$\Delta(1232)$ Resonance in Chiral Perturbation Theory

Marius Hilt

Institut für Kernphysik, Mainz

in collaboration with J. Gegelia and S. Scherer

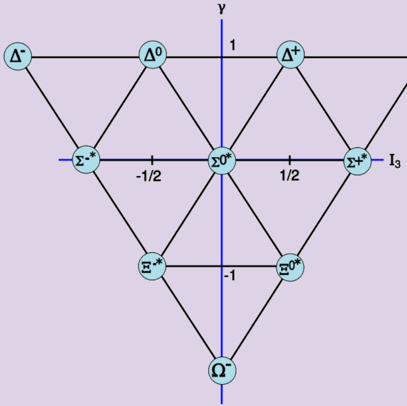
6^{th} Vienna Central European Seminar on Particle Physics and Quantum Field Theory, November 27 - 29, 2009

Overview

1. General Properties of Δ
2. Δ in χPT
3. $\gamma N \to \Delta$ Transition Form Factors
4. Summary

General Properties of Δ

Baryon Decuplet



- Lowest-lying $S = \frac{3}{2}$ and $I = \frac{3}{2}$ baryon resonance
- $m_\Delta = 1232$ MeV $\Rightarrow$ Important in region $\sqrt{s} \approx m_\Delta$

Equation of Motion

- Rarita and Schwinger: Vector spinor ψ_μ
- Each component of ψ_μ is a Dirac spinor
- EOM: $(i\partial\!\!\!/ - m)\psi_\mu = 0$
- But: Too many degrees of freedom

$\Rightarrow$ Constraints:

$\gamma^\mu \psi_\mu = 0$ and $\partial^\mu \psi_\mu = 0$

Point Invariance

- $\mathcal{L}^{(3/2)} = -\bar{\psi}^{\alpha} \Lambda^{A}_{\alpha\beta} \psi^{\beta}$

$$\begin{aligned}\Lambda^{A}_{\alpha\beta} &= (i\partial\!\!\!/ - m) g_{\alpha\beta} + iA(\gamma_\alpha \partial_\beta + \gamma_\beta \partial_\alpha) \\ &+ \frac{i}{n-2}\left[(n-1)A^2 + 2A + 1\right]\gamma_\alpha \partial\!\!\!/ \gamma_\beta \\ &+ \frac{m}{(n-2)^2}\left[n(n-1)A^2 + 4(n-1)A + n\right]\gamma_\alpha\gamma_\beta\end{aligned}$$

$A \neq -\frac{1}{2}, n \neq 2$

- $\mathcal{L}^{(3/2)}$ is invariant under so-called point transformations:

$$\begin{aligned}\psi_\mu &\mapsto \psi_\mu + \frac{4a}{n}\gamma_\mu \gamma_\nu \psi^\nu \\ A &\mapsto \frac{An - 8a}{n(1+4a)}, \qquad a \neq -\frac{1}{4}\end{aligned}$$

Delta Propagator

$$
\begin{aligned}
S_0^{\mu\nu}(p) &= -\frac{\not{p}+m_\Delta}{p^2-m_\Delta^2}\left(g^{\mu\nu} - \frac{\gamma^\mu\gamma^\nu}{n-1} - \frac{\gamma^\mu p^\nu - p^\nu\gamma^\mu}{(n-1)m_\Delta} - \frac{(n-2)p^\mu p^\nu}{(n-1)m_\Delta^2}\right) \\
&\quad - \frac{A+1}{m_\Delta^2(An+2)}\left[\frac{(n-4-nA)m_\Delta\gamma^\mu\gamma^\nu}{(n-2)(nA+2)} + \frac{(n-2)(\gamma^\mu p^\nu + p^\nu\gamma^\mu)}{n-1}\right. \\
&\qquad \left. - \frac{(n-2)(A+1)\gamma^\mu \not{p}\gamma^\nu}{(n-1)(nA+2)}\right]
\end{aligned}
$$

Delta Propagator ($A = -1$)

$$S_0^{\mu\nu}(p) = -\frac{\not{p}+m_\Delta}{p^2-m_\Delta^2}\left(g^{\mu\nu} - \frac{\gamma^\mu\gamma^\nu}{n-1} - \frac{\gamma^\mu p^\nu - p^\nu\gamma^\mu}{(n-1)m_\Delta} - \frac{(n-2)p^\mu p^\nu}{(n-1)m_\Delta^2}\right)$$

