Chiral Perturbation Theory

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INTRODUCTION

Recent reviews on chiral perturbation theory [ChPT]:

G. Ecker:
Status of chiral perturbation theory [42 pages] 2008
http://wwwthep.physik.uni-mainz.de/$\sim$confinement8/site/

G. D'Ambrosio:
Status of Weak ChPT [42 pages] 2009
http://ific.uv.es/eft09

J. Bijnens:
Status of strong ChPT [73 pages] 2009
http://ific.uv.es/eft09

Chiral perturbation theory in the meson sector [91 pages] 2009
http://www.chiral09.unibe.ch

S. Scherer:
Baryon chiral perturbation theory [55 pages] 2009
http://www.chiral09.unibe.ch
$\Rightarrow$ And following talk

These are already more than 300 pages. Many more can easily be found on the arXiv.
Reviews are often structured as follows:

- ChPT: the Lagrangian;power counting
- evaluation of specific processes
- conclusions

At Chiral Dynamics 2009, Bijnens started with a nice historical part, where early articles are listed:

- 50, 40, 35, 30, 25, 20 and 15 years ago
- ChPT: the Lagrangian; power counting
- ...
- renormalization group $\Rightarrow$ L. Carloni, this meeting
- heavy pion ChPT $\Rightarrow$ new stuff
- conclusions

What can I add here to these (more than) 300 pages?

I decided

- to provide a supplement:

Early (and not so early) days of ChPT

[expanding on Hans Bijnens first part]

- to discuss the present status:

What we know today

- to not consider applications in nuclear physics at all. See, e.g.,

Birse; Epelbaum. Status reports at Chiral Dynamics ’09, discussion of power counting

- to concentrate basically on the meson sector

Baryons, nucleons: $\Rightarrow$ talks H.-W. Hammer, M. Hilt, S. König, S. Scherer, A. Vuorinen

η' : $\Rightarrow$ talk P. Masjuan

Content

Introduction

ChPT The early days

ChPT Not so early days

What we know today

Summary

ChPT

The early days

50 years ago

Goldberger-Treiman relation:

$$M_N g_A = F_\pi g_{\pi N}$$

Goldberger and Treiman, '58

where

g_A :	$n \to p e \bar{\nu}_e$	weak interactions
F_π :	$\pi \to \ell \nu_l$	weak interactions
$g_{\pi N}$:	$\pi p \to \pi p$	strong interactions
M_N :	nucleon mass	strong interactions

$\to$ GT-relation looks very surprising

Data around 1970:

$$\Delta_{GT} \doteq 1 - \frac{M_N g_A}{g_{\pi N} F_\pi} = 0.08 \pm 0.02$$

Pagels and Zepeda, '72

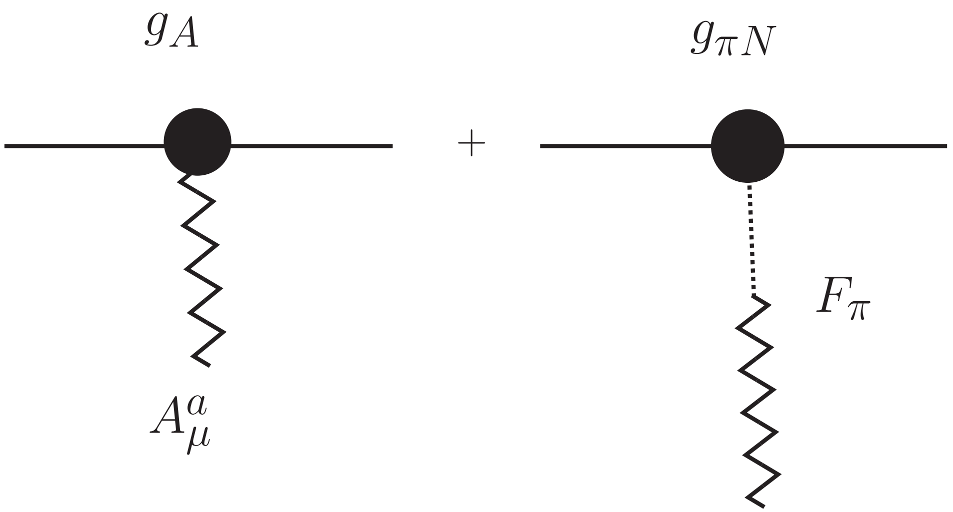
In order to better understand the origin of this relation:
Nambu '60
strong interactions have approximate chiral symmetry, which is spontaneously broken

