Soft-Collinear Effective Theory and B-Meson Decays

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Outline:

Factorization in QCD: Separate short- and long-distance modes

Short-distance modes:

- Heavy massive particles (e.g. electroweak gauge bosons, top quark, Higgs)
- Quantum fluctuations with large virtualities.

⇒ Dynamics encoded in short-distance coefficients (coefficient functions).
Calculable in (RG-improved) Perturbation Theory.

IR-divergences of short-distance coefficients

↑

factorization scale μ

↓

UV-divergences of operator matrix elements

Long-distance modes:

- Particles with small masses/virtualities (light quarks, gluons, photons).

⇒ Dynamics in (hadronic/partonic) matrix elements of composite operators.
Requires non-perturbative methods to deal with masses/virtualities of order $\Lambda \equiv \Lambda_{\rm QCD}$.

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QCD Factorization in *B*-decays

- We are interested in fundamental flavour parameters,
  CKM angles, quark masses, ...
  in the SM or its NP extensions.
- We have to analyze weak decays of *b*-quarks,
  but within a non-perturbative hadronic environment !

The Procedure:

- Integrate out EW gauge bosons (NP particles) at (above) the EW scale:

$\Rightarrow$ Effective Hamiltonian for

  - semi-leptonic decays ("tree")
  - non-leptonic decays, FCNCs and meson-mixings ("penguins", "boxes")

- RG-running of Wilson coefficients from $\mu \sim M_W$ to $\mu \sim m_b$
- QCD factorization to separate
  - perturbative effects (i.e. virtualities scale with $m_b \gg \Lambda_{\rm QCD}$)
  - genuine hadronic properties (i.e. universal for *B*-meson and its decay products)

QCD Factorization in B-decays

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  CKM angles, quark masses, . . .
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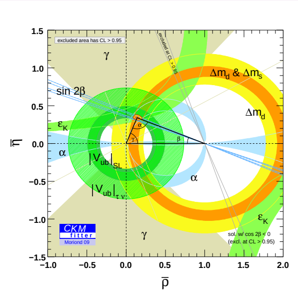
- We have to analyze weak decays of b-quarks,
  but within a non-perturbative hadronic environment !

The result: Precise determination of the CKM triangle

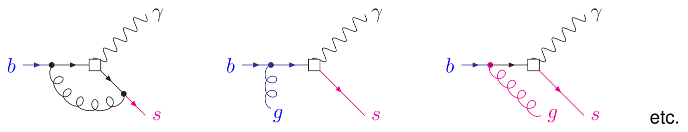


Hadronic parameters:

- decay constants
- transition form factors
- HQET parameters
- shape functions
- light-cone distribution amplitudes
- . . .

Nature of IR divergences:

- for instance, $b \to X_s \gamma$ (X_s : s-quark jet)
  - Large recoil energy: $E_X \sim m_b/2$
  - Invariant mass of "hard-collinear" jet: $p_X^2 \sim \Lambda m_b$



  - *b*-quark interactions with soft gluons described by HQET
  - interactions between soft and collinear jet modes:
    - → Sudakov logarithms in $b \to s$ form factor (involving $\ln^2 p_X^2/m_b^2$)
    - → propagation of *s*-quark in soft background described by jet function
    - → non-trivial dependence on (residual) *b*-quark light-cone momentum:
      - ⇒ *b*-quark PDF ("shape function" for inclusive decays)
      - ⇒ *B*-meson LCDA ("light-cone distribution amplitude" for exclusive decays)

Effective Field Theory Approach: QCD $\to$ SCET ($\to$ HQET)

- Introduce separate field operators for each type of (relevant) IR-mode:
  - collinear fields (light quarks and gluons/photons) for each jet direction.
  - soft fields (light quarks and gluons/photons)
  - quasi-static heavy-quark fields (with soft residual momentum)
- Construct interaction terms, performing multipole-expansion of soft-collinear vertices $\to$ SCET Feynman rules.
- Perturbative matching to QCD at hard scale $\mu_h \sim m_b$:
  $\to$ short-distance coefficient functions (still depend on jet-energy)
- RG running in SCET resums large logs between hard and jet scale (incl. Sudakov logarithms)
- matching onto HQET at jet scale yields non-local operators
  - $\to$ b-quark PDF (for inclusive B decays)
  - $\to$ B-meson LCDA (for exclusive B decays)

[Bauer/Fleming/Luke/Manohar/Pirjol/Rothstein/Stewart/ldots, 2001 -], [Chay/Kim, 2002 -], [Beneke/Diehl/Feldmann/Chapovsky/. . . , 2002 -], [Becher/Hill/Lange/Neubert/Paz/. . . , 2002 -], [. . .]

Formalism:

(see also Andre Hoang's talk . . .)

