# Efficient sources for spectroscopy of excited mesons in lattice QCD

Daniel Mohler

Inst. f. Physik, FB Theoretische Physik
Universität Graz

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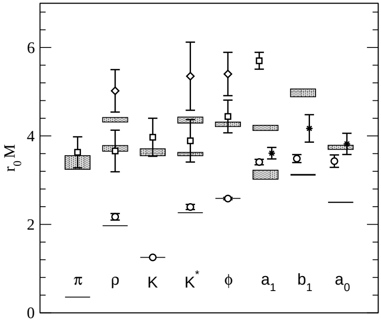
*Advisors:* Christof Gattringer, Christian Lang
*Co-Workers:* Georg Engel, Markus Limmer
Leonid Glozman, Sasa Prelovsek

# Outline

1. Excited states in Lattice QCD
   - Motivation and Introduction
   - Variational method and interpolating fields
   - Construction of efficient quark sources
2. Spectroscopy with derivative sources
   - Excited pions with derivative sources
   - Quenched results for the (isovector) scalar mesons
3. Tetraquark interpolators for scalar mesons
4. Summary & Outlook

# Motivation: Mesons from quenched QCD




Burch et al., PRD 73 (2006) 094505

- Excited states are a lot *more difficult* than ground states!
- Goal: Improving the method for extracting excited states.

# Euclidean correlators

- Euclidean correlator of two Hilbert-space operators $\hat{O}_1$ and $\hat{O}_2$.

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle = \sum_n e^{-tE_n} < 0|\hat{O}_2|n >< n|\hat{O}_1|0 >$$

- Can also be expressed as a Euclidean path integral

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}, U] e^{-S_E} O_2[\psi, \bar{\psi}, U] O_1[\psi, \bar{\psi}, U]$$

- Will show how to apply this to extract the spectrum of mesons from lattice QCD.
- The same applies to other quantities of interest: baryon correlators, three point functions, matrix elements, ...

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# Quenched approximation vs. full QCD

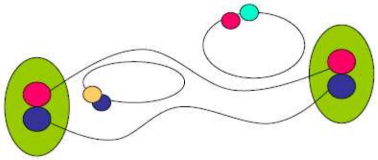
"Full QCD":

$$C(t) \propto \int \mathcal{D}[U]\, \mathcal{D}[\psi,\bar\psi]\, \mathrm{e}^{-S_G[U]-\bar\psi D[U]\psi}\, M(t)\bar M(0)$$

$$= \int \mathcal{D}[U]\, \mathrm{e}^{-S_G[U]} \left(\det D_u \det D_d \ldots\right)$$

$$\times \left[D_u^{-1} D_d^{-1} \ldots + \ldots\right]$$

- Set $\det D \equiv 1$ (no dynamical fermion vacuum, i.e. no sea quarks)
- Gauge field vacuum is fully dynamical (Monte Carlo)
- Consider only the valence quarks
- Hadron correlation functions are built from the quark propagators



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$$C(t) \propto \int \mathcal{D}[U]\,\mathcal{D}[\psi,\bar{\psi}]\,\mathrm{e}^{-S_G[U]-\bar{\psi}D[U]\psi}\,N(t)\bar{N}(0)$$
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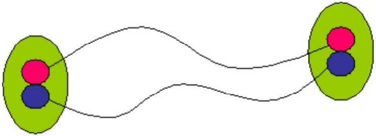

# Variational method

Variational method (C.Michael; Lüscher and Wolff)
Matrix of correlators projected to fixed momentum (will assume 0)

$$C(t)_{ij} = \sum_n e^{-tE_n} \langle 0|O_i|n\rangle \left\langle n|O_j^\dagger|0\right\rangle$$

Solve the generalized eigenvalue problem:

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t)C(t_0)\vec{\psi}^{(k)}$$
$$\lambda^{(k)}(t) \propto e^{-tE_k}\left(1+\mathcal{O}\left(e^{-t\Delta E_k}\right)\right)$$

At large time separation: only a single mass in each eigenvalue.
Eigenvectors can serve as a fingerprint.

