

QCD at finite temperature and density in the strong coupling limit

Philippe de Forcrand
ETH Zürich and CERN

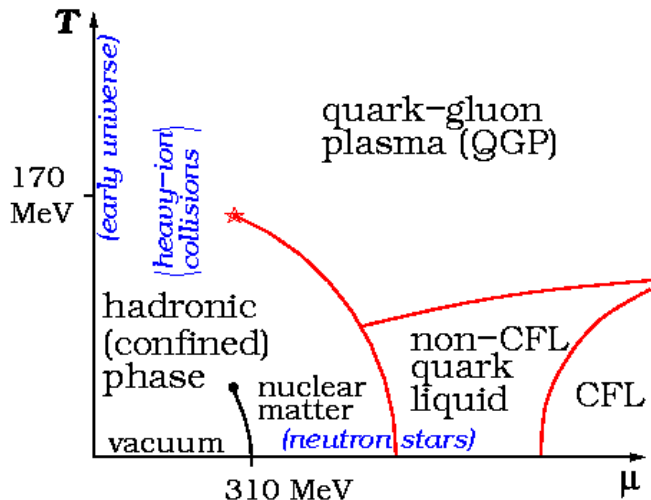
with Michael Fromm (ETH)

arXiv:0811.1931
and in progress

ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

QCD phase diagram according to Wikipedia



This talk is about the **hadron** \leftrightarrow **nuclear matter** transition

Motivation (1)

Strong coupling LQCD: why bother ?

Asymptotic freedom: $a(\beta_{\text{gauge}}) \propto \exp\left(-\frac{\beta_{\text{gauge}}}{4N_c b_0}\right)$

ie. $a \rightarrow 0$ when $\beta_{\text{gauge}} \equiv \frac{2N_c}{g^2} \rightarrow +\infty$. Here $\beta_{\text{gauge}} = 0$:

- Lattice “infinitely coarse”
- Physics not universal

Nevertheless:

- Properties similar to QCD: **confinement** and χ_{SB}
- Include (perhaps) next term in strong coupling expansion, ie. $\beta_{\text{gauge}} > 0$
- When you can't find the answer to a burning question,

change the question

When $\beta_{\text{gauge}} = 0$, sign problem is **manageable** \rightarrow **full phase diagram**

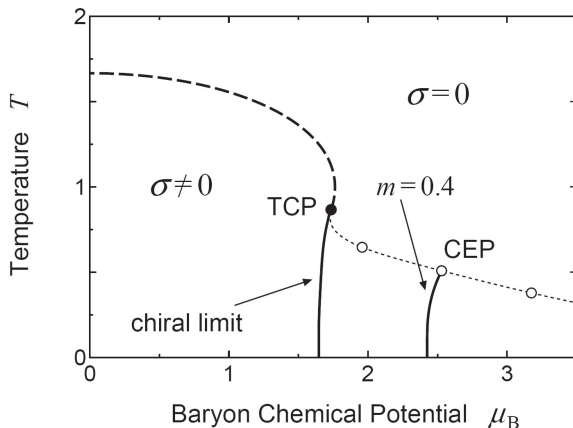
Motivation (2)

- 25⁺ years of analytic predictions:
 - 80's: Kluberg-Stern et al., Kawamoto-Smit, Damgaard-Kawamoto
 $T_c(\mu = 0) = 5/3, \mu_c(T = 0) = 0.66$
 - 90's: Petersson et al., $1/g^2$ corrections
 - 00's: detailed (μ, T) phase diagram: Nishida, Kawamoto,...
 - 08: Ohnishi, Münster & Philipsen,...

How accurate is mean-field ($1/d$) approximation?
- Almost no Monte Carlo crosschecks:
 - 89: Karsch-Mütter \rightarrow MDP formalism $\rightarrow \mu_c(T = 0) \sim 0.63$
 - 92: Karsch et al. $T_c(\mu = 0) \approx 1.40$
 - 99: Azcoiti et al., MDP ergodicity ??
 - 06: PdF-Kim, HMC \rightarrow hadron spectrum $\sim 2\%$ of mean-field

Can one trust the details of analytic phase-diagram predictions?

