

The curvature of the QCD phase transition line

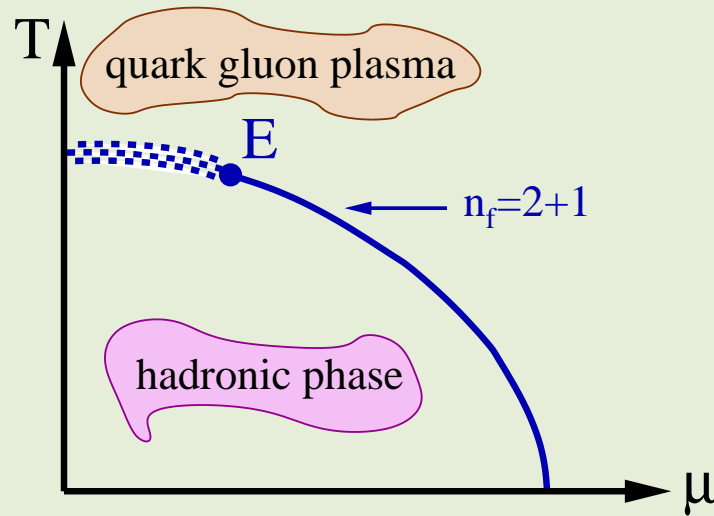
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Introduction, motivation



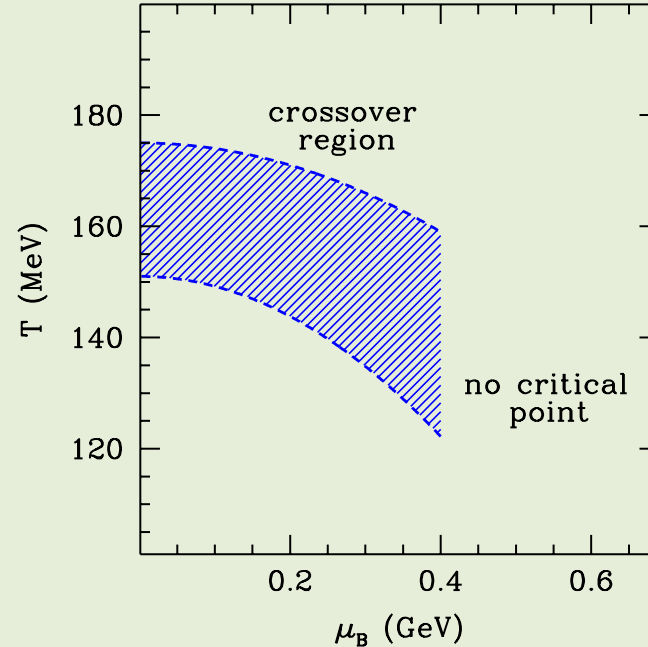
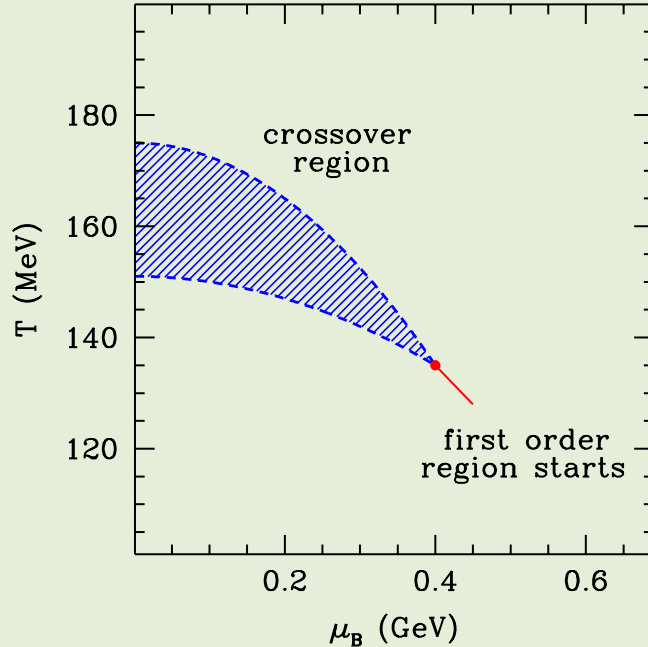
- Zero μ area is relevant for - the early Universe
- high energy collisions
- Nature of QCD transition at zero μ is found to be a crossover [Y. Aoki, G. Endrődi, Z. Fodor, S.D. Katz, K.K. Szabó]
- Different observables give different values for T_c namely, $T_c(\chi_{\bar{\psi}\psi}) \approx 151$ MeV, $T_c(\chi_s) \approx 175$ MeV
[Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabó]