Isospin

- Δ consists of four isospin states
  $\Rightarrow$ One possible description: $\begin{pmatrix} \Delta^{++} & \Delta^{+} & \Delta^{0} & \Delta^{-} \end{pmatrix}^T$
- Our way: Coupling of isospins: $\frac{1}{2} \otimes 1 = \frac{3}{2} \oplus \frac{1}{2}$
  $\Rightarrow$ One needs projection operators:
  $P^{\frac{1}{2}}_{ij,\alpha\beta} = \frac{1}{3}(\tau_i \tau_j)_{\alpha\beta}$,
  $P^{\frac{3}{2}}_{ij,\alpha\beta} = \delta_{ij}\delta_{\alpha\beta} - \frac{1}{3}(\tau_i \tau_j)_{\alpha\beta}, \quad i = 1, 2, 3, \alpha = \pm\frac{1}{2}$
- $\mathcal{L}$ is invariant under local transformations
  $\psi^\mu_i(x) \to \psi^\mu_i(x) + \tau_i \alpha^\mu(x)$
  $\Rightarrow$ Gauge fixing condition: $\tau_i \psi^\mu_i = 0$

Δ in χPT

Basic Idea of χPT

- Theorem of Weinberg: Perturbative description in terms of the most general effective Lagrangian yields the most general S-matrix consistent wih principles of QFT

- Lagrangian contains infinite number of terms $\Rightarrow$ Scheme needed to organize Lagrangian and to estimate importance of terms (Power Counting)
- Power Counting: Rescaling of external momenta ($p \to tp$) and light quark masses ($m_q \to t^2 m_q$)

  $\Rightarrow$ Chiral dimension of given amplitude of diagram:
  $\mathcal{M}(tp, t^2 m_q) = t^D \mathcal{M}(p, m_q)$

S. Weinberg, Physica A **96**, 327 (1979).

Lagrangians without Δ

- $\mathcal{L}_{\pi}^{(2)} = \frac{F^2}{4} Tr(D_\mu U (D^\mu U)^\dagger) + \frac{F^2}{4} Tr(\chi U^\dagger + U \chi^\dagger)$

$$U = exp\left(\frac{i\Phi}{F}\right), \quad \Phi = \begin{pmatrix} \pi_0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi_0 \end{pmatrix}, \quad \chi = 2B(s+ip)$$

- $\mathcal{L}_{N\pi}^{(1)} = \bar{\Psi}\left(i\gamma^\mu D_\mu - m_N + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\Psi$

$$u_\mu = i\left(u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\right), \quad u = \sqrt{U}$$

J. Gasser, H. Leutwyler, Annals. Phys. **158**, 142 (1984).

J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. **B307**, 779 (1988).

Interaction Lagrangians for Δ

- $\mathcal{L}^{(1)}_{\pi N\Delta} = g\bar{\Psi}_{\mu,i} P^{\frac{3}{2}}_{ij} (g^{\mu\nu} + \tilde{z}\gamma^{\mu}\gamma^{\nu})\omega_{\nu,j}\Psi + h.c.$
  $\tau_i \omega_{\nu,i} = u_\nu, \quad \tilde{z} = \frac{1}{2}(1+3A)$
- $\mathcal{L}^{(1)}_{\pi\Delta} = -\bar{\Psi}_{\mu} P^{\frac{3}{2}} \Lambda^{\mu\nu} P^{\frac{3}{2}} \Psi_\nu$

$$\begin{aligned}\Lambda_{\mu\nu} &= (i\not{D} - m_\Delta) g_{\mu\nu} + iA(\gamma_\mu D_\nu + \gamma_\nu D_\mu) \\ &+ \frac{i}{2}(3A^2 + 2A + 1)\gamma_\mu \not{D}\gamma_\nu + m_\Delta(3A^2 + 3A + 1)\gamma_\mu\gamma_\nu \\ &+ \frac{g_1}{2}\not{u}\gamma_5 g_{\mu\nu} + \frac{g_2}{2}(\gamma_\mu u_\nu + u_\mu\gamma_\nu)\gamma_5 + \frac{g_3}{2}\gamma_\mu \not{u}\gamma_5\gamma_\nu\end{aligned}$$

