Coulomb effects in pionless effective field theory

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Outline

- Introduction
- n-d system: Strong interaction only
- p-d system: Including Coulomb effects
- Application and Results
- Summary

Introduction

- Pionless effective field theory
- Effective Lagrangian

Pionless effective field theory

Construct an effective field theory for low-energy few-nucleon systems

Concepts and experimental input:

- At very low energies even pions can be integrated out
  $\longrightarrow$ only nucleons left as effective degrees of freedom
- Nonrelativistic framework
  $\longrightarrow$ demand Galilei-invariance of effective Lagrangian
- Large scattering lengths in NN scattering
  $\longrightarrow$ additional low-energy scale

This program has been carried out quite successfully, already.

e.g. Bedaque, Hammer, van Kolck 2000

Interesting: Study Coulomb effects in the 3-body system

Rupak, Kong 2003

Use nucleon-deuteron system as an example!

Effective Lagrangian

$$\mathcal{L} = N^\dagger \left(\mathrm{i}D_0 + \frac{\vec{D}^2}{2M_N} \right) N + \mathcal{L}_{\text{photon}}$$
$$- d^{i\dagger} \left[\sigma_d + \left(\mathrm{i}D_0 + \frac{\vec{D}^2}{4M_N} \right) \right] d^i - t^{A\dagger} \left[\sigma_t + \left(\mathrm{i}D_0 + \frac{\vec{D}^2}{4M_N} \right) \right] t^A$$
$$- y_d \left[d^{i\dagger} \left(N^T P_d^i N \right) + \text{h.c.} \right] - y_t \left[t^{A\dagger} \left(N^T P_t^A N \right) + \text{h.c.} \right]$$

Contents:

- **Nucleon** field N, doublet in spin and isospin space
- Two auxiliary **dibaryon** fields d^i (spin triplet, isospin singlet) and t^A (spin singlet, isospin triplet) corresponding to the respective channels in NN scattering
- **Coupling constants** y_d and y_t, dibaryon propagators are just constants (σ_d, σ_t) to first order

Couple to photons via covariant derivative: $D_\mu = \partial_\mu + \mathrm{i}e\hat{Q}_{\text{em}} A_\mu$

n-d system: Strong interaction only

"With the lights out, it's less dangerous..."

- Power counting
- Integral equation

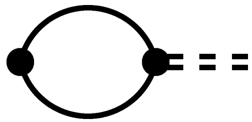
Power counting

Identify scales:

- Low-energy scale: $p \sim \gamma_d \sim \mathcal{O}(Q)$
- Cut-off $\Lambda \sim \mathcal{O}(m_\pi)$, $m_\pi/M_N \sim \mathcal{O}(Q/\Lambda)$
- Assume $y_d^2 \sim y_t^2 \sim 1/\Lambda$ and $\sigma_d \sim \sigma_t \sim Q$

Consequences:

- Integration measure $\int \mathrm{d}^3 q \sim \mathcal{O}(Q^3)$
- Nucleon propagator $\sim \mathcal{O}(M_N/Q^2)$ (non-relativistic!)
- Leading order dibaryon propagator $= -\mathrm{i}/\sigma \sim \mathcal{O}(1/Q)$

 $\sim \mathcal{O}(1)$

$\longrightarrow$ Re-sum propagator!

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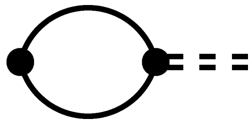
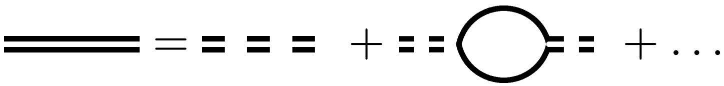
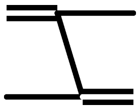
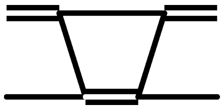


$\sim \mathcal{O}(1)$

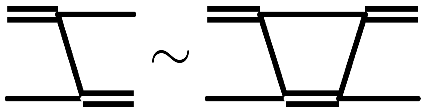
- Bubble chain: $\Delta_d =$ 
- Fix parameters from NN scattering
- Dibaryon kinetic energy $\rightarrow$ range corrections

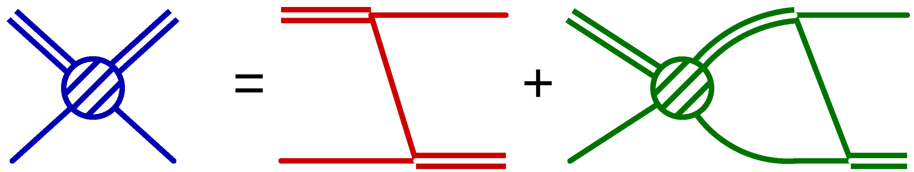
$\rightarrow$ Re-sum propagator!

Integral equation

 $\sim$  $\sim \ldots$ all of same order $\longrightarrow$ Integral equation!

Integral equation

 $\sim \ldots$ all of same order $\longrightarrow$ Integral equation!



