5th Vienna Central European Seminar on Particle Physics and Quantum Field Theory
University of Vienna, 28th November 2008

Automatic dipole subtraction

K. Hasegawa

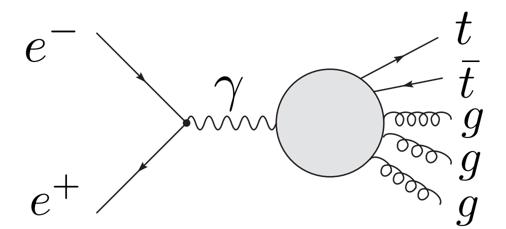
DESY Zeuthen

Collaboration with S. Moch and P. Uwer

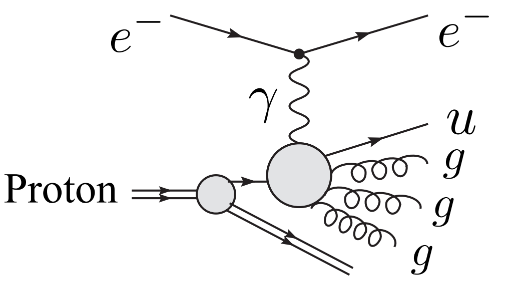
1. Introduction

Processes with many parton legs

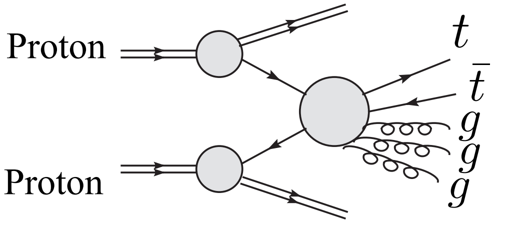
① e^+e^- annihilation into hadrons (LEP, ILC)



② Deep inelastic scattering (SLAC, HERA)



③ Hadron-hadron scattering (Tevatron, LHC)



Leading order (LO) can be calculated automatically using existing software

Typical ones : MadGraph, CompHep, FeynArts, HELAC/PHEGAS, Alpgen, . . .

Challenge: Automatic calculation of Next-to-Leading order (NLO) in QCD

- NLO cross section

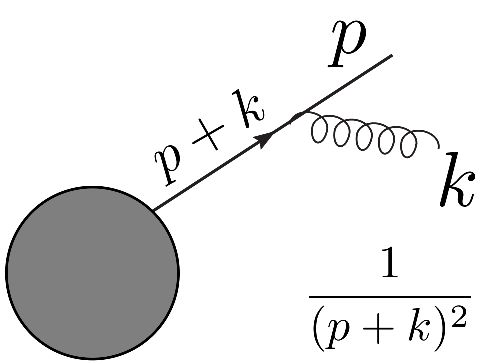
LO : $2 \rightarrow n$

NLO : Real emission correction $+$ Virtual 1-loop correction

$$2 \rightarrow n+1 \qquad\qquad 2 \rightarrow n$$

$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}}$$

- Collinear and soft divergences from real correction



$$\frac{1}{(p+k)^2} = \frac{1}{2E_k E_p(1-\beta\cos\theta_{kp})} \longrightarrow \infty$$

$m_q = 0$ and $\theta_{kq} \rightarrow 0$
: Collinear divergence
(or mass divergence)

$E_k \rightarrow 0$
: Soft divergence
(IR divergence)

Virtual correction also contain singularities

- Collinear and soft divergences are cancelled at σ_{NLO} and becomes finite

$\sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}}$: Infrared safe

Kinoshita-Lee-Nauenberg theorem

- The simplest example : $e^- e^+ \longrightarrow u\bar{u}$

$$\sigma_{\rm NLO} = \quad \sigma_{\rm real} \qquad + \quad \sigma_{\rm virtual}$$

$$= \quad \int {\rm dPS_3} |{\rm M}(e^- e^+ \to u\bar{u}g)|^2 \Big|_{\rm D\text{-}dim} \quad + \int {\rm dPS_2} |{\rm M}(e^- e^+ \to u\bar{u})|^2_{\rm 1loop} \Big|_{\rm D\text{-}dim}$$