Light-cone kinematics

- Specify collinear momentum direction by light-like vectors n_+^μ and n_-^μ,
  (e.g. in rest frame or c.m.s.: $n_+^\mu = (1, 0, 0, 1)$ and $n_-^\mu = (1, 0, 0, -1)$)
- Decomposition of any Lorentz vector:

$$p^\mu = (n_+ p)\,\frac{n_-^\mu}{2} + p_\perp^\mu + (n_- p)\,\frac{n_+^\mu}{2}$$

  - collinear particles: $(n_+ p) \gg p_\perp \gg (n_- p)$
  - soft particles: $(n_+ p) \sim p_\perp \sim (n_- p)$

Physical applications require different variants of SCET

- SCET$_{\mathrm{I}}$: (inclusive reactions (jets); intermediate step for exclusive reactions)

  (hard-)collinear particles: $p_\perp^2 \sim (n_- p)(n_+ p) \sim \Lambda\,(n_+ p)$

- SCET$_{\mathrm{II}}$: (exclusive reactions)

  collinear particles: $p_\perp^2 \sim (n_- p)(n_+ p) \sim \Lambda^2$

Large and small collinear spinor components

- different derivative terms for (massless) collinear quarks scale as

$$\mathcal{L}^{\rm q}_{\rm coll.} = \bar q \left[(in_+D)\,\frac{\not n_-}{2} \;+\; i\not D_\perp \;+\; (in_-D)\,\frac{\not n_+}{2} \right] q$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

$$\mathcal{O}(n_+p) \gg \mathcal{O}(p_\perp) \gg \mathcal{O}(n_-p)$$

- $\to$ large and small spinor components

$$\xi(x) = \frac{\not n_- \not n_+}{4}\, q(x)\,, \qquad \eta(x) = \frac{\not n_+ \not n_-}{4}\, q(x)$$

- solve e.o.m. for small spinor component $\eta(x)$

$$\Rightarrow \quad \mathcal{L}^\xi = \bar\xi \left[(in_-D) + i\not D_\perp \frac{1}{(in_+D)}\, i\not D_\perp \right] \frac{\not n_+}{2}\,\xi$$

(still exact for interactions with collinear gluons)

Multipole Expansion (SCET$_{\mathrm{I}}$)

- For some directions, soft fields have larger wavelengths than collinear ones:

$$\begin{array}{llcl} \text{soft:} & (n_-x,\ x_\perp,\ n_+x) & \sim & (\ \Lambda^{-1}, \quad \Lambda^{-1}, \quad \Lambda^{-1}) \\ \text{collinear:} & (n_-x,\ x_\perp,\ n_+x) & \sim & (\ (n_+p)^{-1}, \quad (n_+p\Lambda)^{-1/2}, \quad \Lambda^{-1}) \end{array}$$

$\Rightarrow$ At soft-collinear vertices, expand all soft fields around $x_-^\mu = (n_+x)\,\frac{n_-^\mu}{2}$.

Field Redefinitions

- Leading interactions with soft gluons via

$$i(n_-D) \to i(n_-\partial) + g(n_-A_c)(x) + g(n_-A_s)(x_-)$$

- Perform *field redefinitions* for collinear quarks and gluons,

$$\xi_c(x) \to Y_s(x_-)\,\xi_c(x)\,, \qquad A_c(x) \to Y_s(x_-)\,A_c(x)\,Y_s^\dagger(x_-)$$

with soft Wilson line

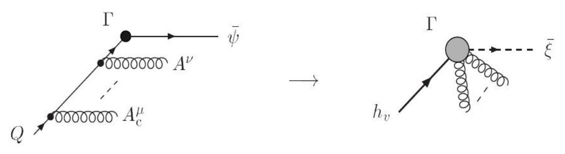
$$Y_s(x_-) = P\,e^{-ig\int_0^\infty dt\, n_-A_s(x_-+tn_-)}\,, \quad (in_-\partial + gn_-A_s)\,Y_s = 0\,.$$

$\Rightarrow$ Soft gluons decouple from collinear fields to first approximation.

Matching of external heavy-to-light currents in $\mathrm{SCET_I}$

Expansion in $1/m_b$ yields:

$$\bar\psi(x)\,\Gamma\,Q(x) \to C_\Gamma(\mu)\,(\bar\xi\,W_c)(x)\,\Gamma\,(Y_s^\dagger h_v)(x_-) + \ldots$$



- Collinear Wilson line W_c with $(in_+\partial + n_+A_c)\,W_c = 0$, from resummation of (unsuppressed) collinear gluon radiation from heavy quark.
- Wilson coefficient C_Γ absorbs short-distance corrections from virtual hard gluons.
- Soft and collinear divergences → Sudakov logarithms:

$$C_\Gamma(\mu) = 1 - \frac{\alpha_s C_F}{4\pi}\left(2\,\ln^2\frac{(n_+p)}{\mu} + \ldots\right) + \ldots$$

(sub-leading currents in the power-expansion can be identified in a similar manner)

Current renormalization in SCET

- Resum logarithms between hard scale (n_+p) and jet scale $\mu \sim |p_\perp| \sim \sqrt{\Lambda m_b}$ using renormalization group in $\mathrm{SCET_I}$

$$\frac{dC(\mu)}{d\ln\mu} = \left(\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{(n_+p)}{\mu} + \gamma(\alpha_s)\right) C(\mu)$$

