Dimensional reduction near the deconfinement transition

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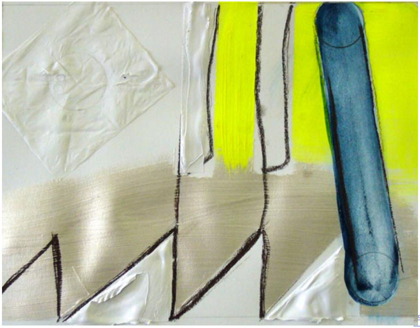
Wien 27.11.2009



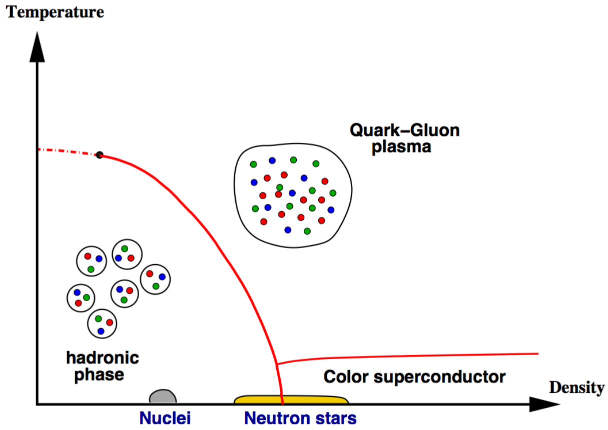
Outline

- Introduction
- Dimensional reduction
- Center symmetry

The deconfinement transition:

QCD has two remarkable properties:

- At small momenta / Large distances: Confinement
  - At $T = 0, \mu_B = 0$: $\longrightarrow$ Hadronic matter
- At large momenta / Small distances: Asymptotic freedom
  - At $T \to \infty$ and/or $\mu \to \infty$: $\longrightarrow$ Quarks and gluons



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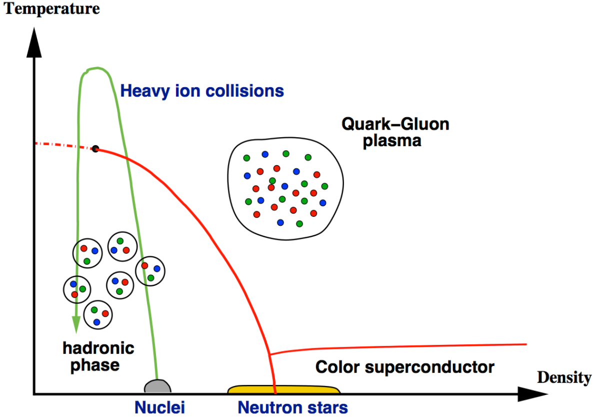
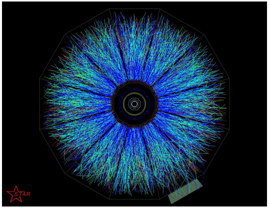
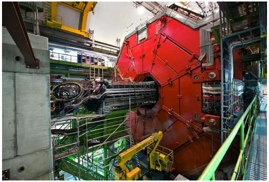






Preliminaries: Quantum fields in thermal equilibrium

Theoretical challenge:

- Fireball described by relativistic hydrodynamics

$$T^{\mu\nu} = \left[p(T) + e(T)\right]u^\mu u^\nu - p(T)g^{\mu\nu}, \quad \partial_\mu T^{\mu\nu} = 0$$

  - Hydrodynamic equations take the equation of state as input
  - $\longrightarrow$ Problem in *equilibrium thermodynamics*
- Quantities of interest:
  - Equation of state
  - Screening lengths
  - Quasi-particle spectral functions
  - Transport coefficients

Preliminaries: Quantum fields in thermal equilibrium

Thermodynamical information is encoded in the partition function:

- Can be written as a path integral (here at zero baryon number density)

$$Z = \mathrm{Tr}\,\exp\left[-\hat{H}/T\right] = \int_{\phi(0,\mathbf{x})=\phi(1/T,\mathbf{x})} \mathcal{D}\phi\,\exp(-S_E)$$

$$S_E = \int_{-\infty}^{\infty} d^3x \int_0^{1/T} d\tau\,\mathcal{L}^E$$

1/T 

- Bosons periodic b.c., fermions anti-periodic b.c.