(Dimensional regularization: ${\rm D} = 4 - 2\epsilon$)

$$= \quad \sigma_0^{(\epsilon)} \frac{\alpha_s}{2\pi} {\rm C_F} \left(\frac{4\pi}{q^2} \right)^\epsilon \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + O(\epsilon) \right] \quad + \quad \sigma_{\rm virtual}$$

$$= \quad \sigma_0 \left[1 + \frac{\alpha_s}{\pi} + O(\alpha_s^2) \right]$$:Infrared finite

This method is not realistic for multi-leg processes like $gg \to t\bar{t}ggg$ because of huge expressions of $|{\rm M}|^2$ in D dimension

■ Dipole subtraction

A general procedure to treat collinear and soft divergences at NLO

S.Catani and M.H.Seymour, Nucl.Phys.B485(1997)291
S.Catani, S.Dittmaier, M.H.Seymour, Z.Trocsanyi, Nucl.Phys.B627(2002)189

$$\sigma_{\rm NLO} = \sigma_{\rm real} + \sigma_{\rm virtual}$$

$$= (\sigma_{\rm real} - \sigma_{\rm a}) + (\sigma_{\rm virtual} + \sigma_{\rm a})$$

$$= \boxed{\int d\Phi_{m+1}\left[|\mathrm{M}_{\rm real}|^2 - \sum_i \mathrm{D}_i\right]\Big|_{\rm D=4}} + \int d\Phi_m \left[\underline{|\mathrm{M}_{\text{1-loop}}|^2 + \int d\Phi_1 \sum_i \mathrm{D}_i}\right]\Big|_{\rm D=4}$$

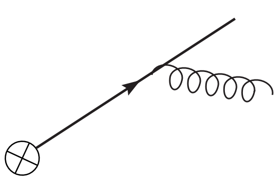
Finite   Finite

-All collinear and soft singularities are subtracted by counter term: $\sigma_a = \int \mathrm{dPS} \sum_i \mathrm{D}_i$

⇨ No regularization for real correction ⟶ $|\mathrm{M}|^2$ in 4 dimension

⇨ Phase space integrals are finite ⟶ Monte Carlo integral

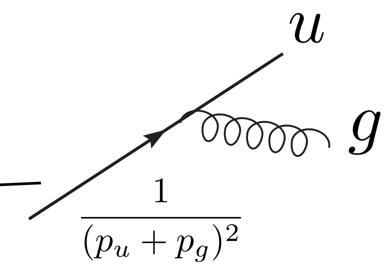
-Factorization of splitting leg and construction of dipole terms in an universal way

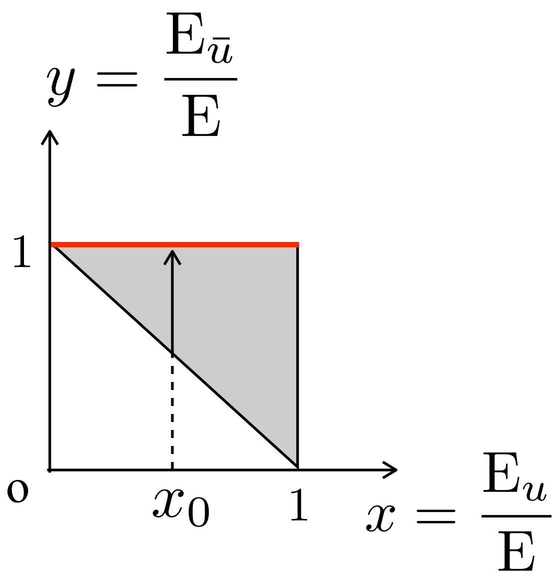


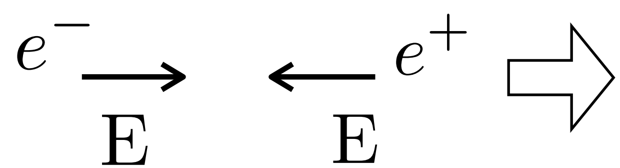
⇨ Algorithm is a combinatorial way ⟶ implemet in a computer code