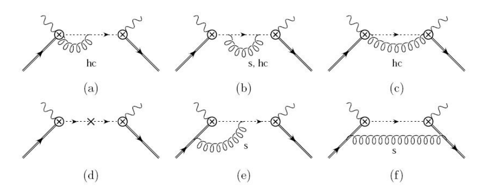
with (universal) "cusp anomalous dimension"

$$\Gamma_{\rm cusp} = 4\,\frac{\alpha_s C_F}{4\pi} + \ldots$$

Jet Function in inclusive reactions

$$J(p^2,\mu) \propto \frac{1}{\pi}\,\mathrm{Im}\left\{ i \int d^4x\, e^{-ipx}\, \langle 0|\,T\,(W_c^\dagger \xi_c)(0)\,(\bar{\xi}_c W_c)(x)|0\rangle \right\}$$

- factorize soft and (hard-)collinear corrections to propagation of energetic quark (e.g. in hadronic tensor for $b \to s\gamma$)



- one-loop result in terms of modified plus distributions:

$$J(p^2,\mu) = \delta(p^2) + \frac{\alpha_s C_F}{4\pi}\left\{ (7-\pi^2)\,\delta(p^2) - 3\left(\frac{1}{p^2}\right)_*^{[\mu^2]} + 4\left(\frac{\ln(p^2/\mu^2)}{p^2}\right)_*^{[\mu^2]} \right\}$$

- two-loop result and solution of RG-equation also known [Becher/Neubert 06]

Applications

1.) $B \to X_u \ell \nu$ $\to$ determination of $|V_{ub}|$

Factorization Theorem (leading power)

$p_X^- = E_X - |\vec{p}_X|$ spectrum in $B \to X_u \ell \nu$:

$$\frac{d\Gamma_u}{dp_X^-} \propto \int_0^1 dy\, y^{1-2a} \underbrace{H_u(y;\mu_h)}_{\text{hard function (QCD)}} U(\mu_h,\mu_i) \int_0^{p_X^-} d\hat{\omega}\, \underbrace{J\left(y m_b(p_X^- - \hat{\omega});\mu_i\right)}_{\text{jet function (SCET)}} \underbrace{\hat{S}(\hat{\omega};\mu_i)}_{\text{shape fct.}}$$

- Cut $p_X^- \leq \Delta < M_D^2/M_B \simeq 0.66$ GeV, in order to suppress charm background
- RG-evolution functions $U(\mu_h,\mu_i)$ and $a = a(\mu_h,\mu_i)$

- similar factorization theorem for $B \to X_s \gamma$ at large photon energy
- factorization at higher orders complicated by resolved photon effects [Paz/Lee/Neubert 09]

Issues:

- Perturbative calculation of hard function(s)
  NLO: [Bosch/Lange/Neubert/Paz 04; Bauer/Manohar 03; Bauer/Fleming/Pirjol/Stewart 00]
  NNLO: [Asatrian et al. 08; Beneke et al. 08; Bell 08]
- Perturbative calculation of jet function
  (massless) NNLO: [Becher/Neubert 05/06]
  (massive) NLO: [Boos/TF/Mannel/Pecjak 05; Fleming/Hoang/Mantry/Stewart 07]
- Shape-function evolution
  2-loop: [Becher/Neubert 05]
- Extracting the shape function from $B \to X_s\gamma$:
  - shape-function independent relations
    [Lange/Neubert/Paz, Lange 05; Hoang/Ligeti/Luke 05; Leibovich/Low/Rothstein 00]
  - model parametrizations and theoretical uncertainties [see below →]
- Sub-leading shape-function effects
  - classification
    [Lee/Stewart 04; Bosch/Neubert/Paz 04; Beneke et al. 04; Tackmann 05]
  - shape-function independent relations (tree-level)
    [K. Lee 08]

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The B-meson shape function (= b-quark pdf in HQET)

Definition and Properties

$$\widehat{S}(\hat{\omega} = \bar{\Lambda} - \omega) = \langle B | \bar{h}_v \, \delta(\omega - i n_- \cdot D) \, h_v | B \rangle \,, \qquad (n_- \cdot v = 1, \; \bar{\Lambda} = M_B - m_b)$$

- support $0 \leq \hat{\omega} < \infty$
- depends on renormalization scheme for b-quark mass
- radiative tail at large $\hat{\omega}$ $\Rightarrow$ positive moments diverge

Phenomenological constraints

- Moments of $B \to X_c \ell \nu$ spectra (HQET parameters $\bar{\Lambda}$, μ_π^2 $\longrightarrow$ SF scheme)
- Photon spectrum in $B \to X_s \gamma$ (through factorization formula)

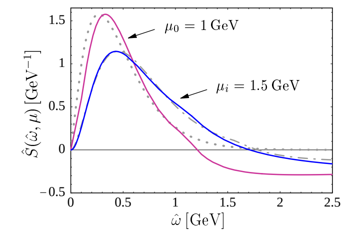
The B-meson shape function (= b-quark pdf in HQET)