The role of the curvature

- Explore the $\mu \neq 0$ region of the phase diagram
- At $\mu \neq 0$ the fermion determinant is complex
→ importance sampling not possible
- Use Taylor-expansion in μ
first term vanishes
second term given by the curvature (κ)
- Aims:
determine the curvature for different observables
 $\chi_{\bar{\psi}\psi}$, $\bar{\psi}\psi$; and for L , χ_s
- Comparison: no continuum limit results; the curvature is in the range of $\kappa = 0.003 \dots 0.01$

[Bielefeld-Swansea; Philipsen, de Forcrand; D'Elia, Lombardo; Fodor, Katz]

Scenarios



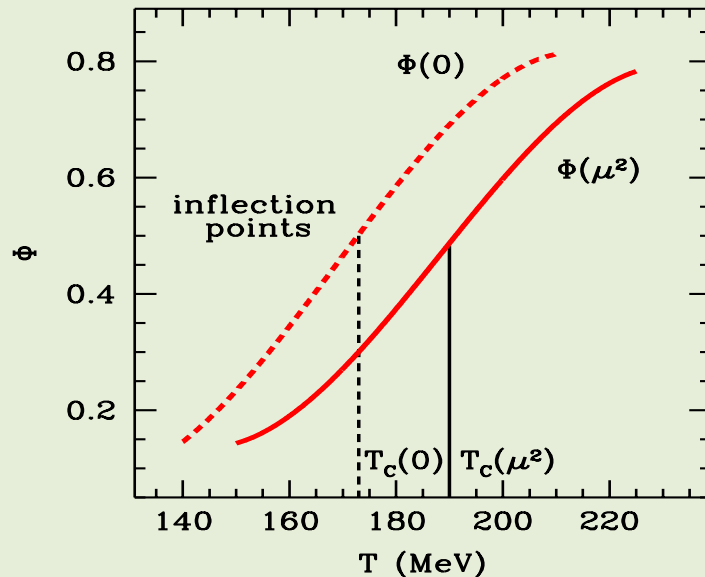
- Does the crossover region shrink or expand?
- The curvature can affect the existence of the critical endpoint
- Estimation: if $\mu_{crit} = 360$ MeV $\rightarrow \Delta\kappa \approx 0.02$
- $\mu \equiv \mu_B$

Curvature determination I.

- Equation of transition line is $T_c(\mu) = T_c \left(1 - \kappa \frac{\mu^2}{T_c^2} \right)$