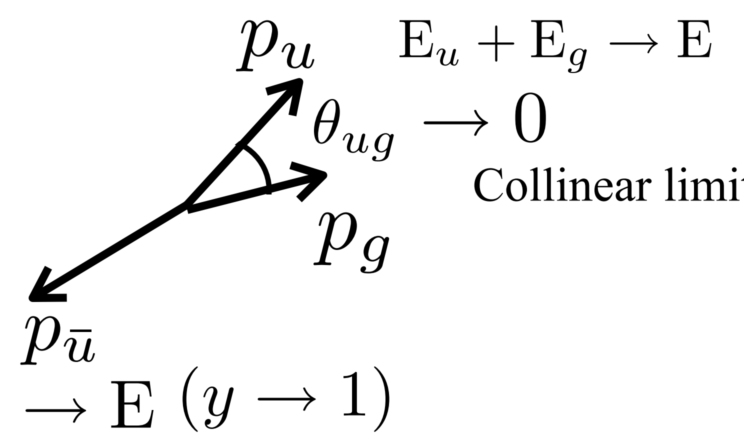
■ The simplest example : $e^-e^+ \to u\bar{u}$

$$\sigma_{\rm real}(e^-e^+ \to u\bar{u}g) = \sigma_0 \frac{\alpha_s}{2\pi} C_F \int dx dy \frac{x^2+y^2}{(1-x)\underline{(1-y)}}$$









$$\left.\frac{d\sigma_{\rm real}}{dx}\right|_{x_0} = \sigma_0 \frac{\alpha_s}{2\pi} C_F \frac{x_0^2+1}{(1-x_0)} \int_0^{x_0} d\delta \frac{1}{\delta} \to -\ln \delta_{\rm cut}$$: Collinear divergence

$(\delta = 1-y)$ (Introduce $\delta = \delta_{cut}$)

$$\left.\frac{d(\sigma_{\rm real}-\sigma_a)}{dx}\right|_{x_0} = \sigma_0 \frac{\alpha_s}{2\pi} C_F \frac{x_0^2+1}{(1-x_0)} \int_0^{x_0} d\delta \left(\frac{1}{\delta} - \frac{1}{\delta} + \cdots\right)$$: Finite

All collinear and soft singularities are subtracted by counter term: $\sigma_a = \int {\rm dPS} \sum_i {\rm D}_i$

- A complex example : $gg \longrightarrow t\bar{t}ggg$ (Real correction to $gg \rightarrow t\bar{t}gg$)

75 dipoles : $\sum_{i=1}^{75} \mathrm{D}(i)$ Each dipole : $\mathrm{D}_i \simeq \mathrm{V} \cdot < gg \rightarrow t\bar{t}gg|\mathrm{T} \cdot \mathrm{T}|gg \rightarrow t\bar{t}gg >$

Factorized part (V) — Square of Born matrix element with two color insertion ($\mathrm{T} \cdot \mathrm{T}$)

- Require the construction of 75 dipole terms

⇨ Automatization of the construction is required

- Calculation capacity is about 75 times of leading order process

⇨ Fast code to treat huge calculation in reasonable run time is mandatory

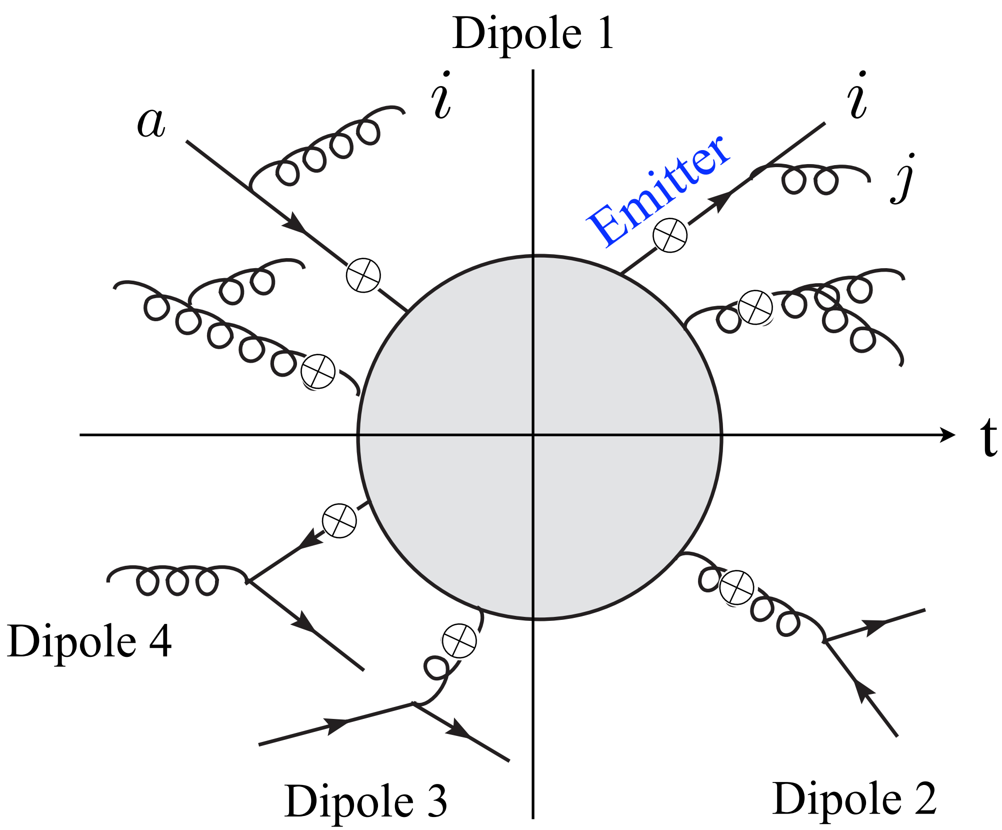
- There is recent work in the same direction

