Leading logarithmic corrections resummed

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Project with Johan Bijnens [arXiv:0909.5086]

- Use RGE $\Rightarrow$ coefficients of the LL series

$$m_\phi^2 = M_0^2[1 + \sum_{n=1} c_n (M^2 \log M^2/\mu^2)^n + ...] + ... \quad (1)$$

which appear in an $n-$th loop calculation.

- At each order LL are potentially the largest correction,
- $\Rightarrow$ check perturbation series convergence.
- Find algorithm to resum the series.

Renormalizable theory:

$$\mathcal{L}_{QED} = \bar{\psi}(\partial\!\!\!/ - ie_0 A\!\!\!/)\psi - m_0\bar{\psi}\psi + \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$+ \quad i\delta e_0 \bar{\psi} A\!\!\!/ \psi + \delta m_0 \bar{\psi}\psi$$

$$e_{phys} = e_0(1 + \frac{e_0^2}{16\pi^2}\log\frac{\Lambda^2}{\mu^2} + \ldots)$$

$$m_{phys} = m_0(1 + \frac{m_0^2}{16\pi^2}\log\frac{\Lambda^2}{\mu^2} + \ldots)$$

- divergences are reabsorbed in the $\mathcal{L}_0$ coupling constants
- the counterterms have the same form as $\mathcal{L}_0$ couplings

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$$e_{phys} = e_0\left(1 + \frac{e_0^2}{16\pi^2}\log\frac{\Lambda^2}{\mu^2} + \ldots\right)$$
$$m_{phys} = m_0\left(1 + \frac{m_0^2}{16\pi^2}\log\frac{\Lambda^2}{\mu^2} + \ldots\right)$$

- the counter term coefficient δe_0 cancels
  - divergence
  - μ dependence

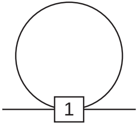
Weinberg´s power counting

Non-Renormalizable theory: expansion p^2/Λ^2_{cut}

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \ldots$$

- order 0 $\quad \mathcal{L}_0^{tree}$
- order 1 $\quad \mathcal{L}_0^{1loop} + \mathcal{L}_1^{tree} \quad \rightarrow \log(M^2/\mu^2)$
- order 2 $\quad \mathcal{L}_0^{2loops} + \mathcal{L}_1^{1loop} + \mathcal{L}_2^{tree} \quad \rightarrow \left[\log(M^2/\mu^2)\right]^2$

Weinberg´s paper: [Physica A 96 (1979) 327]



$$\Rightarrow c_2 \left[\log\left(M^2/\mu^2\right)\right]^2$$

- c_2 is completely determined by a 1 loop calculation,
- c_2 depends only on $\mathcal{L}_0$ coefficients.

Büchler´s-Colangelo paper: [arXiv:hep-ph/0309049]

- Generalization of the results to all orders $\Rightarrow c_n \left[\log\left(M^2/\mu^2\right)\right]^n$

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Alternative Proof:

- How to get LL form 1 loop calculations:

$$\mathcal{L}^{bare} = \sum_n \hbar^n \mathcal{L}_n^{bare} = \sum_n \frac{\hbar^n}{\mu^{\epsilon n}} \left[\mathcal{L}_n^{ren} + \quad \mathcal{L}_n^{div} \right]$$

- choose an operator basis

$$\mathcal{L}_n^{bare} = \sum_i \frac{1}{\mu^{\epsilon n}} \left[c_i^{(n)} + \sum_k \frac{c_{ik}^{(n)}}{\epsilon^k} \right] \cdot \mathcal{O}_i^{(n)}$$

- require μ independence

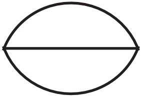
$$\frac{\partial \mathcal{L}^{bare}}{\partial \mu} = 0 \Rightarrow \frac{\partial}{\partial \mu} \left[\mu^{-\epsilon n} \left(\sum_{k=1}^{n} \frac{c_{ik}^{(n)}}{\epsilon^k} + c_i^{(n)} \right) \right] = 0 \;\; \forall \;\; \hbar^n$$