Phase diagram according to Nishida (2004)



- Very similar to conjectured phase diagram of $N_f = 2$ QCD
- But no deconfinement here: high density phase is **nuclear matter**
- Baryon mass = $M_{\text{proton}} \Rightarrow$ lattice spacing $a^{-1} \sim 300$ MeV
- What happens to hadron \leftrightarrow nuclear matter transition as m_π is varied?

Strong coupling $SU(3)$ with staggered quarks

$Z = \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp(-\bar{\psi}(\not{D}(U) + m)\psi)$, no plaquette term ($\beta_{\text{gauge}} = 0$)

- One colored fermion field per site (6 d.o.f. – no Dirac indices)
- $\not{D}(U) = \frac{1}{2} \sum_{x,v} \eta_v(x) (U_v(x) - U_v^\dagger(x - \hat{v}))$, $\eta_v(x) = (-)^{x_1 + \dots + x_{v-1}}$

$U(1)_V \times U(1)_A$ symmetry:

$$\left. \begin{array}{l} \psi(x) \rightarrow e^{i\theta} \psi(x) \\ \bar{\psi}(x) \rightarrow e^{-i\theta} \bar{\psi}(x) \end{array} \right\} \text{unbroken} \Rightarrow \text{quark number} \Rightarrow \text{chem. pot.}$$

$$\left. \begin{array}{l} \psi(x) \rightarrow e^{i\varepsilon(x)\theta} \psi(x) \\ \bar{\psi}(x) \rightarrow e^{i\varepsilon(x)\theta} \bar{\psi}(x) \\ \varepsilon(x) = (-)^{x_0 + x_1 + x_2 + x_3} \end{array} \right\} \text{spont. broken } (m = 0) \Rightarrow \text{quark condensate}$$

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- Chemical potential $\mu \rightarrow \exp(\pm a\mu) U_{\pm 4}$
- **Alternative 1: integrate over fermions**

$$Z = \int \mathcal{D}U \det(\not{D}(U) + m) \rightarrow \text{HMC, etc...}$$

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- **Alternative 2: integrate over links**

Rossi & Wolff

\rightarrow **Color singlet** degrees of freedom:

- **Monomer** (meson $\bar{\psi}\psi$) $M(x) \in \{0, 1, 2, 3\}$
- **Dimer** (meson hopping), non-oriented $n_v(x) \in \{0, 1, 2, 3\}$
- **Baryon** hopping, oriented $\bar{B}B_v(x) \in \{0, 1\} \rightarrow$ self-avoiding loops C

Point-like, hard-core baryons in pion bath

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$$Z(m, \mu) = \sum_{\{M, n_v, C\}} \prod_x \frac{m^{M(x)}}{M(x)!} \prod_{x,v} \frac{(3 - n_v(x))!}{n_v(x)!} \prod_{\text{loops } C} \rho(C)$$

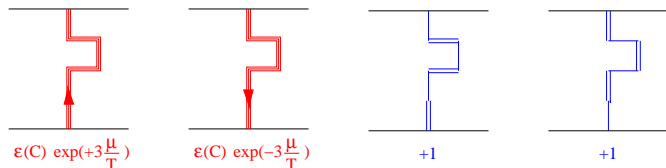
with **constraint** $(M + \sum_{\pm v} n_v)(x) = 3 \quad \forall x \notin \{C\}$

MDP Monte Carlo

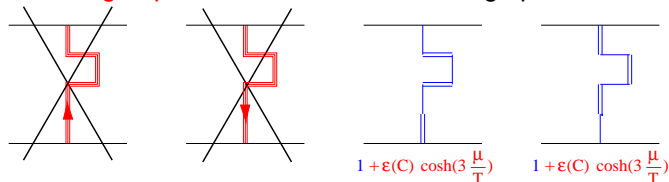
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- **sign** of $\prod_C \rho(C)$: geometric factor $\varepsilon(C) = \pm 1$ for each loop C ; 4 types:



Karsch & Mütter: Regroup into “MDP ensemble” → sign pb. **eliminated** at $\mu = 0$



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 → “MDP ensemble”

Further difficulties:

- changing **monomer number** difficult: weight $\sim m^{\sum_x M(x)}$
 monomer-changing update (**Karsch & Mütter**) restricted to $m \sim o(1)$