  - T. Gleisberg and F. Krauss, Eur.Phys.J.C53(2008)501, arXiv0709.2881
  - M.H. Seymour and C. Tevlin, arXiv0803.2231
  - R. Frederix and T. Gehrmann and N. Greiner, arXiv0808.2128

2. Code structure

■ Algorithm

1. Choose emitter pair



Choose all possible leg-pair which matches one of the seven patterns

(a, i) or (i, j)

Initial parton=a,b
Final parton=i,j,k

2. Choose spectator

Choose a different leg from emitter pair

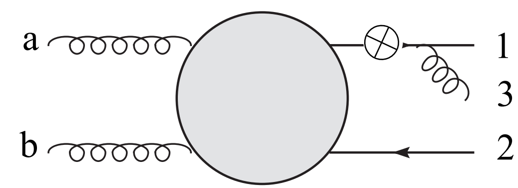
Spectator : $k \neq i, j$ $\quad b \neq a$

emitter pair \ spectator	k	b
(i, j)	$\mathrm{D}_{ij,k}$ $(k \neq i, j)$	$\mathrm{D}_{ij}^{\ b}$
(a, i)	$\mathrm{D}^{ai}_{\ k}$ $(k \neq i)$	$\mathrm{D}^{ai,b}$ $(b \neq a)$

3. Use dipole formulae

$$\mathrm{D}_{ij,k}(p_1,\cdots,p_{m+1}) = -\frac{1}{2p_i\cdot p_j} < 1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1|\frac{\mathrm{T}_k\cdot\mathrm{T}_{ij}}{\mathrm{T}_{ij}^2}\mathrm{V}_{ij,k}|1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1>_m$$

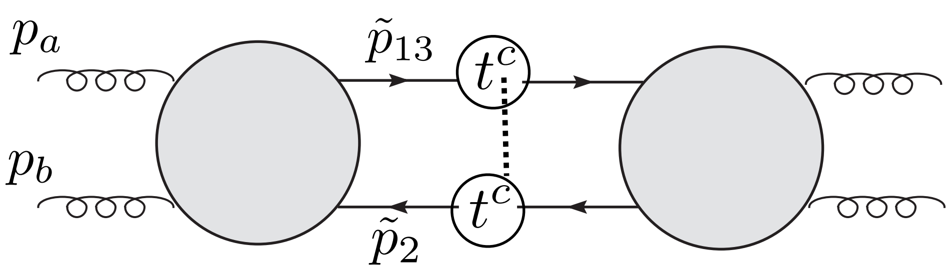
Example : $g(a)g(b)\rightarrow u(1)\bar{u}(2)g(3)$



$$\mathrm{D}_{13,2}(p_1,p_2,p_3,p_a,p_b) = -\frac{1}{2p_1\cdot p_3}\langle gg\rightarrow\tilde{u}\tilde{\bar{u}}|\frac{\mathrm{T}_{\bar{u}}\cdot\mathrm{T}_{ug}}{\mathrm{T}_{ug}^2}\mathrm{V}_{13,2}|gg\rightarrow\tilde{u}\tilde{\bar{u}}\rangle_2$$