- pions have small mass in the real world, because symmetry stays nearly exact
- GT relation is very natural in this framework

Nobel Prize 2008

Consider nucleon matrix element of Axial current

$$\langle p'|A_\mu^a(0)|p\rangle$$



Gell-Mann and Lévy ’60
Linear sigma models
Again, GT relation very natural

Nambu and Lurié ’62:
Application of Nambu’s proposal
Low energy theorems for pion scattering on nucleons. Could relate

$$\pi N \to \pi N \;\Leftrightarrow\; \pi N \to \pi\pi N$$

Emergence of current algebra (CA), PCAC and all that

Weinberg '66: $\pi\pi$ scattering

$$a_0 = \frac{7M_\pi^2}{32\pi F_\pi^2} + O(M_\pi^4) \quad = \quad 0.159 + O(M_\pi^4)$$

a_0: isospin zero, S-wave $\pi\pi$ scattering length

"Greatest defeat of S-matrix theory" Weinberg '97

How is it done? According to LSZ:

$$M(\pi\pi \to \pi\pi) \Leftrightarrow \langle\pi|T\partial^\mu \mathbf{A}_\mu^a(x)\partial^\nu \mathbf{A}_\nu^b(y)|\pi\rangle$$

Pull derivatives outside T-product. Derivatives do not commute:

$$\frac{d}{dx^0} T\mathbf{F}(x)\mathbf{G}(y) = \delta(x^0 - y^0)[\mathbf{F}(x), \mathbf{G}(y)] + T\frac{d}{dx^0}\mathbf{F}(x)\mathbf{G}(y)$$

Use CA to evaluate commutators, like

$$\delta(x^0)[\mathbf{A}_a^0(x), \mathbf{A}_b^\mu(0)] = i\epsilon_{abc}\mathbf{V}_c^\mu(x)\delta^4(x) + S.T.$$

Use assumption

- about extrapolation of matrix element with $\partial_\mu \mathbf{A}_a^\mu$ ["PCAC"]
- about group property of symmetry breaking term

PCAC: Approximation where matrix elements are evaluated in the symmetry limit

Weinberg '67:
It is much easier to use effective Lagrangians at tree level to derive PCAC+CA results, in particular, if many pions are involved

Dashen and Weinstein '69:

Chiral symmetry	$\equiv$	PCAC + current algebra	$\equiv$	effective Lagrangians

Figure à la Pagels '75

Dashen and Weinstein ’69:

$$\mathcal{L}_2 = \frac{1}{2(1+f^{-2}\vec{\pi}^2)^2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi}$$

generates all $\pi\pi \to \pi\pi, \pi\pi\pi\pi, \ldots$ amplitudes in the chiral symmetry limit, at leading order

Develop method to perform perturbation theory with

$$H = H_0 + \epsilon H_1$$

where H_0 is $SU(3) \times SU(3)$ symmetric, symmetry spontaneously broken to diagonal subgroup, and matrix elements are expanded in powers of ϵ, see below.

No Lagrangian used for this step!!

Constructing effective Lagrangians for any group

Coleman, Wess, Zumino; Callan, Coleman, Wess, Zumino ’69

On the use of effective Lagrangians

Weinstein, ’71:

“Clearly, our discussion already makes clear the fact that the Lagrangians are only reliable when used to calculate the low energy behaviour of meson amplitudes ...”

“...in general, there is no compelling reason to believe anything they have to say about terms involving the next order in momenta.”

Charap '70, '71

Considers non-linear sigma model in the chiral limit. Calculates loops, but warns the reader:

"...the loop integrations are so badly divergent as to render the theory non renormalizable, so that we cannot really believe any result of a perturbative calculation anyway."

"Even so, there may be something to be learned from the attempt to calculate higher-order terms in perturbation theory ...So, fully cognizant of the dubious significance of the calculations, we will examine the soft-pion theorem to higher orders for $\pi\pi$ scattering."

If the calculation is done correctly [Charap used a wrong measure first], then

$$
\begin{aligned}
M_\pi &= 0 \\
A(s=0,t=0,u=0) &= 0 \qquad \text{[Adler zero]}
\end{aligned}
$$

also at higher loop level.