# Variational method II

- Interesting observation for $t \leq 2t_0$

$$\Delta E_k \equiv E_{N+1} - E_k$$

Blossier et al. [arXiv: 0808.1017]

- Can also determine couplings:

$$C(t) = \sum_{n=1}^{\infty} v_i^{(n)} v_i^{(n)\star} \mathrm{e}^{-tE^{(n)}} \quad \text{with} \quad v_i^{(n)} = \left\langle 0 | O_i | H^{(n)} \right\rangle$$

$$R(t)_i^{(k)} = \frac{|\sum_j C(t)_{ij} \psi_j^{(k)}|^2}{\sum_k \sum_l \psi_k^{(k)\star} C(t)_{kl} \psi_l^{(k)}} \approx v_i^{(k)} v_i^{(k)\star} \mathrm{e}^{-tE^{(k)}}$$

Burch et al. [arXiv: 0809.1103]

# Interpolating fields for mesons

- We need interpolating field operators with quantum numbers $J^{PC}$
- These are typically build from *smeared quarks*

$$\bar{\psi}\Gamma\psi$$

- Some examples (for Isovector mesons):

| $J^{PC}$ | interpolator |
|---|---|
| $0^{-+}$ | $\bar{u}_s \gamma_5 u_s$ |
| $1^{--}$ | $\bar{u}_s \gamma_i u_s$ |

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- Remark: At finite lattice spacing $a$ interpolators will also couple to higher spin states.

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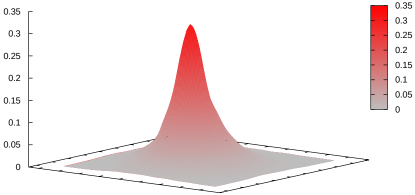
# Quark sources

- Jacobi smeared quark sources, e.g., $u_s \equiv (S\,u)_x$

$$S = M\,S_0 \quad \text{with} \quad M = \sum_{n=0}^{N} \kappa^n H^n$$

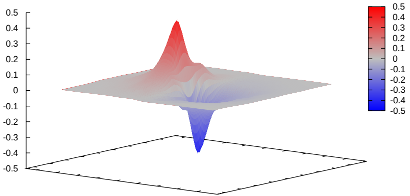
$$H(\vec{n},\vec{m}\,) = \sum_{j=1}^{3} \left( U_j\,(\vec{n},0)\,\delta\left(\vec{n}+\hat{j},\vec{m}\right) + U_j\left(\vec{n}-\hat{j},0\right)^{\dagger}\delta\left(\vec{n}-\hat{j},\vec{m}\right)\right).$$

- Combination allows nodes in the interpolating operators



Derivative quark sources $W_{d_i}$:

$$D_i(\vec{x},\vec{y}) = U_i(\vec{x},0)\delta(\vec{x}+\hat{i},\vec{y}) - U_i(\vec{x}-\hat{i},0)^\dagger\delta(\vec{x}-\hat{i},\vec{y}) \; ,$$
$$W_{d_i} = D_i\, S \; .$$



