QCD Phase Transition and Fermionic Boundary Conditions

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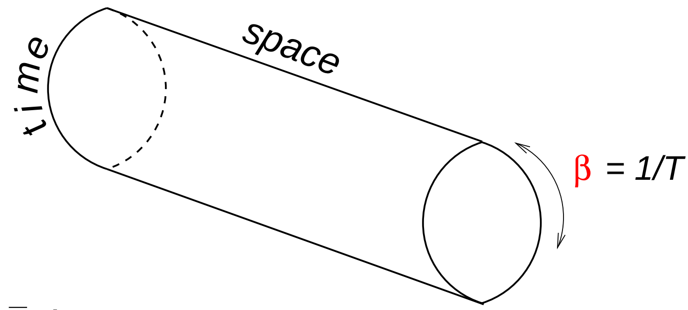
Technical preliminaries: QCD at finite temperature

- Path integral on a 4-dimensional euclidean cylinder with one compact direction of length $\beta = 1/T$.



$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[U,\overline{\psi},\psi]\, e^{-S[U,\overline{\psi},\psi]}\, O[U,\overline{\psi},\psi]$$

$$S = \int d^3x \int_0^{\beta} dx_4\, \mathcal{L}[U,\overline{\psi},\psi]$$

- Roughly speaking: The finite temperature transition occurs when β is small enough such that the system starts to feel the boundary.

A strategy for probing QCD

- The finite temperature transition occurs when β is small enough such that the system starts to feel the boundary.

- We use the boundary conditions in the temporal direction to probe QCD.

- Above T_c (below β_c) the system might behave differently for different boundary conditions.

- Here we allow for varying fermionic boundary conditions:

$$\psi(\vec{x}, \beta) \; = \; e^{i\varphi} \, \psi(\vec{x}, 0)$$

- Motivated through properties of Kraan – van Baal calorons.

Center symmetry and Polyakov loops

- The gauge action is invariant under center transformations ($z \in Z_3$):

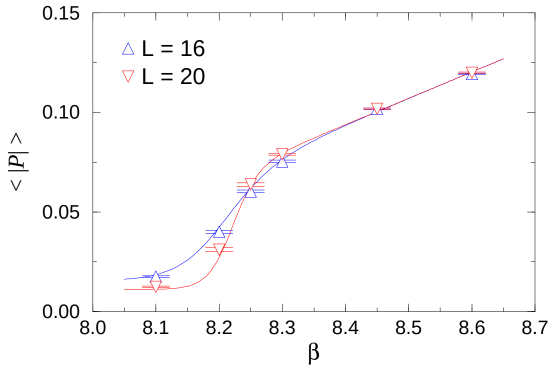
$$U_4(x) \;\rightarrow\; z\, U_4(x) \qquad\qquad \forall\, x_4 = t_0$$

- The deconfinement transition of pure gauge theory can be described as spontaneous breaking of the center symmetry.

- The Polyakov loop transforms non-trivially and is an order parameter.

$$P(\vec{x}) \;=\; \mathrm{tr}_c \prod_{t=1}^{N_t} U_4(\vec{x}, t)$$

$$P(\vec{x}) \;\rightarrow\; z\, P(\vec{x})$$

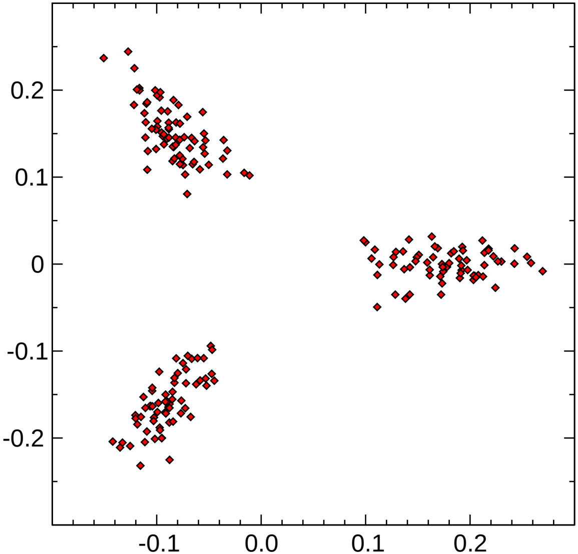


Center symmetry and Polyakov loops

- Above T_c the Polyakov loop spontaneously selects one of three (for SU(3)) possible phases θ.