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Solved with **worm algorithm** (**Prokof'ev & Svistunov**)

Worm algorithm for MDP

Here for chiral limit $m = 0$ (no monomers: $M(x) = 0 \forall x$)

- Break a dimer bond and introduce a pair of adjacent monomers $M(x), M(y)$
- Choose among neighbours of y by local heatbath and move $M(y)$ there
 heatbath: sampling of 2-point function $\frac{1}{Z_{||}} M(x)M(y) \exp(-S_{||})$
- Keep moving “head” y until $y \rightarrow x$, ie. “worm closes” \rightarrow new configuration in $Z_{||}$

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Global change obtained from sequence of local updates

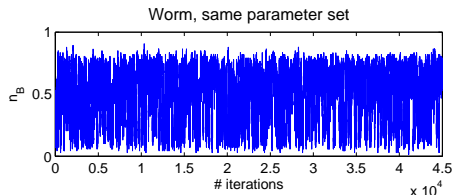
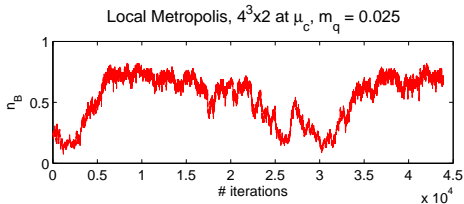
Each local step gives information on 2-point function

cf. Adams & Chandrasekharan for $U(N)$

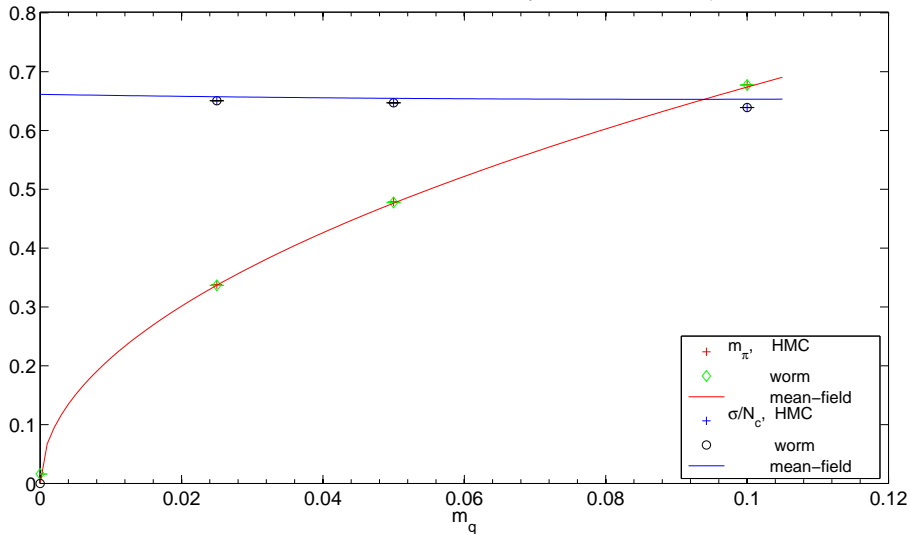
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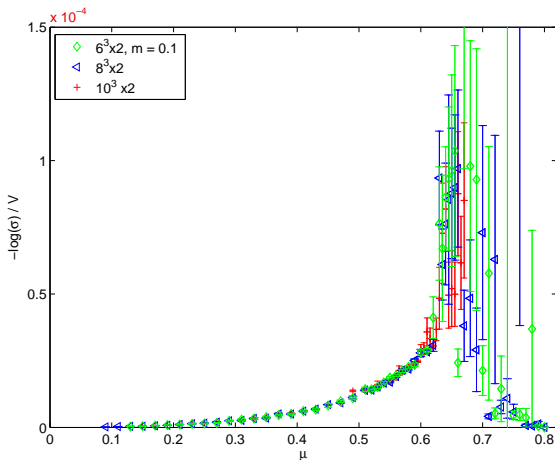
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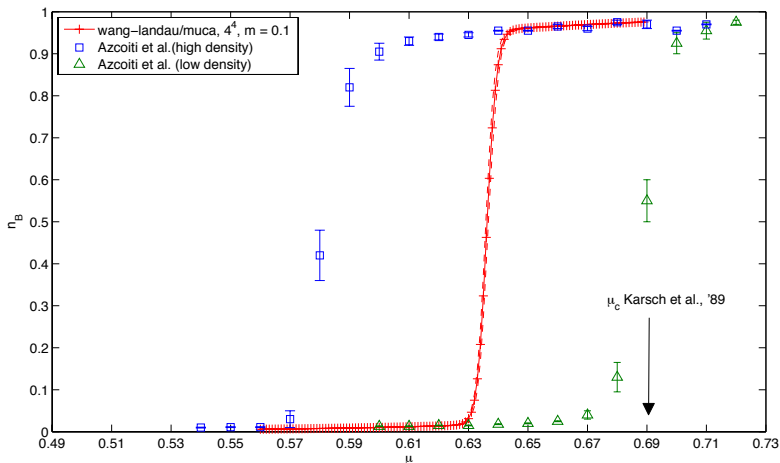


