ATOMINSTITUT

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Center Vortices, Confinement and Chiral Symmetry Breaking

in cooperation with
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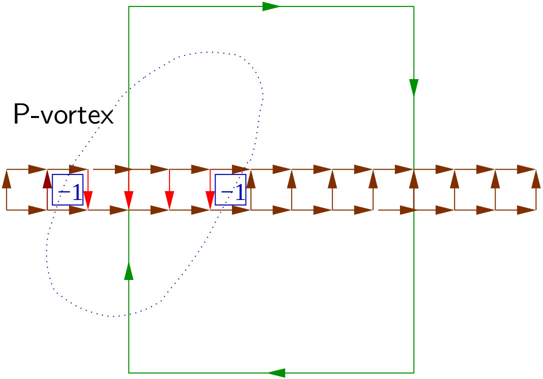
Center Vortices

almost 30 years of vortices

→ *'t Hooft 1979, Nielsen, Ambjorn, Olesen, Cornwall, 1979*
Mack, 1980; Feynman, 1981

- QCD vacuum is a *condensate of closed magnetic flux-lines*, they have topology of tubes (3D) or surfaces (4D),
- magnetic flux corresponds to the *center of the group*,
- Vortex model may explain ...
  - **Confinement** → *piercing of Wilson loop* ≡ crossing of static electric flux tube and moving closed magnetic flux
  - **Topological charge**: vortices carry topological charge at intersection points and writhing points
  - **Spontaneous chiral symmetry breaking ?**

- P-vortex plaquettes
  a plaquette is pierced by a P-vortex, if the product of its center projected links gives -1.



How to Identify Center Vortices?

→ *Del Debbio, Faber, Greensite, Olejnik (1996–1998)*

- Fix thermalized SU(2) lattice configurations to maximal center (adj. Landau) gauge by maximizing the expression:

$$\sum_{x,\mu} \left| \mathrm{Tr}[U_\mu(x)] \right|^2 \quad \text{or} \quad \sum_{x,\mu} \mathrm{Tr}[U^A_\mu(x)]$$

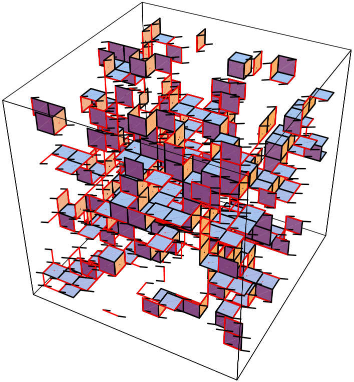
+overrelaxation

- Make center projection by replacing:

$$U_\mu(x) \to Z_\mu(x) \equiv \mathrm{sign}\ \mathrm{Tr}[U_\mu(x)]$$

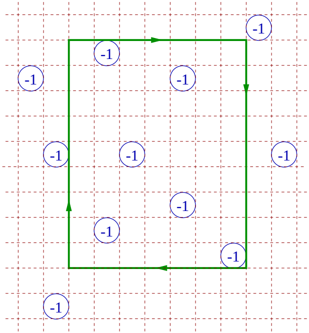
Structure of P-Vortices

In 4D they form closed 2D-surfaces in Dual Space, Random Structure



3-dimensional cut through the dual of a 12^4-lattice.

Area law for center projected loops



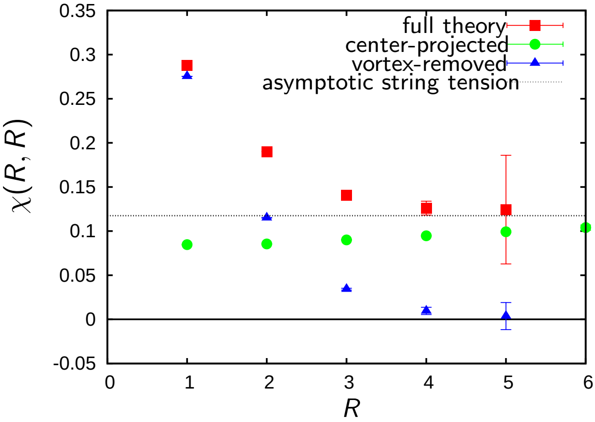
denote f the probability that a plaquette has the value *-1*

$$\langle W(A)\rangle = [f(-1) + (1-f)\cdot 1]^A = \exp[\underbrace{\ln(1-2f)}_{-\sigma} A], =$$

$$= \exp[-\sigma R \times T], \qquad \sigma \equiv -\ln(1-2f) \approx 2f$$

Center Dominance and Precocious Linearity

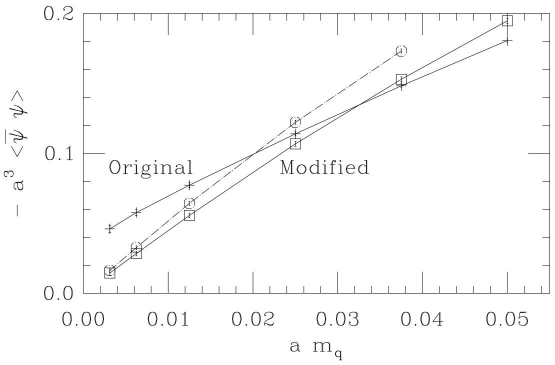
Creutz ratios: $\chi(I,J) = \frac{W(I,J)\,W(I-1,J-1)}{W(I-1,J)\,W(I,J-1)} \to \sigma$



