Bethe-Salpeter Equation Studies of Meson Properties

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Outline

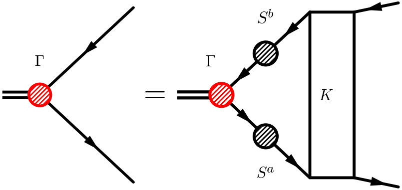
Introduction

- Mesons in QCD: bound states of quarks, anti-quarks and gluons
- Description via Green functions: nonperturbative, covariant
- Bethe-Salpeter equation (BSE) and Dyson-Schwinger equations (DSEs)[1]:
  infinite system of coupled integral equations for Green functions.
- Mesons: integral equation for quark-antiquark channel

[1] Recent reviews: [C. S. Fischer 2006, C. D. Roberts et al. 2007]

Homogeneous BSE

- Inhomogenous integral equation for the $q\bar{q}$ 4-pt. function
- Project on bound-state pole: homogeneous BSE



- In general: describe all possible bound states
- Choose quantum numbers by making the structure of the amplitude explicit[2]
- Unknown quantities (so far):
  - Quark - anti-quark scattering kernel K
  - Dressed quark propagators S^a, S^b

[2] [C. H. Llewellyn-Smith 1969]

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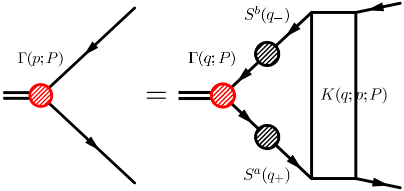
Homogeneous BSE



$$\Gamma(p;P) = \int_q S^a(q+\eta P)\,\Gamma(q;P)\,S^b(q-(1-\eta)P)\,K(q,p;P)$$

- $\Gamma(p;P)$ Bethe-Salpeter Amplitude (BSA)
- $S(k)$ dressed quark propagator
- $K(q,p;P)$ quark-antiquark scattering kernel
- P total momentum, $P^2 = -M^2$
- p relative momentum
- $0 < \eta < 1$ momentum partitioning

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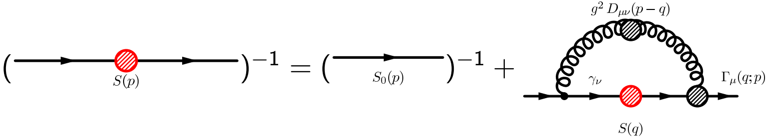
Quark Gap Equation

- Dressed quark propagator

$$S^{-1}(p) = i\,\gamma \cdot p\; A(p^2) + B(p^2)$$

- Obtained from its Dyson-Schwinger equation:
  Quark gap equation



$$S^{-1}(p) = S_0^{-1}(p) + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\; g^2\,D_{\mu\nu}(p-q)\,\gamma_\nu\,S(q)\,\Gamma_\mu(q;p)$$

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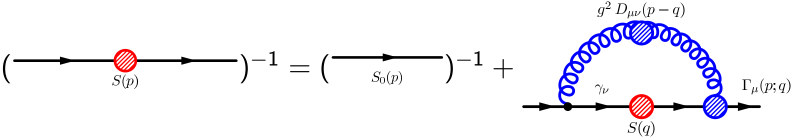
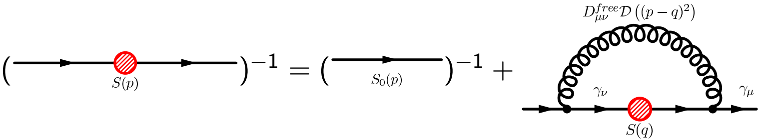
Rainbow-Ladder Approximation



- For numerical studies: employ truncation
- Consistent scheme:
  Axial-Vector Ward-Takahashi Identity is satisfied[3]
  ($\Rightarrow$ e.g., the pion becomes massless in the chiral limit)
- Lowest order in scheme: rainbow truncation for the gap equation

[3] [H. J. Munczek 1995, A. Bender et al. 1996]
[4] [Maris, Tandy 1999]

Rainbow-Ladder Approximation



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  ($\Rightarrow$ e.g., the pion becomes massless in the chiral limit)
- Lowest order in scheme: rainbow truncation for the gap equation
- Effective quark-gluon interaction[4]
  $\mathcal{D}(p^2) = \frac{D}{2} \frac{(2\pi)^2}{\omega^6} p^2 e^{-p^2/\omega^2} + \mathcal{F}_{UV}(p^2)$

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[4] [Maris, Tandy 1999]