[Non-trivial] consistency check with HMC

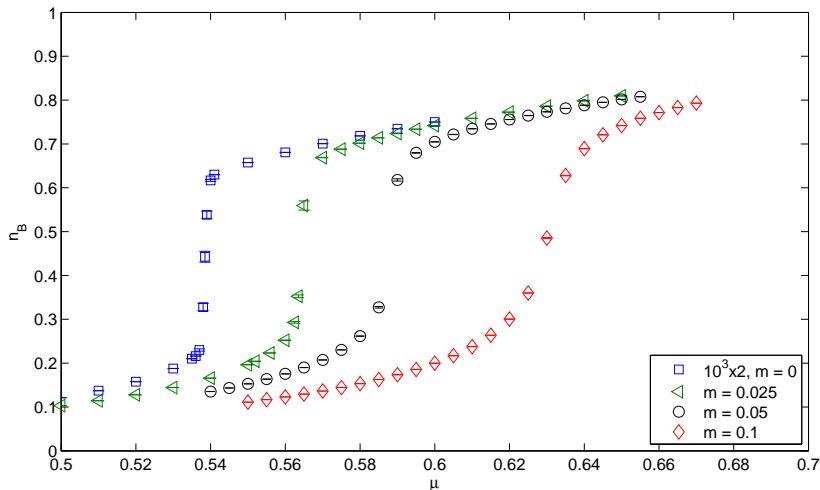
Worm-MDP vs. HMC (Forcrand and Kim '06) $\beta = 0$, same volume ($\mu = T = 0$)

Sign problem? Monitor $-\frac{1}{V} \log \langle \text{sign} \rangle$ 

- $\langle \text{sign} \rangle = \frac{Z_{\parallel}}{Z} \sim \exp(-Vf(\mu^2))$ as expected
- Can reach $\sim 16^3 \times 4 \forall \mu$, ie. adequate

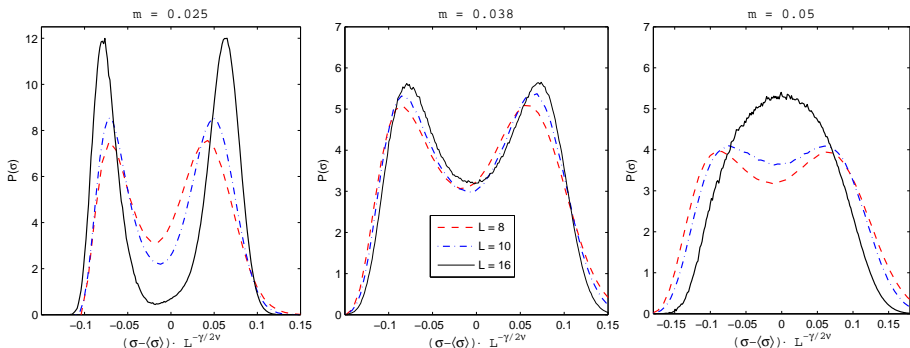
Consistency check with Karsch & Mütter: $T = 1/4, m = 0.1$ 

- High-density phase: **saturation** at 1 baryon per site (cf. nuclear matter)
- Azcoiti was right: ergodicity restored only with multicanonical (Wang-Landau)
- Karsch & Mütter value too large; obtained from metastable branch ?

