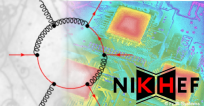


With Automation towards the Four b-Jet Rate at NLO in QCD



5th Vienna Central European Seminar on Particle Physics and QFT, 28–30/11/2008

Overview



Motivation

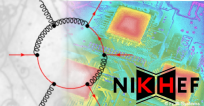
The GOLEM Approach

Summary

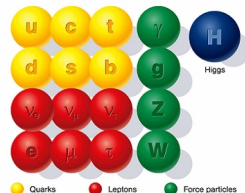
Results for $q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$

Conclusion

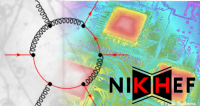
The Missing Link in the Standard Model (SM)



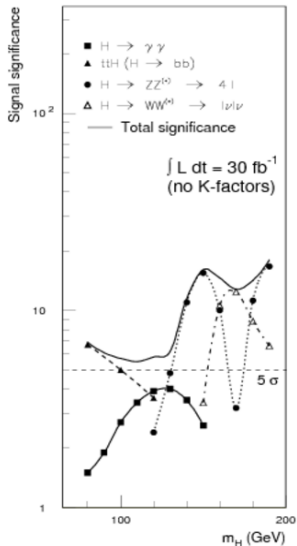
- ▶ SM requires symmetry breaking for mass generation \Rightarrow Higgs boson
- ▶ SM Higgs will be found at LHC
- ▶ SM does not explain everything
 - ▶ Dark matter
 - ▶ Unification of forces
 - ▶ Hierarchy of masses and mixing angles
 - ▶ Gravity
 - ▶ ...
- ▶ Need to look at extensions
- ▶ Promising candidate: Supersymmetry
 - ▶ Testable at LHC (if ≈ 1 TeV)
 - ▶ Solves some (most?) problems



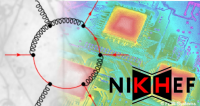
The Missing Link in the Standard Model (SM)



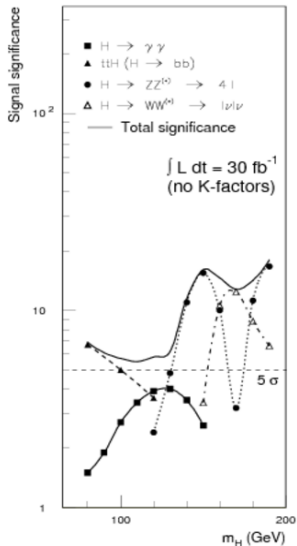
- ▶ SM requires symmetry breaking for mass generation \Rightarrow Higgs boson
- ▶ SM Higgs will be found at LHC
- ▶ SM does not explain everything
 - ▶ Dark matter
 - ▶ Unification of forces
 - ▶ Hierarchy of masses and mixing angles
 - ▶ Gravity
 - ▶ ...
- ▶ Need to look at extensions
- ▶ Promising candidate: Supersymmetry
 - ▶ Testable at LHC (if ≈ 1 TeV)
 - ▶ Solves some (most?) problems



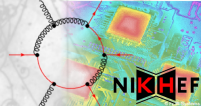
The Missing Link in the Standard Model (SM)



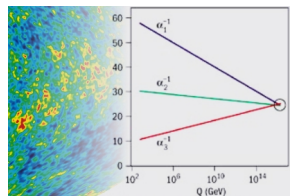
- ▶ SM requires symmetry breaking for mass generation \Rightarrow Higgs boson
- ▶ SM Higgs will be found at LHC — if it exists
- ▶ SM does not explain everything
 - ▶ Dark matter
 - ▶ Unification of forces
 - ▶ Hierarchy of masses and mixing angles
 - ▶ Gravity
 - ▶ ...
- ▶ Need to look at extensions
- ▶ Promising candidate: Supersymmetry
 - ▶ Testable at LHC (if ≈ 1 TeV)
 - ▶ Solves some (most?) problems



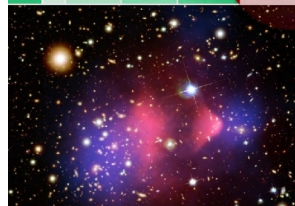
The Missing Link in the Standard Model (SM)



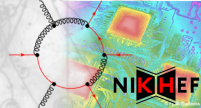
- ▶ SM requires symmetry breaking for mass generation \Rightarrow Higgs boson
- ▶ SM Higgs will be found at LHC — if it exists
- ▶ SM does not explain everything
 - ▶ Dark matter
 - ▶ Unification of forces
 - ▶ Hierarchy of masses and mixing angles
 - ▶ Gravity
 - ▶ ...
- ▶ Need to look at extensions
- ▶ Promising candidate: Supersymmetry
 - ▶ Testable at LHC (if ≈ 1 TeV)
 - ▶ Solves some (most?) problems



	Charge	First generation	Second generation	Third generation
Leptons	0	Electron neutrino	Muon neutrino	Tau neutrino 77
	-1e	Electron 0.511	Muon 105.7	Top 1777
Quarks	+2/3q	Up 2	Charm 1302	Top 180 Top 180
	0			
	-1/3q	Down 1/3	Strange 163	Bottom 4180



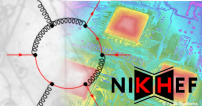
The Missing Link in the Standard Model (SM)



- ▶ SM requires symmetry breaking for mass generation \Rightarrow Higgs boson
- ▶ SM Higgs will be found at LHC — if it exists
- ▶ SM does not explain everything
 - ▶ Dark matter
 - ▶ Unification of forces
 - ▶ Hierarchy of masses and mixing angles
 - ▶ Gravity
 - ▶ ...
- ▶ Need to look at extensions
- ▶ Promising candidate: Supersymmetry
 - ▶ Testable at LHC (if ≈ 1 TeV)
 - ▶ Solves some (most?) problems

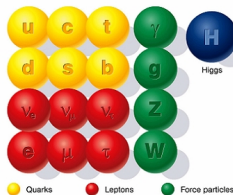


The Missing Link in the Standard Model (SM)

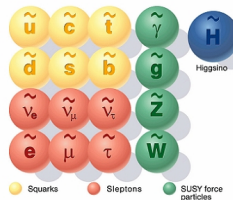


- ▶ SM requires symmetry breaking for mass generation \Rightarrow Higgs boson
- ▶ SM Higgs will be found at LHC — if it exists
- ▶ SM does not explain everything
 - ▶ Dark matter
 - ▶ Unification of forces
 - ▶ Hierarchy of masses and mixing angles
 - ▶ Gravity
 - ▶ ...
- ▶ Need to look at extensions
- ▶ Promising candidate: Supersymmetry
 - ▶ Testable at LHC (if ≈ 1 TeV)
 - ▶ Solves some (most?) problems

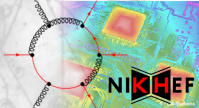
Standard particles



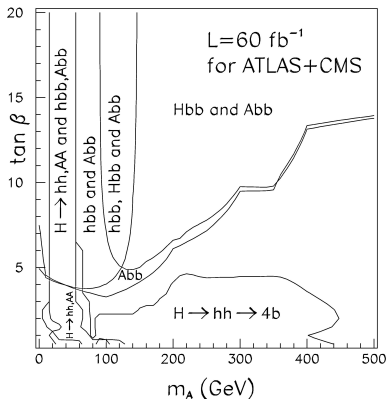
SUSY particles



MSSM Higgs at large $\tan\beta$



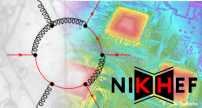
If the MSSM is realized in nature: Higgs sector (h^0, H^0, H^\pm, A)



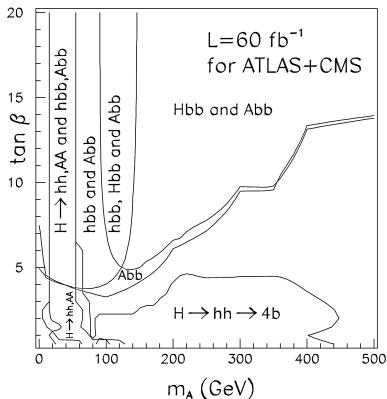
- ▶ no a priori knowledge of MSSM parameters ($m_A, \tan\beta$)
- ▶ still want to find a Higgs boson
- ▶ for large values of $\tan\beta$:
 - ▶ $H^0 \rightarrow b\bar{b}$ important channel
 - ▶ $pp \rightarrow b\bar{b}b\bar{b}$ irreducible background
 - ▶ precise knowledge of signal and background crucial
- ▶ In other parameter regions $H^0 \rightarrow h^0 h^0 \rightarrow b\bar{b}b\bar{b}$

[Dai, Gunion, Vega]

MSSM Higgs at large $\tan\beta$



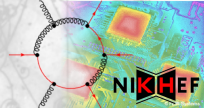
If the MSSM is realized in nature: Higgs sector (h^0, H^0, H^\pm, A)



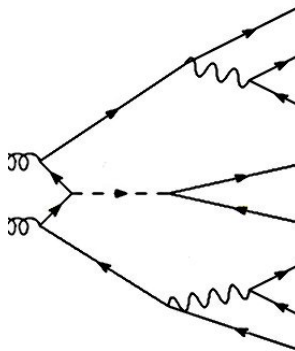
- ▶ no a priori knowledge of MSSM parameters ($m_A, \tan\beta$)
- ▶ still want to find a Higgs boson
- ▶ for large values of $\tan\beta$:
 - ▶ $H^0 \rightarrow b\bar{b}$ important channel
 - ▶ $pp \rightarrow b\bar{b}b\bar{b}$ irreducible background
 - ▶ precise knowledge of signal and background crucial
- ▶ In other parameter regions $H^0 \rightarrow h^0 h^0 \rightarrow b\bar{b}b\bar{b}$

[Dai, Gunion, Vega]

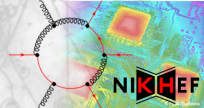
Is NLO really necessary?