- $\Rightarrow$ set of equations for every $\hbar^n$, solved recursively example: $\mathcal{M}^{(2)}$

- pick a *complete enough* $\{O_i\}$ to describe it at this order

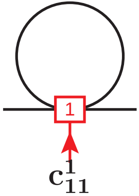
$$\rightarrow \mathcal{L}^{bare} = \sum_{n\leq 2} \frac{\hbar^n}{\mu^{\epsilon n}} \left[c_i^{(n)}(\mu) + \sum_k \frac{c_{ik}^{(n)}}{\epsilon^k} \right] \cdot \mathcal{O}_i^{(n)}$$

- check which divergences can come from having $\ell-$loops



$$\sim \frac{1}{\epsilon^2} \qquad \rightarrow \mathcal{M}_\ell^{(2)} = \sum_{k=0}^{\ell\leq 2} \frac{\mathcal{M}_{\ell k}^{(2)}}{\epsilon^k}$$

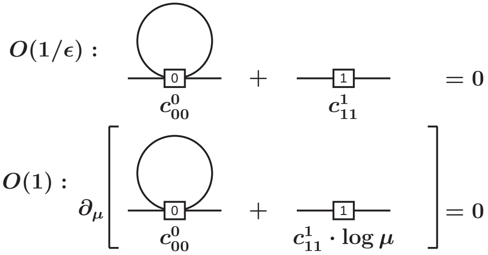
- check which divergences can come from having diverging vertices



$$c_{11}^1 \sim \frac{1}{\epsilon} \qquad \rightarrow \mathcal{M}_{\ell k}^{(2)}(\{c\}_{kj}^{(m<2)})$$

- expand $\mu^{-\epsilon} \simeq 1 - \epsilon \log \mu$ and solve recursively:

$$\hbar^1 : \mathcal{L}^{bare} = \mathcal{L}_0^{bare} + \hbar^1 \mathcal{L}_1^{bare}$$

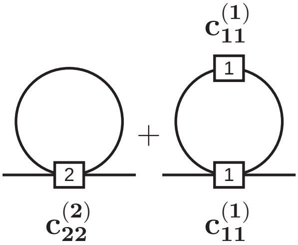


- c_{11}^1 is the coefficient of the LL
- completely determined by 1 loop calculation.

bottom line:

- LL coefficient $c_n \left[\log\left(M^2/\mu^2\right)\right]^n \Leftrightarrow c^n_{nn}$,
- $c^n_{nn} \Leftarrow 1$ loop calculations,
- substitute each vertex by corresponding c^m_{mm}.

$$\mathbf{3 \cdot c_{33}^{(3)}} \quad =$$



Generic N

- Apply this method to m_ϕ, F_ϕ, $\mathcal{A}_{\phi\phi\to\phi\phi}$, and *form factors*

- The Model: massive $O(N+1)/O(N)$ non-linear σ-model

$$\mathcal{L}_0^{N+1} = \frac{F^2}{2}\partial_\mu\Phi\partial^\mu\Phi + \underbrace{F^2\chi^T\Phi}$$

explicit $O(N+1)$
symmetry breaking
$\chi^T = (M^2, 0, ..., 0)$

- spontaneously broken down by $\langle\Phi^T\rangle = (1, 0, ..., 0)$

$$\Phi = \frac{1}{\sqrt{1+\frac{\phi\cdot\phi}{F^2}}}\begin{pmatrix} 1 \\ \frac{\phi_1}{F} \\ \vdots \\ \frac{\phi_N}{F} \end{pmatrix}$$

- if $M^2 = 0 \Rightarrow N$ GB $\phi_1, ..., \phi_N$
- if $M^2 \neq 0 \Rightarrow N$ PGB $m_\phi^2 \neq 0$.