Approach 1: [Becher/Lange/Neubert/Paz]

- Parametrization at low input scales, e.g.

$$S(\omega, \mu_0) = \frac{N}{\Lambda} \left(\frac{\hat{\omega}}{\Lambda} \right)^{b-1} \exp\left(-b \, \frac{\hat{\omega}}{\Lambda} \right) + \frac{\alpha_s(\mu_0)}{\pi} \times \text{[radiative tail]}$$

  - adjust to HQET parameters $\bar{\Lambda}$, μ_π^2
  - RG evolution to intermediate scale μ_i
  - compare with $B \to X_s \gamma$ spectrum
  - predict $B \to X_u \ell \nu$ spectrum



The B-meson shape function (= b-quark pdf in HQET)

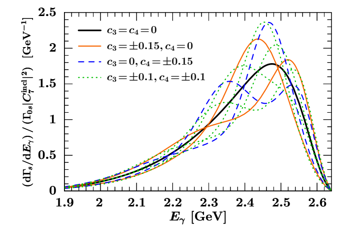
Approach 2: [Ligeti/Stewart/Tackmann, arXiv:0807.1926]

- Calculate partonic matrix element: $\widehat{S}_{\rm part.}(\hat\omega,\mu_0) = \delta(\hat\omega) + \frac{\alpha_s(\mu_0)}{\pi}\,[\ldots]$
- Generate model shape function via convolution

$$\widehat{S}(\hat\omega,\mu_0) := \int dk\,\widehat{S}_{\rm part.}(\hat\omega - k,\mu_0)\,\widehat{F}(k)$$

- $\widehat{F}(k)$ normalized to HQET parameters
- can be expanded in terms of suitable basis functions

→ systematic studies of theoretical uncertainties in global fits [. . . to be done]



Determination of $|V_{ub}|$ [update of BLNP-method, Greub/Neubert/Pecjak 09]

Values of $|V_{ub}|$ determined at NLO and NNLO. In the columns labeled $|V_{ub}|$ the first error is experimental, the second is the sum of all theoretical and parametric errors *except* for that from m_b^*, and the third is that from m_b^*.

Method	$\Delta\mathcal{B}^{\rm exp}$ $[10^{-4}]$	$|V_{ub}|$ $[10^{-3}]$ NLO	$|V_{ub}|$ $[10^{-3}]$ NNLO
$E_l > 2.1\,\text{GeV}$ CLEO	$3.3 \pm 0.2 \pm 0.7$	$3.56 \pm 0.40^{+0.48}_{-0.27}{}^{+0.31}_{-0.26}$	$3.81 \pm 0.43^{+0.33}_{-0.21}{}^{+0.31}_{-0.26}$
$E_l > 2.0\,\text{GeV}$ BABAR	$5.7 \pm 0.4 \pm 0.5$	$3.97 \pm 0.22^{+0.37}_{-0.23}{}^{+0.26}_{-0.25}$	$4.30 \pm 0.24^{+0.26}_{-0.20}{}^{+0.28}_{-0.27}$
$E_l > 1.9\,\text{GeV}$ BELLE	$8.5 \pm 0.4 \pm 1.5$	$4.27 \pm 0.39^{+0.32}_{-0.19}{}^{+0.25}_{-0.22}$	$4.65 \pm 0.43^{+0.27}_{-0.18}{}^{+0.27}_{-0.24}$
$M_X < 1.7\,\text{GeV}$ BELLE	$12.3 \pm 1.1 \pm 1.2$	$3.55 \pm 0.24^{+0.22}_{-0.13}{}^{+0.21}_{-0.19}$	$3.87 \pm 0.26^{+0.21}_{-0.13}{}^{+0.21}_{-0.19}$
$M_X < 1.55\,\text{GeV}$ BABAR	$11.7 \pm 0.9 \pm 0.7$	$3.67 \pm 0.18^{+0.29}_{-0.17}{}^{+0.26}_{-0.24}$	$3.96 \pm 0.19^{+0.20}_{-0.13}{}^{+0.26}_{-0.24}$
$P_+ < 0.66\,\text{GeV}$ BELLE	$11.0 \pm 1.0 \pm 1.6$	$3.56 \pm 0.31^{+0.30}_{-0.17}{}^{+0.27}_{-0.23}$	$3.84 \pm 0.33^{+0.21}_{-0.13}{}^{+0.26}_{-0.22}$
$P_+ < 0.66\,\text{GeV}$ BABAR	$9.4 \pm 1.0 \pm 0.8$	$3.30 \pm 0.23^{+0.27}_{-0.16}{}^{+0.25}_{-0.22}$	$3.55 \pm 0.24^{+0.19}_{-0.13}{}^{+0.24}_{-0.21}$

- NNLO effects important ($\sim$ 10% shift in $|V_{ub}|$)