- We will show that the Polyakov loop phase θ acts like an additional boundary condition.

Polyakov loop in the complex plane above T_c

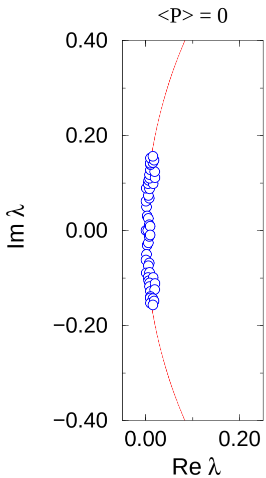


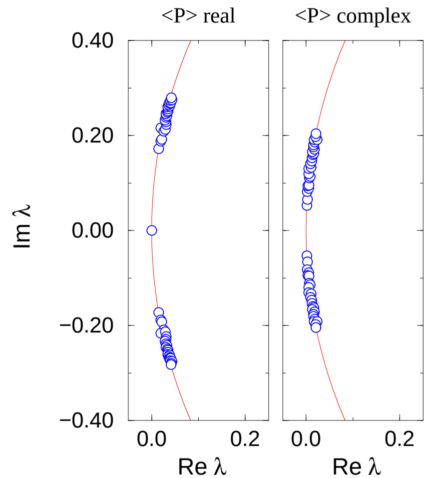
Chiral symmetry breaking and Dirac spectrum

- The Banks Casher formula relates the chiral condensate to the spectral density of the Dirac operator at the origin.

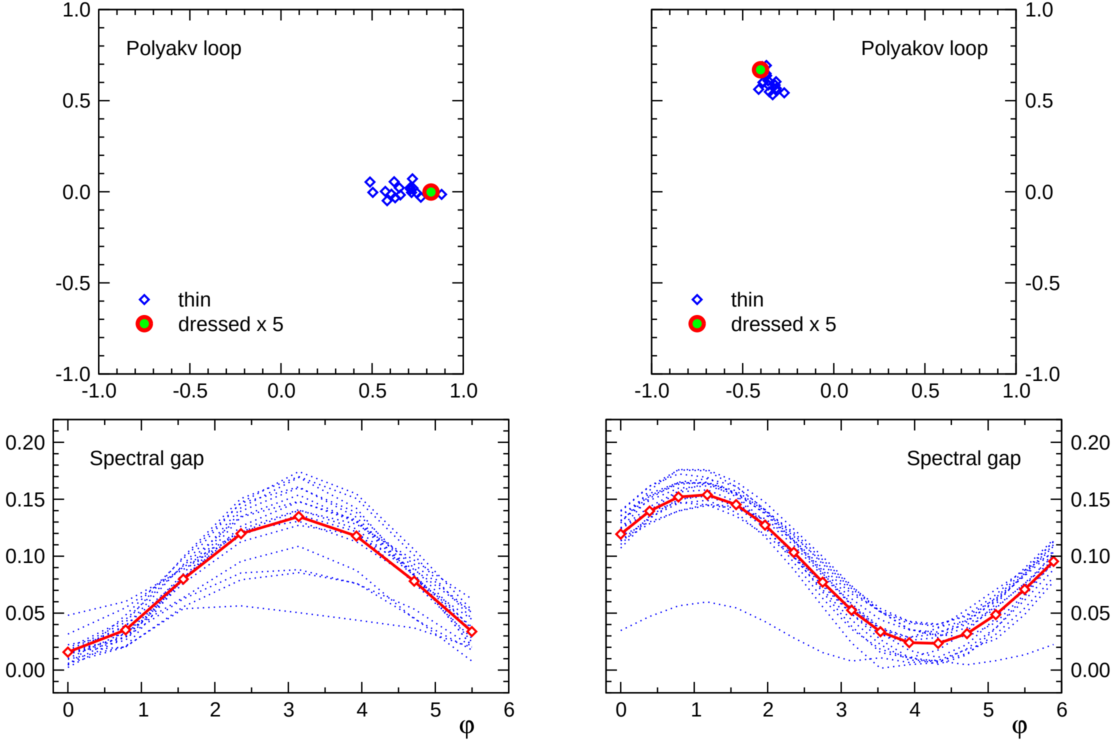
$$\langle \overline{\psi} \psi \rangle = -\pi \rho(0)$$

- At the QCD phase transition a gap opens up in the spectrum and the chiral condensate vanishes.