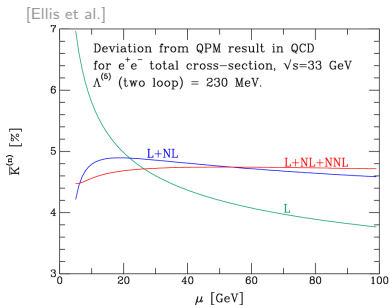
- ▶ “Traditional” approach:
 - ▶ use LO + parton shower for the shape and
 - ▶ fix normalisation with experimental data.
- ▶ not always enough:
 - ▶ In QCD: large renormalisation scale dependency at LO
 - ▶ LO and NLO may differ in shape \Rightarrow no global K -factor
- ▶ Therefore: For LHC many processes needed at \geq NLO.
- ▶ Automated tools as for LO desirable.



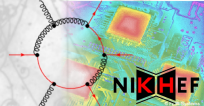
Is NLO really necessary?



- ▶ “Traditional” approach:
 - ▶ use LO + parton shower for the shape and
 - ▶ fix normalisation with experimental data.
- ▶ not always enough:
 - ▶ In QCD: large renormalisation scale dependency at LO
 - ▶ LO and NLO may differ in shape \Rightarrow no global K -factor
- ▶ Therefore: For LHC many processes needed at \geq NLO.
- ▶ Automated tools as for LO desirable.

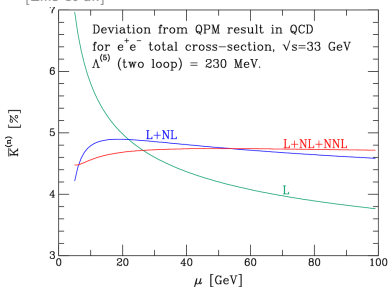


Is NLO really necessary?

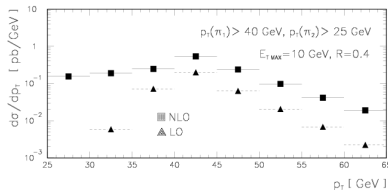


- ▶ “Traditional” approach:
 - ▶ use LO + parton shower for the shape and
 - ▶ fix normalisation with experimental data.
- ▶ not always enough:
 - ▶ In QCD: large renormalisation scale dependency at LO
 - ▶ LO and NLO may differ in shape
⇒ no global K -factor
- ▶ Therefore: For LHC many processes needed at \geq NLO.
- ▶ Automated tools as for LO desirable.

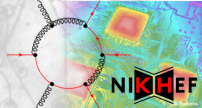
[Ellis et al.]



[Binoth, Guillet, Pilon, Werlen]



Is NLO really necessary?



- ▶ “Traditional” approach:
 - ▶ use LO + parton shower for the shape and
 - ▶ fix normalisation with experimental data.
- ▶ not always enough:
 - ▶ In QCD: large renormalisation scale dependency at LO
 - ▶ LO and NLO may differ in shape
⇒ no global K -factor
- ▶ Therefore: For LHC many processes needed at \geq NLO.
- ▶ Automated tools as for LO desirable.

$$pp \rightarrow VVb\bar{b}$$

$$pp \rightarrow VV + 2\text{jets}$$

$$pp \rightarrow V + 3\text{jets}$$

$$pp \rightarrow t\bar{t}b\bar{b} \quad [\text{hep-ph}/0807.1453]$$

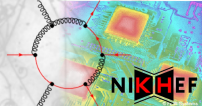
$$pp \rightarrow t\bar{t} + 2\text{jets}$$

$$pp \rightarrow b\bar{b}b\bar{b}$$

[hep-ph/0604120, hep-ph/0803.0494]

After after first LHC data:
more to come

Is NLO really necessary?



- ▶ “Traditional” approach:
 - ▶ use LO + parton shower for the shape and
 - ▶ fix normalisation with experimental data.
- ▶ not always enough:
 - ▶ In QCD: large renormalisation scale dependency at LO
 - ▶ LO and NLO may differ in shape
⇒ no global K -factor
- ▶ Therefore: For LHC many processes needed at \geq NLO.
- ▶ Automated tools as for LO desirable.

$$pp \rightarrow VVb\bar{b}$$

$$pp \rightarrow VV + 2\text{jets}$$

$$pp \rightarrow V + 3\text{jets}$$

$$pp \rightarrow t\bar{t}b\bar{b} \quad [\text{hep-ph}/0807.1453]$$

$$pp \rightarrow t\bar{t} + 2\text{jets}$$

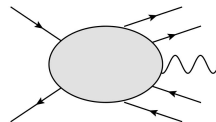
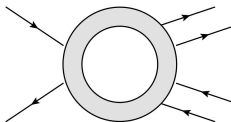
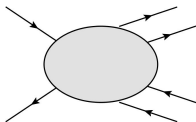
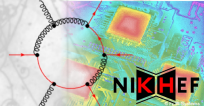
$$pp \rightarrow b\bar{b}b\bar{b}$$

[hep-ph/0604120, hep-ph/0803.0494]

After after first LHC data:
more to come

GOLEM:

General One Loop Evaluator for Matrix Elements



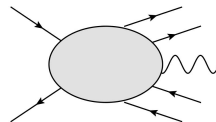
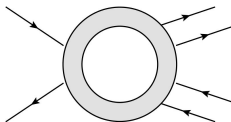
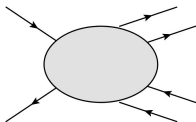
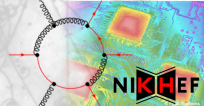
	LO	NLO
Method	standard tool	hand crafted code
Time scale	hours – days	months – years

The (main) problems with NLO calculations:

- ▶ bottle neck: loop diagrams
- ▶ complexity: $\#$ diagrams \times $\#$ terms per diagram
- ▶ tensor integrals, cancellations \leftrightarrow numerical (in-)stability
- ▶ computational challenge: memory and run time

GOLEM:

General One Loop Evaluator for Matrix Elements



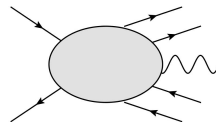
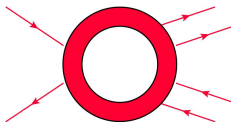
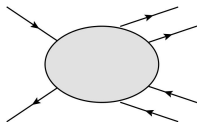
	LO	NLO
Method	standard tool	hand crafted code
Time scale	hours – days	months – years

The (main) problems with NLO calculations:

- ▶ bottle neck: loop diagrams
- ▶ complexity: $\#$ diagrams \times $\#$ terms per diagram
- ▶ tensor integrals, cancellations \leftrightarrow numerical (in-)stability
- ▶ computational challenge: memory and run time

GOLEM:

General One Loop Evaluator for Matrix Elements



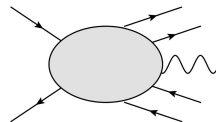
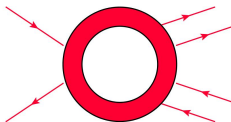
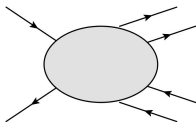
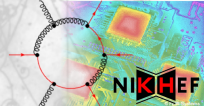
	LO	NLO
Method	standard tool	hand crafted code
Time scale	hours – days	months – years

The (main) problems with NLO calculations:

- ▶ bottle neck: loop diagrams
- ▶ complexity: $\#\text{diagrams} \times \#\text{terms per diagram}$
- ▶ tensor integrals, cancellations \leftrightarrow numerical (in-)stability
- ▶ computational challenge: memory and run time

GOLEM:

General One Loop Evaluator for Matrix Elements

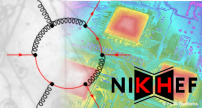


	LO	NLO
Method	standard tool	hand crafted code
Time scale	hours – days	months – years

The (main) problems with NLO calculations:

- ▶ bottle neck: loop diagrams
- ▶ complexity: $\#$ diagrams \times $\#$ terms per diagram
- ▶ tensor integrals, cancellations \leftrightarrow numerical (in-)stability
- ▶ computational challenge: memory and run time