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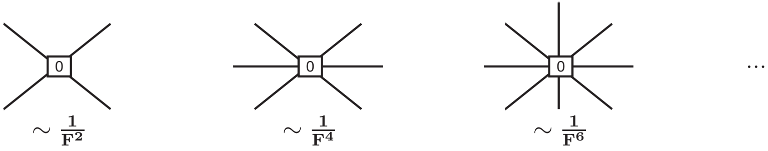
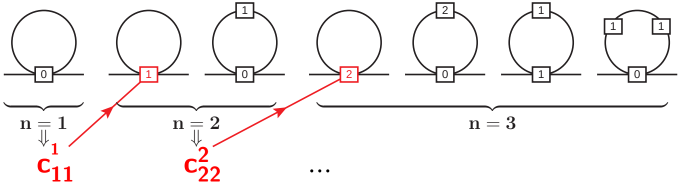
- When you expand the square root $\sqrt{1+\frac{\phi\phi}{F^2}} = 1 + \frac{1}{2}\frac{\phi\phi}{F^2} + \ldots$



- to find the LL



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Results:

- $m_\phi^2 = M^2(1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \ldots)$

i	a_i for $N = 3$	a_i for general N
1	$-1/2$	$1 - \frac{N}{2}$
2	$17/8$	$\frac{7}{4} - \frac{7N}{4} + \frac{5\ N^2}{8}$
3	$-103/24$	$\frac{37}{12} - \frac{113N}{24} + \frac{15\ N^2}{4} - N^3$
4	$24367/1152$	$\frac{839}{144} - \frac{1601\ N}{144} + \frac{695\ N^2}{48} - \frac{135\ N^3}{16} + \frac{231\ N^4}{128}$
5	$-8821/144$	$\frac{33661}{2400} - \frac{1151407\ N}{43200} + \frac{197587\ N^2}{4320} - \frac{12709\ N^3}{300} + \frac{6271\ N^4}{320} - \frac{7\ N^5}{2}$

- $F_\phi = F(1 + b_1 L_M + b_2 L_M^2 + b_3 L_M^3 + \ldots)$

i	b_i for $N = 3$	b_i for general N
1	1	$\frac{N}{2} - \frac{1}{2}$
2	$-\frac{5}{4}$	$-\frac{1}{2} + \frac{7N}{8} - \frac{3N^2}{8}$
3	$\frac{83}{24}$	$-\frac{7}{24} + \frac{21N}{16} - \frac{73N^2}{48} + \frac{1N^3}{2}$
4	$-\frac{3013}{288}$	$\frac{47}{576} + \frac{1345N}{864} - \frac{14077N^2}{3456} + \frac{625N^3}{192} - \frac{105N^4}{128}$
5	$\frac{2060147}{51840}$	$-\frac{23087}{64800} + \frac{459413N}{172800} - \frac{189875N^2}{20736} + \frac{546941\ N^3}{43200} - \frac{1169\ N^4}{160} + \frac{3\ N^5}{2}$

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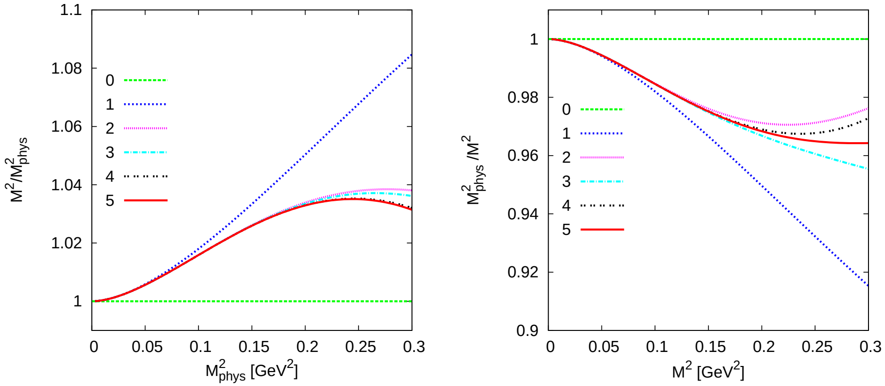
- convergence is much better for
  $M^2 = m_\phi^2(1 + b_1 L_{m_\phi} + b_2 L_{m_\phi}^2 + b_3 L_{m_\phi}^3 + \dots)$