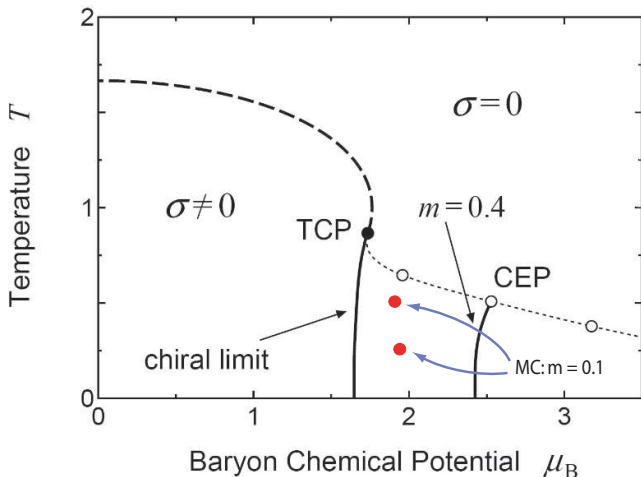
Varying the quark mass at $T = 1/2$ fixed

As $m \rightarrow 0$, μ_c decreases and transition becomes stronger

critical mass m_c ?

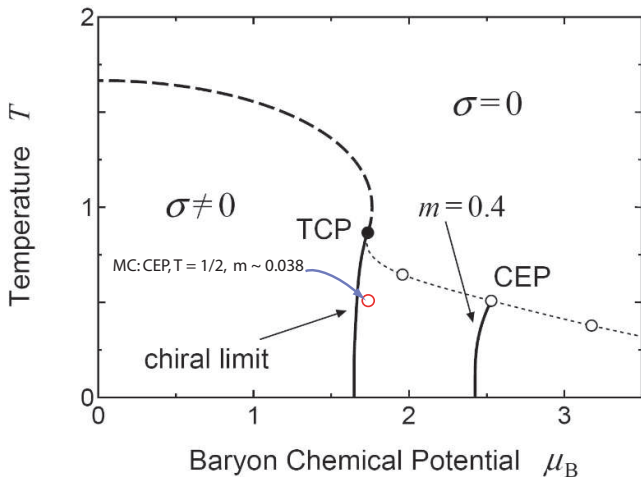
Critical mass $m_c(T = 1/2)$?Distribution of quark condensate: **finite-size scaling** ?

- **Critical mass:** $m_c(T = 1/2) \sim 0.038$
- **Universality class:** γ/ν consistent with 3d Ising

Compare with Nishida (2004): transition line for $m = 0.1$ 

Clausius-Clapeyron: $\frac{dT}{d\mu} = -\frac{\Delta n}{\Delta S} \rightarrow$ vertical at $T = 0$ (non-deg. ground-state)

- Nishida: at low T , $S|_{\text{high}} < S|_{\text{low}}$, because $n_B|_{\text{high}} \approx$ saturation
- sensitive to $n_B|_{\text{low}}$ vs $(1 - n_B|_{\text{high}})$: not borne out by MC

Compare with Nishida (2004): CEP at $T = 1/2$ 

$m_{CEP} \approx 0.038$ vs prediction ≈ 0.4 : **wrong by $O(10)$!**

Beware of quantitative mean-field predictions for phase diagram

Transition to nuclear matter: $T = 0, \mu = \mu_c$

Puzzle:

- Mean-field baryon mass is $\approx 3 \Rightarrow$ expect $\mu_c = \frac{1}{3} F_B(T=0) \approx 1$
- Mean-field estimate $\mu_c \sim 0.55 - 0.66$ much smaller, ie. $\mu_c^B \sim 600$ MeV !