2.) SCET in Exclusive B Decays

Factorization Theorems for Decay Amplitudes

[Beneke/Buchalla/Neubert/Sachrajda 99; Korchemsky/Pirjol/Yan 99; Beneke/Feldmann 00/03; Bauer/Pirjol/Stewart 01; Lunghi/Pirjol/Wyler 02; Bosch/Hill/Lange/Neubert 03; Becher/Hill/Neubert 05]

$$\begin{aligned}
\mathcal{A}_i(B \to \gamma + \text{lept.}) &= && + T_i^{\mathrm{II}}(\mu) \otimes \phi_B(\mu) \\
\mathcal{A}_i(B \to M + \text{lept.}) &= \xi_M(\mu) \cdot T_i^{\mathrm{I}}(\mu) && + T_i^{\mathrm{II}}(\mu) \otimes \phi_B(\mu) \otimes \phi_M(\mu) \\
\mathcal{A}_i(B \to MM') &= \xi_M(\mu) \cdot T_i^{\mathrm{I}}(\mu) \otimes \phi_{M'}(\mu) && + T_i^{\mathrm{II}}(\mu) \otimes \phi_B(\mu) \otimes \phi_M(\mu) \otimes \phi_{M'}(\mu)
\end{aligned}$$

- universal transition form factor ξ_M (non-perturbative input)
- two-particle LCDAs for B-meson and light hadrons M (non-perturbative input)
- perturbative coefficient functions: T_i^{I} ("non-factorizable")

  $T_i^{\mathrm{II}} = H_i \otimes J$ ("factorizable" in $\mathrm{SCET_I} \to \mathrm{SCET_{II}}$)

- Λ/m_b Power-corrections lead to more factorizable and non-factorizable terms !

Recent Perturbative Results

- NNLO vertex corrections in non-leptonic B decays (T_i^{I}):
  - imaginary part [Bell 07]
  - real part [Bell 09; Beneke/Huber/Li 09]
- NLO Spectator scattering in non-leptonic B decays (T_i^{II}):
  - tree amplitudes [Beneke/Jäger 05; Kivel 06; Pilipp 07]
  - leading penguin amplitudes [Beneke/Jäger 06]
- $\mathcal{O}(\alpha_s^2)$ corrections in $B \to V\gamma$ decays:
  - Contributions from $\mathcal{O}_7^\gamma$ and $\mathcal{O}_8^g$ [Ali/Greub/Pecjak 07]

Endpoint divergencies and non-factorizable effects

- Collinear modes in <u>exclusive</u> final state have the same virtualities (i.e. same transverse momenta) as soft spectators in B-meson.
- Dimensional regularization not sufficient to render integration over <u>individual</u> soft and collinear momentum regions IR-finite.

  $\Rightarrow$ Soft and collinear dynamics entangled in a non-factorizable manner.
- Still, in the endpoint region certain symmetry relations hold, similar to those known from the Isgur-Wise function in HQET.

$\Rightarrow$ Universal non-factorizable matrix elements in $\mathrm{SCET_I}$, e.g.

$$\langle \pi(p) | (\bar\xi_{\rm hc} W_{\rm hc}) \, \Gamma \, (Y_s h_v) | B(v) \rangle \; \propto \; \xi_\pi(n_+p, \mu) \, \mathrm{tr} \left[\frac{\not n_+ \not n_-}{4} \, \Gamma \, \frac{1+\not v}{2} \right]$$

- Corrections to symmetry relations are factorizable (!) (or power-suppressed)

  (similar statement for corrections to “naive” factorization in non-leptonic B decays)

A toy integral:

[Beneke/TF]

- Consider (UV-finite) integral in $D = 4 - 2\epsilon$ dimensions

$$\mathcal{I} = \int [\widetilde{dk}] \, \frac{1}{[(k-l)^2][k^2 - m^2][(p-k)^2 - m^2]} \,,$$

with $l^2 = m^2 \,,\ p^2 = 0 \,,$ and large momentum transfer $p \cdot l \gg m^2$

- Integral decomposes into 3 momentum regions: [see Beneke/Smirnov]

    - hard-collinear:

$$\mathcal{I}_{\rm hc} = \int [\widetilde{dk}] \, \frac{1}{[k^2 - (n_+k)(n_-l)][k^2][k^2 - (n_-k)(n_+p)]}$$
$$= -\frac{1}{(n_+p)(n_-l)} \left\{ \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2}{(n_+p)(n_-l)} + \frac{1}{2} \ln^2 \frac{\mu^2}{(n_+p)(n_-l)} - \frac{\pi^2}{12} \right\} ,$$

well-defined in dim-reg – can be reproduced by $\mathrm{SCET_I}$ Feynman rules.

