^* \left(\tau^j\right)^T \left(\tau^i\right)^T\right] \\
&\times \int_k\int_l\int_r \frac{\mathrm{Tr}\left[P_\sigma\, \bar\sigma\cdot\mathbf{k}\,(P_\sigma)^*\,\bar\sigma^T\cdot\mathbf{k}\right]}{((\mathbf{k}+\mathbf{r})^2-p^2))((\mathbf{k}+\mathbf{l}+\mathbf{r})^2+x_1^2)(r^2+x_2^2)(k^2+m_1^2)(l^2+m_2^2)} \\
&= \frac{(d-2)^2 m_2^2}{\ldots}\int\int\int \frac{1}{\ldots}
\end{aligned}
$$

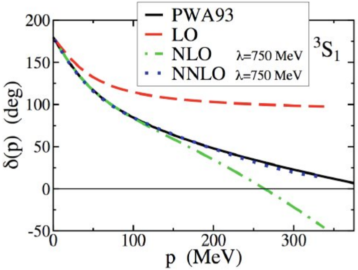
$$
\begin{aligned}
c_1 &= -\frac{3}{512\pi}\Bigg\{\frac{24 i m_2^2 m_1^2}{p^3} - \frac{16 m_2 m_1^2}{p^2} + \frac{2\log\left(\frac{4p^2}{(m_1+m_2)^2}+1\right) m_2^2 (m_1+m_2)(4p^2+3m_1m_2) m_1^2}{p^6} \\
&+ \frac{4i\log\left(1-\frac{2ip}{m_1}\right)\left(m_1^2(2p^2-3m_2^2)-4p^2m_2^2\right)m_1^2}{p^5} \\
&- \frac{2\,\mathrm{Im}\left(\mathrm{Li}_2\left(\frac{2ip-m_2}{m_1}\right)+\mathrm{Li}_2\left(\frac{2ip-m_1}{m_2}\right)+\mathrm{Li}_2\left(-\frac{2ip+m_1}{m_2-2ip}\right)\right)m_2^2\left(8p^4+4m_2^2p^2+m_1^2(4p^2+3m_2^2)\right)m_1^2}{p^7} \\
&+ \frac{i\log\left(1-\frac{2ip}{m_1}\right)\log\left(\frac{4p^2}{m_2^2}+1\right)m_2^2\left(8p^4+4m_2^2p^2+m_1^2(4p^2+3m_2^2)\right)m_1^2}{p^7} \\
&+ \frac{\arctan\left(\frac{2p}{m_2}\right)\log\left(\frac{4p^2+m_2^2}{m_1^2}\right)m_2^2\left(8p^4+4m_2^2p^2+m_1^2(4p^2+3m_2^2)\right)m_1^2}{p^7} - \frac{16 i m_1^2}{p} - \frac{16 m_2^2 m_1}{p^2} - 64 m_1 \\
&- \frac{16 i m_2^2}{p} + 32 i p - 64 m_2 - \frac{4\tan^{-1}\left(\frac{2p}{m_1+m_2}\right)(4p^2+m_1^2+m_2^2)\left(8p^4+2m_2^2p^2+m_1^2(2p^2-3m_2^2)\right)}{p^5} \\
&- \frac{4i\log\left(1-\frac{2ip}{m_2}\right)m_2^2\left(m_1^2(4p^2+3m_2^2)-2p^2m_2^2\right)}{p^5}\Bigg\}, \qquad (64)
\end{aligned}
$$

$$
\begin{aligned}
a_2 &= -\frac{3 i m_2^2}{256\pi^3 p}\Bigg\{p^2\left[\log\left(\frac{4\mu^2}{(m_2-2ip)^2}\right)-\frac{7}{3}\right] - m_1^2\Bigg[\log\left(1-\frac{2ip}{m_2}\right)\log\left(\frac{\sqrt{m_2(m_2-2ip)}}{m_1-2ip}\right) \\
&- \mathrm{Li}_2\left(\frac{2ip-m_1}{m_2}\right) + \mathrm{Li}_2\left(-\frac{m_1}{m_2-2ip}\right)\Bigg]\Bigg\}, \qquad (65)
\end{aligned}
$$

$$
b_2 = -\frac{3m_2^2}{\cdots}\Big\{2p^2\log\left(1-\frac{2ip}{\cdots}\right) - m_1^2\Big[\log\left(1-\frac{2ip}{\cdots}\right)\log\left(\frac{\sqrt{m_2(m_2-2ip)}}{\cdots}\right)
$$

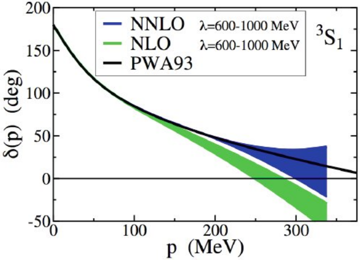
...and after 35 pages:

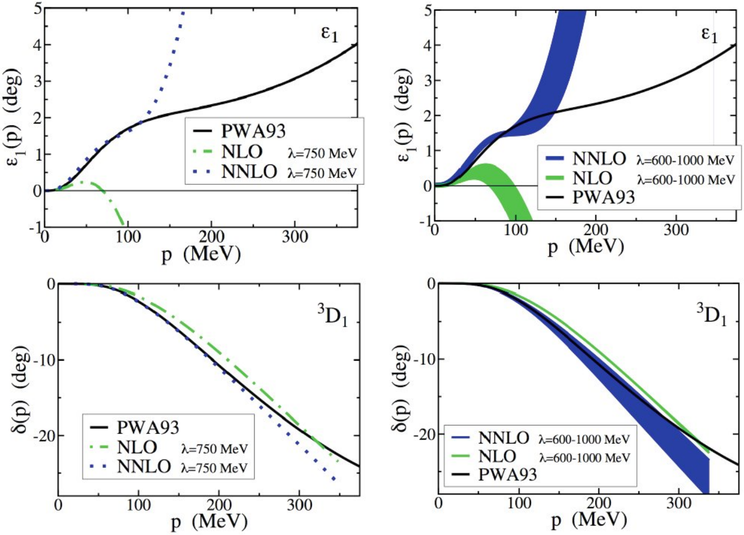
# $^3S_1$ channel with PV fields




Sensitivity to $\lambda$

$^3S_1$ greatly improved




ε1
ε1(p) (deg)
p (MeV)
PWA93
NLO λ=750 MeV
NNLO λ=750 MeV
ε1
ε1(p) (deg)
p (MeV)
NNLO λ=600-1000 MeV
NLO λ=600-1000 MeV
PWA93
3D1
δ(p) (deg)
p (MeV)
PWA93
NLO λ=750 MeV
NNLO λ=750 MeV
3D1
δ(p) (deg)
p (MeV)
NNLO λ=600-1000 MeV
NLO λ=600-1000 MeV
PWA93

And more to follow (soon)...

# Table of contents

# Conclusions and outlook

- Effective theories offer controlled and physically transparent way to do low energy nuclear physics
- KSW power counting leads to renormalizable theory with few fit parameters
  - Deformation of pion propagators cures problems due to singular tensor potential
- Lots to do:
  - Higher partial waves (partially completed; ${}^3P_0$ problematic)
  - $\mathrm{N}^3$LO amplitudes
  - Applications: N-body processes, lattice implementation (in progress), ...