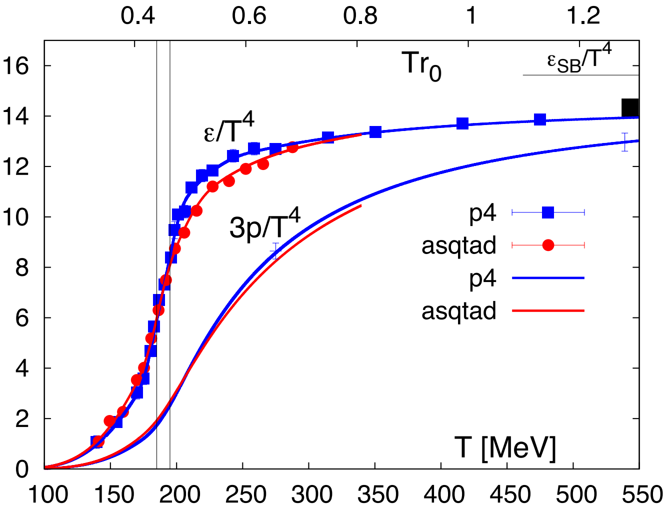
Quantites of interest:

$$p(T) = \frac{\partial(T\ln Z)}{\partial V} = T/V\ln Z$$

$$s(T) = \frac{\partial(T\ln Z)}{\partial T}$$

$$E(T) = -PV + TS$$

The lattice approach:



Bazavov et al. 0903.4379

The perturbative approach:

High T: Renormalized coupling $g(T) \sim 1/\ln T$

- Periodicity in coordinate space makes p_0 discrete

$$\frac{1}{p^2} \longrightarrow \frac{1}{\mathbf{p}^2 + \omega_n^2},$$

  - Bosons: $\omega_n = (2n)\pi T$ *Static mode:* $n = 0$
  - Fermions: $\omega_n = (2n+1)\pi T$

- 4d integrals become 3+1d sum-integrals

$$\int \frac{d^4p}{(2\pi)^4} \to T \sum_n \int \frac{d^3p}{(2\pi)^3}$$

- To get $(\ln Z)$: Sum of 1PI vacuum diagrams

The perturbative approach:

General structure of the perturbative expansion $g^2(T) \sim 1/\log(T)$:

- Leading order: Gas of non-interacting quarks and gluons

$$p_{\rm SB}/T^4 = \frac{\pi^2}{45}\left(N_c^2 - 1 + \frac{7N_cN_f}{4}\right)$$

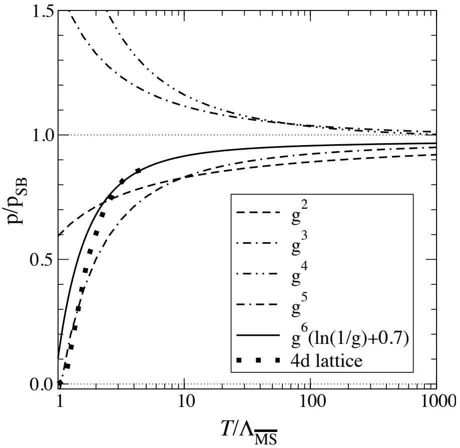
- Recipe to compute higher order corrections: Vacuum diagrams

$$p(T) \sim p_{\rm SB}(T)(1 + \#g^2 + \#g^3 + \#g^4\ln g + \#g^4 + \#g^5 + \#g^6\ln g + \ldots)$$

- Non-analytic terms originate from IR divergences of *static modes* $\rightarrow$ resummations

Note: Number of loops $\neq$ power in g^2

The perturbative approach: Kajantie et al. hep-ph/0211321



$$p(T) \sim p_{\rm SB}(T)\left(1 + g^2 + g^3 + g^4 \ln g + g^4 + g^5 + g^6 \ln g + g^6 + \ldots\right)$$

- Not straightforward to improve!! g^6-term non-perturbative.
- n-loop diagram $\sim g^6(g^2T/m)^{(4-l)}$ with $m \sim g^2T$

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Dimensional reduction

- At high T: For long distance properties ($\Delta x \gg 1/T$), the system looks 3d.