Precocious linearity of center projected Creutz ratios.
String tension sweeps away the $1/r$-potential.

Vortex removal restores chiral symmetry

→ *De Forcrand and D'Elia (1999)*



Chiral condensate in quenched lattice configurations before ("Original") and after ("Modified") vortex removal.

Banks-Casher relation

Chiral symmetry breaking $\Longrightarrow$

$\Longrightarrow$ Low-lying eigenmodes of Dirac operator

$$\bar{\psi}\psi = - \lim_{m \longrightarrow 0} \lim_{V \longrightarrow \infty} \left\langle \frac{1}{V} \sum_n \frac{1}{\mathrm{i}\lambda_n + m} \right\rangle$$

Non-zero eigenvalues appear in pairs $\pm \mathrm{i}\lambda_n$

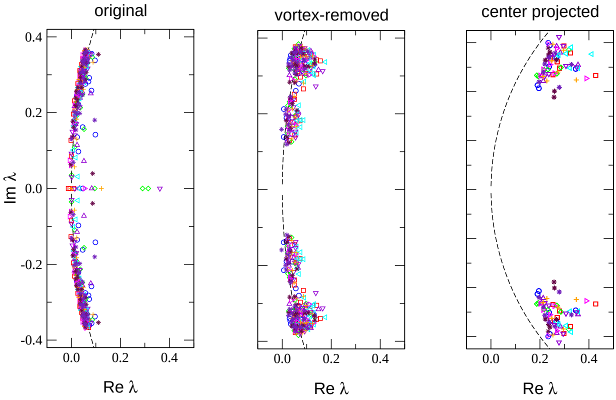
$$\lim_{m \longrightarrow 0} \frac{2m}{\lambda_n^2 + m^2} \longrightarrow \pi\delta(0)$$

Chiral condensate $\Longrightarrow$ Density of Near-Zero modes.

$$\bar{\psi}\psi = \frac{\pi\rho(0)}{V}$$

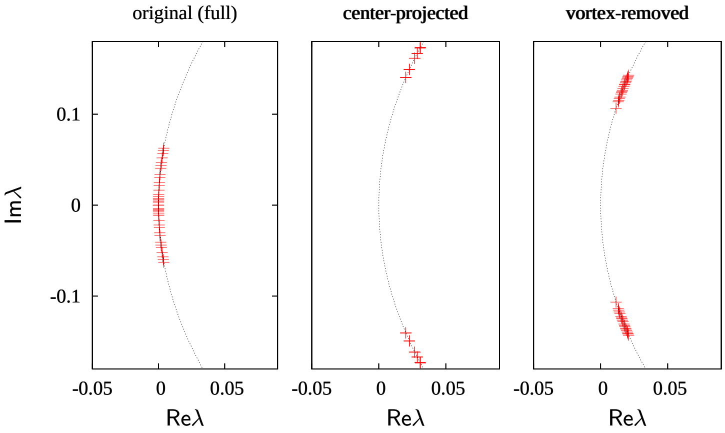
➜ *Banks, Casher, 1980*

Chiral Improved Fermions



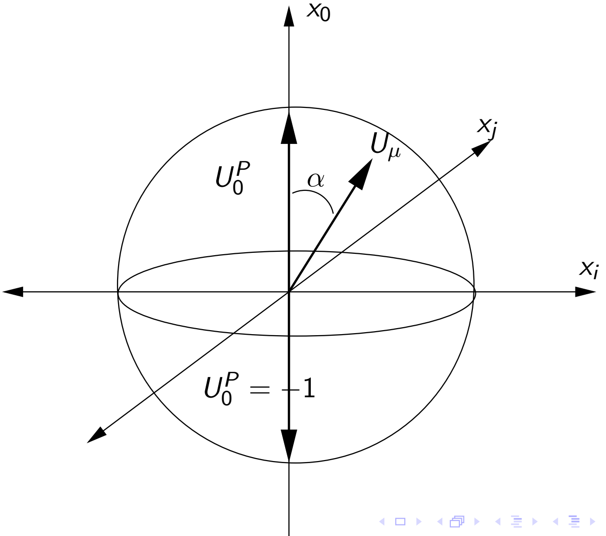
➜ *From J. Gattnar et al., Nucl. Phys. B716 (2005)105.*