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- Integral decomposes into 3 momentum regions: [see Beneke/Smirnov]
  - collinear:

$$\mathcal{I}_c = \int [\widetilde{dk}] \, \frac{[-\nu^2]^\delta}{[-(n_+k)(n_-l)]^{1+\delta} \, [k^2 - m^2][k^2 - m^2 - (n_-k)(n_+p)]}$$

$$= -\frac{1}{(n_+p)(n_-l)} \left(-\frac{1}{\delta} + \ln \frac{(n_+p)(n_-l)}{\nu^2} \right) \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right) \, ,$$

not defined in dim-reg – requires additional IR regulator (here analytic reg.)

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with $l^2 = m^2\,,\;\; p^2 = 0\,,$ and large momentum transfer $p \cdot l \gg m^2$

- Integral decomposes into 3 momentum regions: [see Beneke/Smirnov]
  - soft:

$$\mathcal{I}_s = \int [\widetilde{dk}] \, \frac{[-\nu^2]^\delta}{[(k-l)^2]^{1+\delta}\,[k^2 - m^2][-(n_- k)(n_+ p)]}$$

$$= -\frac{1}{(n_+ p)(n_- l)} \left(\left[\frac{1}{\delta} - \ln \frac{m^2}{\nu^2} \right] \left[\frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right] - \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{\mu^2}{m^2} - \frac{1}{2} \ln^2 \frac{\mu^2}{m^2} + \frac{5\pi^2}{12} \right) .$$

not defined in dim-reg – requires additional IR regulator (here analytic reg.)

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$$\mathcal{I} = \int [\widetilde{dk}] \, \frac{1}{[(k-l)^2][k^2 - m^2][(p-k)^2 - m^2]} \,,$$

with $l^2 = m^2$, $p^2 = 0$, and large momentum transfer $p \cdot l \gg m^2$

- Integral decomposes into 3 momentum regions: [see Beneke/Smirnov]
- additional IR regulator drops out in sum,

$$\mathcal{I} = \mathcal{I}_{\rm hc} + \underbrace{(\mathcal{I}_{\rm c} + \mathcal{I}_{\rm s})}_{\text{non-perturbative / non-factorizable}} + \mathcal{O}\left(\frac{m^2}{p \cdot l}\right)$$

Recent ideas to solve the issue by so-called "zero-bin" subtractions [Manohar/Stewart]
have been shown not to work consistently in exclusive decays [Beneke/Vernazza]

SCET Sum Rules for Heavy-to-light Form Factors

[De Fazio/Feldmann/Hurth 05/07]

- Consider correlation function in SCET_I :
  exclusive final state (e.g. pion) is replaced by (off-shell) interpolating current.

$\Rightarrow$ Factorization theorem for correlation function (soft $\otimes$ hard-collinear)

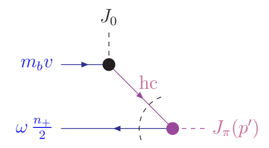
- Dispersion relation between
  - (unphysical) region of large (hc) space-like momenta
  - physical spectral function, containing the hadronic state

$\Rightarrow$ Sum rule for non-factorizable form factor in SCET_I:

$\xi_\pi(q^2, \mu)$ in terms of light-cone distribution amplitudes of B meson

- correlation function at tree level:

$$\Pi_0(n_- p') = f_B m_B \int_0^\infty d\omega \, \frac{\phi_-^B(\omega)}{\omega - n_- p' - i\eta}$$



- $\phi_-^B(\omega)$: distribution amplitude for light-cone momentum of B-meson spectator.

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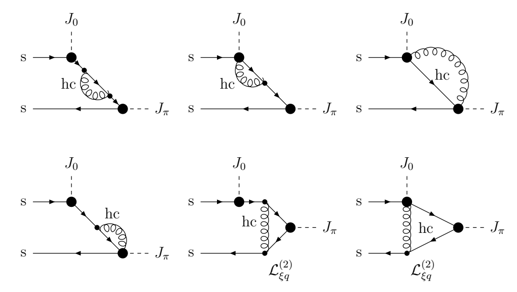
- Sum rule after continuum subtraction (ω_s) and Borel trafo (ω_M) :

$$m_b\, f_\pi\, \xi_\pi = \frac{1}{\pi} \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \mathrm{Im}\left[\Pi_0(\omega')\right]$$

[ω_s, ω_M: intrinsic sum-rule parameters to be optimized]

Radiative corrections

- Corrections involving $\phi_-^B(\omega)$:



  - Explicit calculation: Factorization works (on the level of correlator) √
  - But $\mathcal{O}(\alpha_s)$ result for *form factor* contains (non-resummed) large logarithms involving sum rule parameters. (!)