Dipole splitting function : $\mathrm{V}_{13,2}(z,y) = \delta_{ss'}8\pi\alpha C_F\left[\frac{2}{1-z_i(1-y_{ij,k})}-(1+z_i)\right]$

$\left\langle gg\rightarrow\tilde{u}\tilde{\bar{u}}\middle|\mathrm{T}_{\bar{u}}\cdot\mathrm{T}_{ug}\middle|gg\rightarrow\tilde{u}\tilde{\bar{u}}\right\rangle_2$: Color linked Born squared (CLBS)



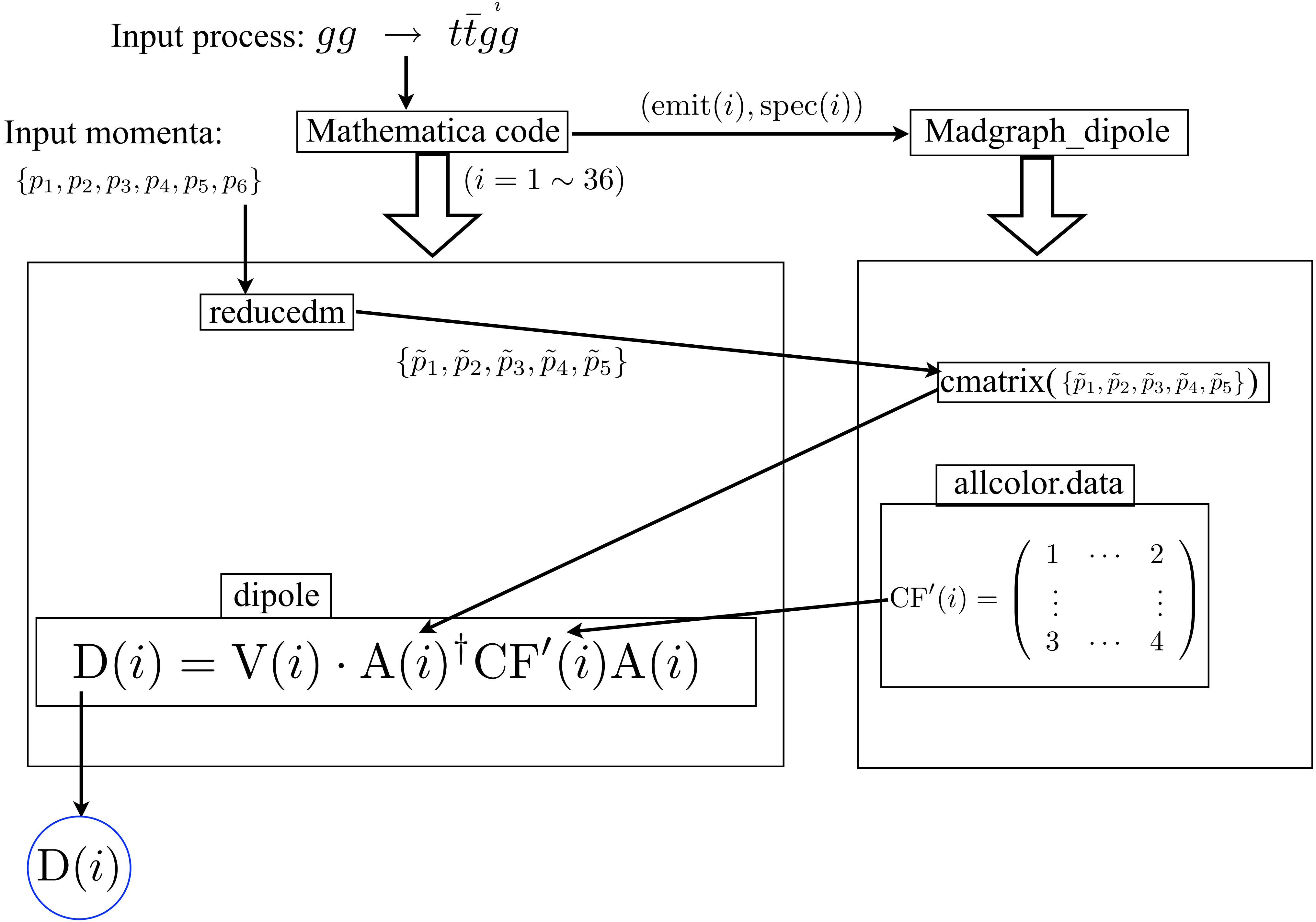
$$\mathrm{T}_{\mathrm{X}}^a = \begin{cases} t^a & (\mathrm{X} = \text{quark}) \\ f^a & (\mathrm{X} = \text{gluon}) \end{cases}$$

$$z_i = \frac{p_i\cdot p_k}{p_j\cdot p_k + p_i\cdot p_k} \qquad y_{ij,k} = \frac{p_i\cdot p_j}{p_i\cdot p_j + p_j\cdot p_k + p_k\cdot p_i}$$