Chiral Perturbation Theory I

Li and Pagels ’71

Perturbation Theory about a Goldstone Symmetry

(comment on Dashen/Weinstein method)

Consider

$$H = H_0 + \epsilon H_1$$

H_0 : SU(2)$\times$ SU(2)$\rightarrow$ SU(2)$_V$
H_1 : isospin symmetric
ϵ : strength of symmetry breaking

Main observation of Li and Pagels:

S-matrix elements are not analytic at $\epsilon = 0$
Standard perturbation theory breaks down

“S-matrix elements contain $\epsilon \log \epsilon$ terms”

[now known as chiral logarithms]

Proof for scalar form factor:

$$\langle\pi(p')\pi(p)|j(0)|0\rangle = F(s)\,;\; s = (p+p')^2$$

Dispersion relation:

$$F(s) = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'-s}\text{Im}F(s') = F(0)\{1+\frac{\langle r^2\rangle}{6}s + O(s^2)\} \qquad (1)$$

with

$$\langle r^2\rangle = \frac{6}{\pi F(0)}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^2}\text{Im}F(s')$$

Use unitarity near threshold:

$$\text{Im}F(s) = t_0(s)\bar{F}(s)\sqrt{1-\frac{4M_\pi^2}{s}}\,;\; t_0 : \pi\pi \to \pi\pi$$

Insert into Eq. (1)

Conclusions

- $\langle r^2 \rangle$ is logarithmically divergent as $M_\pi \to 0$
- Coefficient of logarithm is given by matrix element of symmetric Hamiltonian H_0

Questions

- Are low-energy theorems affected by these singularities? e.g., is $a_0 \simeq M_\pi^2 \log M_\pi^2$? [Later: No!]
- How can one determine the non analytic terms in general?

Li and Pagels worked out correction to GT relation in this first article:

$$\Delta_{GT} = C M_\pi^2 \log \frac{M_\pi}{\Lambda} \; ; \; C \text{ known}$$

Bace, Bég, Langacker, Li, Pagels, Pardee, Zepeda,... 1971-1975

Long series of long papers. The general method is developed and outlined in

Langacker and Pagels: Chiral perturbation Theory
Phys. Rev. D8 (1973) 4595

Technique:

- Assume quantity Q has term $\epsilon \log \epsilon$
- Then $|\frac{dQ}{d\epsilon}| \to \infty$ as $\epsilon \to 0$
- Write field theory expression for this derivative. Write dispersion relation in various channels
- Saturate with pion intermediate states. Single out divergent part

- Procedure is complicated: a mixture of Quantum Field Theory, of (formal) operator commutation relations, and of dispersion techniques. Results are (very) difficult to reproduce with this technique. E.g. chiral logarithms in GT relation

Application

- Logarithmic singularity in F_π:

$$F_\pi = F\{1 - \frac{M_\pi^2}{16\pi^2 F^2} \log \frac{M_\pi^2}{\Lambda^2} + O(M_\pi^4)\}$$

  Result agrees with modern ChPT. However: Scale Λ is not worked upon, relation to scalar radius not seen in this manner [Ward identities are not solved]

- Evaluation of corrections to various current algebra predictions

Langacker and Pagels, ’73:
“In general, we find perturbation theory around SU(2)$\times$ SU(2) to be quite good, perturbation theory around SU(3)$\times$SU(3) to be marginal.”

Further developments

Lehmann, Trute '72, '73: Evaluate $\pi\pi \to \pi\pi$ at order p^4 and vector form factor of pion at order p^2 in chiral limit, using $\mathcal{L}_2$ from page 13

Ecker and Honerkamp '72,'73: Evaluate $\pi\pi \to \pi\pi$ using superpropagators

Pagels '75:
"Departures from Chiral Symmetry" Physics Reports '75
Summary of methods used and of results obtained so far

Pagels comments on $\pi\pi \to \pi\pi$:

"If the experimental scattering lengths turn out to be much larger than the ones predicted by CA, we would have to rethink our ideas about chirality."
Is correct, sounds very modern

A quiet period for ChPT followed - presumably due to

Gross, Politzer, Wilczek ’73:
“Asymptotic freedom” Nobel Prize 2004

Fritzsch, Gell-Mann and Leutwyler ’73:
“ Advantages Of The Color Octet Gluon Picture”

The birth of QCD

Gasser and Zepeda '80 (preprint '79):
Evaluation of leading non analytic contributions (LNAC) in the quark mass expansion of particles of any spin

- framework: QCD
- technique: Rayleigh-Schrödinger perturbation theory

E.g. for a heavy fermion F, isospin I, spin s:

$$M_F^2 = M_{F0}^2 + \sigma_F + C_I \frac{G_A^2}{F^2} \sigma_\pi^{3/2} + \cdots$$

where

$$\sigma_F = \frac{1}{2s+1} \sum_{s_z} \langle F(\mathbf{p}=\mathbf{0}) | m_u \bar{u} u + m_d \bar{d} d | F(\mathbf{p}=\mathbf{0}) \rangle_0$$

and similarly for σ_π. C_I is a Clebsch-Gordan coefficient, G_A the axial coupling of F.