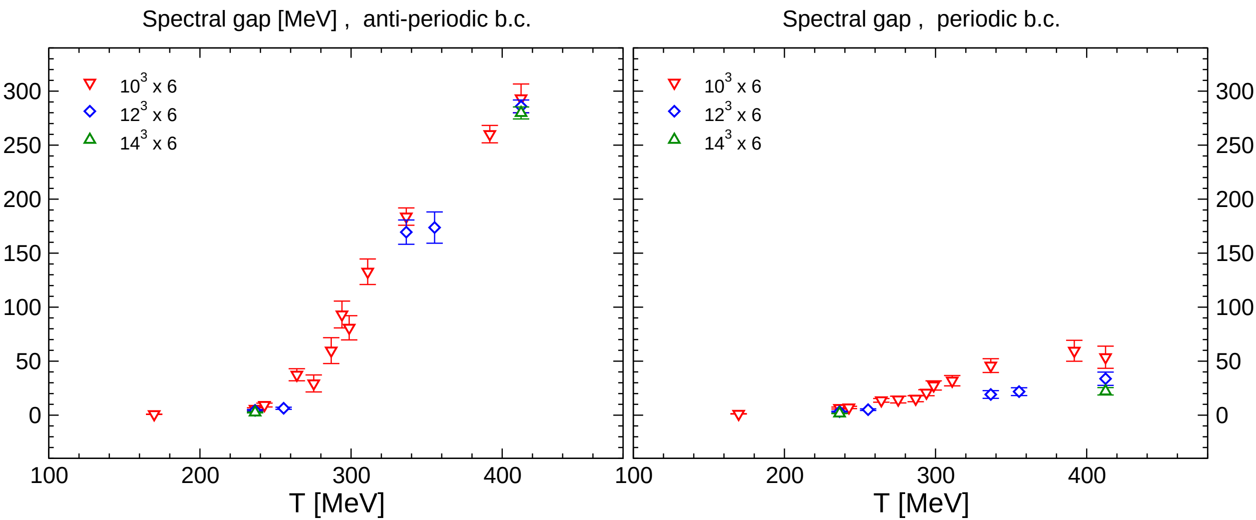


Spectral gap above T_c



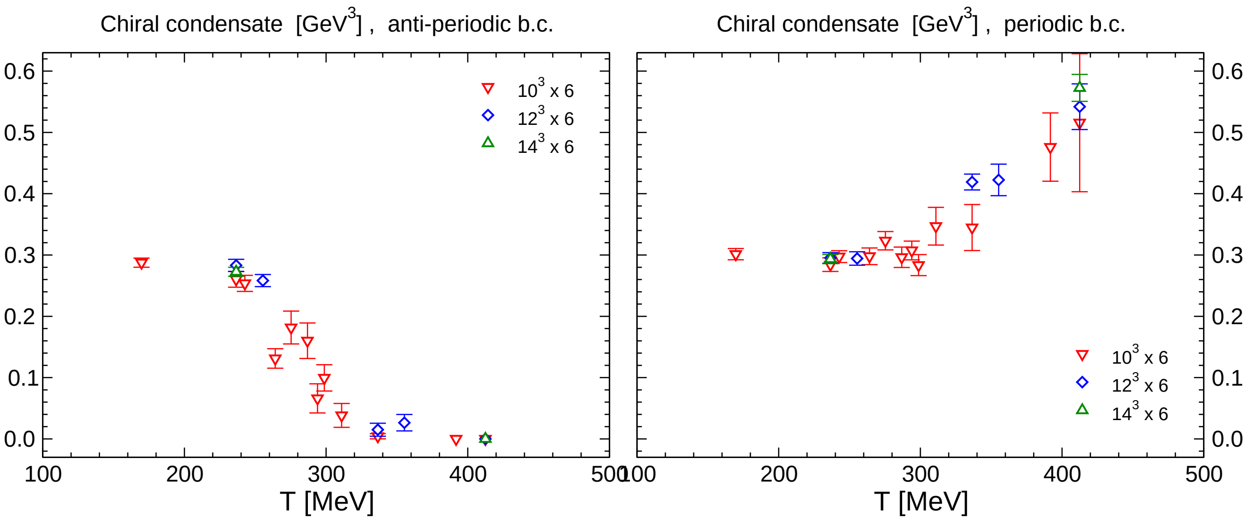
Spectral gap depends on the relative phase between b.c. and Polyakov loop.