- Corrections involving 3-particle LCWF:

(only tree-level available so far [Khodjamirian/Mannel/Offen])

Predictions for non-leptonic B decays:

- Specify hadronic input (lattice, sum rules, ... or data).
- Guesstimate size of power corrections.
- Calculate corrections to naive factorization to sufficient accuracy (including renormalization running in $\mathrm{SCET_{II}}$)
- Compare with experiment. [here: for tree-dom. decays at NNLO, Beneke/Huber/Li 09]

	Theory I *(lattice,SR)*		Theory II *(data)*		Experiment
$B^- \to \pi^-\pi^0$	$5.43^{+0.06\,+1.45}_{-0.06\,-0.84}$	$(\star)$	$5.82^{+0.07\,+1.42}_{-0.06\,-1.35}$	$(\star)$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.37^{+0.86\,+1.22}_{-0.69\,-0.97}$	$(\star)$	$5.70^{+0.70\,+1.16}_{-0.55\,-0.97}$	$(\star)$	5.16 ± 0.22
$\bar{B}^0_d \to \pi^0\pi^0$	$0.33^{+0.11\,+0.42}_{-0.08\,-0.17}$		$0.63^{+0.12\,+0.64}_{-0.10\,-0.42}$		1.55 ± 0.19
$B^- \to \pi^-\rho^0$	$8.68^{+0.42\,+2.71}_{-0.41\,-1.56}$	$(\star\star)$	$9.84^{+0.41\,+2.54}_{-0.40\,-2.52}$	$(\star\star)$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38^{+0.90\,+2.18}_{-0.77\,-1.41}$	$(\star)$	$12.13^{+0.85\,+2.23}_{-0.73\,-2.17}$	$(\star)$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\rho^-$	$17.80^{+0.62\,+1.76}_{-0.56\,-2.10}$	$(\star)$	$13.76^{+0.49\,+1.77}_{-0.44\,-2.18}$	$(\star)$	15.7 ± 1.8
$\bar{B}^0 \to \pi^-\rho^+$	$10.28^{+0.39\,+1.37}_{-0.39\,-1.42}$	$(\star\star)$	$8.14^{+0.34\,+1.35}_{-0.33\,-1.49}$	$(\star\star)$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08^{+0.27\,+3.82}_{-0.19\,-3.50}$	$(\dagger)$	$21.90^{+0.20\,+3.06}_{-0.12\,-3.55}$	$(\dagger)$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.04\,+1.11}_{-0.03\,-0.43}$		$1.49^{+0.07\,+1.77}_{-0.07\,-1.29}$		2.0 ± 0.5
$B^- \to \rho_L^-\rho_L^0$	$18.42^{+0.23\,+3.92}_{-0.21\,-2.55}$	$(\star\star)$	$19.06^{+0.24\,+4.59}_{-0.22\,-4.22}$	$(\star\star)$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho_L^+\rho_L^-$	$25.98^{+0.85\,+2.93}_{-0.77\,-3.43}$	$(\star\star)$	$20.66^{+0.68\,+2.99}_{-0.62\,-3.75}$	$(\star\star)$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho_L^0\rho_L^0$	$0.39^{+0.03\,+0.83}_{-0.03\,-0.36}$		$1.05^{+0.05\,+1.62}_{-0.04\,-1.04}$		$0.55^{+0.22}_{-0.24}$

CP-averaged branching fractions in units of 10^{-6} of tree-dominated $B \to \pi\pi$, $\pi\rho$ and $\rho_L\rho_L$ decays. The first error on a quantity comes from the CKM parameters, while the second one stems from all other parameters added in quadrature. ($\star$, $\star\star$, $\dagger$: additional overall uncertainty from form factors)

Summary

SCET helps:

- to separate effects associated to different dynamical scales appearing in processes involving soft and energetic particles,
- to establish the corresponding factorization theorems,
- to define/identify process-independent non-perturbative input parameters/functions.
- to resum large logarithms in RG-improved perturbation theory.

Summary

SCET Applications:

- Inclusive B decays:
  - Factorization theorems (→ B-meson shape function)
  - Precise determination of $|V_{ub}|$ from $B \to X_u \ell \nu$ (NNLO, using HQET moments)
  - Not discussed: SM Precision Tests in $B \to X_s \gamma$ [Lee/Neubert/Paz 09]
  - Not discussed: Shape-function effects in $B \to X_s \ell^+ \ell^-$ [Lee/Ligeti/Stewart/Tackmann 06]
- Exclusive B decays:
  - Factorization theorems (→ hadronic LCDAs)
  - Endpoint divergencies → non-factorizable dynamics (→ form factor, power corr.)
  - QCD corrections to non-leptonic B decays ((N)NLO, depending on hadronic input)
  - SCET sum rules for (soft) form factors
- Collider Physics (QCD + EW): [see A. Hoang's talk]

Backup Slides:

Modified plus distributions

$$\int_{\leq 0}^{M} du\, F(u) \left(\frac{1}{u}\right)_*^{[m]} = \int_0^M du \frac{F(u) - F(0)}{u} + F(0) \ln\left(\frac{M}{m}\right),$$

$$\int_{\leq 0}^{M} du\, F(u) \left(\frac{\ln(u/m)}{u}\right)_*^{[m]} = \int_0^M du \frac{F(u) - F(0)}{u} \ln \frac{u}{m} + \frac{F(0)}{2} \ln^2\left(\frac{M}{m}\right).$$

satisfying

$$-\frac{1}{\pi}\mathrm{Im}\left[\ln\left(-\frac{u}{m}\right)\frac{1}{u}\right] = \left(\frac{1}{u}\right)_*^{[m]}$$

$$-\frac{1}{\pi}\mathrm{Im}\left[\ln^2\left(-\frac{u}{m}\right)\frac{1}{u}\right] = 2\left(\frac{\ln(u/m)}{u}\right)_*^{[m]} - \frac{\pi^2}{3}\delta(u),$$

Theoretical accuracy in $B \to X_s\gamma$ (large photon energy)

Phenomenological Importance:

- Good understanding of $d\Gamma/dE_\gamma$ important for $|V_{ub}|$ extraction (see above)
- $\Gamma(B \to X_s\gamma)$ sensitive to New Physics.