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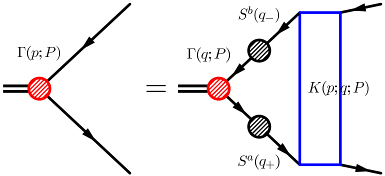

- Ladder truncation for the BSE - kernel

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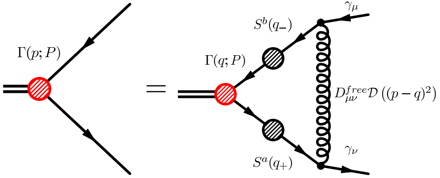
Rainbow-Ladder Approximation



- Ladder truncation for the BSE - kernel
- Using the same effective quark-gluon interaction as in gap equation

$$K(p;q;P) \Rightarrow \gamma_\nu \mathcal{D}((p-q)^2) D^{free}_{\mu\nu}(p-q)\gamma_\mu$$

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Gap Equation

- 2 coupled nonlinear inhomogeneous integral equations for the components of the inverse quark propagator, i.e. the dressing functions A and B

$$S^{-1}(p) = i\,\gamma \cdot p\, A(p^2) + B(p^2)$$

- Solved iteratively on the real p^2 axis
- In BSE: $S^a(q + \eta P)$, $S^b(q - (1 - \eta)P)$
- Argumets are complex (on shell amplitude: $P^2 = -M^2$)
- Numerical analytical continuation of $A(p^2)$ and $B(p^2)$ into the complex plane needed

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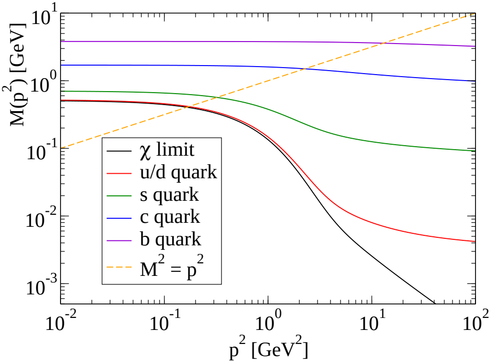
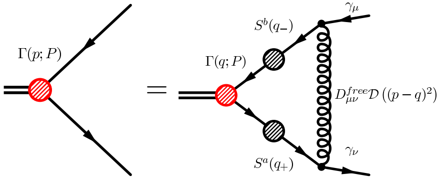
Gap Equation

- Obtain, e.g., quark mass function: $M(p^2) = B(p^2)/A(p^2)$



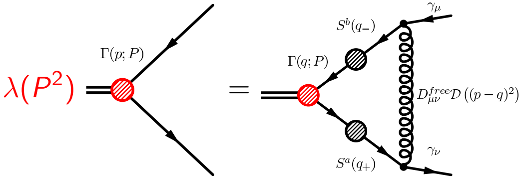
- Dynamical chiral symmetry breaking: $M(p^2) \neq 0$ even in the chiral limit!

Homogeneous BSE



- Homogenous linear integral equation
- Only valid if $P^2 = -M^2$ (M mass of bound state)

Homogeneous BSE



- Homogenous linear integral equation
- Only valid if $P^2 = -M^2$ (M mass of bound state)
- Introduce an eigenvalue $\lambda(P^2)$
- The equation has a solution at mass M iff $\lambda(P^2 = -M^2) = 1$

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Homogeneous BSE

- Can (numerically) be written as an eigenvalue equation for a matrix
$$\lambda(P^2)\Gamma(p;P) = \tilde{K}(p;q;P)\Gamma(q;P)$$
($\tilde{K}(p;q;P)$ contains the dressed quark propagators)
- Strategy: solve for the largest few eigenvalues $\lambda_i(P^2)$, $i = 0, 1, \dots$ for various P^2
- Obtain curves $\lambda(P^2)$ for chosen eigenvalues:
- $\lambda_0(P^2) = 1 \Rightarrow P^2 = -M_0^2$ mass of the ground state
- $\lambda_1(P^2) = 1 \Rightarrow P^2 = -M_1^2$ mass of the first excited state
- ...