Spectral gap as a function of T



With periodic b.c. also above T_c the gap remains closed.
(for the real Polyakov sector)

Chiral condensate as a function of T



With periodic b.c. the chiral condensate persists also above T_c
(for the real Polyakov sector).

An important physics question

- At zero temperature QCD shows two characteristic features:
  - Quarks are confined.
  - Chiral symmetry is broken: $\langle\overline{\psi}\psi\rangle \neq 0$.

- QCD has a finite temperature transition where:
  - Quarks become deconfined.
  - Chiral symmetry is restored: $\langle\overline{\psi}\psi\rangle = 0$.

Is there an underlying mechanism that links the two key features of QCD?

What role is played by the boundary conditions? Can we use them?

A possible approach

- Confinement and chiral symmetry breaking both should leave a trace in properties of the Dirac operator D, since D^{-1} describes the propagation of quarks.
- For chiral symmetry breaking the Banks-Casher formula connects the order parameter $\langle \overline{\psi} \psi \rangle$ to IR properties of the Dirac spectrum.
- Concerning confinement it is not even clear where to look in the spectrum, in the UV or the IR part.
- Maybe through analyzing spectral properties of D one can find a link between confinement and chiral symmetry breaking.
- The lattice formulation provides a suitable framework (rigorously defined) which allows for both, analytical and numerical approaches.

Lattice fermions and loops

- Discretized Dirac operator on the lattice

$$D \;=\; \frac{1}{2a} \sum_{\mu=1}^{4} \gamma_\mu(x) \left[U_\mu(x)\, \delta_{x+\hat{\mu}\,,\,y} \;-\; U_\mu(x-\hat{\mu})^\dagger\, \delta_{x-\hat{\mu}\,,\,y} \right]$$

- The chiral condensate corresponds to a sum of loops:

$$\langle \overline{\psi}\psi \rangle \;=\; -\frac{1}{V} \mathrm{Tr}[m+D]^{-1} \;=\; \sum_{l \in \mathcal{L}} c(l)\; \mathrm{Tr}_c \prod_{(x,\mu)\in l} U_\mu(x)$$

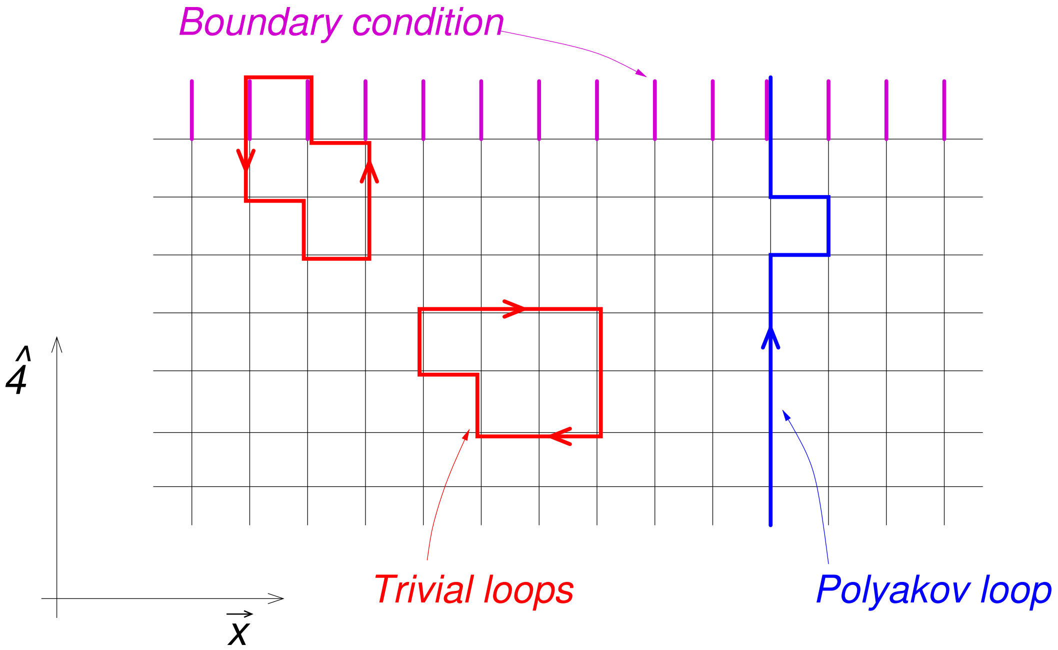
- A change of the temporal boundary conditions

$$U_4(\vec{x}, N_t) \;\longrightarrow\; z\, U_4(\vec{x}, N_t) \quad , \qquad z \;=\; e^{i\varphi} \;\in\; \mathrm{U}(1)$$

  affects only loops that wind non-trivially around compact time.