In addition, we realized:
A one-loop calculation with the nonlinear sigma model, including a symmetry breaking term, generates the correct LNAC in M_π, F_π

J.G. '81:

- LNAC to meson and baryon octet
- electromagnetic contributions estimated
- worked out quark mass ratios at NLO
- a pion-nucleon sigma term of 60 MeV requires the nucleon mass in the chiral limit to be 600 MeV. Would be a strange world
- there is an effective Lagrangian whose one-loop contributions reproduce the LNAC in the quark mass expansion of the baryon octet

Status around ’80

Status of calculations a la Pagels et al.

- Chiral limit is approached in non a analytic manner (“chiral logarithms”)
- In masses and decay constants, LNAC can be evaluated in a safe manner. Strength of singularity given by Clebsch Gordan coefficient of symmetric theory.

However

- Higher order terms in the quark mass expansion?
- Ward identities?
- Non analytic terms in general?

How ?

ChPT

Not so early days

This is where most talks on ChPT start

Joseph-Louis Lagrange 1736 -1813

“Phenomenological Lagrangians”

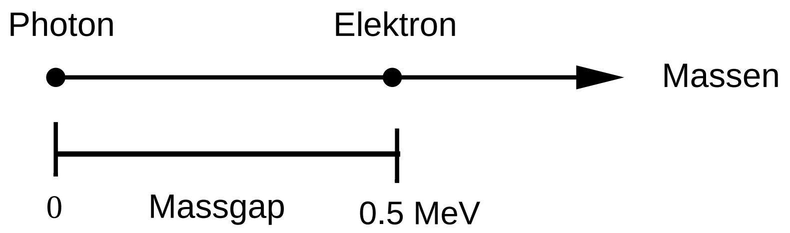
Weinberg ’79:

$$\mathcal{L}_2 = \frac{1}{2(1+f^{-2}\vec{\pi}^2)^2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi}$$

- tree graphs with $\mathcal{L}_2$ reproduce CA results for $\pi\pi \to \pi\pi, \pi\pi\pi\pi, \dots$
  $f = f_\pi$ at $m_u = m_d = 0$.
- no CA needed
- higher orders in momentum expansion: add higher derivative terms, evaluate loops with $\mathcal{L}_2 + \cdots$
- effect of loops is suppressed by powers of energy (power counting), scale is $4\pi F_\pi$
- mass terms constructed at order p^2, p^4
- renormalization discussed (is a non renormalizable QFT)
- renormalization group used to calculate leading logarithms at two-loop order in $\pi\pi \to \pi\pi$

Compare with photon–photon interactions in QED

Consider interaction between photons at very low energies



- Effective Lagrangian for photon interactions: Write all terms allowed by symmetry (gauge, Lorentz, P, C, T)

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e_1}{m_e^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{e_2}{m_e^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \cdots$$

- Amounts to an expansion in powers of ∂_μ/m_e and $F_{\mu\nu}/m_e^2$
- Scale: electron mass
- Low energy constants (LECs) e_i fixed through QED

Euler, Heisenberg '36

- $\mathcal{L}_{\rm eff}$ is a non renormalizable QFT

With properly chosen coefficients e_i, the above Lagrangian reproduces the matrix elements

$$n\gamma \rightarrow m\gamma$$

in full QED, to any order in α and in momenta/m_e (for small momenta).

$$\mathcal{L}_2 = \frac{1}{2(1+f^{-2}\vec{\pi}^2)^2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi}$$

Is Heisenberg-Euler Lagrangian for $\pi\pi$ interactions. Generates exact leading term for any number of pions: more powerful than the Heisenberg-Euler Lagrangian.