Reduced momenta:

$$\tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}p_k^\mu \qquad \tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}p_k^\mu$$

■ Scheme to calculate $|\mathrm{M}|^2 - \sum_i \mathrm{D}_i$



3. Results

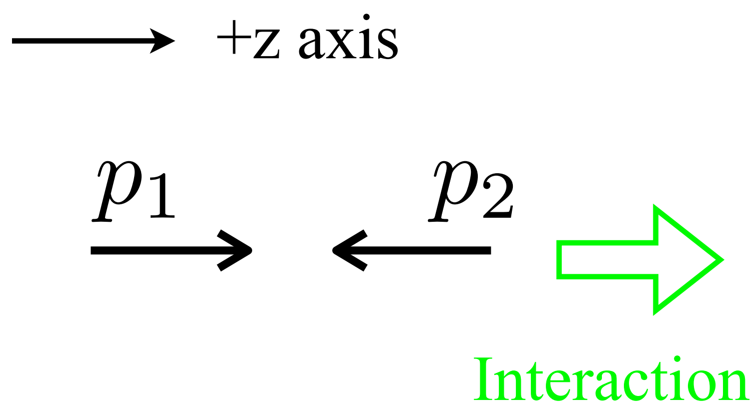
Checked processes

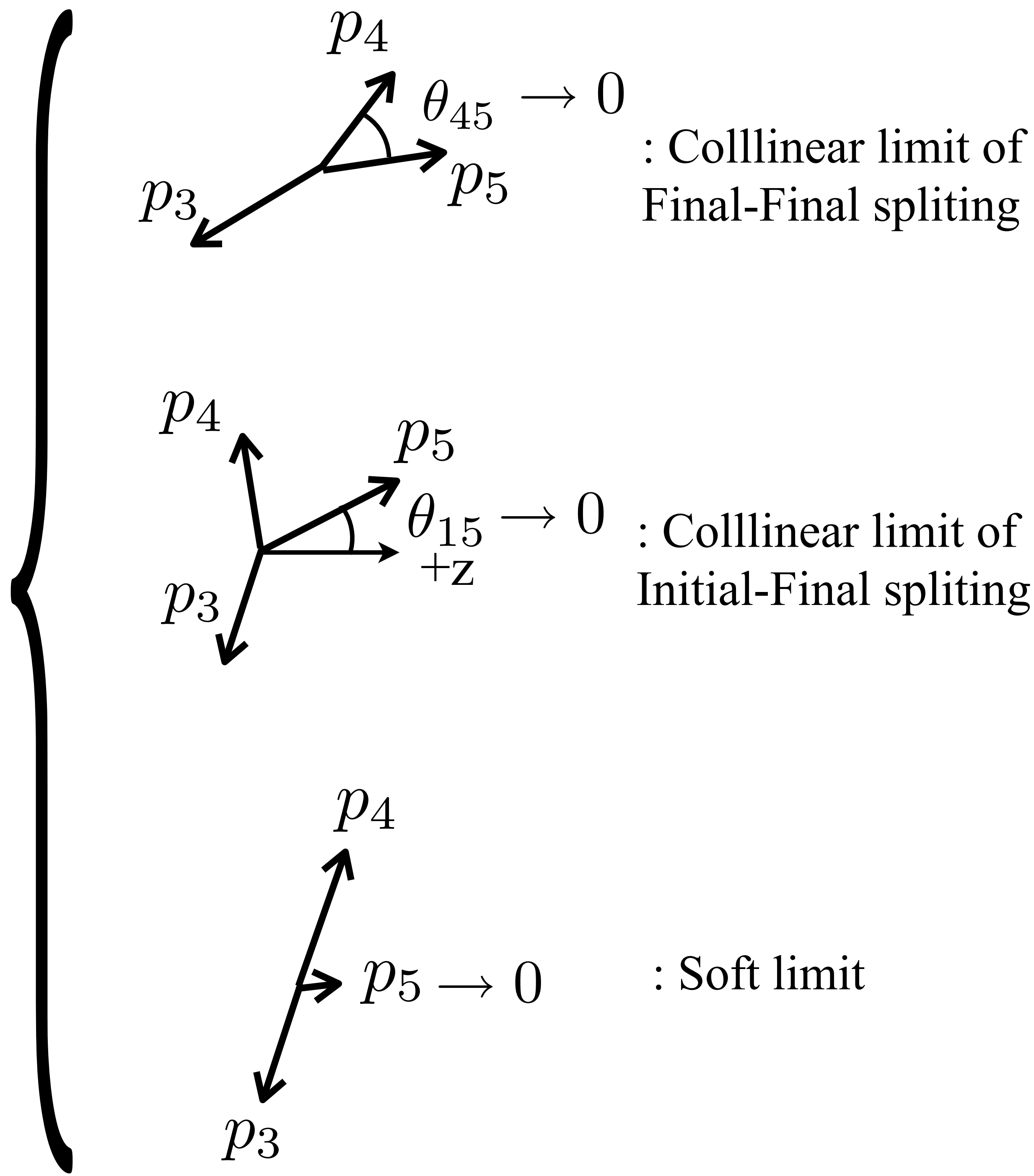
Real correction process	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
Massless				
(Including lepton)	$e^+e^- \to u\bar{u}g$			
	$e^-u \to e^-ug$			
	$e^-g \to e^-u\bar{u}$			
	$u\bar{u} \to e^+e^-g$			
(Parton only)	$gg \to u\bar{u}g$	$gg \to u\bar{u}gg$	$u\bar{u} \to d\bar{d}ggg$	
	$gg \to 3g$	$gg \to 4g$		
	$u\bar{u}g \to d\bar{d}g$			
Massive				
(Including lepton)	$e^+e^- \to t\bar{t}g$			
(Parton only)	$gg \to t\bar{t}g$	$gg \to t\bar{t}gg$	$gg \to t\bar{t}ggg$	$u\bar{u} \to t\bar{t}b\bar{b}gg$
		$u\bar{u} \to t\bar{t}gg$	$gg \to t\bar{t}b\bar{b}g$	
		$ug \to t\bar{t}ug$		
		$\bar{u}g \to t\bar{t}\bar{u}g$		
		$gg \to t\bar{t}u\bar{u}$		
		$u\bar{u} \to t\bar{t}u\bar{u}$		