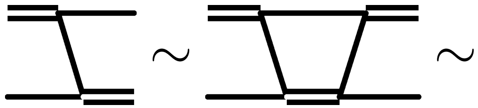
Formal structure:

$$\mathcal{T}(E;k,p) = K(E;k,p) + \frac{1}{\pi}\int_0^\Lambda \mathrm{d}q\, K(E;q,p)\, D(E;q)\, \mathcal{T}(E;k,q)$$

for S-waves, incoming momentum k, outgoing momentum p

- Quartet channel $\rightarrow$ quite simple

Integral equation



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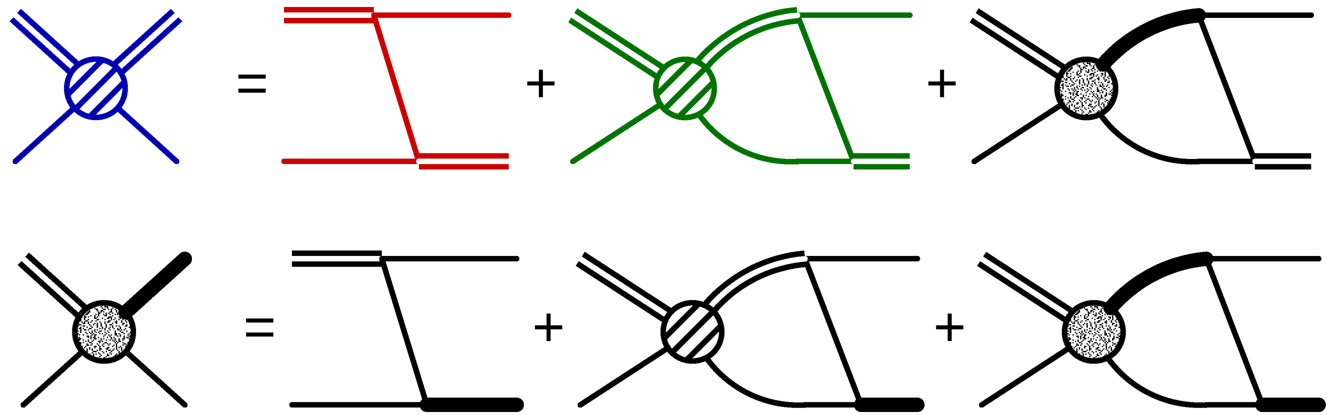

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- Quartet channel $\rightarrow$ quite simple
- Doublet channel $\rightarrow$ coupled channels, 3N-force $H(\Lambda)$ at leading order

p-d system: Including Coulomb effects

"...here we are now, entertain us!"

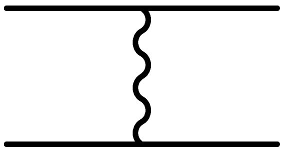
- Coulomb photons
- Modified power counting
- New integral equations
- Numerical methods

Coulomb photons

Consider $\mathcal{L}_{\text{photon}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}\Big(\underbrace{\partial_\mu A^\mu - \eta_\mu\eta_\nu\partial^\nu A^\mu}_{=\vec{\nabla}\cdot\mathbf{A} \text{ for } \eta^\mu=(1,0,0,0)}\Big)^2 - e\,j_\mu A^\mu$

$\rightarrow$ quantization in Coulomb gauge

- One finds: Field component A_0 does not propagate
  $\rightarrow$ eliminate with equation of motion
- Poisson equation: $\Delta A^0 = -e\,j^0 \iff (\mathrm{i}\mathbf{k})^2 A^0 = -e\,j^0$
- Re-insert into Lagrangian: $\mathrm{i}\mathcal{L}_{\text{int}}(\mathbf{k}) \supset (\mathrm{i}e)\,j_0(\mathbf{k})\,\frac{\mathrm{i}}{\mathbf{k}^2}\,(\mathrm{i}e)\,j_0(\mathbf{k})$



$$\sim \mathrm{i}e^2\frac{1}{(\mathrm{i}\mathbf{k})^2} = (\mathrm{i}e)\,\frac{\mathrm{i}}{\mathbf{k}^2}\,(\mathrm{i}e) \quad .$$

$\rightarrow$ exchange of Coulomb photons

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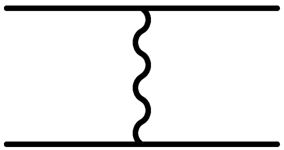

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$\longrightarrow$ exchange of Coulomb photons

Compared to this, transverse photons are suppressed by powers of momenta and/or α / M_N.

Power counting revisited

Coulomb effects $\sim \alpha M_N/p$ are dominant at very low momenta!

$\longrightarrow$ we can no longer assume $p \sim \gamma_d, \gamma_t \sim Q$

Need simultaneous expansion in Q/Λ and $p/(\alpha M_N)$!

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Two scales in the loop integrations:

- Either $\int \mathrm{d}^3q \sim \mathcal{O}(Q^3)$ or $\int \mathrm{d}^3q \sim \mathcal{O}(p^3)$
- Full dibaryon propagator either $\sim \mathcal{O}(1/Q)$ or $\sim \mathcal{O}(Q/p^2)$
- Same for the photon propagator: $\sim \mathcal{O}(1/Q^2)$ or $\sim \mathcal{O}(1/p^2)$

depending on which contribution is picked up after the $\mathrm{d}q_0$-integration.