The GOLEM and the Tensor Integrals



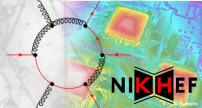
Problem: solve $\int \frac{d^n k}{i\pi^{n/2}} \frac{q_{a_1}^{\mu_1} \cdots q_{a_r}^{\mu_r}}{\prod_{j=1}^N (q_j^2 - m_j^2 + i\delta)}, \quad q_a^\mu = k^\mu + r_a^\mu$

- ▶ tensor reduction separates tensor structure from integrals
- ▶ usually: expression in terms of scalar integrals $I_N^n()$
 \Rightarrow (Gram determinants)⁻¹
- ▶ in GOLEM: allow for certain numerators $I_N^n(l_1, \dots, l_p)$
- ▶ switch between numerical evaluation and algebraic reduction

$$I_N^n(l_1, \dots, l_p) = (-1)^N \Gamma(N - \frac{n}{2}) \int \delta(1 - \sum_{i=1}^N z_i) \frac{z_1 \cdots z_p}{(-\frac{1}{2} z^T S z - i\delta)^{N-n/2}} d^N z$$

Implementation: [golem95](#) — publicly available [arXiv:0810.0992]

The GOLEM and the Tensor Integrals



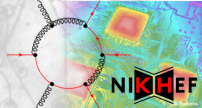
Problem: solve
$$\int \frac{d^n k}{i\pi^{n/2}} \frac{q_{a_1}^{\mu_1} \cdots q_{a_r}^{\mu_r}}{\prod_{j=1}^N (q_j^2 - m_j^2 + i\delta)}, \quad q_a^\mu = k^\mu + r_a^\mu$$

- ▶ tensor reduction separates tensor structure from integrals
- ▶ usually: expression in terms of scalar integrals $I_N^n()$
⇒ (Gram determinants)⁻¹
- ▶ in GOLEM: allow for certain numerators $I_N^n(l_1, \dots, l_p)$
- ▶ switch between numerical evaluation and algebraic reduction

$$I_N^n(l_1, \dots, l_p) = (-1)^N \Gamma(N - \frac{n}{2}) \int \delta(1 - \sum_{i=1}^N z_i) \frac{z_1 \cdots z_p}{(-\frac{1}{2} z^T S z - i\delta)^{N-n/2}} d^N z$$

Implementation: [golem95](#) — publicly available [arXiv:0810.0992]

The GOLEM and the Tensor Integrals



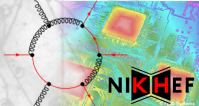
Problem: solve $\int \frac{d^n k}{i\pi^{n/2}} \frac{q_{a_1}^{\mu_1} \cdots q_{a_r}^{\mu_r}}{\prod_{j=1}^N (q_j^2 - m_j^2 + i\delta)}$, $q_a^\mu = k^\mu + r_a^\mu$

- ▶ tensor reduction separates tensor structure from integrals
- ▶ usually: expression in terms of scalar integrals $I_N^n()$
 \Rightarrow (Gram determinants) $^{-1}$
- ▶ in GOLEM: allow for certain numerators $I_N^n(l_1, \dots, l_p)$
- ▶ switch between numerical evaluation and algebraic reduction

$$I_N^n(l_1, \dots, l_p) = (-1)^N \Gamma(N - \frac{n}{2}) \int \delta(1 - \sum_{i=1}^N z_i) \frac{z_{l_1} \cdots z_{l_p}}{(-\frac{1}{2} z^T S z - i\delta)^{N-n/2}} d^N z$$

Implementation: [golem95](#) — publicly available [arXiv:0810.0992]

The GOLEM and the Tensor Integrals



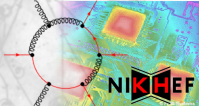
Problem: solve $\int \frac{d^n k}{i\pi^{n/2}} \frac{q_{a_1}^{\mu_1} \cdots q_{a_r}^{\mu_r}}{\prod_{j=1}^N (q_j^2 - m_j^2 + i\delta)}$, $q_a^\mu = k^\mu + r_a^\mu$

- ▶ tensor reduction separates tensor structure from integrals
- ▶ usually: expression in terms of scalar integrals $I_N^n()$
 \Rightarrow (Gram determinants)⁻¹
- ▶ in GOLEM: allow for certain numerators $I_N^n(l_1, \dots, l_p)$
- ▶ switch between numerical evaluation and algebraic reduction

$$I_N^n(l_1, \dots, l_p) = (-1)^N \Gamma(N - \frac{n}{2}) \int \delta(1 - \sum_{i=1}^N z_i) \frac{z_{l_1} \cdots z_{l_p}}{(-\frac{1}{2} z^T S z - i\delta)^{N-n/2}} d^N z$$

Implementation: [golem95](#) — publicly available [arXiv:0810.0992]

The GOLEM and the Tensor Integrals



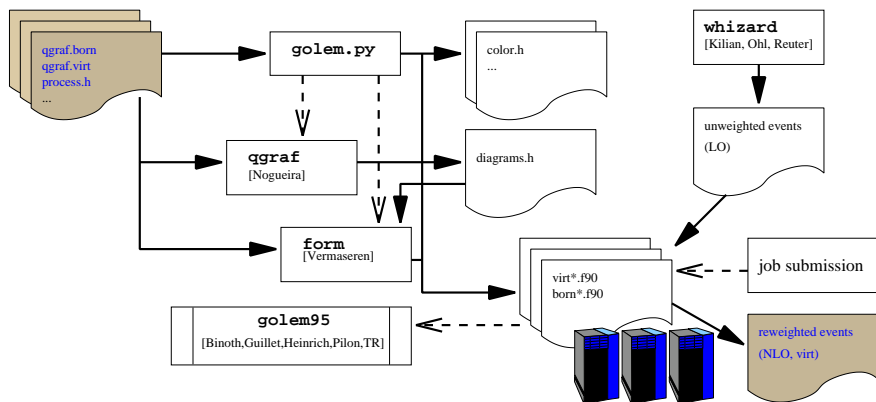
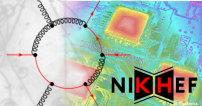
Problem: solve $\int \frac{d^n k}{i\pi^{n/2}} \frac{q_{a_1}^{\mu_1} \cdots q_{a_r}^{\mu_r}}{\prod_{j=1}^N (q_j^2 - m_j^2 + i\delta)}$, $q_a^\mu = k^\mu + r_a^\mu$

- ▶ tensor reduction separates tensor structure from integrals
- ▶ usually: expression in terms of scalar integrals $I_N^n()$
 \Rightarrow (Gram determinants)⁻¹
- ▶ in GOLEM: allow for certain numerators $I_N^n(l_1, \dots, l_p)$
- ▶ switch between numerical evaluation and algebraic reduction

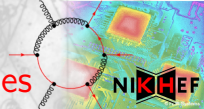
$$I_N^n(l_1, \dots, l_p) = (-1)^N \Gamma(N - \frac{n}{2}) \int \delta(1 - \sum_{i=1}^N z_i) \frac{z_{l_1} \cdots z_{l_p}}{(-\frac{1}{2} z^T S z - i\delta)^{N-n/2}} d^N z$$

Implementation: **golem95** — publicly available [arXiv:0810.0992]

GOLEM: Preliminary Setup



Phase Space Integration of One-Loop Amplitudes



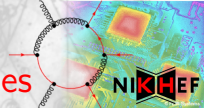
- ▶ at LO: adaptive MC integration over phase space
- ▶ at NLO: adaption can become unstable in presence of integrable singularities.
- ▶ alternative: adapt to LO matrix element but integrate one-loop amplitude
- ▶ in practice: reweighting of unweighted LO events:

$$\langle O \rangle_{\text{one-loop}} = \frac{\sigma_{\text{LO}}}{|U|} \sum_{e \in U} O(e) \frac{d\sigma_{\text{one-loop}}(e)}{d\sigma_{\text{LO}}(e)}$$

- ▶ corresponds to importance sampling with

$$p(e) \propto \frac{1}{\sigma_{\text{LO}}} \frac{d\sigma_{\text{LO}}(e)}{d\Phi(N)}$$

Phase Space Integration of One-Loop Amplitudes



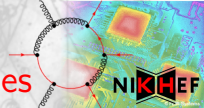
- ▶ at LO: adaptive MC integration over phase space
- ▶ at NLO: adaption can become unstable in presence of integrable singularities.
- ▶ alternative: adapt to LO matrix element but integrate one-loop amplitude
- ▶ in practice: reweighting of unweighted LO events:

$$\langle O \rangle_{\text{one-loop}} = \frac{\sigma_{\text{LO}}}{|U|} \sum_{e \in U} O(e) \frac{d\sigma_{\text{one-loop}}(e)}{d\sigma_{\text{LO}}(e)}$$

- ▶ corresponds to importance sampling with

$$p(e) \propto \frac{1}{\sigma_{\text{LO}}} \frac{d\sigma_{\text{LO}}(e)}{d\Phi(N)}$$