- Fourier transformation with respect to φ allows one to project to the equivalence class of loops winding exactly once: *Dressed Polyakov Loops*

Graphical representation



Dual chiral condensate = dressed Polykov loop

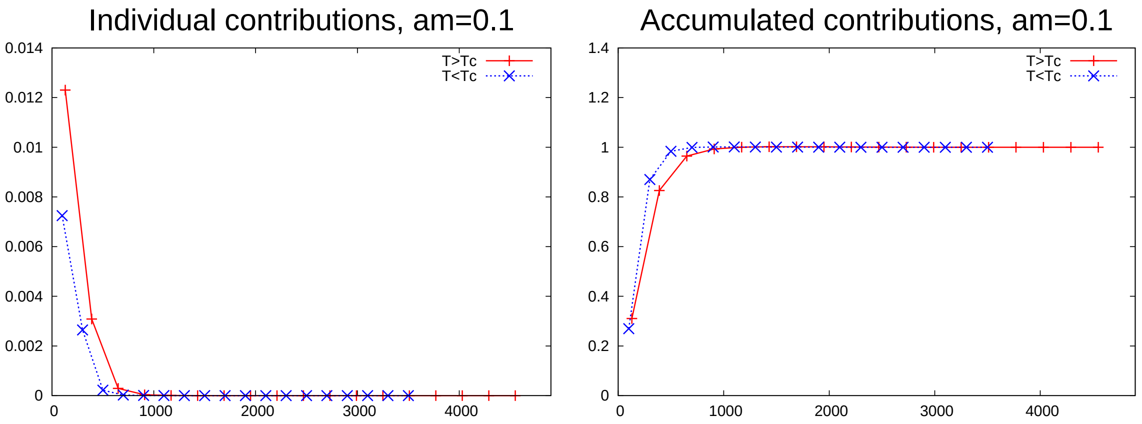
- Fourier transformation with respect to the boundary condition connects the order parameters for confinement and for chiral symmetry breaking:

$$\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} = \int_0^{2\pi} \frac{d\varphi\, e^{-i\varphi}}{2\pi} \left\langle \overline{\psi}\psi \right\rangle_\varphi = \sum_{l \in \mathcal{L}_{\mathbf{1}}} c(l) \left\langle \mathrm{Tr}_c \prod_{(x,\mu)\in l} U_\mu(x) \right\rangle$$

$$\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} = -\int_0^{2\pi} \frac{d\varphi\, e^{-i\varphi}}{2\pi\, V} \mathrm{Tr}[m + D_\varphi]^{-1} = -\int_0^{2\pi} \frac{d\varphi\, e^{-i\varphi}}{2\pi\, V} \sum_k \left\langle \frac{1}{m + \lambda_\varphi^{(k)}} \right\rangle_\varphi$$

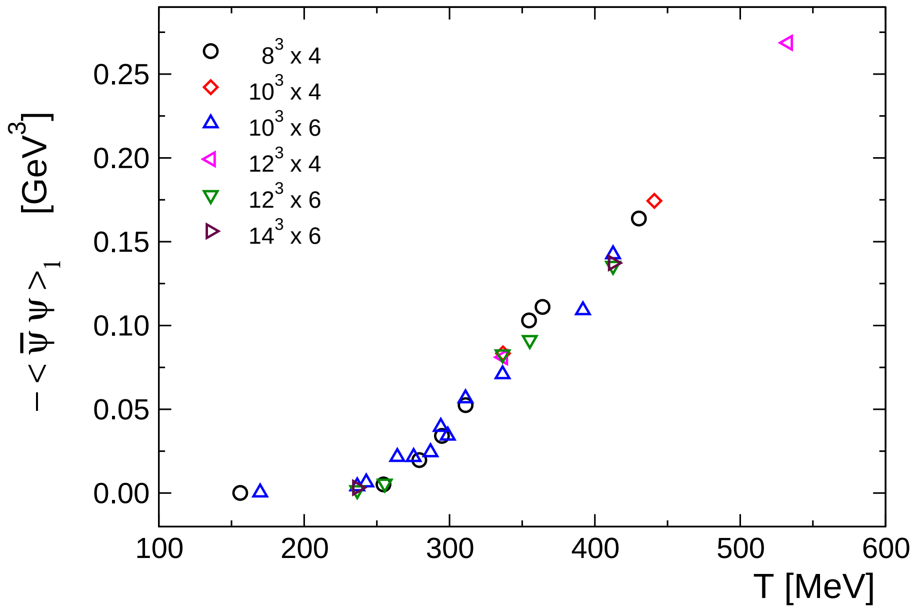
- The representation as a spectral sum of Dirac eigenvalues allows one to study the role of IR and UV eigenmodes for the mechanisms of confinement and chiral symmetry breaking.