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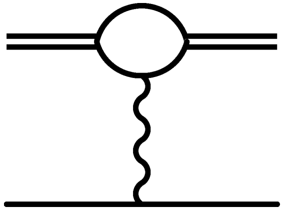
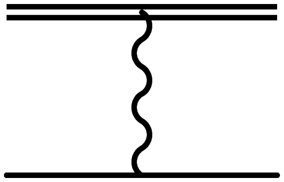
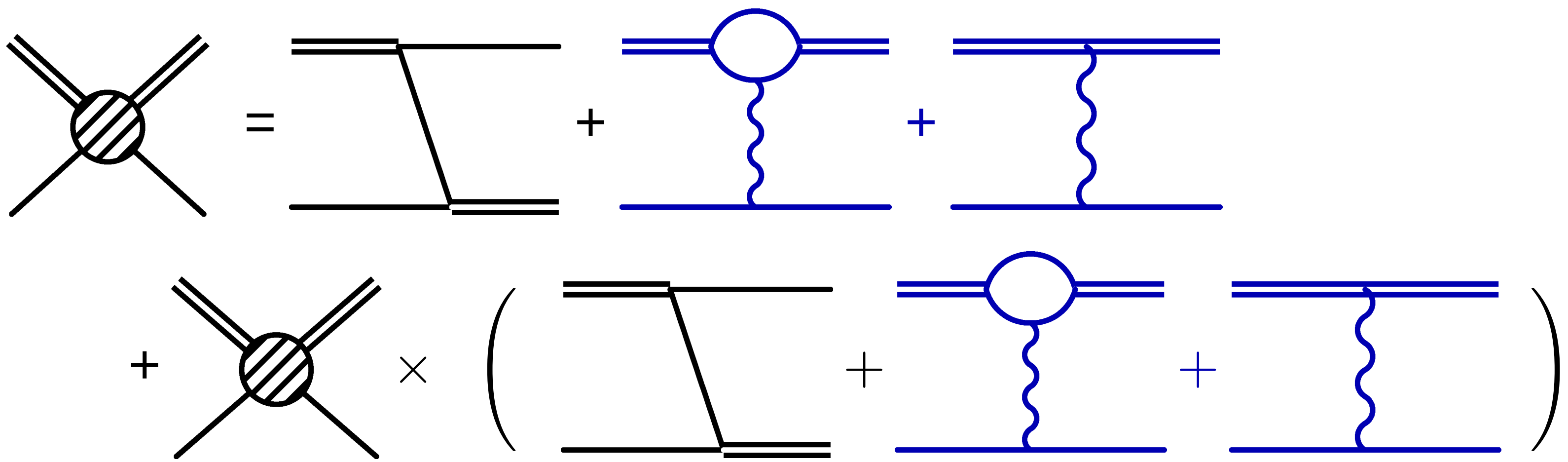
Bottom line:

Iterate  and  !

Rupak, Kong 2003

Integral equation with Coulomb photons

Include photons into the integral equation:



$\longrightarrow$ full scattering amplitude $\mathcal{T}_{\text{full}}$

Leave out nucleon exchange diarams $\longrightarrow$ Coulomb amplitude $\mathcal{T}_{\text{c}}$

1. Solve both equations
2. Get phase shifts from forward amplitude: $\delta = \frac{1}{2\mathrm{i}} \log\left(1 + \frac{2\mathrm{i}kM_N}{3\pi} Z_0 \mathcal{T}\right)$
3. Compare to experiment: $\delta_{\text{diff}}(k) \equiv \delta_{\text{full}}(k) - \delta_{\text{c}}(k)$

c.f. Jackson, Blatt 1950
Harrington 1965

This means that we remove the initial and final state Coulomb interaction!

Numerical methods (I)

There are two types of singularities in the integral equations:

- Poles in the propagators (deuteron bound state!)

$$D_{d,t}(E;q) = \frac{1}{-\gamma_{d,t} + \sqrt{3q^2/4 - M_N E - \mathrm{i}\epsilon}}$$

- Logarithmic singularities in the kernels (from S-wave projection)

$$K(E;q,p) \sim Q_0\left(\frac{q^2+p^2-M_N E - \mathrm{i}\epsilon}{qp}\right) \quad \text{with} \quad Q_0(a) = \frac{1}{2}\log\left(\frac{a+1}{a-1}\right)$$

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Could deform integration contour, but not very convenient. . .

Instead:

- Principal value integration: $\frac{1}{x\pm\mathrm{i}\epsilon} = \mathrm{PV}\frac{1}{x} \mp \mathrm{i}\pi\delta(x)$
- Log-singularities are integrable! However, numerically delicate. . .

Discretisation yields a simple matrix equation!

Numerical methods (II)

- Photon propagator is singular at zero momentum transfer! Regularize this with a small photon mass: $\frac{\mathrm{i}}{\mathbf{q}^2} \longrightarrow \frac{\mathrm{i}}{\mathbf{q}^2+\lambda^2}$
- In the integral equation: $K(E;k,p) \sim Q_0\left(-\frac{k^2+p^2+\lambda^2}{2kp}\right)$

Major numerical difficulty turns out to be the peak in the inhom. terms!

$\longrightarrow$ Re-shuffle integration mesh points!