Phase Space Integration of One-Loop Amplitudes



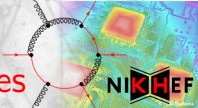
- ▶ at LO: adaptive MC integration over phase space
- ▶ at NLO: adaption can become unstable in presence of integrable singularities.
- ▶ alternative: adapt to LO matrix element but integrate one-loop amplitude
- ▶ in practice: reweighting of unweighted LO events:

$$\langle O \rangle_{\text{one-loop}} = \frac{\sigma_{\text{LO}}}{|U|} \sum_{e \in U} O(e) \frac{d\sigma_{\text{one-loop}}(e)}{d\sigma_{\text{LO}}(e)}$$

- ▶ corresponds to importance sampling with

$$p(e) \propto \frac{1}{\sigma_{\text{LO}}} \frac{d\sigma_{\text{LO}}(e)}{d\Phi(N)}$$

Phase Space Integration of One-Loop Amplitudes



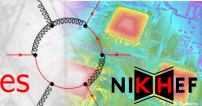
- ▶ at LO: adaptive MC integration over phase space
- ▶ at NLO: adaption can become unstable in presence of integrable singularities.
- ▶ alternative: adapt to LO matrix element but integrate one-loop amplitude
- ▶ in practice: reweighting of unweighted LO events:

$$\langle O \rangle_{\text{one-loop}} = \frac{\sigma_{\text{LO}}}{|U|} \sum_{e \in U} O(e) \frac{d\sigma_{\text{one-loop}}(e)}{d\sigma_{\text{LO}}(e)}$$

- ▶ corresponds to importance sampling with

$$p(e) \propto \frac{1}{\sigma_{\text{LO}}} \frac{d\sigma_{\text{LO}}(e)}{d\Phi(N)}$$

Phase Space Integration of One-Loop Amplitudes



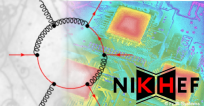
- ▶ at LO: adaptive MC integration over phase space
- ▶ at NLO: adaption can become unstable in presence of integrable singularities.
- ▶ alternative: adapt to LO matrix element but integrate one-loop amplitude
- ▶ in practice: reweighting of unweighted LO events:

$$\langle O \rangle_{\text{one-loop}} = \frac{\sigma_{\text{LO}}}{|U|} \sum_{e \in U} O(e) \frac{d\sigma_{\text{one-loop}}(e)}{d\sigma_{\text{LO}}(e)}$$

- ▶ corresponds to importance sampling with

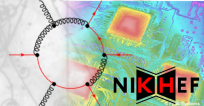
$$p(e) \propto \frac{1}{\sigma_{\text{LO}}} \frac{d\sigma_{\text{LO}}(e)}{d\Phi(N)}$$

Summary I: The GOLEM Project



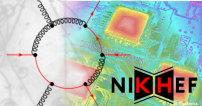
- ▶ GOLEM is
 - ▶ an **algorithm** for the reduction of tensor integrals **plus its implementation** (`golem95`)
 - ▶ an **algorithm** for the automatic algebraic simplification and numerical evaluation of one-loop matrix elements **plus its implementation** (under development)
- ▶ Work in progress:
 - ▶ building a reliable, general tool (based on existing frameworks)
 - ▶ extending `golem95` to massive propagators
 - ▶ doing phenomenology, e.g. $pp \rightarrow b\bar{b}b\bar{b}$
[Binoth, Guffanti, TR, Reuter]
- ▶ GOLEM in the past:
 - ▶ $pp \rightarrow VV + \text{jet}$ [Karg, Sanguinetti] also [Dittmaier et al.; Campbell et al.]
 - ▶ $\gamma\gamma\gamma\gamma \rightarrow 0$ [Binoth, Heinrich, Gehrmann, Mastrolia]; [Bernicot, Guillet]
 - ▶ Rational Polynomials in one-loop amplitudes [Binoth, Guillet, Heinrich]

Summary I: The GOLEM Project



- ▶ GOLEM is
 - ▶ an **algorithm** for the reduction of tensor integrals **plus its implementation** (`golem95`)
 - ▶ an **algorithm** for the automatic algebraic simplification and numerical evaluation of one-loop matrix elements **plus its implementation** (under development)
- ▶ Work in progress:
 - ▶ building a reliable, general tool (based on existing frameworks)
 - ▶ extending `golem95` to massive propagators
 - ▶ doing phenomenology, e.g. $pp \rightarrow b\bar{b}b\bar{b}$
[Binoth, Guffanti, TR, Reuter]
- ▶ GOLEM in the past:
 - ▶ $pp \rightarrow VV + \text{jet}$ [Karg, Sanguinetti] also [Dittmaier et al.; Campbell et al.]
 - ▶ $\gamma\gamma\gamma\gamma \rightarrow 0$ [Binoth, Heinrich, Gehrmann, Mastrolia]; [Bernicot, Guillet]
 - ▶ Rational Polynomials in one-loop amplitudes [Binoth, Guillet, Heinrich]

Summary I: The GOLEM Project

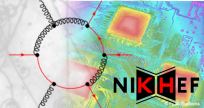


- ▶ GOLEM is
 - ▶ an **algorithm** for the reduction of tensor integrals **plus its implementation** (`golem95`)
 - ▶ an **algorithm** for the automatic algebraic simplification and numerical evaluation of one-loop matrix elements **plus its implementation** (under development)
- ▶ Work in progress:
 - ▶ building a reliable, general tool (based on existing frameworks)
 - ▶ extending `golem95` to massive propagators
 - ▶ doing phenomenology, e.g. $pp \rightarrow b\bar{b}b\bar{b}$

[Binoth, Guffanti, TR, Reuter]

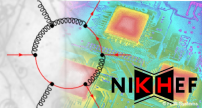
- ▶ GOLEM in the past:
 - ▶ $pp \rightarrow VV + \text{jet}$ [Karg, Sanguinetti] also [Dittmaier et al.; Campbell et al.]
 - ▶ $\gamma\gamma\gamma\gamma \rightarrow 0$ [Binoth, Heinrich, Gehrmann, Mastrolia]; [Bernicot, Guillet]
 - ▶ Rational Polynomials in one-loop amplitudes [Binoth, Guillet, Heinrich]

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Setup



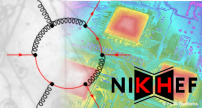
- ▶ software setup as described before
- ▶ check through second, independent implementation (FeynArts, Form, Maple — no code in common)
- ▶ improved IR Dipole subtraction [Catani, Seymour, Nagy]
- ▶ **here**: only $d\sigma_{\text{virtual}} + d\sigma_{\text{LO}} + \langle I(\epsilon, \alpha) \rangle$ (full result soon)
- ▶ cuts and parameters
 - ▶ $m_b = 0$
 - ▶ $p_T > 25 \text{ GeV}$, $|\eta| < 2.5$, $\Delta R > 0.4$
 - ▶ $\mu_R = \mu_F = \sum_{i=1}^4 p_T^{(i)} / 4$
 - ▶ pdfs CTEQ6.5
 - ▶ Nagy's $\alpha = 0.1$ (for the shown results)
- ▶ performance on ECDF cluster
 - ▶ one phase-space point per node (Xeon5450 quad-core 3 GHz)
 - ▶ 8.9 s in double precision
 - ▶ 17.6 s in quadruple precision

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Setup



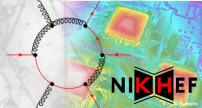
- ▶ software setup as described before
- ▶ check through second, independent implementation (FeynArts, Form, Maple — no code in common)
- ▶ improved IR Dipole subtraction [Catani, Seymour, Nagy]
- ▶ here: only $d\sigma_{\text{virtual}} + d\sigma_{\text{LO}} + \langle I(\epsilon, \alpha) \rangle$ (full result soon)
- ▶ cuts and parameters
 - ▶ $m_b = 0$
 - ▶ $p_T > 25$ GeV, $|\eta| < 2.5$, $\Delta R > 0.4$
 - ▶ $\mu_R = \mu_F = \sum_{i=1}^4 p_T^{(i)} / 4$
 - ▶ pdfs CTEQ6.5
 - ▶ Nagy's $\alpha = 0.1$ (for the shown results)
- ▶ performance on ECDF cluster
 - ▶ one phase-space point per node (Xeon5450 quad-core 3 GHz)
 - ▶ 8.9 s in double precision
 - ▶ 17.6 s in quadruple precision

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Setup



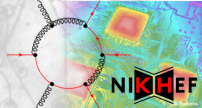
- ▶ software setup as described before
- ▶ check through second, independent implementation (FeynArts, Form, Maple — no code in common)
- ▶ improved IR Dipole subtraction [Catani, Seymour, Nagy]
- ▶ here: only $d\sigma_{\text{virtual}} + d\sigma_{\text{LO}} + \langle I(\epsilon, \alpha) \rangle$ (full result soon)
- ▶ cuts and parameters
 - ▶ $m_b = 0$
 - ▶ $p_T > 25 \text{ GeV}$, $|\eta| < 2.5$, $\Delta R > 0.4$
 - ▶ $\mu_R = \mu_F = \sum_{i=1}^4 p_T^{(i)} / 4$
 - ▶ pdfs CTEQ6.5
 - ▶ Nagy's $\alpha = 0.1$ (for the shown results)
- ▶ performance on ECDF cluster
 - ▶ one phase-space point per node (Xeon5450 quad-core 3 GHz)
 - ▶ 8.9 s in double precision
 - ▶ 17.6 s in quadruple precision