1/T

- Degrees of freedom are static modes $\phi_0(\mathbf{x})$

  $\phi(\mathbf{x}, \tau) = T \sum_{n=-\infty}^{\infty} \exp(i\omega_n \tau)\phi_n(\mathbf{x})$

- Effective action: Integrate out non-static modes

$$Z = \int \mathcal{D}\phi_0 \mathcal{D}\phi_n \exp(-S_0(\phi_0) - S_n(\phi_0, \phi_n)) \quad \text{4d theory}$$

$$= \int \mathcal{D}\phi_0 \exp(-S_0(\phi_0) - S_{\text{eff}}(\phi_0)) \quad \text{3d theory}$$

- In practice: Need a scale separation between static and non-static modes to give a truncation to the effective action

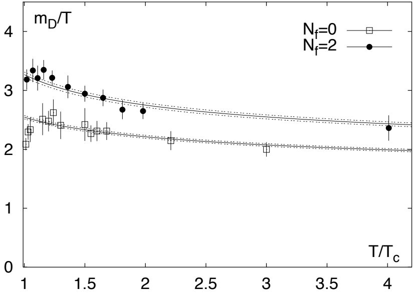
Where does Dimensional Reduction work?

Scales in hot Yang-Mills:

- Perturbatively:
  - Hard scale: $2\pi T$ Typical thermal momentum, non-static modes
  - Soft scale: $m_E \sim gT$ Debye screening, static electric modes, A_0
  - Ultra-soft scale: $m_M \sim g^2 T$ color magnetic screening, static magnetic modes, A_i

  $\Rightarrow$ Asymptotic dimensional reduction

- Non-perturbatively: $m_E(T_c) \sim 3T_c \overset{?}{\ll} 2\pi T_c$



Kaczmarek,Zantow

- No scale separtion below T_c

Electrostatic QCD, Magnetostatic QCD Braaten & Nieto

Integrate out the hard scale to get EQCD ($\Delta\mathbf{x} \gg 1/T$)
(=3d Yang-Mills + adjoint Higgs)

$$S_{\rm EQCD} = \underbrace{\frac{1}{g_3^2}}_{\sim g^2 T} \int \mathrm{d}^3x \Big[-p_E + \underbrace{\frac{1}{2}\mathrm{Tr}F_{ij}^2}_{\text{spatial gluons}} + \underbrace{\mathrm{Tr}(D_i A_0)^2}_{\text{adjoint kinetic}} + \underbrace{\frac{1}{2}\overbrace{m_{\rm E}^2}^{g^2T^2}\mathrm{Tr}A_0^2 + \frac{1}{4}\lambda_E \mathrm{Tr}A_0^4}_{\text{interactions from integration out}} + \ldots \Big]$$

- Higher order terms suppressed by powers of the scale difference
- Eff. theory parameters $g_3(T), m_E(T), \lambda_E(T)$ via perturbative matching, no resummations needed.
- After integrating out the soft modes A_0, ($\Delta\mathbf{x} \gg 1/(gT)$):

$$S_{\rm MQCD} = \frac{1}{g_3^2} \int \mathrm{d}^3x \left[\frac{1}{2}\mathrm{Tr}F_{ij}^2\right]$$

- Philosophy: Integrate out heavy modes analytically, simulate low-energy theory numerically.

Example: g^6-coefficient from MQCD

The g^6-resummation can be done *numerically* in eff. theory framework:

- Match QCD $\rightarrow$ EQCD to sufficient depth in g
  - $g_3^2(T) \ = T(\#g^2 + \#g^4)$
  - $m_E^2(T) = T^2(\#g^2 + \#g^4)$
  - $\lambda_E(T) \ = \#g^4$
  - $p_E(T) \ = T^4(1 + \#g^2 + \#g^4 + \#g^6)$
    - Requires a 4-loop computation in full QCD
    - The matching coefficient at g^6 known only in the $N_f \rightarrow \infty$ limit

    Ipp&Rebhan hep-ph/0305030; Gynther, AK, Vuorinen 0909.3521
- Match EQCD $\rightarrow$ MQCD to sufficient depth in g