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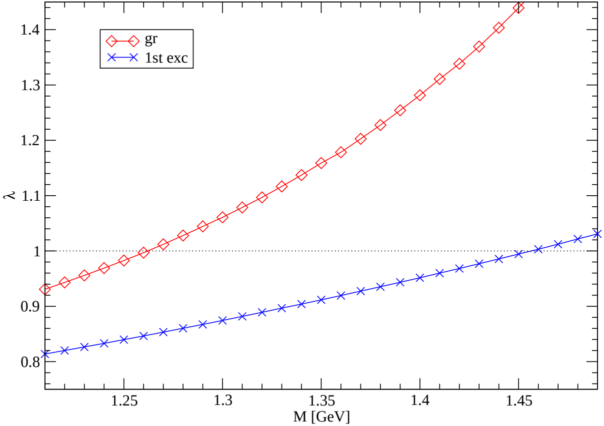
Homogeneous BSE

Illustration: curves for ground and first excited state



What is the Bethe-Salpeter Equation (BSE)?

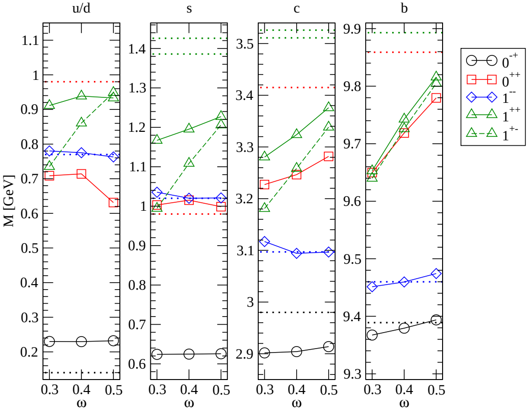
Solution Strategies

Applications

Summary & Outlook

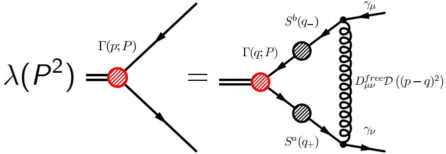
Meson Masses

- Procedure applicable from the chiral limit to $m_q = m_b$

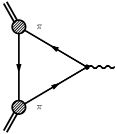


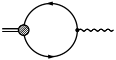
Observables

- Solution of the eigenvalue equation

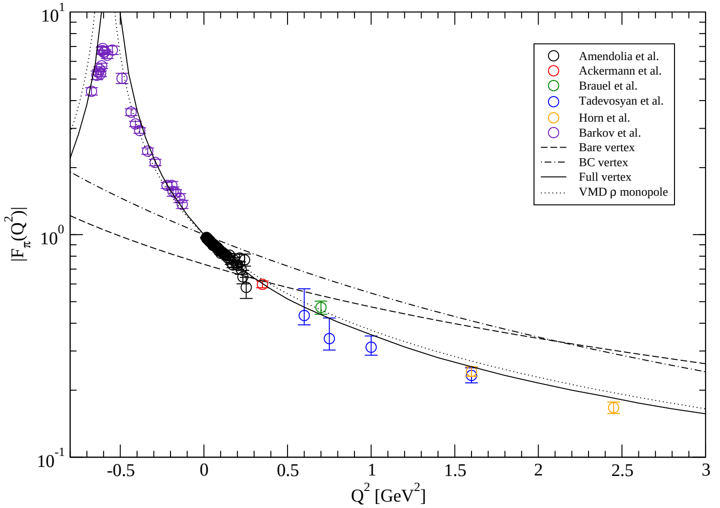


  gives eigenvalues and eigenvectors
- Eigenvector $\Rightarrow$ Bethe-Salpeter amplitude (BSA) of the corresponding state
- Can be used in various ways to compute observables
- E.g., electromagnetic form factors, leptonic decay constants





Pion Electromagnetic Form Factor



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Summary

- BSE and DSEs provide a nonperturbative, covariant decription of mesons as bound states in QCD
- For numerical studies: work in symmetry preserving trucation, e.g., rainbow-ladder
  (lowest order in nonperturbative truncation scheme)
- Ingredients: gap equation and homogeneous BSE

Gap equation: Solve iteratively on the real axis
Numerical analytical continuation to the complex plane

Hom. BSE: Solve as eigenvalue equation
$\lambda(P^2) = 1 \Rightarrow P^2 = -M^2$

- Obtain meson masses from eigenvalues
- Use eigenvectors (BSA) and quark propagators to compute further observables like form factors

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Outlook

- Comprehensive analysis of meson hadronic decays
- Reliable study of states with exotic quantum numbers
- Sophisticated meson model beyond RLT
- Good description of axial-vector mesons
- Reliable study of radial excitations
- Treatment of meson excitations as resonances in the BSE

Other Directions:

- Study of baryons
- Extension to finite temperature and chemical potential

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