Eigenvalues of the Overlap Dirac operator on the Ginsparg-Wilson circle

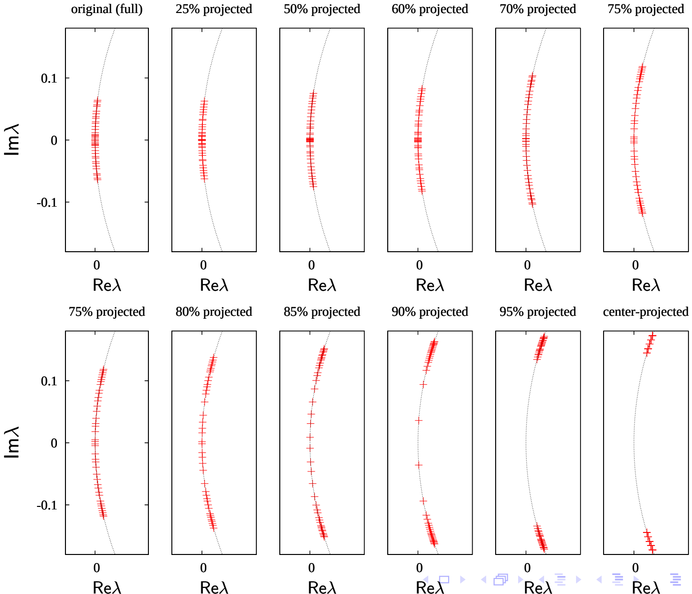


Interpolated gauge fields

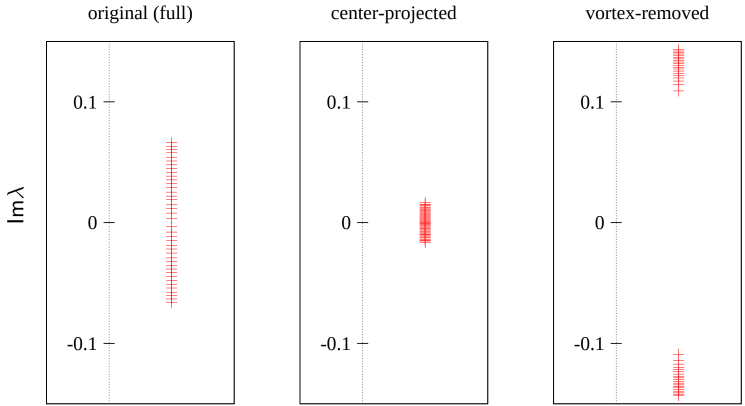
Interpolation between full U_μ and center-projected $U_0^P = \pm 1$.



Overlap eigenvalues on interpolated gauge fields



Asqtad Staggered Fermions



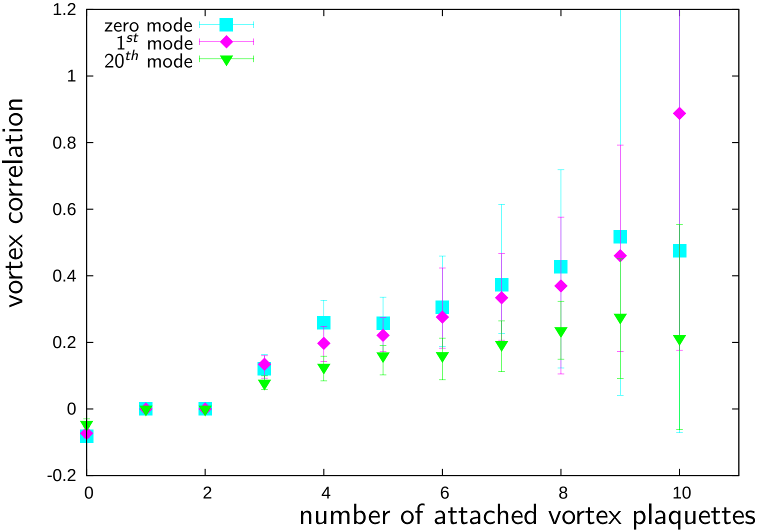
Correlator

$$C_\lambda = \frac{\sum_{P_i} \sum_{x \in H} (V\rho_\lambda(x) - \langle V\rho_\lambda(x) \rangle)}{\sum_{P_i} \sum_{x \in H} 1}$$

➜ *Kovalenko, Morozov, Polikarpov and Zakharov 2005*

- vortex points P_i on the dual lattice
- scalar eigenmode density $\rho_\lambda(x)$, averaged over the vertices x of the 4d hypercube H, dual to P_i
- strongly depends on the number of the vortex plaquettes, attached to a point P_i