$\mu = 1GeV$

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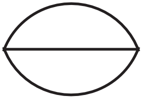
Large N limit: resumming LL

- Power counting
  - pick $\mathcal{L}$ extensive in N $\Rightarrow F^2 \sim N$
  - a vertex with $2n$ legs $\Leftrightarrow F^{2-2n} \sim \frac{1}{N^{n-1}}$
  - each loop $\Leftrightarrow N$
  - 1PI diagrams

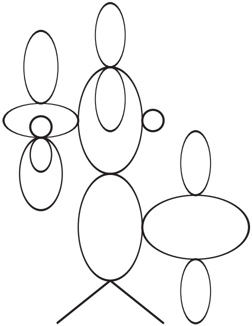
$$\left.\begin{array}{l} N_L = N_I - \sum_n N_{2n} + 1 \\ 2N_I + N_E = \sum_n 2nN_{2n} \end{array}\right\} \Rightarrow N_L = \sum_n (n-1)N_{2n} - \frac{1}{2}N_E + 1$$

  - diagram suppression factor $N^{N_L - N_E/2+1}$

- diagrams with shared lines are suppressed



- in the large N limit only "*cactus*" diagrams survive:



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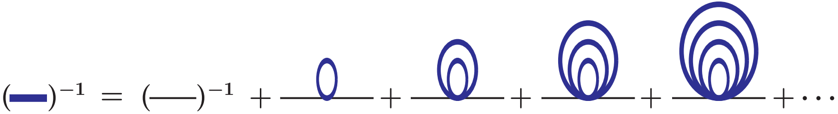
- these diagrams can all be generated recursively via Gap equation



- $\Rightarrow$ resum the series

$$M^2 = m_\phi^2 \sqrt{1 + \frac{N}{F^2} \mathcal{A}(m_\phi^2)}$$

- LL come from $\mathcal{A}(m_\phi^2) = \frac{m^2{}_\phi}{16\pi^2} \log \frac{\mu^2}{m_\phi^2}$.

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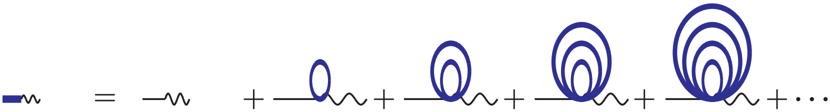
- analogously for the decay constant F_ϕ



- we can resum the series

$$\boxed{F_\phi = F\sqrt{1 + \frac{N}{F^2}\mathcal{A}(m_\phi^2)}}$$

- again LL come from $\mathcal{A}(m_\phi^2) = \frac{{m^2}_\phi}{16\pi^2}\log\frac{\mu^2}{m_\phi^2}$.

- The LL series

$$m_\phi^2 = M^2(1 + \frac{-1}{2} N L_M + \frac{5}{8} N^2 L_M^2 - N^3 L_M^3 + \frac{231}{128} N^4 L_M^4 + \frac{-7}{2} N^5 L_M^5 + ...)$$
$$M^2 = m_\phi^2(1 + \frac{1}{2} N L_{m_\phi} + \frac{-1}{8} N^2 L_{m_\phi}^2 + \frac{1}{16} N^3 L_{m_\phi}^3 + \frac{-5}{128} N^4 L_{m_\phi}^4 + ...)$$

- unfortunately the large N approximation does not work too well. compare with the generic N results:

$$\text{5-loop coeff.:} \qquad ... + 6271/320 N^4 - 7/2 N^5$$

to be negligible (10% correction) $N \sim 20$.

1. Alternative, more intuitive proof that you can get LL coefficients from 1 loop calculations
2. m_ϕ^2 and F_ϕ up to $n = 5$, work in progress for $\mathcal{A}_{\phi\phi\to\phi\phi}$ and form factors.
3. We can resum the whole series in the large N limit.

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