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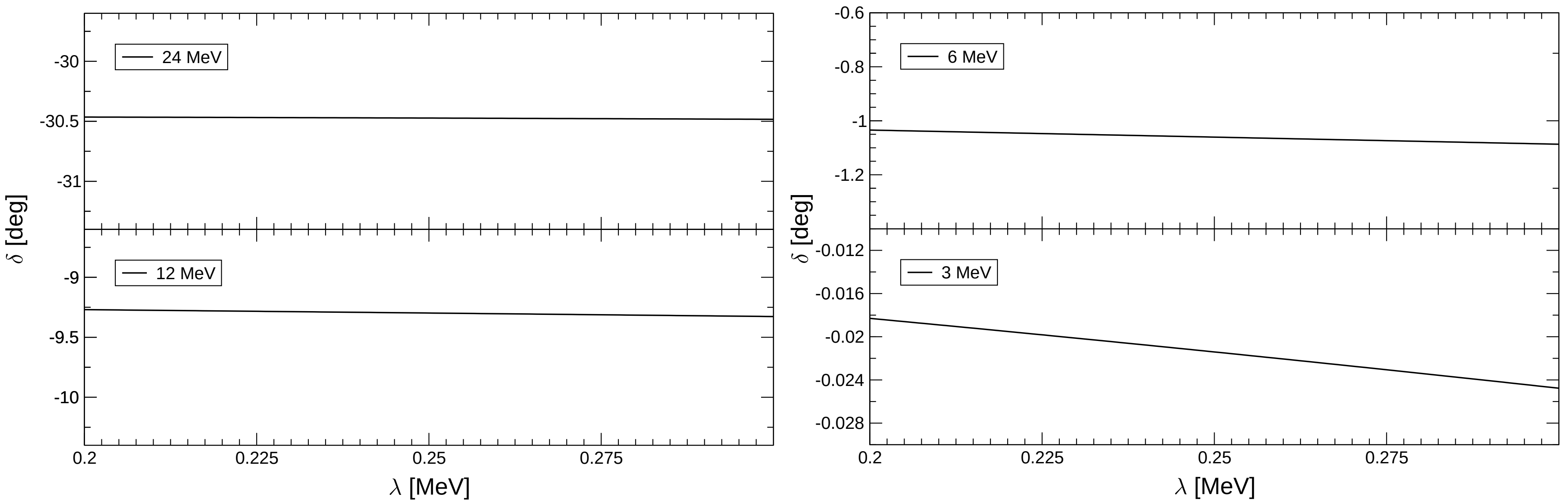


Application and Results

- Quartet channel
- Doublet channel
- He-3 binding energy

Quartet channel

Couple deuteron and nucleon spins to spin $^3/_2$

All three nucleon spins aligned $\longrightarrow$ Pauli principle

Consequences:

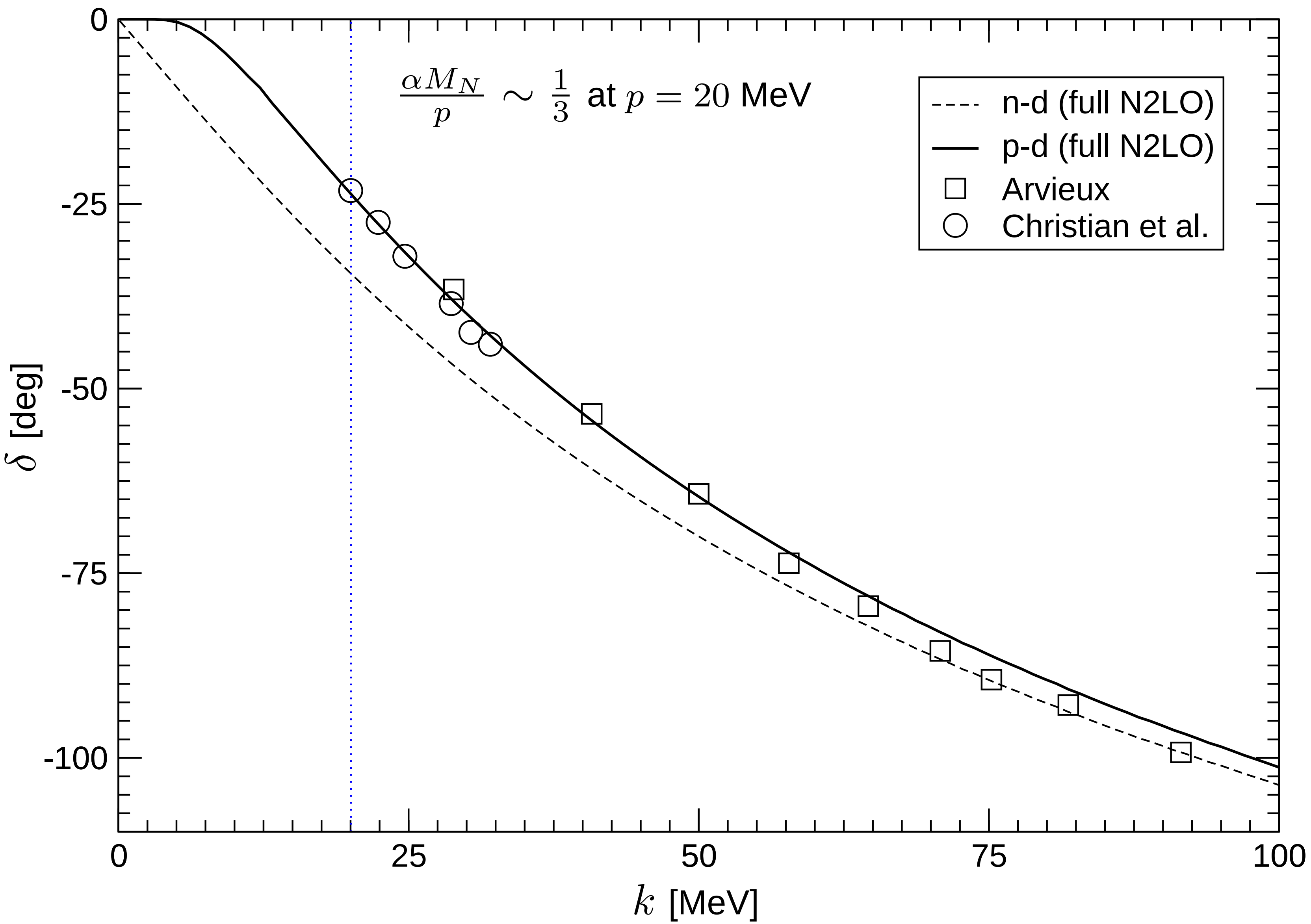
- Not very sensitive to short-range physics
- No bound state
- Only one channel, simple integral equation