ChPT for $\pi\pi \to \pi\pi$ at any order now boils down to

- construction of the effective Lagrangian [linear algebra]
- calculation of loops [mathematics]
- determination of the low energy constants [difficult]
- comparison with data [difficult]

Weinberg: “Symmetry of the Lagrangian is symmetry of the S-matrix”

Systematizing

JG, Leutwyler '84

Consider QCD, with external c-number fields v, a, s, p:

$$e^{iZ(v,a,s,p)} = \langle O_{\rm out} | O_{\rm in} \rangle_{v,a,s,p}$$

with

$$\begin{aligned} \mathcal{L} &= \mathcal{L}^{\rm QCD}_{m_q=0} + \bar{q}\gamma^\mu (v_\mu + a_\mu \gamma_5) q - \bar{q}(s - i\gamma_5 p) q \\ \bar{q} &= (\bar{u}, \bar{d}) \end{aligned}$$

$\mathcal{L}$ invariant under

$$\begin{aligned} q &\longrightarrow \frac{1}{2}((1+\gamma_5)V_R + (1-\gamma_5)V_L)q \\ {v_\mu}' + {a_\mu}' &\longrightarrow V_R(v_\mu + a_\mu)V_R^\dagger + iV_R \partial_\mu V_R^\dagger \\ &\text{etc} \end{aligned}$$

$$V_{R,L} \in SU(2)$$

$$Z(v', a', s', p') = Z(v, a, s, p)$$

contains all Ward identities associated with vector, axial vector, scalar and pseudoscalar currents in QCD.

Effective Theory: $\mathcal{L} \to \mathcal{L}_{\rm eff}$

with

$$\begin{aligned} \mathcal{L}_{\rm eff} &= \mathcal{L}_2 + \mathcal{L}_4 + \cdots \\ \mathcal{L}_2 &= \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle \end{aligned}$$

where

$$\begin{aligned} U &= e^{i\phi/F}\,;\ D_\mu U = \partial_\mu U - i(v+a)_\mu U + iU(v-a)_\mu \\ \chi &= 2B(s+ip)\ ;\ \text{pion fields} \in \phi \end{aligned}$$

F, B low energy constants (LECs), not fixed by chiral symmetry

Tree graphs with $\mathcal{L}_2$ generate leading order term of all Green functions of vector, axial vector, scalar and pseudoscalar currents, and satisfy all Ward identities, by construction.

Higher order Lagrangians

$$\mathcal{L}_4 = \sum_{i=1}^{10} \ell_i Q_i \; ; \; \mathcal{L}_6 = \sum_{i=1}^{56} c_i P_i \; ; \; \ldots$$

- $\mathcal{L}$ describes physics of pions at low energies
- LECs ℓ_i, c_i not fixed by symmetry. Local polynomials Q_i, P_i (expressed in meson fields) are known. J.G., Leutwyler '84; Bijnens, Colangelo, Ecker '99
- Calculations with $\mathcal{L}_{\rm eff}$ generate an expansion (ChPT) in powers of quark masses and of external momenta. Effect of loops is suppressed by powers of p (power counting). Scale: $4\pi F \simeq$ 1 GeV.
- ChPT generates relations between Green functions

Examples

pion mass

$$M_\pi^2 = M^2 \left[1 - x\bar{\ell}_3 + O(x^2)\right] \; ; \; (MF)^2 = -(m_u + m_d)\langle 0|\bar{u}u|0\rangle$$

$\pi\pi$ scattering length: (isospin zero, S-wave)

$$a_0 = \frac{7}{32\pi}\frac{M_\pi^2}{F_\pi^2}\left\{1 + xL + O(x^2)\right\}$$ Isospin =0, S-wave

$$L = \frac{40}{21}\left(\bar{l}_1 + 2\bar{l}_2 - \frac{3}{8}\bar{l}_3 + \frac{21}{10}\bar{l}_4 + \frac{21}{8}\right)$$

$$x = \frac{M^2}{32\pi^2 F^2} \simeq 0.007$$

First term in a_0: Weinberg '66

$\bar{l}_i$ are renormalized LECs ; $\bar{l}_i = \log \frac{\Lambda_i^2}{M_\pi^2}$

Terms of the order x^2 are also known

Bürgi '96 [M_π]; Bijnens, Colangelo, Ecker, JG, Sainio '96 [M_π, a_0]

Advantages/disadvantages

Pros

Calculating with this Lagrangian and properly chosen LECs, one reproduces S-matrix elements of QCD, at low energy, in a systematic manner

Weinberg ’79; Leutwyler ’94

Contras

- Limited energy range of validity
- Many LECs

Note, however:

- LECs are fixed by QCD
- Can be determined
  - from experiment
  - or using lattice calculations, see below