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Setup



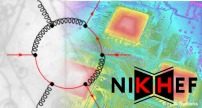
- ▶ software setup as described before
- ▶ check through second, independent implementation (FeynArts, Form, Maple — no code in common)
- ▶ improved IR Dipole subtraction [Catani, Seymour, Nagy]
- ▶ **here:** only $d\sigma_{\text{virtual}} + d\sigma_{\text{LO}} + \langle I(\varepsilon, \alpha) \rangle$ (full result soon)
- ▶ cuts and parameters
 - ▶ $m_b = 0$
 - ▶ $p_T > 25 \text{ GeV}$, $|\eta| < 2.5$, $\Delta R > 0.4$
 - ▶ $\mu_R = \mu_F = \sum_{i=1}^4 p_T^{(i)} / 4$
 - ▶ pdfs CTEQ6.5
 - ▶ Nagy's $\alpha = 0.1$ (for the shown results)
- ▶ performance on ECDF cluster
 - ▶ one phase-space point per node (Xeon5450 quad-core 3 GHz)
 - ▶ 8.9 s in double precision
 - ▶ 17.6 s in quadruple precision

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Setup



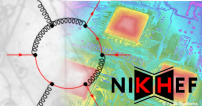
- ▶ software setup as described before
- ▶ check through second, independent implementation (FeynArts, Form, Maple — no code in common)
- ▶ improved IR Dipole subtraction [Catani, Seymour, Nagy]
- ▶ **here:** only $d\sigma_{\text{virtual}} + d\sigma_{\text{LO}} + \langle I(\varepsilon, \alpha) \rangle$ (full result soon)
- ▶ cuts and parameters
 - ▶ $m_b = 0$
 - ▶ $p_T > 25$ GeV, $|\eta| < 2.5$, $\Delta R > 0.4$
 - ▶ $\mu_R = \mu_F = \sum_{i=1}^4 p_T^{(i)} / 4$
 - ▶ pdfs CTEQ6.5
 - ▶ Nagy's $\alpha = 0.1$ (for the shown results)
- ▶ performance on ECDF cluster
 - ▶ one phase-space point per node (Xeon5450 quad-core 3 GHz)
 - ▶ 8.9 s in double precision
 - ▶ 17.6 s in quadruple precision

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Setup

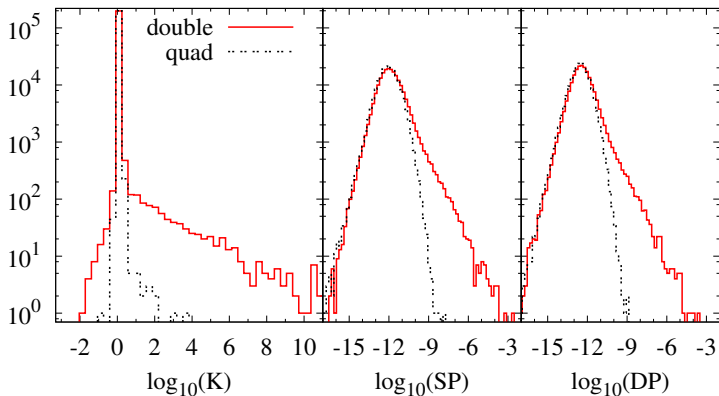


- ▶ software setup as described before
- ▶ check through second, independent implementation (FeynArts, Form, Maple — no code in common)
- ▶ improved IR Dipole subtraction [Catani, Seymour, Nagy]
- ▶ **here:** only $d\sigma_{\text{virtual}} + d\sigma_{\text{LO}} + \langle I(\varepsilon, \alpha) \rangle$ (full result soon)
- ▶ cuts and parameters
 - ▶ $m_b = 0$
 - ▶ $p_T > 25$ GeV, $|\eta| < 2.5$, $\Delta R > 0.4$
 - ▶ $\mu_R = \mu_F = \sum_{i=1}^4 p_T^{(i)} / 4$
 - ▶ pdfs CTEQ6.5
 - ▶ Nagy's $\alpha = 0.1$ (for the shown results)
- ▶ performance on ECDF cluster
 - ▶ one phase-space point per node (Xeon5450 quad-core 3 GHz)
 - ▶ 8.9 s in double precision
 - ▶ 17.6 s in quadruple precision

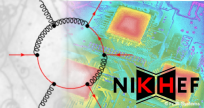
$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Quality Management (QM)



200,000 random phase space points



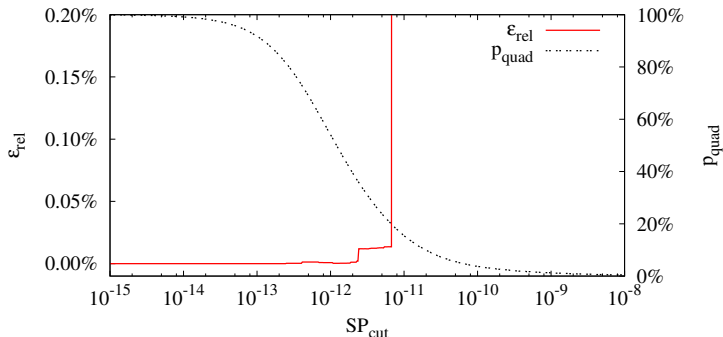
► Need to look at integrated results



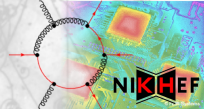
$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Finding Good QM Criteria

Switch to quadruple precision if single pole $> SP_{\text{cut}}$:

$$\varepsilon_{\text{rel}} = |\sigma(SP_{\text{cut}}) - \sigma(0)| / \sigma(0).$$

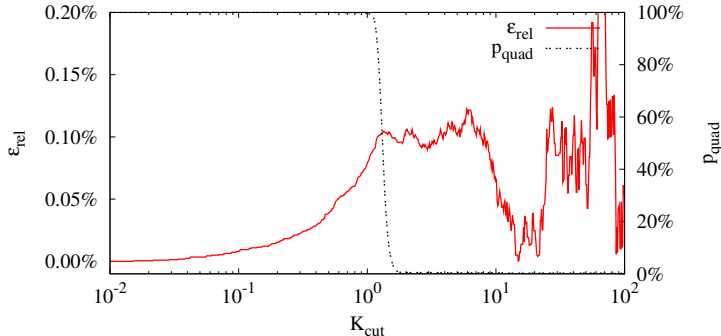


- ▶ precision goal $\varepsilon_{\text{rel}} \leq 0.1\%$
⇒ requires 20% of evaluations in quadruple precision
- ▶ nearly identical plot for double pole



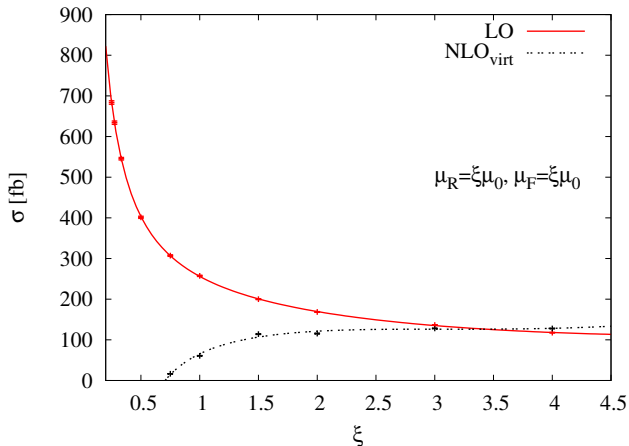
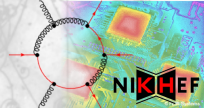
$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Finding Good QM Criteria

Switch to quadruple precision if $d\sigma_{1\text{-loop}}/d\sigma_{\text{LO}} > K_{\text{cut}}$:



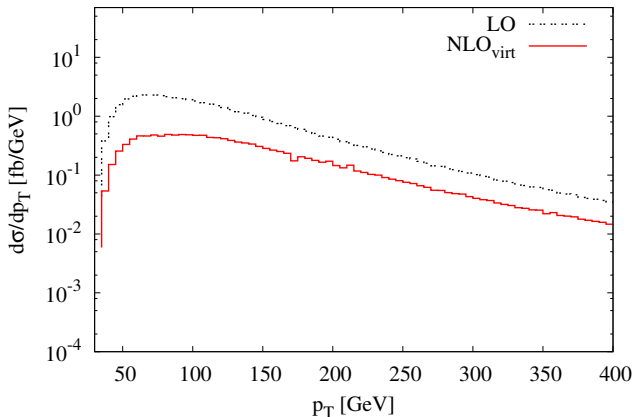
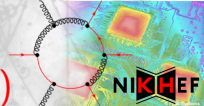
- ▶ precision goal $\varepsilon_{\text{rel}} \leq 0.1\%$
⇒ requires 0.5% of evaluations in quadruple precision
- ▶ better efficiency than by looking at the pole part