Mesons with derivative sources: Pion channel

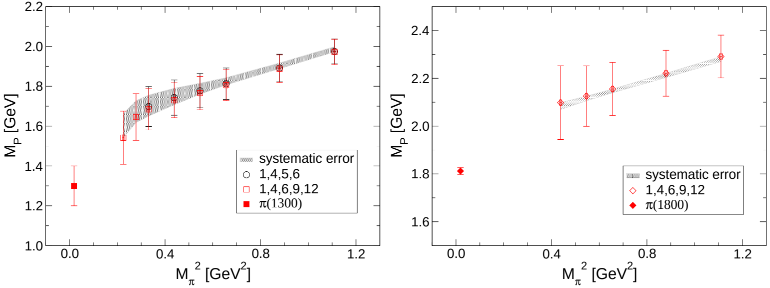



Figure: 1st and 2nd excitation of $\pi$

Gattringer et al., ArXiv:0802.2020 [hep-lat], PRD 78, 034501, 2008

# Mesons from quenched ensemble

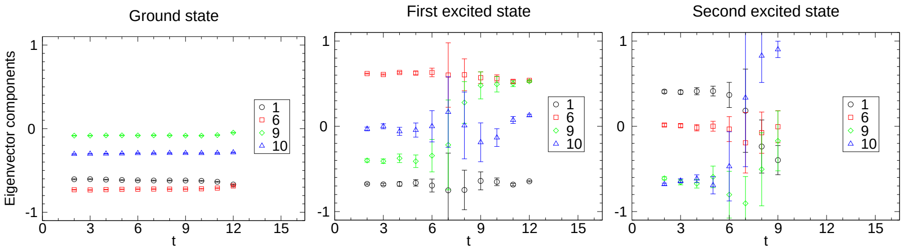



Figure: Eigenvector components for ground state and lowest excitations

Gattringer et al., ArXiv:0802.2020 [hep-lat], PRD 78, 034501, 2008

Mesons with derivative sources: Isovector Scalar 1 $0^{++}$

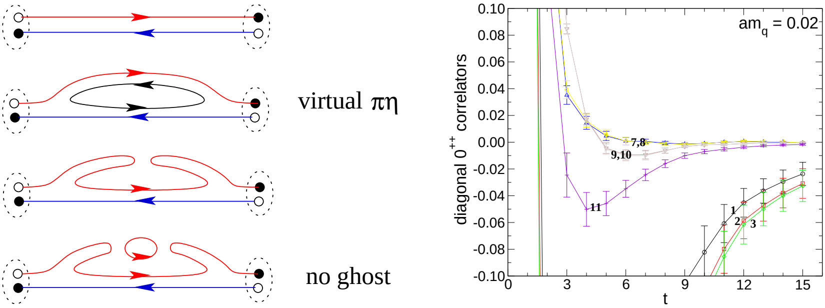



Figure: Diagonal correlators and ground state in the $0^{++}$ channel

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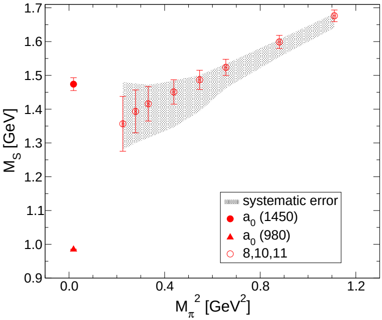


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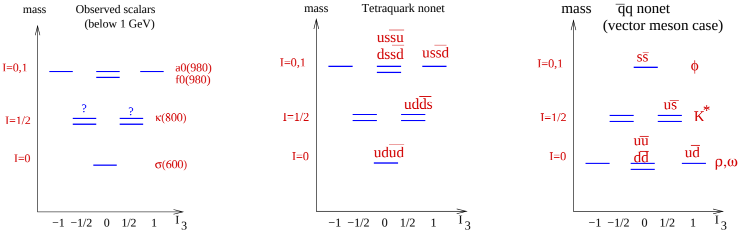
Other quenched results (also) extrapolate to $a_0(1450)$

# The scalar meson puzzle

- Low lying scalars could be tetraquark states




- quark models would place $\bar{q}q$ with $L = 1$ above 1GeV
- $m_\kappa < m_{a_0}$ hard to reconcile with $\bar{u}s$ and $\bar{u}d$
- $a_0(980)$ couples well with $K\bar{K}$

# Tetraquarks in the scalar channel

Tetraquarks with a diquark - anti-diquark structure

$$[qQ]_a \equiv \epsilon_{abc}[q_b^T C\gamma_5 Q_c - Q_b^T C\gamma_5 q_c]$$
$$[\bar{q}\bar{Q}]_a \equiv \epsilon_{abc}[\bar{q}_b C\gamma_5 \bar{Q}_c^T - \bar{Q}_b C\gamma_5 \bar{q}_c^T] \ ,$$

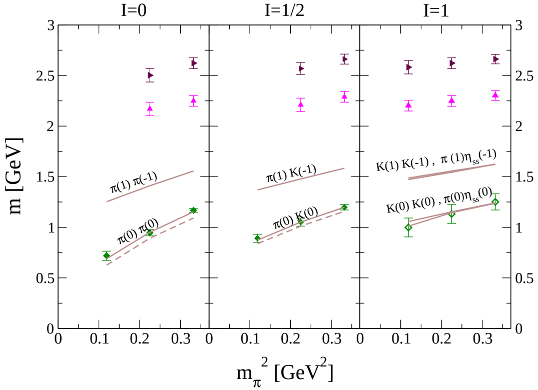
We simulate

$$\mathcal{O}^{I=0} = [ud][\bar{u}\bar{d}] \ , \quad \mathcal{O}^{I=1/2} = [ud][\bar{d}\bar{s}] \ , \quad \mathcal{O}^{I=1} = [us][\bar{d}\bar{s}]$$

with different source/sink smearings and use the variational method.

S.Prelovsek and D.M., arXiv:0810.1759

# Scalar tetraquarks - results




Main conclusion: No evidence of low lying scalar tetraquarks for $M_\pi$ in the range $344 \ldots 576\mathrm{MeV}$

S.Prelovsek and D.M., arXiv:0810.1759

# Summary and Outlook

- Excited states are difficult on the lattice.
- Variational method can serve to reliably extract excited states.
- Presented results from an improved basis containing interpolators with derivatives.
- Presented results for scalar mesons using tetraquark interpolators.

- Analysis of dynamical data is ongoing.
- Improved basis can also be used for baryons and tetraquarks.
- Scattering states and resonances further complicate the analysis.
- Want to apply methods to charmonium states using MILC lattices (With C. DeTar and T. Burch).

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