We can include the deuteron kinetic energy operator to all orders!

$$D_d(E;q) = -\frac{4\pi}{M_N y_d^2} \frac{1}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\epsilon} - \frac{\rho_d}{2}(3q^2/4 - M_N E - \gamma_d^2)}$$

Cut-off stays below unphysical second pole in the propagator.

Scattering phase shifts - Quartet channel



data from Arvieux 1973; Christian, Gammel 1953

Doublet channel

Now couple deuteron and nucleon spins to spin $^1/_2$

Consequences:

- Pauli principle does not apply here $\longrightarrow$ need higher cut-off
- Singlet dibaryon in the intermediate state $\longrightarrow$ coupled channels
- Cannot easily use re-summed N2LO propagators...

Furthermore:

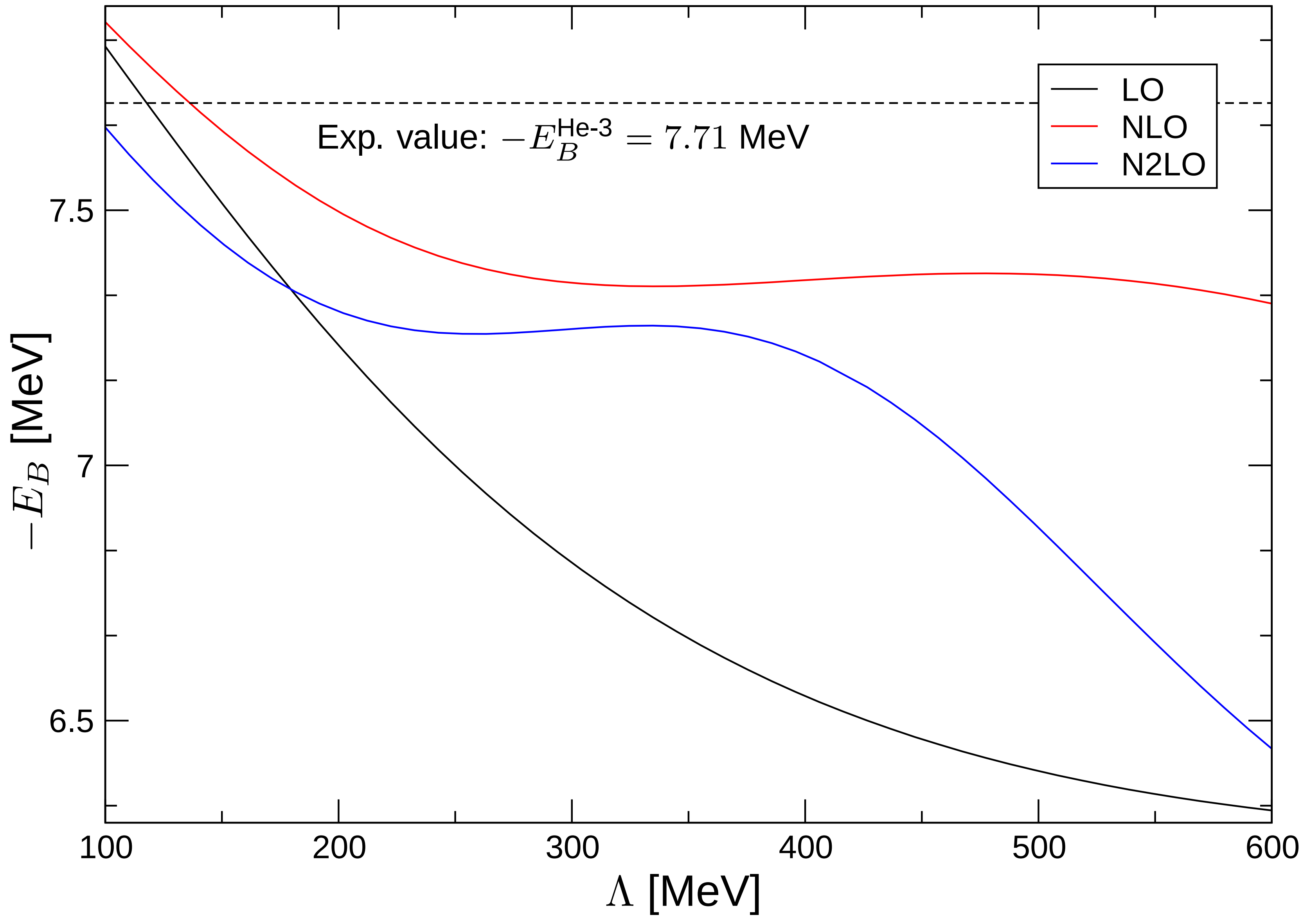
Fix leading-order 3-Nucleon force from bound-state equation!