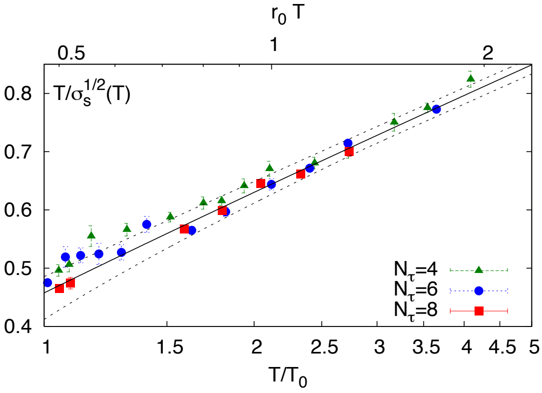
  Kajantie et al. hep-ph/0211321
- Match the 3d continuum $\overline{\text{MS}}$ theory with lattice theory to order a^0
  - done using 4-loop Numerical Stochastic Perturbation Theory

  Di Renzo et al. 0808.0557
- Measure pressure of lattice theory numerically

  Hietanen et al. hep-lat/0509107, Hietanen & AK hep-lat/0609015

Example: Spatial string tension from MQCD

Spatial string tension (magnetic screening) to $\sim T_c$



Karsch et al., 0806.3264

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Center symmetry

At finite T the full symmetry group of Yang-Mills:

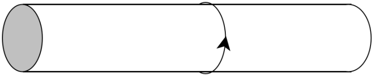
- Gauge symmetry

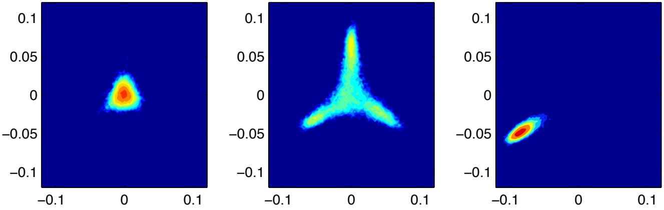
$$A_\mu \longrightarrow \lambda(\tau,\mathbf{x})(A_\mu + i\partial_\mu)\lambda(\tau,\mathbf{x})^{-1}, \quad s \in \mathrm{SU}(N_\mathrm{c})$$

- *Global* center symmetry

$$\lambda(\tau + 1/T, \mathbf{x}) = z\lambda(\tau,\mathbf{x}), \qquad z \in Z_{N_\mathrm{c}}$$

Order parameter: $\Omega(\mathbf{x}) = \mathrm{Tr}W(\mathbf{x}) = \mathrm{Tr}\left[P\exp\left(ig\int \mathrm{d}\tau A_0(\tau,\mathbf{x})\right)\right]$

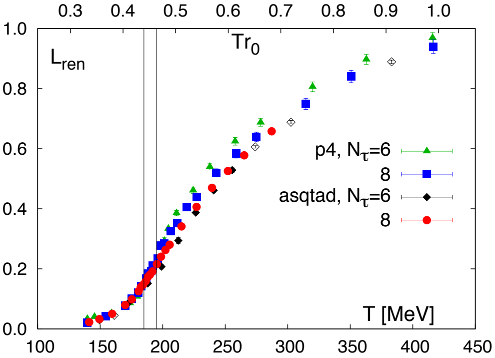




$T < T_c$ $\quad$ $T \approx T_c$ $\quad$ $T > T_c$

Light fermions break the center symmetry softly

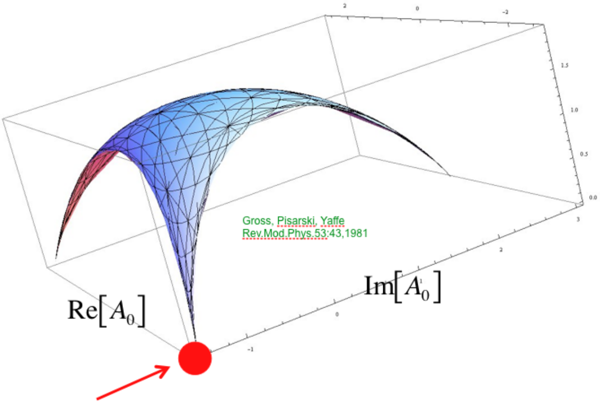
- Polyakov-loop still good *approximate* order parameter:



Bazavov et al. 0903.4379

Center symmetry

Effective potential in perturbation theory:



- EQCD obtained by expanding A_0 around *one* of the N_c deconfining minima.
  - EQCD breaks the center symmetry explicitly.
    - No chance for correct phase structure
    - Quantities sensitive to A_0 not well described near T_c