$$\rightarrow \kappa = -T_c \left. \frac{dT_c(\mu)}{d\mu^2} \right|_{\mu=0}$$

- Determination of κ demonstrated on $\chi_s/T^2 \equiv \Phi$



- Procedure #1: shape of Φ may get distorted at finite μ

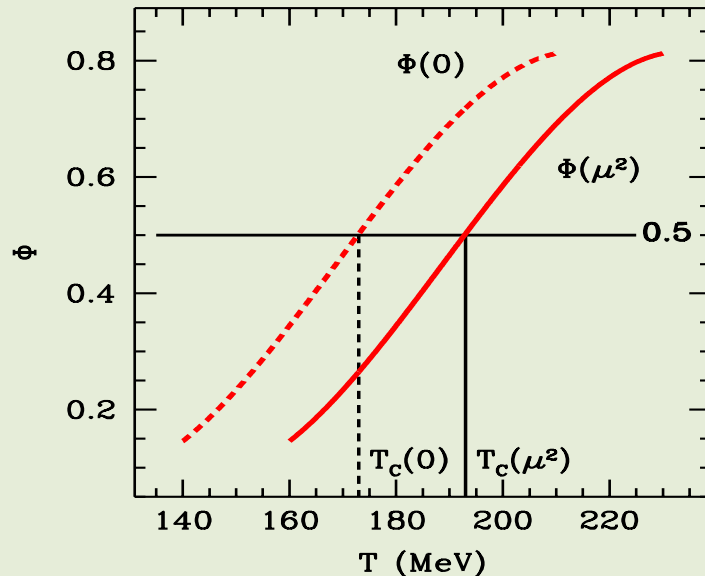
- T_c defined as inflection point of Φ

$$\rightarrow \frac{dT_c}{d\mu^2} = \frac{T_c(\mu^2) - T_c(0)}{\Delta\mu^2}$$

- Very computer time-demanding

Curvature determination II.

- Procedure #2: suppose shape of Φ unchanged
- Let us define T_c as $\Phi(T_c) = 0.5$
- For $\Phi(T, \mu^2)$: $d\Phi = \frac{\partial\Phi}{\partial T} \cdot dT + \frac{\partial\Phi}{\partial\mu^2} \cdot d\mu^2$



- along the $T_c(\mu)$ line
 $d\Phi = 0$ by definition

$$\rightarrow \frac{dT_c}{d\mu^2} = - \left(\frac{\partial\Phi}{\partial\mu^2} \right) / \left(\frac{\partial\Phi}{\partial T} \right)$$

- This method is much cheaper in CPU-time
- For $\frac{\partial\Phi}{\partial\mu^2}$ we need to measure new operators

Technique

- Consider $\mathcal{Z} = \int \mathcal{D}U e^{-S_g(U)} \det M^{N_f/4}$
- $\frac{\partial \log \mathcal{Z}}{\partial \mu_{u,d}} = \langle n_{u,d} \rangle$; $\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_{u,d}^2} = \langle \chi_{u,d} \rangle$, where ($' \equiv \frac{\partial}{\partial \mu_{u,d}}$)
$$n_{u,d} = \frac{N_f}{4} \text{Tr} (M^{-1} M')$$
 and
$$\chi_{u,d} = n_{u,d}^2 + \frac{N_f}{4} \text{Tr} (M^{-1} M'' - M^{-1} M' M^{-1} M')$$
- Observables Φ that don't depend on $\mu_{u,d}$ (L, χ_s):

$$\frac{\partial \langle \Phi \rangle}{\partial (\mu_{u,d}^2)} = \frac{1}{2} \frac{\partial^2 \langle \Phi \rangle}{\partial \mu_{u,d}^2} = \langle \Phi \chi_{u,d} \rangle - \langle \Phi \rangle \langle \chi_{u,d} \rangle$$

- Observables Φ that depend on $\mu_{u,d}$ ($\bar{\psi}\psi, \chi_{\bar{\psi}\psi}$):

$$\frac{\partial \langle \Phi \rangle}{\partial (\mu_{u,d}^2)} = \frac{1}{2} \frac{\partial^2 \langle \Phi \rangle}{\partial \mu_{u,d}^2} = \langle \Phi \chi_{u,d} \rangle - \langle \Phi \rangle \langle \chi_{u,d} \rangle + \langle 2\Phi' n_{u,d} + \Phi'' \rangle$$

Simulation details

- Symanzik improved gauge and stout-link improved staggered fermionic lattice action
- Physical masses for $m_{u,d}$ and for m_s
- LCP determined by fixing m_K/f_K and m_K/m_π
- Scale set by f_K
- Lattice spacings used: $N_T = 4, 6, 8, 10$
- with aspect ratios $N_S/N_T = 4$ and 3
- Measurements carried out with 80 random vectors (measurements and config. production balanced)
- Derivatives ϕ' and ϕ'' calculated numerically using a purely imaginary chemical potential

Observables I.