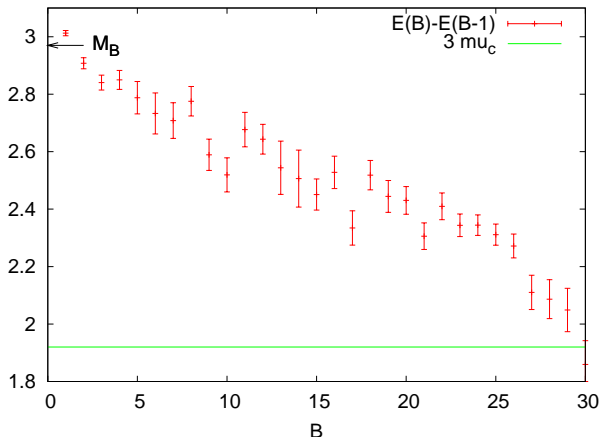
Wrong M_B ? wrong μ_c ?

- Baryon mass ≈ 3 checked by HMC
- $\mu_c \approx 0.64$ (see earlier) for $m = 0.1, T = 1/4$

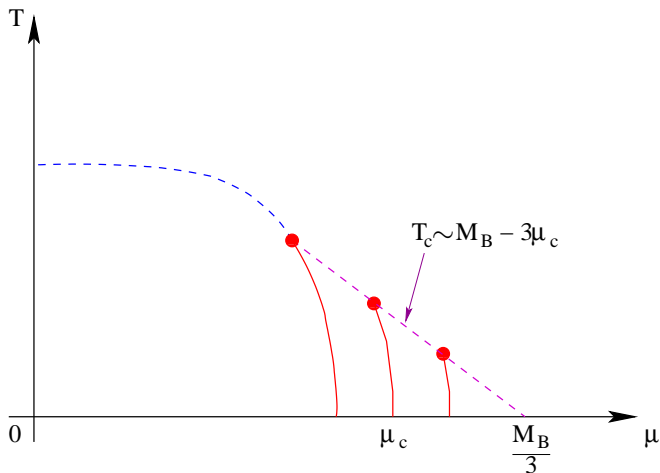
PdF & Kim

Explanation: nuclear attraction $\sim 1/3$ baryon mass ??

Monitor $T = 0$ energy as a function of baryon number, ie. $(E(B) - E(B-1))$ vs B

Internal energy vs baryon number at $T = 1/4, m = 0.1$ 

- $E(B=1) \sim M_B$: temperature is low ($T \sim 75$ MeV, $m_\pi \sim 200$ MeV)
- $E(B=2) - 2E(B=1) \sim -0.1$, ie. “deuteron” binding energy ca. 30 MeV
- Further binding with ca. 20 “nearest-neighbours” to give μ_c ca. 600 MeV
- “Magic numbers” with increased stability?

Hadron \leftrightarrow nuclear matter transition vs pion mass

- Transition becomes weaker for heavier pions
- In strong coupling LQCD, for physical m_π/M_B , interactions **enhanced by $\mathcal{O}(10)$** \rightarrow good laboratory

Conclusions

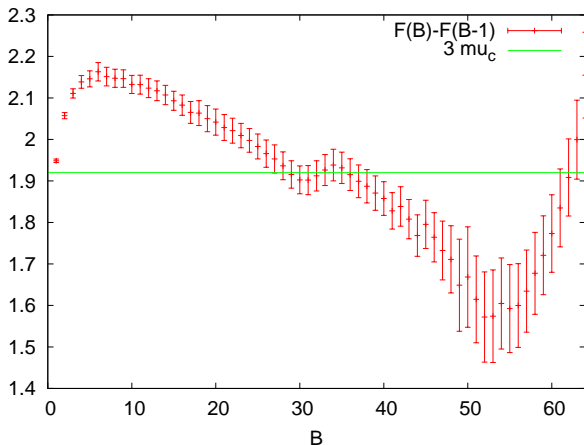
Summary

- Take mean-field results with a grain of salt
- “Clean-up” of phase diagram justified
- **Nuclear matter** from QCD