We can:

- Fix $H(\Lambda)$ from triton binding energy in the strong system...
- ...then calculate bound state from equation with Coulomb effects

$\longrightarrow$ Predict He-3 binding energy!

He-3 binding energy



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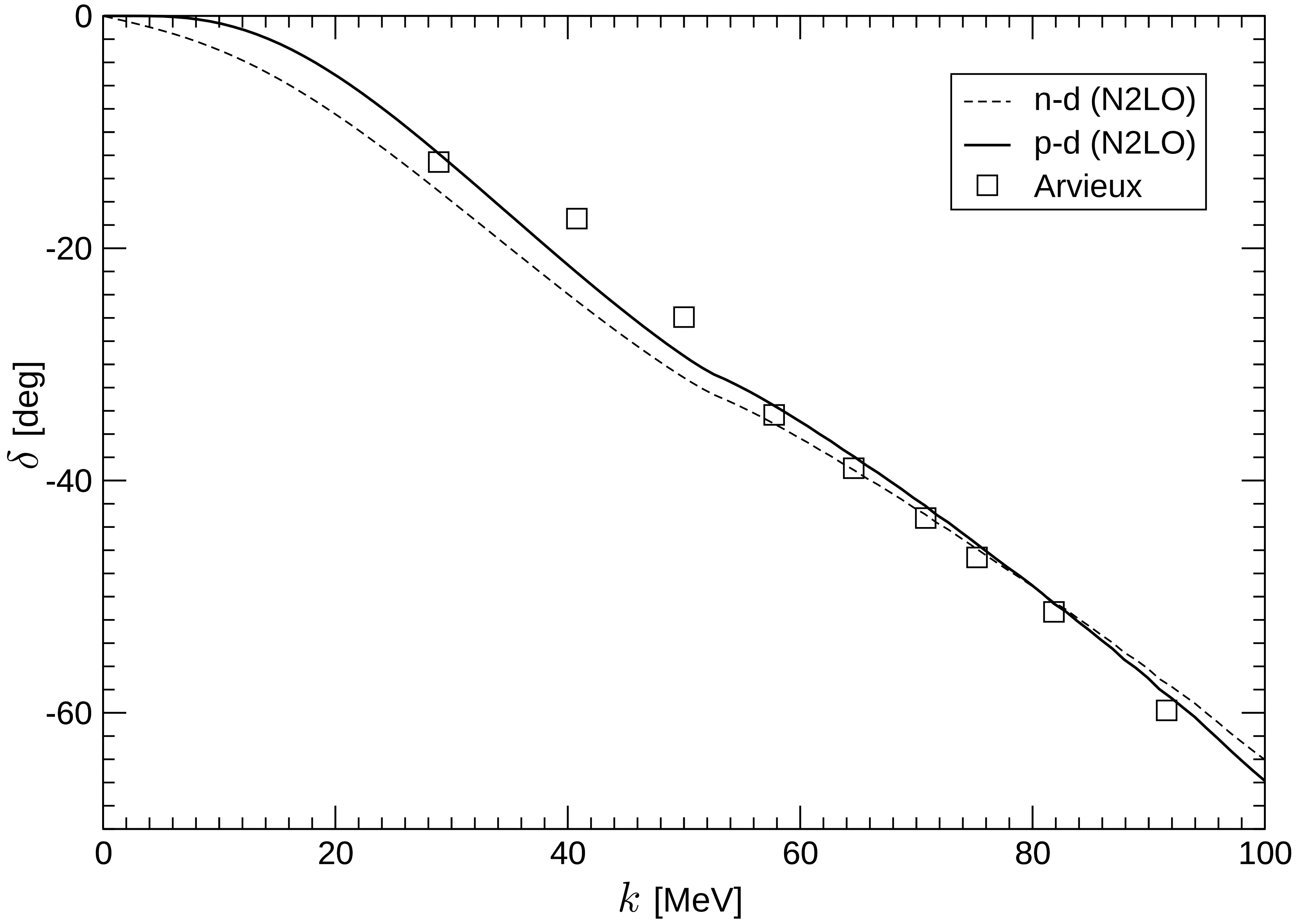


data from Arvieux 1973

Summary

It was demonstrated that. . .

- Coulomb effects can be included in pionless effective field theory
- the static Coulomb potential is dominant at low momenta
- power counting needs to be modified in the presence of Coulomb effects
- one needs to implement numerical methods carefully
- taking the screening limit is possible
- we can predict p-d observables from n-d experimental input

***

Thanks for your attention!