- Polyakov loop $L = \frac{1}{N_S^3} \sum_x \text{Tr} \prod_{t=0}^{N_T-1} U_4(x, t)$

renormalization: $L_r = L \exp(V(r_0)/2T)$

- Strange susceptibility $\chi_s = \frac{T}{V} \frac{\partial^2 \log \mathcal{Z}}{\partial \mu_s^2}$

no renormalization necessary

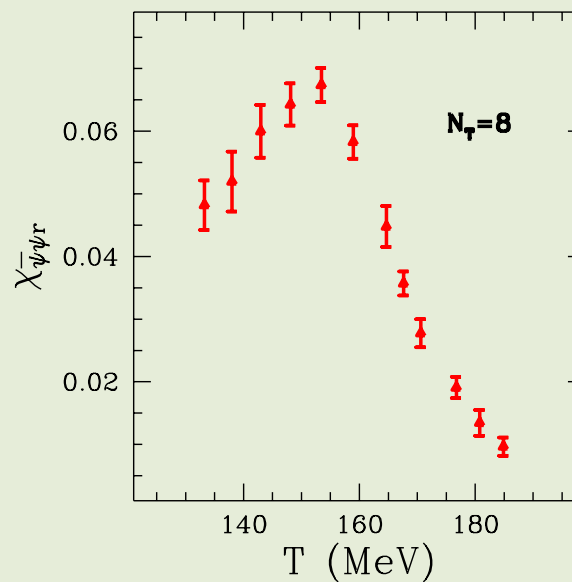
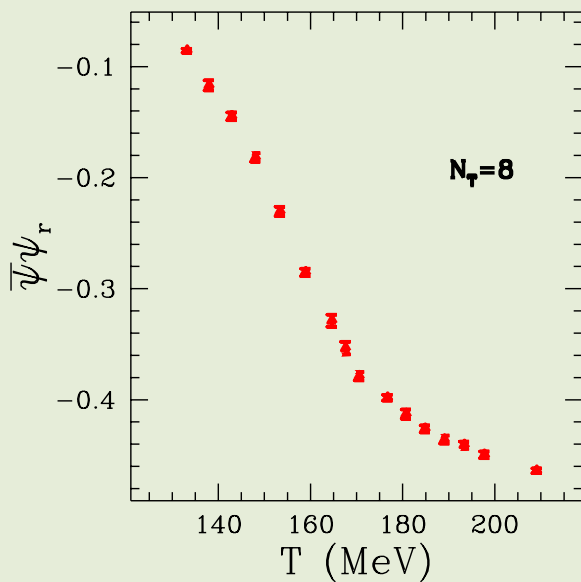
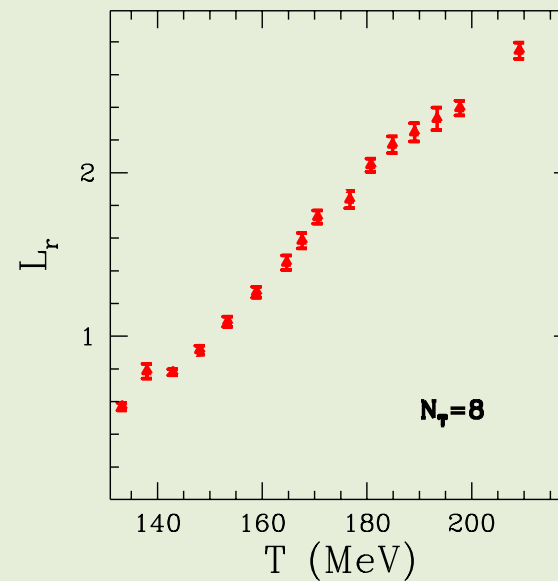
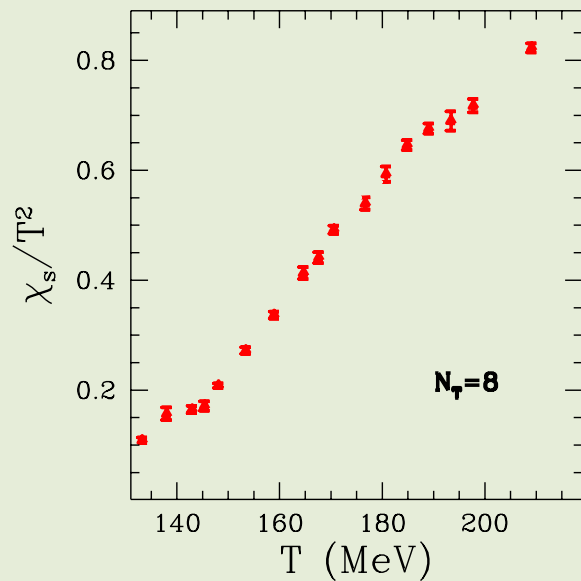
- Chiral condensate $\bar{\psi}\psi = \frac{T}{V} \frac{\partial \log \mathcal{Z}}{\partial m}$

renormalization: $\bar{\psi}\psi_r = (\bar{\psi}\psi - \bar{\psi}\psi(T=0)) \cdot m \cdot \frac{1}{m_\pi^4}$