Complication:

Operators in weak effective Hamiltonian for $b \to s$ transitions contribute differently to hadronization process:

- electromagnetic operator $\mathcal{O}_7(b \to s\gamma)$
- chromomagnetic operator $\mathcal{O}_8(b \to sg)$
- 4-quark operators $\mathcal{O}_{1-6}(b \to s\, q\bar{q})$

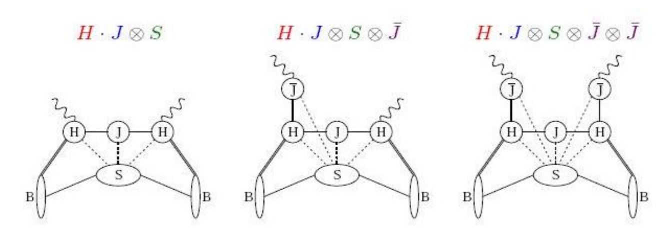
New effects at sub-leading order in $1/m_b$ expansion:

- Photon does not couple directly to short-distance $b \to s$ transition.

$\Rightarrow$ New Factorization Theorem

[Paz/Lee/Neubert 09]

Structure of New Factorization Formula



Features of "resolved" photon contribution:

- Involves new jet function $\bar{J}$ in opposite direction to X_s
- New soft functions from operators that are non-local in 2 light-cone directions
- Potential mechanism to observe CP Violation in $B \to X_s\gamma$
- Leading mechanism for Isospin Violation in $B \to X_s\gamma$
- Difficult to estimate – Vacuum Insertion Approximation $\sim 5\%$

Light-Cone Distribution Amplitudes

B-mesons:

[Grozin/Neubert 96]

- 2-particle LCDAs defined from HQET matrix elements:

$$\langle 0|\bar{q}(z)_\beta \, [z,0] \, h_v(0)_\alpha |B(v)\rangle \qquad \text{(with } z^2 = 0\text{)}$$

- 2 independent Dirac structures $\longrightarrow \phi_B^+(\omega), \phi_B^-(\omega)$, with light-cone momentum ω of the light quark (after Fourier transform.)

Properties:

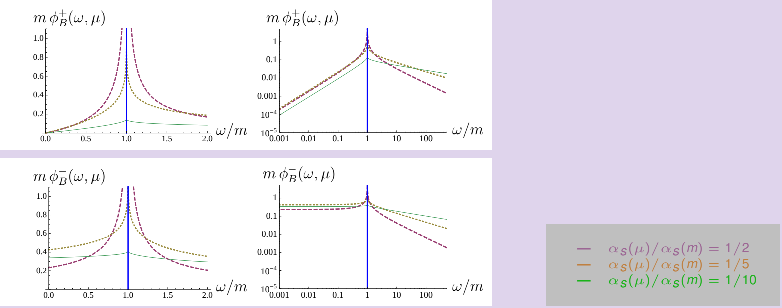
- $1/\omega$ moment of $\phi_B^+(\omega)$ relevant for leading contribution in factorization theorem.
- 1-loop evolution equation for $\phi_B^+(\omega,\mu)$ [Lange/Neubert 03]
- phenomenological parametrizations:
  - sum rules: $\langle\omega^{-1}\rangle_{\mu=1\ \mathrm{GeV}} = (2.15 \pm 0.5)/\mathrm{GeV}$ [Braun/Ivanov/Korchemsky 03]
  - moment analysis : $\langle\omega^{-1}\rangle_{\mu=1\ \mathrm{GeV}} = (2.09 \pm 0.24)/\mathrm{GeV}$ [Lee/Neubert 05]

Non-relativistic Toy-Model for B-meson LCDAs:

- Light constituent quark mass m (assumption: $m \gg \Lambda$)

$$\Rightarrow \quad \phi_B^\pm(\omega, \mu \sim m) \simeq \delta(\omega - m)$$

- Study evolution towards relativistic scales $\mu \gg m$: (WW approx. for $\phi_B^-(\omega)$)



- $\phi_B^+(\omega) \propto \omega$ for $\omega \to 0$
- $\phi_B^-(\omega) \propto \text{const.}$ for $\omega \to 0$
- radiative tail for $\phi_B^\pm(\omega)$: positive moments do not exist (analogous to SF)

[Bell/Feldmann 08, Talk by G. Bell at SCET'08]