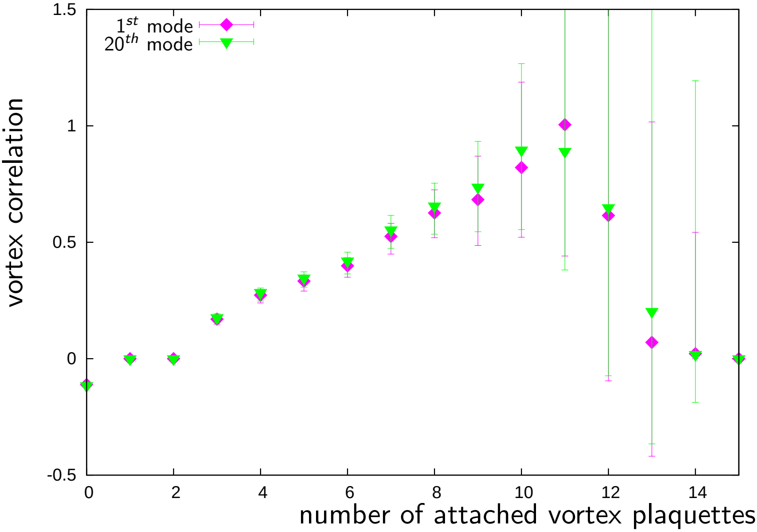
Vortex correlation for overlap modes



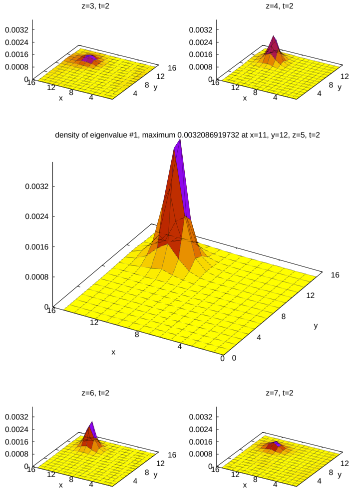
Vortex correlation for asqtad staggered modes

Center Vortices, Confinement and Chiral Symmetry Breaking

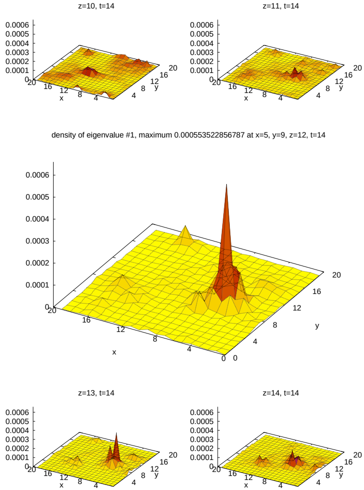
Roman Höllwieser



Dirac Eigenmode Density Peaks

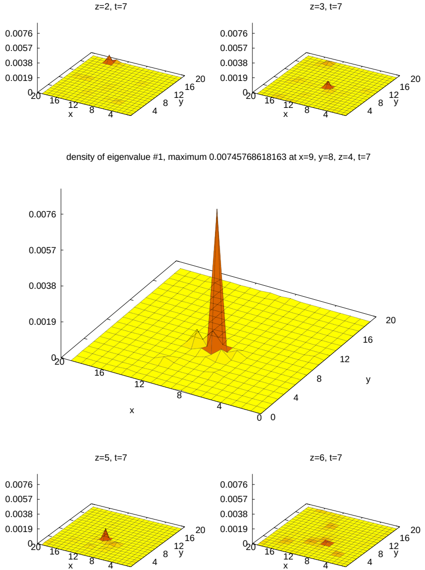


overlap eigenmode

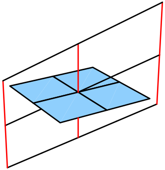


asqtad staggered

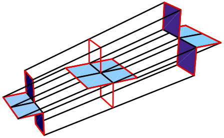
Center-projected eigenmode and topological charge



- intersections



- writhing points



(Engelhardt, Reinhardt)

Conclusions

- Confining Disorder $\equiv$ Center Disorder
- P-vortices locate center vortices $W_n / W_0 = (-1)^n$
- Center Dominance: The projected string tension is close to the asymptotic string tension σ of full Monte-Carlo configurations $\chi_{cp}(R, R) \approx \sigma \quad (R \geq 2)$
- Upon abelian projection, center vortices appear as chains of monopoles and antimonopoles.
- Vortex removal restores chiral symmetry
- Asqtad staggered fermions show confinement and chiral symmetry breaking also for center-projected configurations
- Strong correlations between Dirac eigenmodes and center vortices
- Dirac eigenmodes show sharp peaks at intersection and writhing points

Thank you for your attention!

Questions?