More particles, more interactions

Can introduce more particles, more interactions

- K, η, η'
- baryons $\Rightarrow$ Scherer
- photons
- weak interactions
- resonances
- $\ldots$

effective Lagrangian is enlarged

Effective chiral Lagrangian (meson sector)

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}^{\text{odd}}_{p^4}(0) + \mathcal{L}^{\Delta S=1}_{G_F p^2}(2) + \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^0}(1)$ $+ \mathcal{L}^{\text{em}}_{e^2 p^0}(1) + \mathcal{L}^{\text{leptons}}_{\text{kin}}(0)$	$L = 0$
$+ \mathcal{L}_{p^4}(10) + \mathcal{L}^{\text{odd}}_{p^6}(23) + \mathcal{L}^{\Delta S=1}_{G_8 p^4}(22) + \mathcal{L}^{\Delta S=1}_{G_{27} p^4}(28)$ $+ \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^2}(14) + \mathcal{L}^{\text{em}}_{e^2 p^2}(13) + \mathcal{L}^{\text{leptons}}_{e^2 p^2}(5)$	$L \leq 1$
$+ \mathcal{L}_{p^6}(90)$	$L \leq 2$

LECs $\equiv$ low energy constants

in red: effective Lagrangian of early 80s

courtesy of G. Ecker

Applications

We are now at the point where we can go back to the more than 2500 articles which are concerned with aspects of ChPT

- quark mass ratios
- realization of SSB
- scattering processes
- decays: leptonic, semileptonic, non leptonic
- form factors
- precision statements about scattering lengths
- resonances in $\pi\pi$ scattering (σ, ρ, κ-pole)
- statements about structure of matrix elements (e.g. photo production of π^0)
- applications in $(g-2)_{\mathrm{Muon}}$
- extrapolations in lattice calculations ($M_\pi \to M_{\pi|\mathrm{physical}}, V \to \infty$).
- ...

Consult the reviews on page 3

What we know today

about

1. The effective Lagrangian
2. The underlying theory
3. ChPT as used in data analysis
4. Information from lattice calculations
5. Problems in sight

1. The effective Lagrangian

• Classification of the various terms: mesons, baryons (strong + weak interactions)	$\surd$
• power counting	$\surd$ ⇒talk S. Scherer
• loop calculations	$\surd$ [1,2]
• RG and its applications	substantial progress [3] ⇒talk L. Carloni
• inclusion of resonances	⇒talk S. Scherer
• LECs	$((\surd))$ [4]

[1] For a review, see Bijnens,hep-ph/0604043

[2] Ecker and Unterdorfer, Mathematica program for one-loop graphs '05

[3] Weinberg '79; Colangelo '95; Colangelo, Bijnens, Ecker '98; Büchler, Colangelo 2003; Bissegger, Fuhrer 2007; Kivel, Polyakov, Vladimirov 2008; Bijnens, Carloni 2009

[4] For reviews, see e.g. Ecker '07; Necco '08; Necco CD09 (lattice) ; Colangelo (FLAVIAnet, Bari) '09

2. The underlying theory

Information from EFT and from lattice QCD

- QCD: SU(3)$\times$ SU(3) symmetry which is spontaneously broken to SU(3) [no mathematical proof available (yet)]
- chiral limit is approached in non analytic manner
- information on quark mass ratios (EFT, lattice) and on absolute values of quark masses (lattice)
- $\langle 0|\bar{u}u|0\rangle$ is of “normal size”: Gell-Mann – Oakes-Renner picture

  Gell-Mann, Oakes and Renner ’68
- positions of resonances on second Riemann sheet in $\pi\pi \to \pi\pi$ from first principles: σ, ρ, κ

  Caprini, Colangelo, Leutwyler 2006; Moussallam 2006

Many statements on specific processes and on specific matrix elements, consult review talks on page 3. For example,

σ term of size $\sim$ 60 MeV would be very difficult to understand

3. ChPT as used in data analysis

ChPT can be used to extract interesting parameters, like CKM matrix elements, scattering lengths, from data. In particular, to perform radiative corrections, ChPT is a very convenient and systematic method [1]

[1] Cirigliano, Giannotti, Kastner, Knecht, Neufeld, Pichl, Rupertsberger, Talavera,Urech. . .

Examples

- K_{l3} decays to extract $|f_+(0)V_{us}|$
- pionium decay $|a_0 - a_2|$
- cusp analysis a_0, a_2
- K_{e4} a_0, a_2

$-\Gamma(P \to e\nu(\gamma))/\Gamma(P \to \mu\nu(\gamma))$: physics beyond SM?