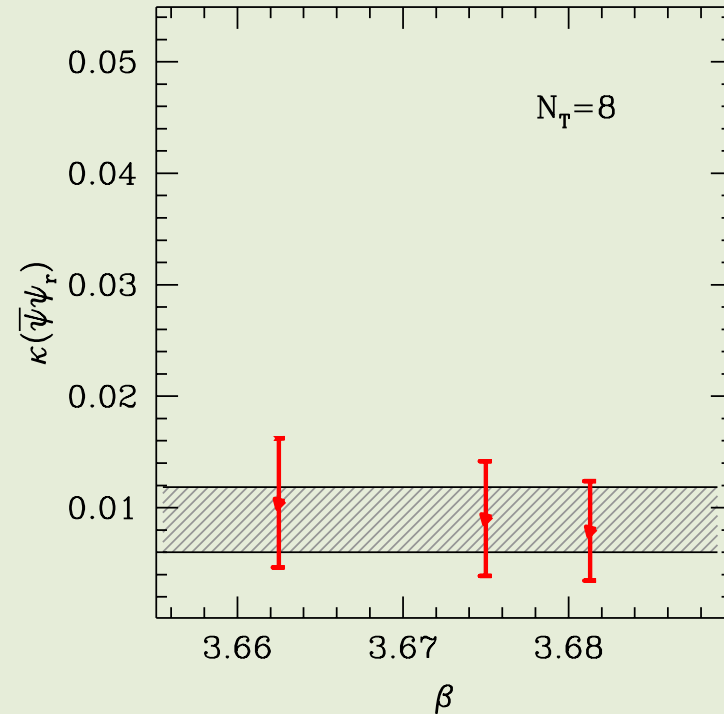
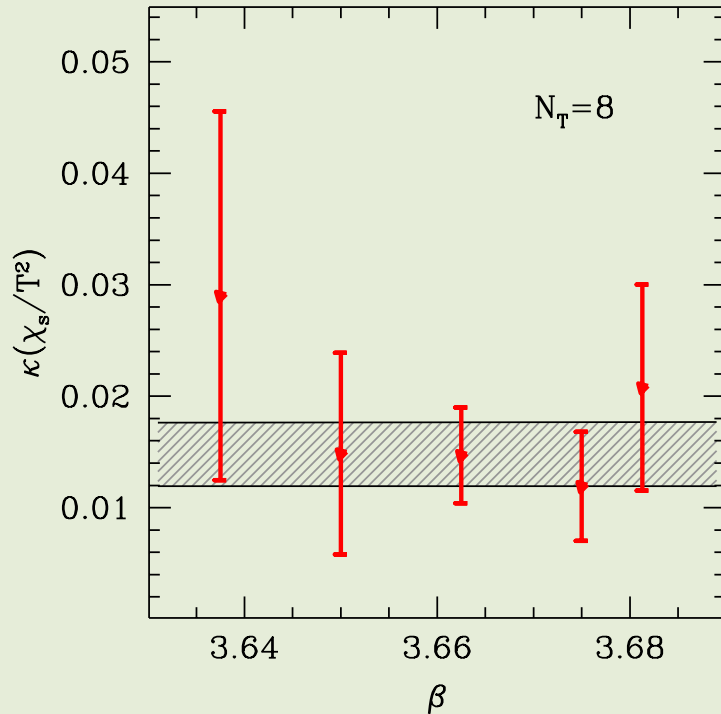
- Chiral susceptibility $\chi_{\bar{\psi}\psi} = \frac{T}{V} \frac{\partial^2 \log \mathcal{Z}}{\partial m^2}$

renormalization: $\chi_{\bar{\psi}\psi_r} = (\chi_{\bar{\psi}\psi} - \chi_{\bar{\psi}\psi}(T=0)) \cdot m^2 \cdot \frac{1}{T^4}$

Observables II.