Outlook

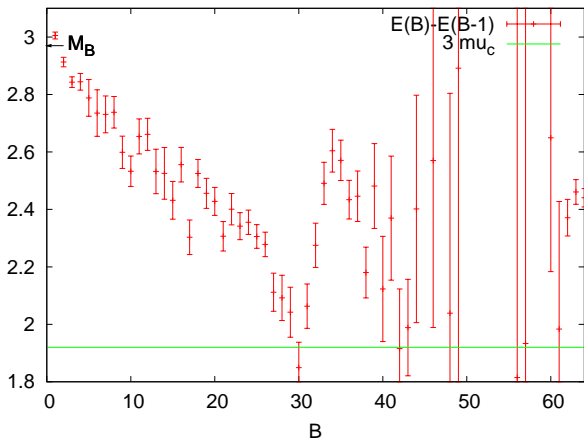
- Improve systematics:
 - Asymmetry γ in Dirac coupling to vary T continuously
 - Compare real and imaginary μ
- Determine phase diagram:
 - Tricritical point for $m = 0$
 - Critical end-point as a function of m
- Include second quark species \rightarrow **isospin**
- Include $\mathcal{O}(\beta)$ effects ?

Backup slide: Maxwell construction



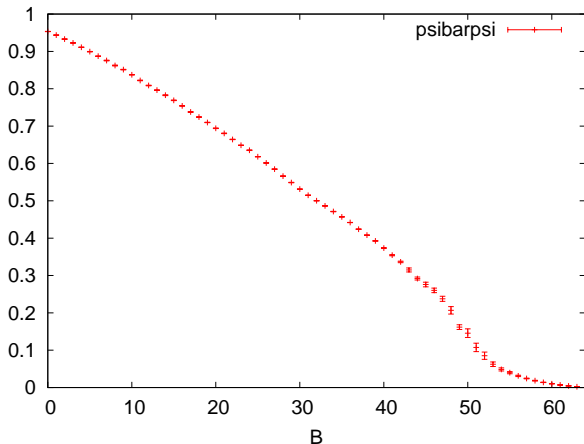
- μ_c consistent with Maxwell construction
- Area gives interface tension of nuclear matter

Backup slide: energy for each additional baryon



The energy of an additional baryon in the dense phase is $> 3\mu_c$

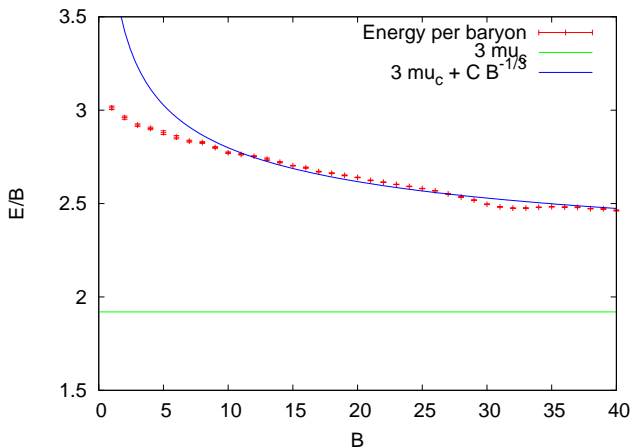
Backup slide: chiral symmetry restoration



$\langle \bar{\psi}\psi \rangle$ decreases \sim linearly with B

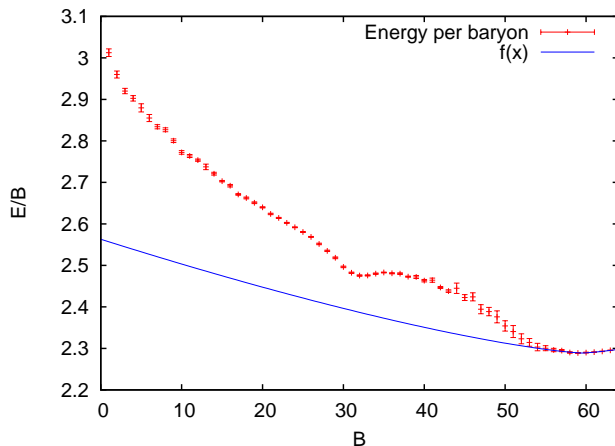
\rightarrow chiral symm. restored in nuclear matter

Backup slide: energy per baryon



$E(B)$ well described by $(a_V + a_S B^{2/3})$, ie. (bulk + surface tension) (Weizsäcker)

Backup slide: nuclear matter compressibility



$$K = p_F^2 \frac{d^2 E/B}{dp_F^2} \rightarrow K \sim 160 \text{ MeV}$$

This is the end...

Dear Helmuth,



Merci Viel Mal !