$-$small quark mass, large volume extrapolations in lattice calculations

etc

4. Information from lattice calculations

Numerical simulations have made breathtaking progress in recent years

See e.g. report by Colangelo in Bari, Nov. ’09

The FLAG working group: status report

FLAG = FLAVIAnet lattice Averaging Group

FLAG aims to provide a summary of lattice results relevant for the phenomenology accessible to non-experts

- light quark masses
- LECs
- decay constants
- form factors

Will be made public soon. Watch out!

Two examples

i) Pion mass: the chiral logarithm

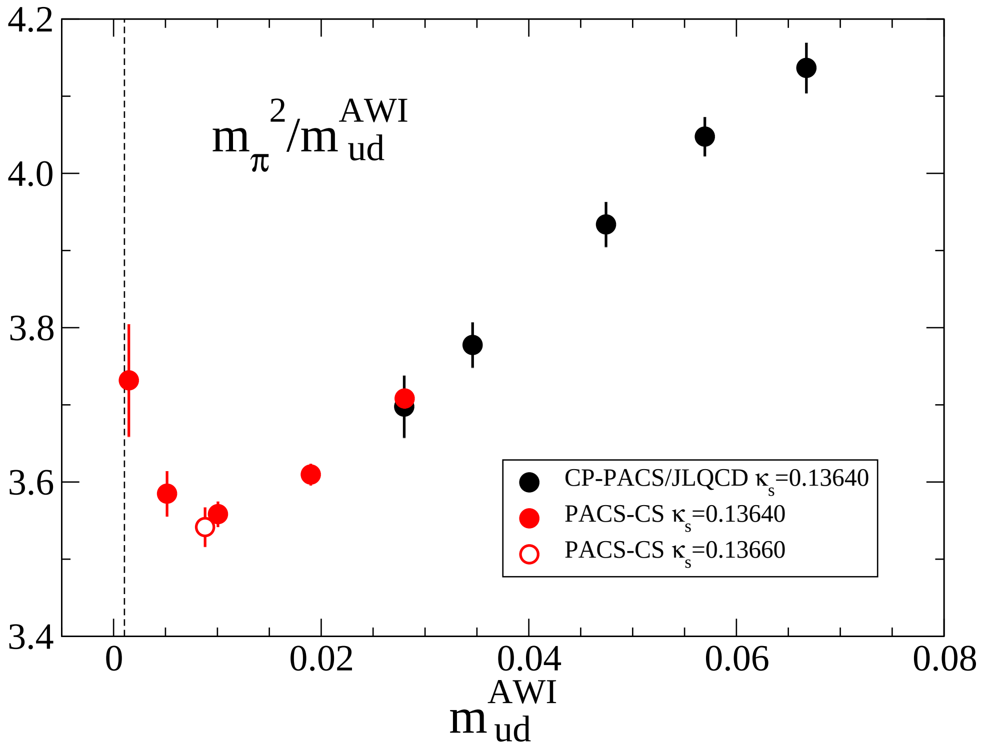


Figure taken from

S. Aoki *et al.* [PACS-CS Collaboration]
Phys. Rev. D **79** (2009) 034503
arXiv:0807.1661 [hep-lat]

slope: see p. 35

ii) The coupling $\bar{l}_3$

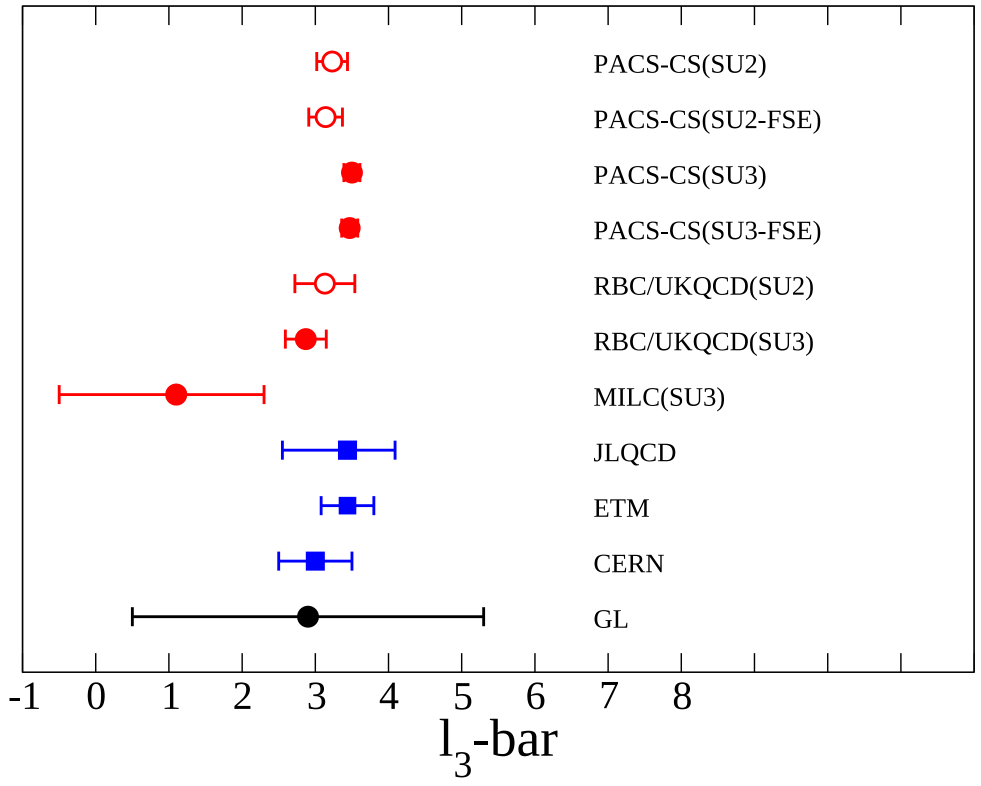


Figure taken from

S. Aoki *et al.* [PACS-CS Collaboration]
Phys. Rev. D **79** (2009) 034503
arXiv:0807.1661 [hep-lat]

5. Problems in sight

- Consensus: ChPT works well for SU(2)$\times$ SU(2)
- Seems less so for SU(3) $\times$ SU(3). Convergence?
  my attitude:
  - Meson sector: wait and see [a]
  - Baryon sector: more of a problem [which already Pagels and Langacker were aware of]
- $\eta \to 3\pi$ decays? (rate, slope α)
- Charged pion polarizabilities?
- $f_+(0)$?
- what did I miss?

---

[a] This was a good attitude in many cases in the life of ChPT. An example is the Goldberger-Treiman discrepancy:

$$\Delta_{GT} \doteq 1 - \frac{M_N g_A}{g_{\pi N} F_\pi} = 0.08 \; [1970] \text{ not understandable}$$

$$= 0.01 \; [2009] \; \surd \text{ (preliminary value)}$$

To do, meson sector

By definition, list is incomplete; order of issues is random