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Scale Variation (virtual part)



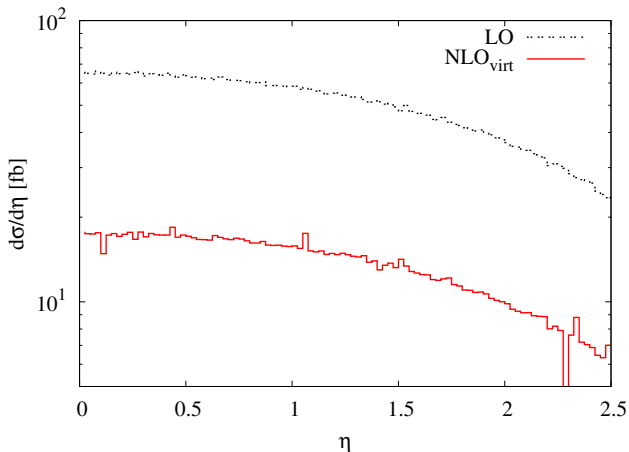
- ▶ simultaneous parallel variation of μ_R and μ_F
- ▶ $\text{NLO}_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: p_T Distributions (virtual part)



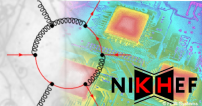
- ▶ p_T of the hardest jet
- ▶ $NLO_{virt} = \sigma_{LO} + \sigma_{one-loop} + I(\epsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Rapidity Distributions (virtual part)

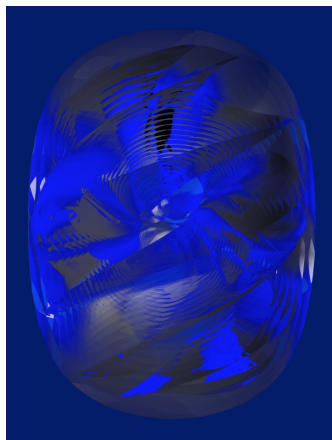


- ▶ rapidity of the hardest jet
- ▶ $\text{NLO}_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$

Summary II



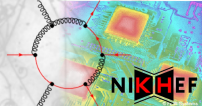
- ▶ Reliable precision (\geq NLO) predictions needed to interpret LHC results, for SM and beyond
- ▶ Only feasible if NLO calculations can be done at shorter time scales
- ▶ GOLEM project important step towards fully automated NLO tool
- ▶ $pp \rightarrow b\bar{b}b\bar{b}$ important irreducible background for BSM signatures
- ▶ one of many examples that GOLEM approach works



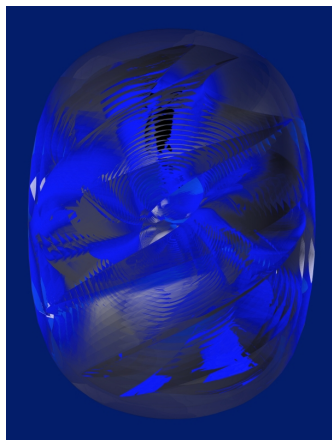
[Jan-Henrik Andersen]

<http://lappweb.in2p3.fr/lapth/Golem/golem95.html>

Summary II

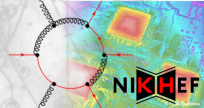


- ▶ Reliable precision (\geq NLO) predictions needed to interpret LHC results, for SM and beyond
- ▶ Only feasible if NLO calculations can be done at shorter time scales
- ▶ GOLEM project important step towards fully automated NLO tool
- ▶ $pp \rightarrow b\bar{b}b\bar{b}$ important irreducible background for BSM signatures
- ▶ one of many examples that GOLEM approach works



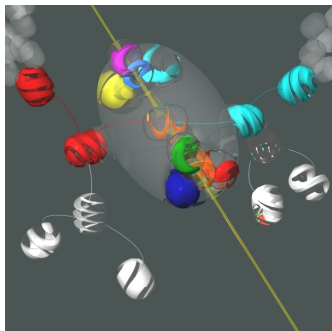
[Jan-Henrik Andersen]

<http://lappweb.in2p3.fr/lapth/Golem/golem95.html>

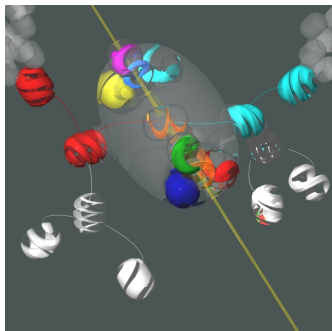
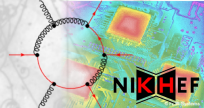


Some work remains to be done:

- ▶ extend GOLEM for massive integrals
- ▶ better user interface
preferably linked into existing frameworks
- ▶ finish $pp \rightarrow b\bar{b}b\bar{b}$ program
 - ▶ real corrections + dipoles
 - ▶ gluon induced subprocess
- ▶ apply GOLEM technology to other interesting $2 \rightarrow 4$ processes
- ▶ explore how far we can get beyond 4 final state particles



[Jan-Henrik Andersen]

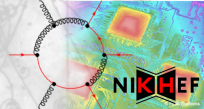


[Jan-Henrik Andersen]

Some work remains to be done:

- ▶ extend GOLEM for massive integrals
- ▶ better user interface
preferably linked into existing frameworks
- ▶ finish $pp \rightarrow b\bar{b}b\bar{b}$ program
 - ▶ real corrections + dipoles
 - ▶ gluon induced subprocess
- ▶ apply GOLEM technology to other interesting $2 \rightarrow 4$ processes
- ▶ explore how far we can get beyond 4 final state particles

Backup Slides

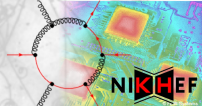


Amplitude Organisation

Integral Reduction

More Results

Amplitude Organisation

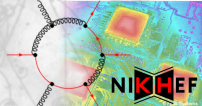


$$\sigma = \int_n d\sigma^{\text{LO}}$$

The full NLO amplitude calculation consists of

- ▶ LO (tree level) diagrams
- ▶ virtual corrections (loop diagrams)
- ▶ real corrections ($(n + 1)$ -particle tree diagrams)
- ▶ subtraction terms (for explicit cancellation of IR poles)

Amplitude Organisation

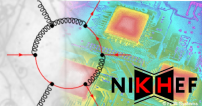


$$\sigma = \int_n d\sigma^{\text{LO}} + \int_n \left(d\sigma^{\text{V}} \right)$$

The full NLO amplitude calculation consists of

- ▶ LO (tree level) diagrams
- ▶ virtual corrections (loop diagrams)
- ▶ real corrections (($n + 1$)-particle tree diagrams)
- ▶ subtraction terms (for explicit cancellation of IR poles)

Amplitude Organisation

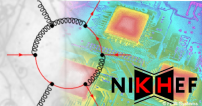


$$\sigma = \int_n d\sigma^{\text{LO}} + \int_n \left(d\sigma^{\text{V}} \right) + \int_{n+1} \left(d\sigma^{\text{R}} \right)$$

The full NLO amplitude calculation consists of

- ▶ LO (tree level) diagrams
- ▶ virtual corrections (loop diagrams)
- ▶ real corrections (($n + 1$)-particle tree diagrams)
- ▶ subtraction terms (for explicit cancellation of IR poles)

Amplitude Organisation

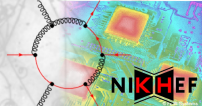


$$\sigma = \int_n d\sigma^{\text{LO}} + \int_n \left(d\sigma^{\text{V}} + \int_1 d\sigma^{\text{A}} \right) + \int_{n+1} (d\sigma^{\text{R}} - d\sigma^{\text{A}})$$

The full NLO amplitude calculation consists of

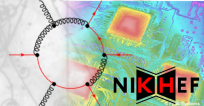
- ▶ LO (tree level) diagrams
- ▶ virtual corrections (loop diagrams)
- ▶ real corrections ($(n + 1)$ -particle tree diagrams)
- ▶ subtraction terms (for explicit cancellation of IR poles)

Amplitude Organisation



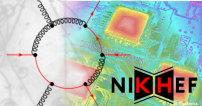
$$\mathcal{M}_{c_1 \dots c_n}(p_1, \dots, p_n) = \sum_{\lambda} \mathcal{A}_{c_1 \dots c_n}(p_1^{\lambda_1}, \dots, p_n^{\lambda_n})$$

- ▶ gauge invariant helicity subamplitudes: independent, no cancellations expected
- ▶ color subamplitudes: minimal basis by choosing irreducible representation
- ▶ subamplitudes are sums of diagrams



$$\mathcal{M}_{c_1 \dots c_n}(p_1, \dots, p_n) = \sum_{\lambda} \sum_c C_{c_1 \dots c_n}^c \mathcal{A}^c(p_1^{\lambda_1}, \dots, p_n^{\lambda_n})$$