Center-symmetric effective theories

- Goal: Want to construct an effective theory that
  - Preserves the Z_N center symmetry
  - Reduces to EQCD at high T
  - Is superrenormalizable
- Effective theory of Wilson lines not (super)renormalizable

  Pisarski hep-ph/0608242 $\sim$ non-linear sigma model

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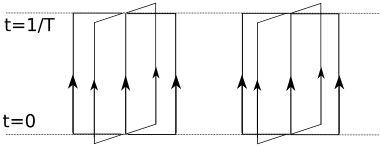
Idea: Construct effective theory for *coarse grained* Wilson loop

Vuorinen&Yaffe hep-ph/0604100

$$\mathcal{Z}(\mathbf{x}) = \frac{T}{V_{\text{Block}}} \int_V \mathrm{d}^3 y U(\mathbf{x},\mathbf{y}) W(\mathbf{y}) U(\mathbf{y},\mathbf{x}), \notin SU(N_c)$$



Center-symmetric effective theory for SU(2)

de Forcrand, AK, Vuorinen arXiv:0801.1566

- For SU(2), sum of matrices proportional to SU(2)

$$\mathcal{Z} = \lambda\Omega, \quad \Omega \in \mathrm{SU}(2), \quad \lambda > 0$$

$$\mathcal{Z} = \frac{1}{2}\Big\{ \underbrace{\Sigma \mathbb{1}}_{\text{Singlet}} + i \underbrace{\Pi_a \sigma_a}_{\text{Adjoint scalar}} \Big\} = \begin{pmatrix} \frac{1}{2}\Sigma + i\Pi_1 & i\Pi_2 - \Pi_3 \\ i\Pi_2 + \Pi_3 & \frac{1}{2}\Sigma - i\Pi_1 \end{pmatrix}$$

- Transforms exactly like Wilson line

$$\begin{aligned} \mathcal{Z} &\longrightarrow \lambda^{-1}(\mathbf{x})\mathcal{Z}\lambda(\mathbf{x}) && \text{gauge} \\ \mathcal{Z} &\longrightarrow -\mathcal{Z} && \text{center } Z_2 \end{aligned}$$

- ...but note: (3 d.o.f in $W = e^{igTA_0}$) $\neq$ (4 d.o.f in $\mathcal{Z}$)

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- Most general superrenormalizable Lagrangian with A_i and $\mathcal{Z}$:

$$\mathcal{L}_{Z(2)} = \underbrace{\frac{1}{2}\mathrm{Tr}\, F_{ij}^2}_{\text{spatial gluons}} + \underbrace{\mathrm{Tr}\left(D_i \mathcal{Z}^\dagger D_i \mathcal{Z}\right)}_{\text{Adjoint Kinetic}} + V(\mathcal{Z})$$

$$V(\mathcal{Z}) = \underbrace{b_1 \Sigma^2 + b_2 \Pi_a^2 + c_1 \Sigma^4 + c_2 (\Pi_a^2)^2 + c_3 \Sigma^2 \Pi_a^2}_{\text{interactions from integration out}}$$

- Higher order terms suppressed by scale difference m_E/T.

Perturbative matching of the parameters

Parameters of the effective theory $\{g_3, b_1, b_2, c_1, c_2, c_3\}$ can be matched to the full theory parameters $\{g, T\}$ at high T:

- Split the potential in to “hard” and “soft” $V = V_h + g_3^2 V_s$
- Hard potential describes $\sim T$ scales, coarse graining $\rightarrow$ Forces $\mathcal{Z} \in \mathrm{SU}(2)$

$$V_h = h_1 \mathrm{Tr}\,(\mathcal{Z}^\dagger \mathcal{Z}) + h_2 (\mathrm{Tr}\,\mathcal{Z}^\dagger \mathcal{Z})^2$$

- Soft potential encodes the physics of small fluctuations, EQCD

$$V_s = s_1 \mathrm{Tr}\,\Pi^2 + s_2 \left(\mathrm{Tr}\,\Pi^2\right)^2 + s_3 \Sigma^4$$