The Dressed Polyakov Loop is dominated by IR modes



$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} = \sum_k \frac{1}{2\pi V} \int_0^{2\pi} d\varphi \, e^{-i\varphi} \left\langle \frac{1}{m+\lambda_\varphi^{(k)}} \right\rangle_\varphi$$

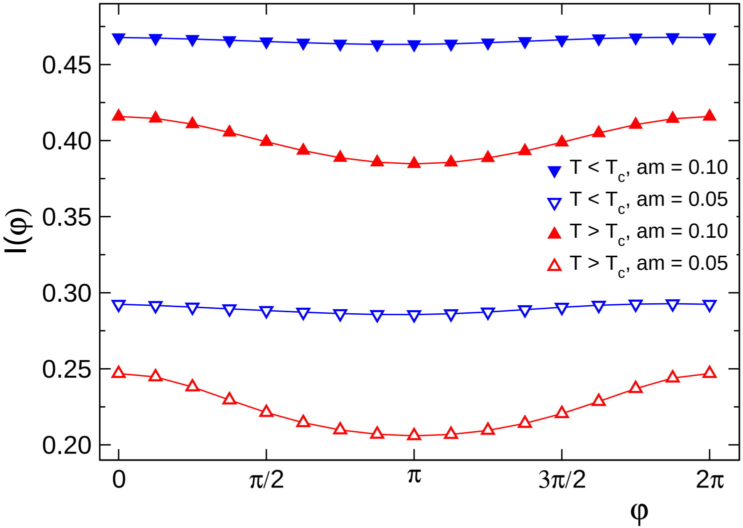
The Dressed Polyakov Loop is an order parameter



Results from different lattices fall on a universal curve.

$\rightarrow$ Good scaling and renormalization properties.

Spectral properties at the phase transition



$$I(\varphi) = \frac{1}{V} \sum_k \left\langle \frac{1}{m + \lambda_\varphi^{(k)}} \right\rangle$$

The confined and deconfined phases give rise to a different response of the IR part of the Dirac spectrum to changing boundary conditions.

Generalization of the Banks-Casher formula

- Having identified the connection between spectral properties and the dressed Polyakov loops, we can now formulate the physical picture in terms of a generalized Banks-Casher relation.

- Performing $\lim_{m\to 0}\lim_{V\to\infty}$ we find:

$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} \;=\; \frac{1}{2}\int_0^{2\pi} d\varphi\, e^{-i\varphi}\,\rho(0)_\varphi$$

- How does the spectral density $\rho(0)_\varphi$ at the origin have to behave as a function of φ such that:

$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} \;=\; 0 \qquad \text{below}\, T_c$$

$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} \;>\; 0 \qquad \text{above}\, T_c$$

Emerging picture for the generalized Banks-Casher formula

- The spectral density at the origin, $\rho(0)_\varphi$, behaves as (θ denotes the phase of the Polyakov loop):

$$\begin{aligned} \rho(0)_\varphi &= \text{const} && \text{below } T_c \\ \rho(0)_\varphi &\propto \delta(\varphi + \theta) && \text{above } T_c \end{aligned}$$

- The dual chiral condensate is given by:

$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} = \frac{1}{2}\int_0^{2\pi} d\varphi\, e^{-i\varphi}\, \rho(0)_\varphi$$

- And behaves correctly as:

$$\begin{aligned} -\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} &= 0 && \text{below } T_c \\ -\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} &= \rho_0 \exp(i\theta) && \text{above } T_c \end{aligned}$$

Summary

- We use the fermionic temporal boundary conditions to probe QCD at finite temperature.