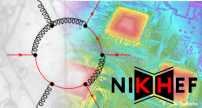
- ▶ gauge invariant helicity subamplitudes: independent, no cancellations expected
- ▶ color subamplitudes: minimal basis by choosing irreducible representation
- ▶ subamplitudes are sums of diagrams



$$\mathcal{M}_{c_1 \dots c_n}(p_1, \dots, p_n) = \sum_{\lambda} \sum_c C_{c_1 \dots c_n}^c \left(\sum_{\mathcal{D}} \mathcal{D}^c(p_1^{\lambda_1}, \dots, p_n^{\lambda_n}) \right)$$

- ▶ gauge invariant helicity subamplitudes: independent, no cancellations expected
- ▶ color subamplitudes: minimal basis by choosing irreducible representation
- ▶ subamplitudes are sums of diagrams

Integral Reduction



- ▶ One-loop diagrams lead to integrals like

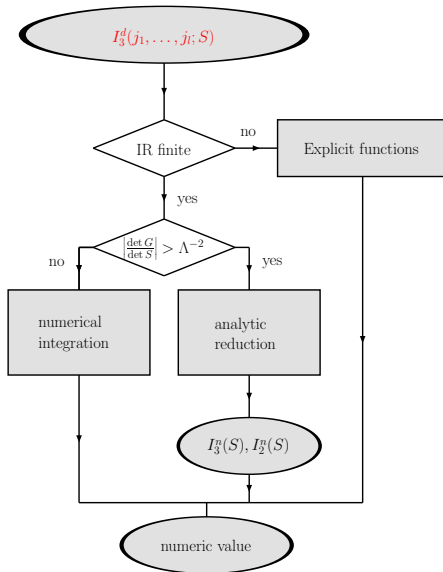
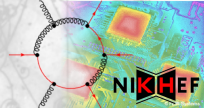
$$I_N^{d;\mu_1\dots\mu_r}(a_1, \dots, a_r; S) \equiv \int \frac{d^d k}{i\pi^{d/2}} \frac{q_{a_1}^{\mu_1} \cdots q_{a_r}^{\mu_r}}{\prod_{j=1}^N (q_j^2 - m_j^2 + i\delta)},$$

$$q_j = k + p_j - p_{j-1}.$$

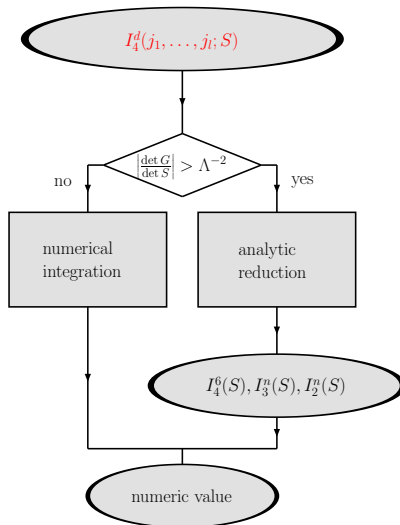
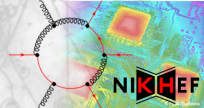
- ▶ tensor integrals have to undergo tensor reduction.
- ▶ form factors can be written in terms of Feynman-parameter integrals.
- ▶ relations between Feynman-parameter integrals lead to reduction formalism for integrals.
- ▶ Tensor Reduction by Subtraction

[Binoth, Guillet, Heinrich, Pilon, Schubert: JHEP 0510:015,2005]

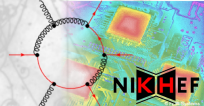
Integral Reduction: Three-Point Functions



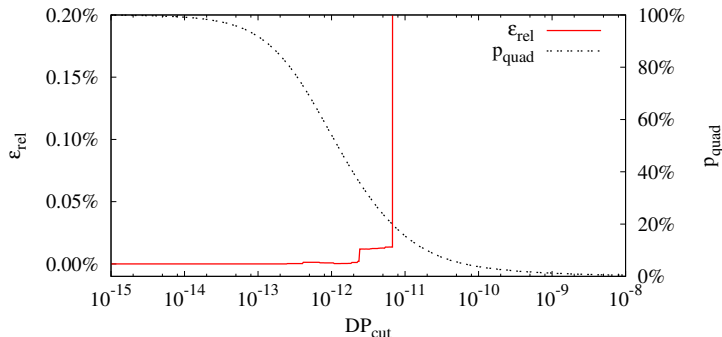
Integral Reduction: Four-Point Functions



$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Finding Good QM Criteria

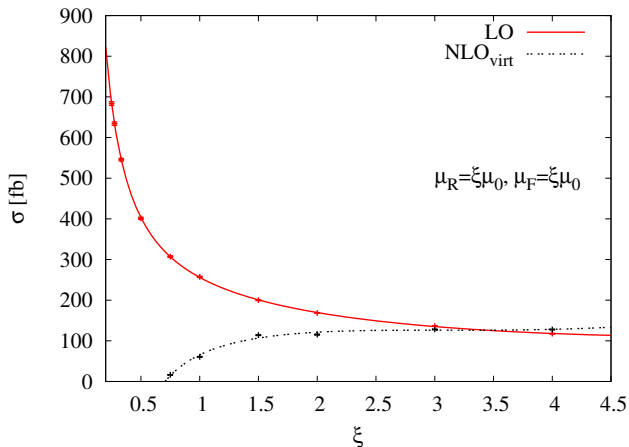
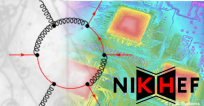


Switch to quadruple precision if single pole $> DP_{\text{cut}}$:



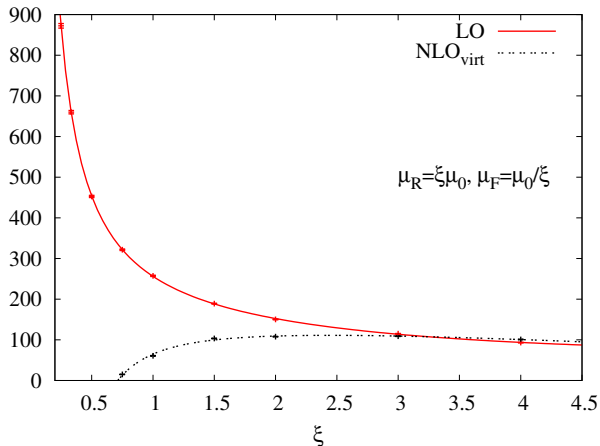
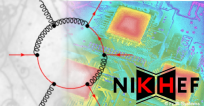
- ▶ each point evaluated at either double or quad precision
- ▶ error defined over the integral of 200,000 phase space points

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Scale Variation (virtual part)



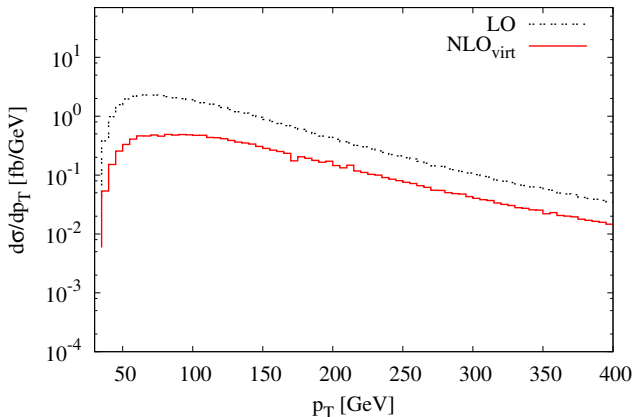
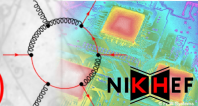
- ▶ simultaneous parallel variation of μ_R and μ_F
- ▶ $NLO_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Scale Variation (virtual part)



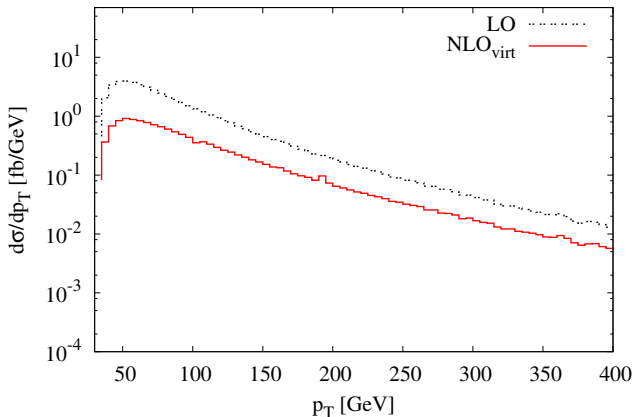
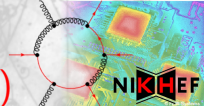
- ▶ simultaneous anti-parallel variation of μ_R and μ_F
- ▶ $\text{NLO}_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: p_T Distributions (virtual part)