- read the reviews listed on page 3
- improve knowledge of LECs at order p^6 in meson sector
- update Cottingham formula for e.m. contributions$\Leftrightarrow \frac{m_d}{m_u}$
- discuss matching for $\alpha_{\rm QED} \neq 0$ [1]
- complete matching SU(2) $\times$ SU(2) $\Leftrightarrow$ SU(3)$\times$ SU(3) [2]
- matching SU(N-1) $\times$ SU(N-1) $\Leftrightarrow$ SU(N)$\times$ SU(N)
- work out manageable analytic form (for some of the) p^6 terms, e.g. $f_+(0)$ [3]
- work on hard pion ChPT [4]
- familiarize yourself with lattice calculations, read FLAG

[1] See Rusetsky for nice discussion at CD09

[2] JG, Haefeli, Ivanov, Schmid '07, '09

[3] Bijnens and Talavera '03; see Kaiser '07 for M_π^2

[4] Bijnens and Celis '09

SUMMARY

Summary

- ChPT is the effective theory of the SM. It enjoys a very healthy life
- It took a long time to reach its present perfection, both, in technical terms, and in understanding various aspects of the underlying theory
- It has proven to be an indispensable tool in many areas
- We wrote in 1984 that "...one will have to wait a long time before lattice calculations achieve the accuracy we are aiming at in ChPT"

  JG, Leutwyler '84
- It appears that we will soon arrive at this point, after 25 years

SPARES

$\pi\pi$ scattering length

Connection with the vacuum structure [simplified picture]:

Consider isospin zero S-wave scattering length in $\pi\pi$ scattering, at one loop:

$$a_0 = \frac{12M_\pi^2 - 5M^2}{32\pi F^2} + O(M^4)\,;\; M^2 = \frac{m_u + m_d}{F^2}|\langle 0|\bar{u}u|0\rangle|\,.$$

Weinberg '66

For small value of $|\langle 0|\bar{u}u|0\rangle|$, the scattering length becomes larger.