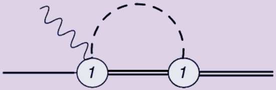
$g_2 = A g_1, \quad g_3 = -\frac{1+2A+A^2(n-1)}{n-1} g_1$

L. M. Nath, B. Etemadi, J. D. Kimel, Phys. Rev. D **3**, 2153 (1971).

T. R. Hemmert, B. R. Holstein, J. Kambor, J. Phys. G **24**, 1831 (1998).

N. Wies, J. Gegelia, S. Scherer, Phys. Rev. **D73**, 094012 (2006).

Small Scale Expansion



- Treat M_π, q and $\Delta m := m_\Delta - m_N$ as same scale:

$$\epsilon = \frac{M_\pi}{\Lambda} = \frac{q}{\Lambda} = \frac{\Delta m}{\Lambda} \Rightarrow M_\pi, q, \Delta m = \mathcal{O}(\epsilon^1)$$

- Example: $\epsilon =$

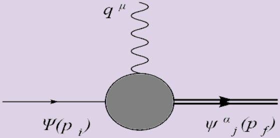
$$\underbrace{1}_{Vertex1} + \underbrace{1}_{Vertex2} - \underbrace{1}_{\Delta Propagator} - \underbrace{2}_{\pi Propagator} + \underbrace{n}_{\#Dimensions} \overset{n \to 4}{=} 3$$

E. E. Jenkins and A. V. Manohar, Phys. Lett. B **259**, 353 (1991).

T. R. Hemmert, B. R. Holstein, J. Kambor, J. Phys. G **24**, 1831 (1998).

$\gamma N \rightarrow \Delta$ Transition Form Factors

$\gamma N \to \Delta$ Transition Form Factors



- Parametrisation:
  $\langle \Delta | J^\mu(0) | N \rangle = e\bar{\psi}_\alpha(p_f)\Gamma^{\alpha\mu}\psi(p_i)$

  $\Gamma^{\alpha\mu} = \left(C_1 g^{\alpha\mu} + C_2 q^\alpha p_i^\mu + C_3 q^\alpha q^\mu + C_4 q^\alpha \gamma^\mu \right) \not{q}\gamma_5$

- Current conservation: $q_\mu \Gamma^{\alpha\mu}|_{p_f^2 = m_\Delta^2} = 0$

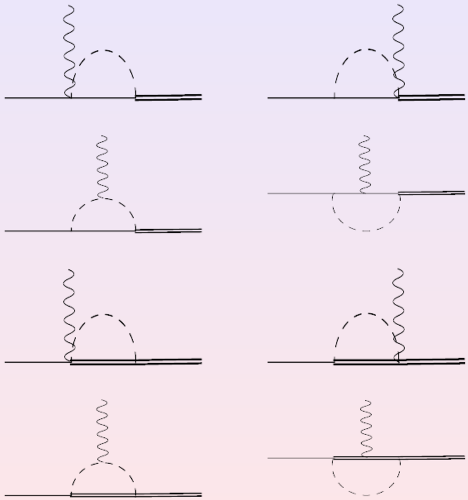
  $\Rightarrow \Gamma^{\alpha\mu} = G_M^*(q^2)\mathcal{K}_1^{\alpha\mu} + G_E^*(q^2)\mathcal{K}_2^{\alpha\mu} + G_C^*(q^2)\mathcal{K}_3^{\alpha\mu}$

H.F. Jones, M.D. Scadron, Annals Phys.81,1 (1973).

Tree Level Diagrams

- Two free parameters from $\gamma N \rightarrow \Delta$ tree level diagram
- $\Rightarrow$ Description of form factors is not so good
- Include ρ-Meson (tree level) $\Rightarrow$ Four free parameters
  $\Rightarrow$ Improved description

Loop Diagrams



Extraction of Form Factors

- Parametrisation (only if current conservation fulfilled):

$$
\begin{aligned}
i\mathcal{M}^{\mu}_{N\gamma\to\Delta} &= \sqrt{\frac{2}{3}}\frac{e}{2m_N}\bar{\psi}_\alpha(p_f)\Big\{G_1^\dagger\left[(m_\Delta+m_N)g^{\alpha\mu}-\gamma^\mu q^\alpha\right] \\
&+ \frac{G_2^\dagger}{2m_N}(p_i^\mu q^\alpha - p_i q g^{\alpha\mu}) \\
&+ \frac{G_3^\dagger}{2(m_\Delta-m_N)}(q^\alpha q^\mu - q^2 g^{\alpha\mu})\Big\}\gamma_5\psi(p_i) \\
&= i\bar{\psi}_\alpha(p_f)(C_1 g^{\alpha\mu} + C_2 q^\alpha p_i^\mu \\
&+ C_3 q^\alpha q^\mu + C_4 q^\alpha \gamma^\mu)\not{q}\gamma_5\psi(p_i)
\end{aligned}
$$