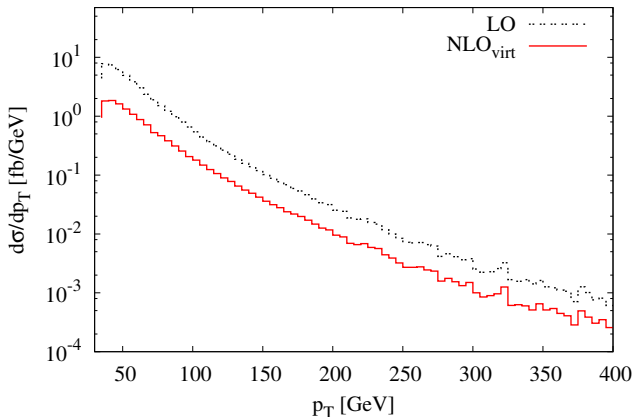
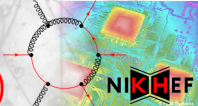
- ▶ p_T of the hardest jet
- ▶ $NLO_{virt} = \sigma_{LO} + \sigma_{one-loop} + I(\epsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: p_T Distributions (virtual part)



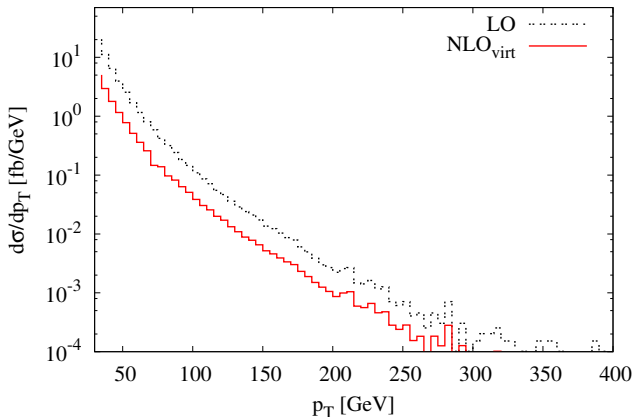
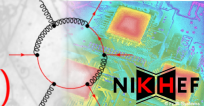
- ▶ p_T of the second hardest jet
- ▶ $\text{NLO}_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: p_T Distributions (virtual part)



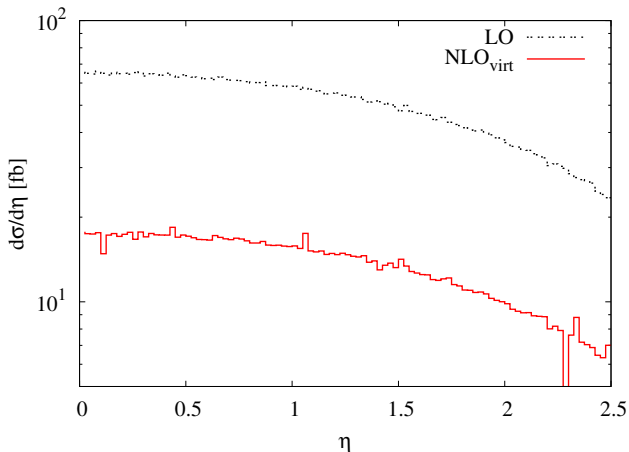
- ▶ p_T of the third hardest jet
- ▶ $\text{NLO}_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: p_T Distributions (virtual part)



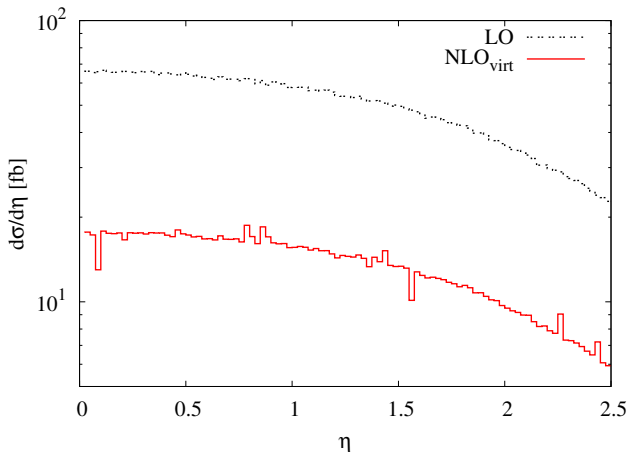
- ▶ p_T of the softest jet
- ▶ $\text{NLO}_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Rapidity Distributions (virtual part)



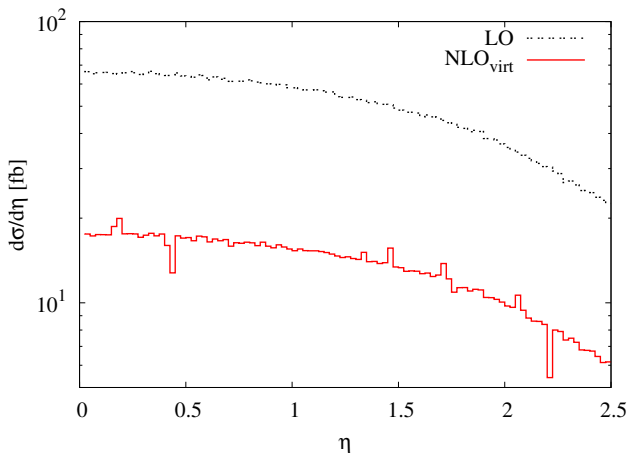
- ▶ rapidity of the hardest jet
- ▶ $\text{NLO}_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Rapidity Distributions (virtual part)



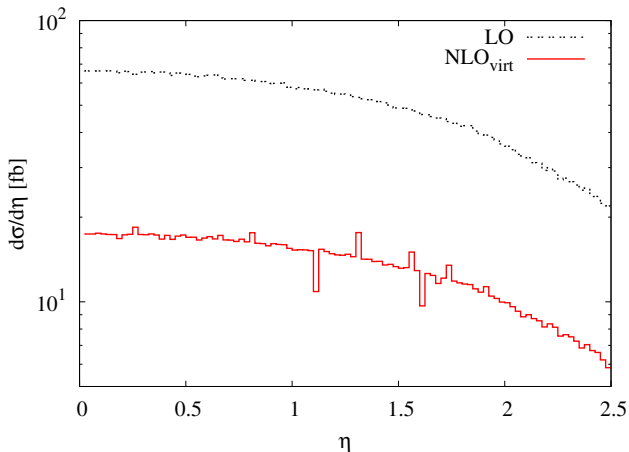
- ▶ rapidity of the second hardest jet
- ▶ $NLO_{virt} = \sigma_{LO} + \sigma_{one-loop} + I(\epsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Rapidity Distributions (virtual part)



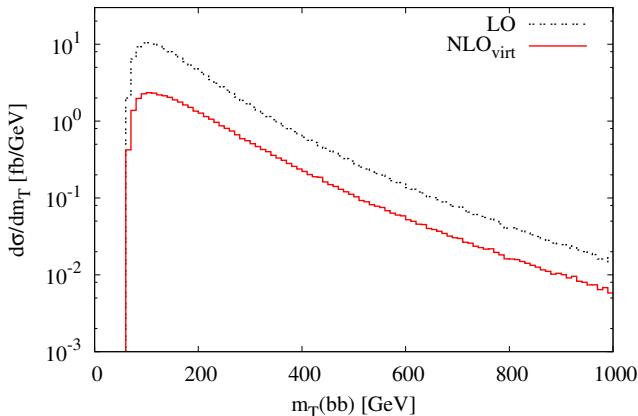
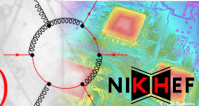
- ▶ rapidity of the third hardest jet
- ▶ $NLO_{virt} = \sigma_{LO} + \sigma_{one-loop} + I(\epsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Rapidity Distributions (virtual part)



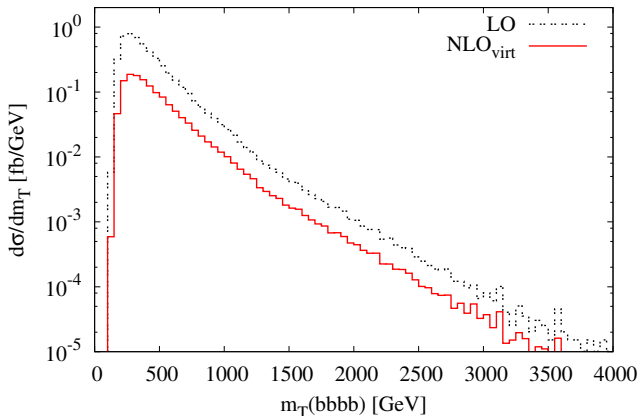
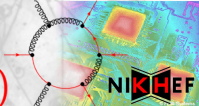
- ▶ rapidity of the softest jet
- ▶ $NLO_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Transverse Mass (virtual part)



- ▶ Sum of m_T of all b -quark pairings
- ▶ $NLO_{virt} = \sigma_{LO} + \sigma_{one-loop} + I(\epsilon, \alpha)$

$q\bar{q} \rightarrow b\bar{b}b\bar{b} + X$: Transverse Mass (virtual part)



- ▶ Sum of m_T of the four- b system
- ▶ $\text{NLO}_{\text{virt}} = \sigma_{\text{LO}} + \sigma_{\text{one-loop}} + I(\varepsilon, \alpha)$