Spares

Spin- and S-wave projection

- Use

$$P_d^i = \frac{1}{\sqrt{8}}\,\sigma_2\sigma^i\tau_2 \quad \text{and} \quad P_t^A = \frac{1}{\sqrt{8}}\,\sigma_2\tau_2\tau^A$$

  to project onto the $^3S_1, I=0$ and $^1S_0, I=1$ states

- Couple the spins in the nucleon-deuteron system according to

$$\mathbf{1}\otimes\frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{3}}{\mathbf{2}}\oplus\frac{\mathbf{1}}{\mathbf{2}} \implies$$ 2 channels for amplitude $(\mathcal{T}^{ij})_{\alpha a}^{\beta b}(E;\mathbf{k},\mathbf{p})$

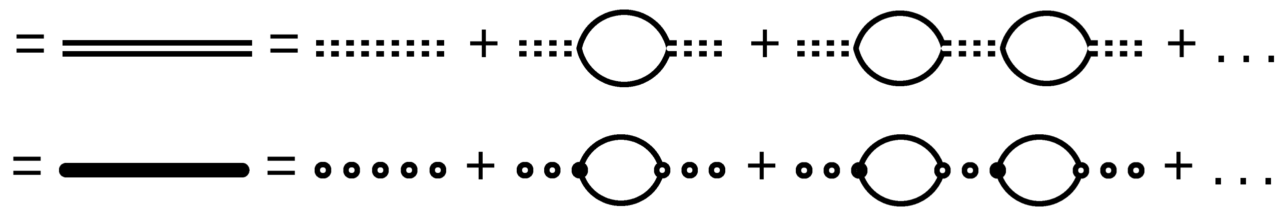
- Quartet: Set $i,j=(1\mp \mathrm{i}2)/\sqrt{2}$, $\alpha=\beta=1$, $a=b=1 \rightarrow \mathcal{T}^{\mathrm{q}}$
- Doublet: $\mathcal{T}^{\mathrm{d}} = \frac{1}{3}(\sigma^i)_\alpha^{\alpha'}(\mathcal{T}^{ij})_{\alpha' a}^{\beta' b}(\sigma^j)_{\beta'}^{\beta}$ with $\alpha=\beta=1$, $a=b=1$

Project onto S-waves with $\mathcal{T}_{l=0}(E;k,p) = \frac{1}{2}\int\limits_{-1}^{1} \mathrm{d}\cos\theta\,\mathcal{T}(E;\mathbf{k},\mathbf{p})$

Dibaryon propagators

We have to re-sum the dibaryon propagators to all orders:

$\Delta_d =$, $\Delta_t =$



Fix parameters from NN scattering:

$$\mathrm{i}\mathcal{A}_{d,t}(k) = -y_{d,t}^2 \Delta_{d,t}\left(p_0 = \frac{\mathbf{k}^2}{2M_N}, \mathbf{p} = 0\right) = \frac{4\pi}{M_N} \frac{\mathrm{i}}{k \cot \delta_{d,t} - \mathrm{i}k}$$

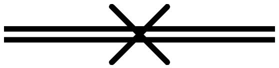
- $k \cot \delta_d = -\gamma_d + \frac{\rho_d}{2}(k^2 + \gamma_d^2) + \ldots \implies y_d, \sigma_d$
- $k \cot \delta_t = -\gamma_t + \frac{r_{0t}}{2} k^2 + \ldots$ with $\gamma_t \equiv \frac{1}{a_t} \implies y_t, \sigma_t$

Residue of pole in $\Delta_d \rightarrow$ deuteron wave function renormalization Z_0

Include dibaryon kinetic energy operator $\rightarrow$ effective range corrections

Effective range corrections

Include dibaryon kinetic energy operators:



$$\sim \; \mathrm{i}\Delta^{\mathrm{LO}}(p) \times -\mathrm{i}\left(p_0 - \frac{\mathbf{p}^2}{4M_N}\right) \times \mathrm{i}\Delta^{\mathrm{LO}}(p)$$

- Treat this as a perturbation $\rightarrow$ NLO, N2LO
- Possible to re-sum geometric series, e.g.

$$\mathrm{i}\Delta_d^{ij}(p) = -\frac{4\pi\mathrm{i}}{M_N y_d^2} \frac{\delta^{ij}}{\frac{4\pi\sigma_d}{M_N y_d^2} - \mu + \sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - \mathrm{i}\epsilon} + \frac{4\pi}{M_N y_d^2}\left(p_0 - \frac{\mathbf{p}^2}{4M_N}\right)}$$

but still only N2LO (other contributions neglected!)

- Fix parameters by reproducing effective range expansions up to $\mathcal{O}(k^2)$
- Unphysical second pole in re-summed propagator!