Matching at $T \to \infty$

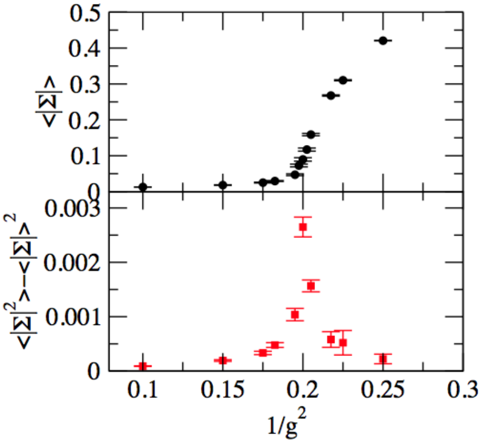
Parameters can be mached in perturbation theory (series in $\frac{g^2(7T)}{16\pi^2}$!):

$$
\begin{aligned}
b_1 &= -\frac{1}{4} r^2 T^2, \\
b_2 &= -\frac{1}{4} r^2 T^2 + 0.441841 g^2 T^2, \\
c_1/g_3^2 &= 0.0311994 r^2 + 0.0135415 g^2, \\
c_2/g_3^2 &= 0.0311994 r^2 + 0.008443432 g^2, \\
c_3/g_3^2 &= 0.0623987 r^2, \\
g_3^2 &= g^2 T
\end{aligned}
$$

- Parameters functions of full theory parameters (g, T) and r
  - rT: mass of fluctuation away from SU(2) manifold
- rT = Cutoff of the effective theory
- "continuum limit" $= r \to \infty$

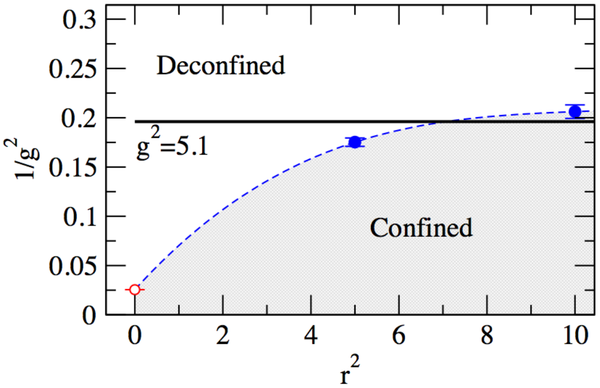
Results from simulations:

Z_2-restoring phase transition



$$\beta = 6, n = 64, r^2 = 5$$

Results from simulations:



- Phase diagram resembles the full theory (unlike in EQCD).
- Insensitive to $r > 1$
- Phase transition at correct $g(T)$!

Effective theory for SU(3)

- For SU(3) no special relations
  $\Rightarrow$ degree of freedom $\mathcal{Z} \in$GL(3,$\mathbb{C}$)

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}F_{ij}F_{ij} + \mathrm{Tr}(D_i Z^\dagger D_i Z) + V_0(Z) + g_3^2 V_1(Z)$$

with

$$\begin{aligned} V_0 &= c_1 \mathrm{Tr}\mathcal{Z}^\dagger\mathcal{Z} + c_2\left(\det\mathcal{Z} + \det\mathcal{Z}^\dagger\right) + c_3\mathrm{Tr}(\mathcal{Z}^\dagger\mathcal{Z})^2 \\ g_z^2 V_1 &= d_1\mathrm{Tr}M^\dagger M + d_2\mathrm{Tr}(M^3 + M^{\dagger 3}) + d_3\mathrm{Tr}(M^\dagger M)^2 \end{aligned}$$

where $M = \mathcal{Z} - \frac{1}{3}\mathbb{1}\mathrm{Tr}\mathcal{Z}$

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  where $M = \mathcal{Z} - \frac{1}{3}\mathbb{1}\mathrm{Tr}\mathcal{Z}$
- The operator list is not exhaustive
- Similar splitting of action
  - Hard potential keeps $\mathcal{Z}$ near unitary, has superfluous symmetry
  - Soft potential encodes gT physics

Summary

- Dimensional reduction provides a bridge between lattice computations and perturbation theory in the deconfined phase.
- In DR setup, one can analytically deal with the heavy modes (non-static bosons and fermions) to get a theory which is more amenable to numerical simulations.
- Incorporating the (approximate) center symmetry to EQCD leads to correct phase diagram
- Lots of things to do:
  - Check accuracy near T_c:
    - Domain wall tension
    - Spatial string tension
    - Screening masses
  - Make predictions:
    - Quarks: Z_N breaking terms
    - Finite chemical potential (Correct phase transitions?)