G. C. Gellas, T. R. Hemmert, C. N. Ktorides and G. I. Poulis, Phys. Rev. D **60**, 054022 (1999).

Extraction of Form Factors

- Matching the coefficients:

$$
\begin{aligned}
C_1 &= -i\sqrt{\frac{2}{3}}\frac{e}{2m_N}\left[G_1^\dagger(m_\Delta + m_N) - \frac{G_2^\dagger}{2m_N}p_i q - \frac{G_3^\dagger}{2(m_\Delta - m_N)}q^2\right] \\
C_2 &= -i\sqrt{\frac{2}{3}}\frac{e}{2m_N}\frac{G_2^\dagger}{2m_N} \\
C_3 &= -i\sqrt{\frac{2}{3}}\frac{e}{2m_N}\frac{G_3^\dagger}{2(m_\Delta - m_N)} \\
C_4 &= i\sqrt{\frac{2}{3}}\frac{e}{2m_N}G_1^\dagger
\end{aligned}
$$

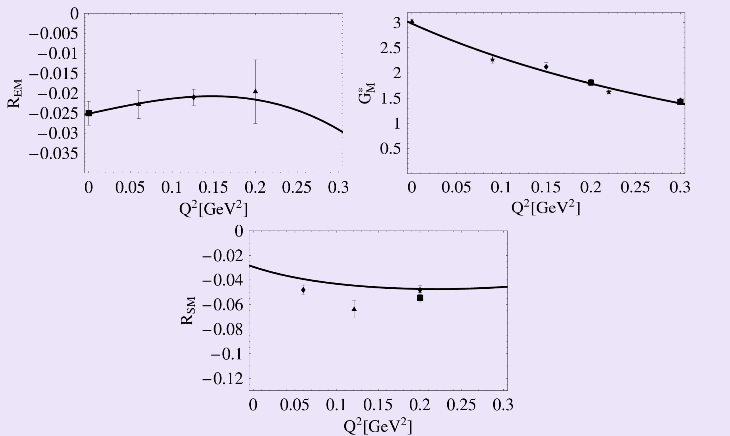
Extraction of Form Factors

- Connection to form factors:

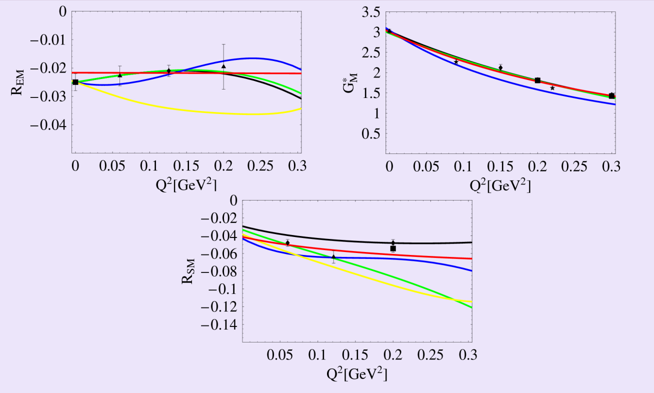
$$
\begin{aligned}
G_M^* &= \frac{m_N}{3(m_N+m_\Delta)}\Big\{\big[(3m_\Delta+m_N)(m_\Delta+m_N)-q^2\big]\frac{G_1^\dagger}{2m_N m_\Delta} \\
&- (m_\Delta^2-m_N^2-q^2)\frac{G_2^\dagger}{4m_N^2}-\frac{q^2 G_3^\dagger}{2m_N(m_\Delta-m_N)}\Big\} \\
G_E^* &= \frac{m_N}{3(m_N+m_\Delta)}\Big[(m_\Delta^2-m_N^2+q^2)\frac{G_1^\dagger}{2m_N m_\Delta} \\
&- (m_\Delta^2-m_N^2-q^2)\frac{G_2^\dagger}{4m_N^2}-\frac{q^2 G_3^\dagger}{2m_N(m_\Delta-m_N)}\Big] \\
G_C^* &= \frac{2m_N}{3(m_N+m_\Delta)}\Big[\frac{m_\Delta G_1^\dagger}{m_N}-(m_\Delta^2+m_N^2-q^2)\frac{G_2^\dagger}{4m_N^2} \\
&- (m_\Delta^2-m_N^2+q^2)\frac{G_3^\dagger}{4m_N(m_\Delta-m_N)}\Big]
\end{aligned}
$$