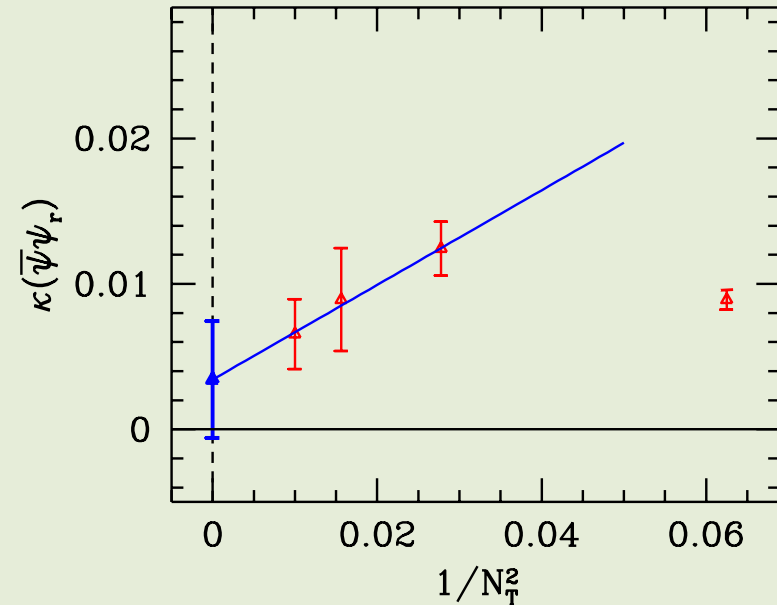
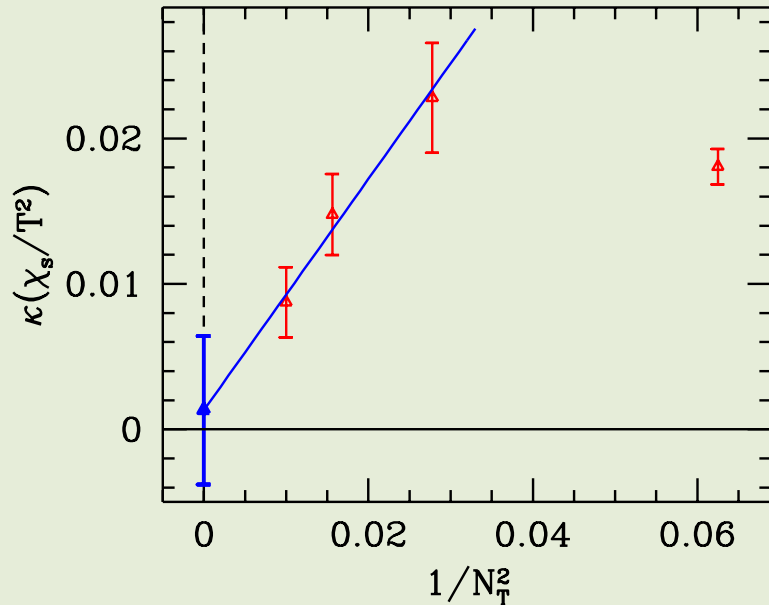


Results I.



- Using procedure #2, κ can be calculated at arbitrary T
- Independent measurements at different β values

Results II.



- Continuum extrapolated results are preliminary
- Notice large errors
→ for $N_T = 6, 8, 10$ more statistics is needed

$$\kappa(\chi_{\bar{\psi}\psi_r}) = -0.0018(34), \quad \kappa(\bar{\psi}\psi_r) = 0.0034(40)$$

$$\kappa(L_r) = -0.0095(93), \quad \kappa(\chi_s/T^2) = 0.0013(51)$$

Summary

- Although $\kappa(\bar{\psi}\psi_r) < \kappa(\chi_s/T^2)$ for every N_T , they become consistent in the continuum limit
- For that reason we cannot make an estimate about the location of the critical endpoint
- We need to increase statistics to achieve a reliable continuum limit
- Procedure #1 is to be used to determine κ