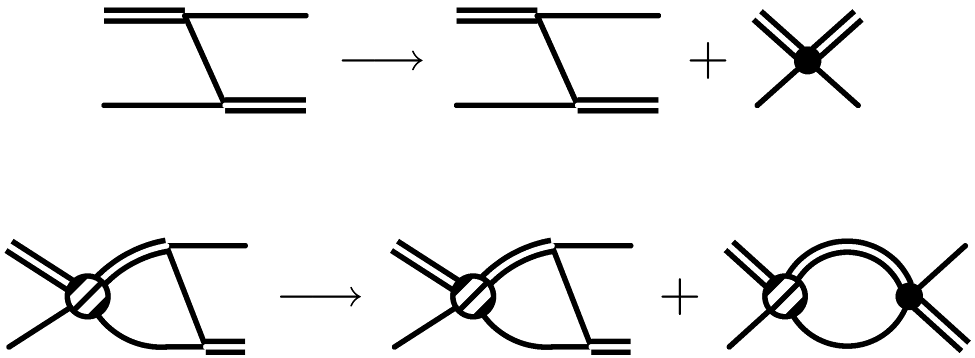
Three-nucleon force

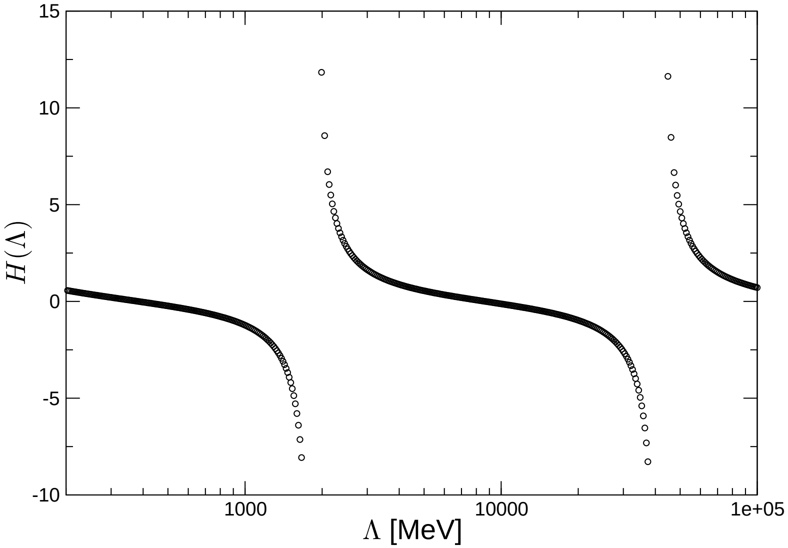
One finds: Strong cut-off dependence in the doublet channel!

$\rightarrow$ Renormalize with leading order three-nucleon force ($SU(4)$-symmetric)

$$\mathcal{L}_3 = -M_N \frac{H(\Lambda)}{\Lambda^2}\left(y_d^2 N^\dagger(\vec{d}\cdot\vec{\sigma})^\dagger(\vec{d}\cdot\vec{\sigma})N + \ldots\right)$$

Substitute:





Fix $H(\Lambda)$ with three-body input $\rightarrow$ e.g. Triton binding energy

Bound state equation

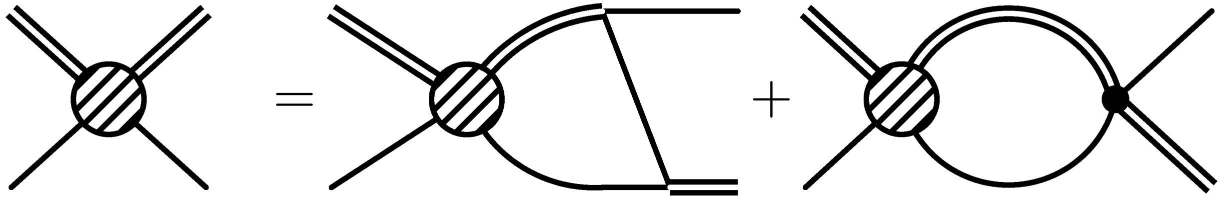
Bound state $\to$ pole in T-matrix

$$\mathcal{T}(E;k,p) = \frac{\mathcal{B}(k)\mathcal{B}(p)}{E+E_{\rm B}} \quad \text{for} \quad E \to -E_{\rm B}$$

$\to$ homogeneous integral equation for $\mathcal{B}(p)$

$$\mathcal{B}(E,p) = \frac{1}{\pi}\int_0^{\Lambda} \mathrm{d}q \; K(E;q,p)\,D(E;q)\,\mathcal{B}(E,q)$$

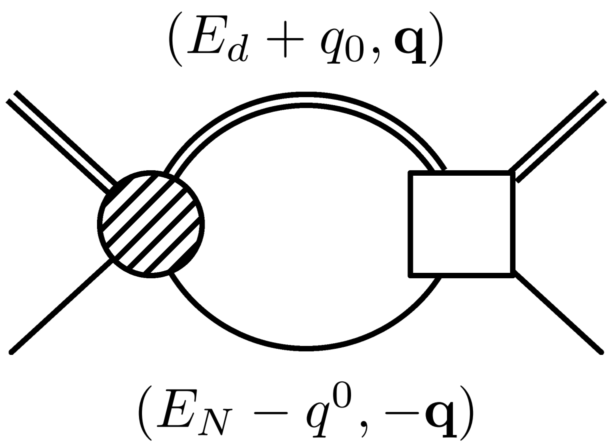
Diagrammatic:



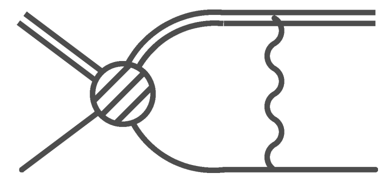
Fix $E = -E_{\rm B}$ and cut-off Λ, find suitable $H(\Lambda)$

Scaling of the deuteron propagator

Consider a diagram of the form



e.g.



Integrate over $q_0 \ldots$

$$\to \Delta_d\left(E_d + E_N - \frac{\mathbf{q}^2}{2M_N}, \mathbf{q}\right) \sim \frac{\gamma_d + \sqrt{\frac{3}{4}\left(\mathbf{q}^2 - \mathbf{p}^2\right) + \gamma_d^2}}{\frac{3}{4}\left(\mathbf{q}^2 - \mathbf{p}^2\right)} \sim \frac{Q}{q^2}$$

- Deuteron pole enhanced for $q \sim p \ldots$
- $\ldots$ but typically suppressed by $\mathrm{d}^3 q \sim p^3$
- Except when we also have a Coulomb photon propagator $\sim 1/p^2$!

Rupak, Kong 2003