Results

$$R_{EM} = -\Re\left(\frac{G_E^*}{G_M^*}\right),\ R_{SM} = -\frac{\sqrt{((m_\Delta+m_N)^2-q^2)((m_\Delta-m_N)^2-q^2)}}{4m_\Delta^2}\Re\left(\frac{G_C^*}{G_M^*}\right)$$



Comparison with other Calculations



EOMS: black, IR: green, GH: blue, PV: yellow, MAID2007: red

T. A. Gail, T. R. Hemmert, Eur. Phys. J. A **28**, 91 (2006).

V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. D **73**, 034003 (2006).

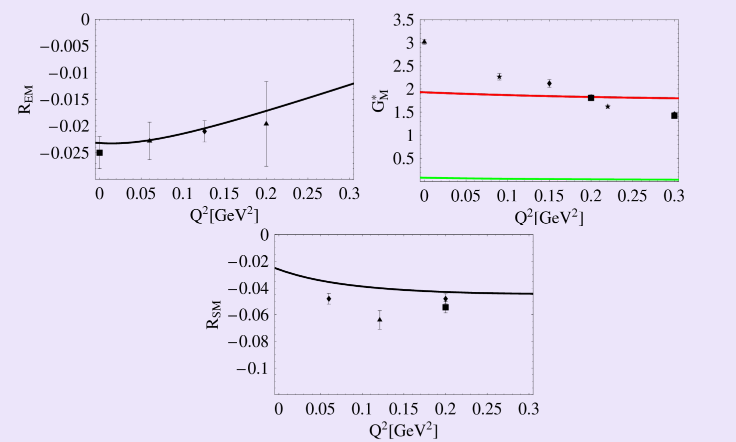
D. Drechsel, S.S. Kamalov, L. Tiator, Eur. Phys. J. A **34**, 69 (2007).

Summary

Summary

- Δ explicitly needed for $\sqrt{s} \approx m_\Delta$
- Field theory: Rarita/Schwinger, Isospin
- Interaction Lagrangians
- Power Counting: SSE
- Application: Form Factors

Results without ρ



SSE

- Treat M_π, q and Δm as same scale:

$$\epsilon = \frac{M_\pi}{\Lambda} = \frac{q}{\Lambda} = \frac{\Delta m}{\Lambda}$$

$$\Rightarrow M_\pi, q, \Delta m = \mathcal{O}(\epsilon^1)$$

δ-Expansion

- Expansion parameter: $\delta = \frac{\Delta m}{\Lambda} \approx \frac{M_\pi}{\Delta m}$
- Assign δ-counting index $\alpha \rightarrow$ Graph counts as δ^α

$$\Rightarrow M_\pi = \mathcal{O}(\delta^2), \quad (\Delta m) = \mathcal{O}(\delta^1)$$

$$q \approx M_\pi : q = \mathcal{O}(\delta^2), \qquad q \approx (\Delta m) : q = \mathcal{O}(\delta^1)$$

Parametrisation of tree level contributions

$$\mathcal{B}^{\alpha}_{\Delta N\gamma} = i\sqrt{\frac{2}{3}}\frac{e}{2m_N}\left([A(m_\Delta + m_N) - B\frac{p_i\cdot q}{4m_N}]g^{\alpha\mu} - \frac{B}{2m_N}q^\alpha p_i^\mu + Aq^\alpha\gamma^\mu\right)\gamma_5\epsilon_\mu .$$

Renormalisation scheme	A [GeV]	B [GeV]	$\tilde{A}$ [GeV^3]	$\tilde{B}$ [GeV^3]
EOMS (Fit I)	-1.92	5.21	0	0
EOMS (Fit II)	3.49	10.48	4.23	3.96
IR	3.08	1.93	3.64	-10.34