- At the phase transition the behavior of the low-lying eigenvalues changes:

  1. In the confined phase we have a non-vanishing spectral density $\rho(0)_\varphi$ at the origin which is independent of the boundary condition.

  2. Above T_c the spectral gap has a sine-like dependence on the phase between boundary condition and Polyakov loop and $\rho(0)_\varphi \propto \delta(\varphi+\theta)$.

- The phase of the Polyakov loop acts like a background phase, but does not play a dynamical role.

- The center of the gauge group does not seem to play a major role in the microscopic dynamical process.

Summary (continued)

- Fourier transforming the chiral condensate with respect to the fermionic boundary condition we define the *Dual Chiral Condensate*.
- The dual chiral condensate is an order parameter for center symmetry, interpreted as *Dressed Polyakov Loops*.
- The dual condensate can be represented as a spectral sum of Dirac eigenvalues which is dominated by the IR modes.
- Most elegantly the results are expressed as a generalized Banks-Casher formula for the dual condensate:

$$- \widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} \;=\; \frac{1}{2} \int_0^{2\pi} d\varphi \, e^{-i\varphi} \, \rho(0)_\varphi$$

Chiral symmetry breaking and confinement are, via a duality transformation, connected to closely related spectral properties of the IR Dirac spectrum.

Link between confinement and chiral symmetry breaking?

Bonus material: The centerless gauge group G_2

- What role does the center play in our picture?
  $\Rightarrow$ Study a gauge group with trivial center.

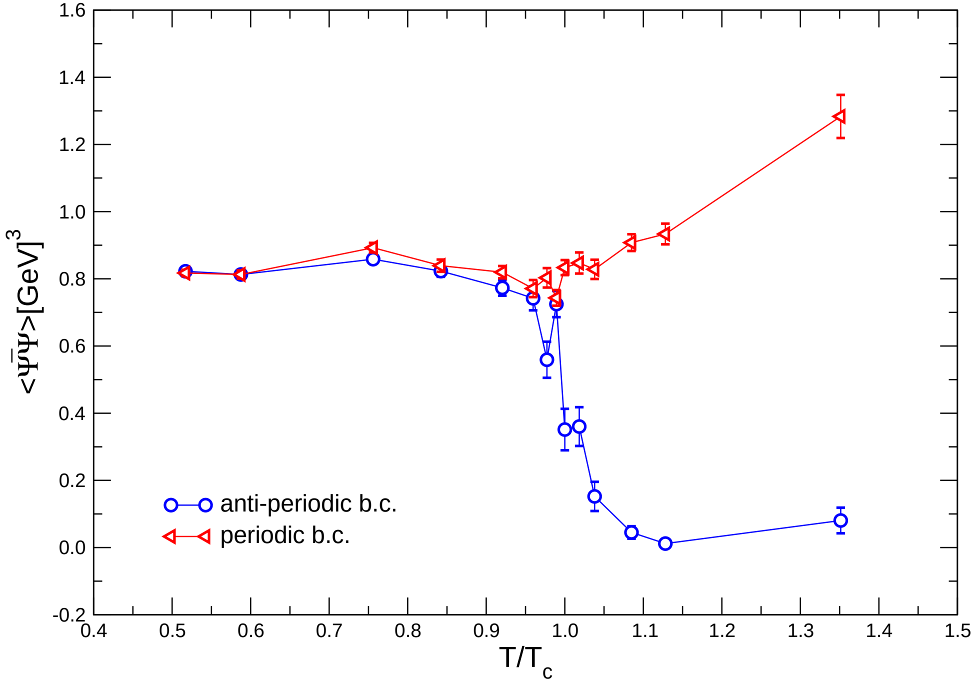
- Analyze the Dirac spectrum and its response to changing boundary conditions using quenched G_2 configurations.

- Finding:

  Behavior is exactly the same as for SU(3) in the real Polyakov sector.

- Another piece of evidence that the picture developed here is universal are the recent results in SU(2): Bornyakov *et al.*

G_2 : *Chiral condensate* $(12^3 \times 6)$



G_2 : *Spectral gap* $(12^3 \times 6)$