Complete agreement with
S. Dittmaier, P. Uwer and S. Weinzierl, arXiv:0810.0452

- Collinear and Soft limits

$$g(1)g(2) \to u(3)\bar{u}(4)g(5)$$





■ $2 \rightarrow 3$ process

$gg \rightarrow u\bar{u}g$

Contents of dipoles

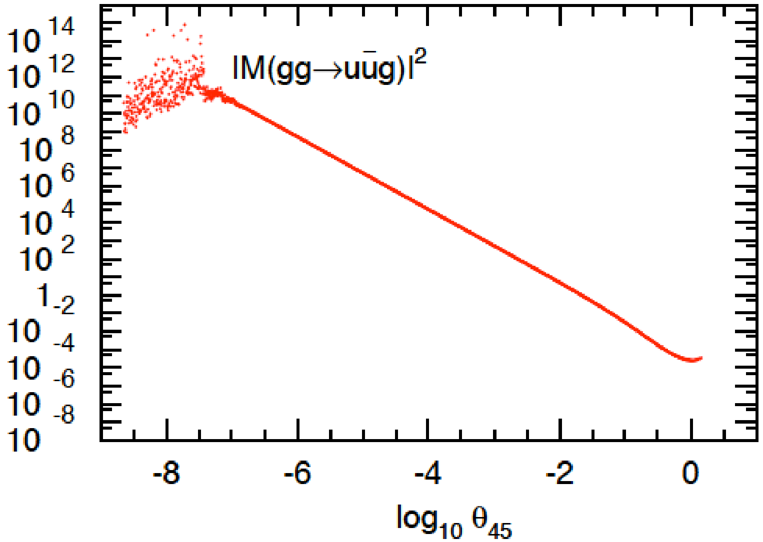
```
Number of dipoles

[Dipole1] : 12

 B1 : 12

{Splitting (1):(i,j)=(f,g)}:  6 (0)

                [1.(ij,k)=(fg,k): Dij,k]  2 (0)

                [2.(ij,a)=(fg,a): Dij^a]  4 (0)

{Splitting (2):(i,j)=(g,g)}:  0 (0)

                [3.(ij,k)=(gg,k): Dij,k]  0 (0)

                [4.(ij,a)=(gg,a): Dij^a]  0 (0)

{Splitting (3):(a,i)=(f,g)}:  0 (0)

                [5.(ai,k)=(fg,k): D^ai,k]  0 (0)

                [6.(ai,b)=(fg,b): D^ai,b]  0 (0)

{Splitting (4):(a,i)=(g,g)}:  6 (0)

                [7.(ai,k)=(gg,k): D^ai,k]  4 (0)

                [8.(ai,b)=(gg,b): D^ai,b]  2 (0)

-------------------

[Dipole2] : 3

{Splitting (5):(i,j)=(f,fbar)}

  (u-ubar splitting) B2u : 3

                [9-u.(ij,k)=(u ubar,k): Dij,k]  1 (0)

                [10-u.(ij,a)=(u ubar,a): Dij^a]  2 (0)

  (d-dbar splitting) B2d : 0

                [9-d.(ij,k)=(u ubar,k): Dij,k]  0 (0)

                [10-d.(ij,a)=(u ubar,a): Dij^a]  0 (0)

  (b-bbar splitting) B2b : 0
```

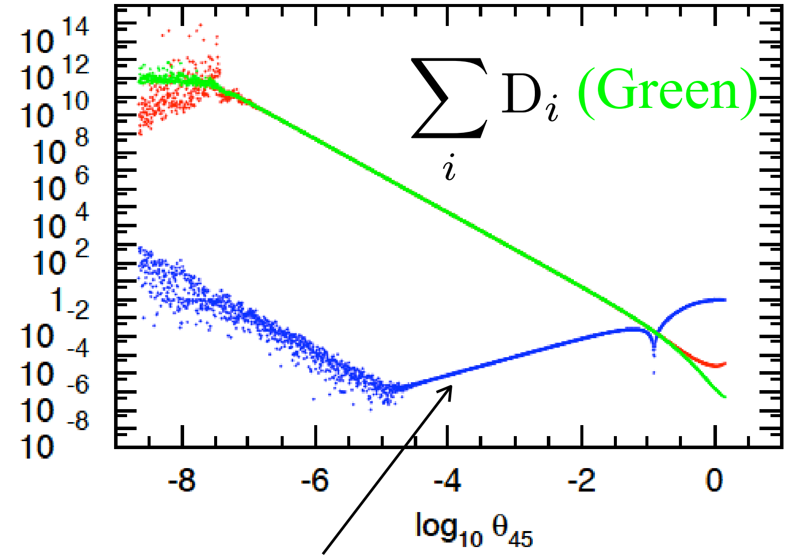
```
[Dipole4] : 12

{Splitting (7):(a,i)=(g,f) or (g,fbar)}

 ((a,i)=(g,u) splitting)  B4u : 6

                [13-u.(ai,k)=(gu,k): D^ai,k]  4 (0)

                [14-u.(ai,b)=(gu,b): D^aib]  2 (0)

 ((a,i)=(g,ubar) splitting)   B4ubar : 6

                [13-ubar.(ai,k)=(g ubar,k): D^ai,k]  4 (0)

                [14-ubar.(ai,b)=(g ubar,b): D^aib]  2 (0)

 ((a,i)=(g,d) splitting) B4d : 0

                [13-d.(ai,k)=(gd,k): D^ai,k]  0 (0)

                [14-d.(ai,b)=(gd,b): D^aib]  0 (0)

 ((a,i)=(g,dbar) splitting) B4dbar : 0

                [13-dbar.(ai,k)=(g ubar,k): D^ai,k]  0 (0)

                [14-dbar.(ai,b)=(g ubar,b): D^aib]  0 (0)

-------------------

[Total] : 27

 (Massive dipoles : 0)

-------------------

END
```

- Collinear limit

  - Final-Final splitting

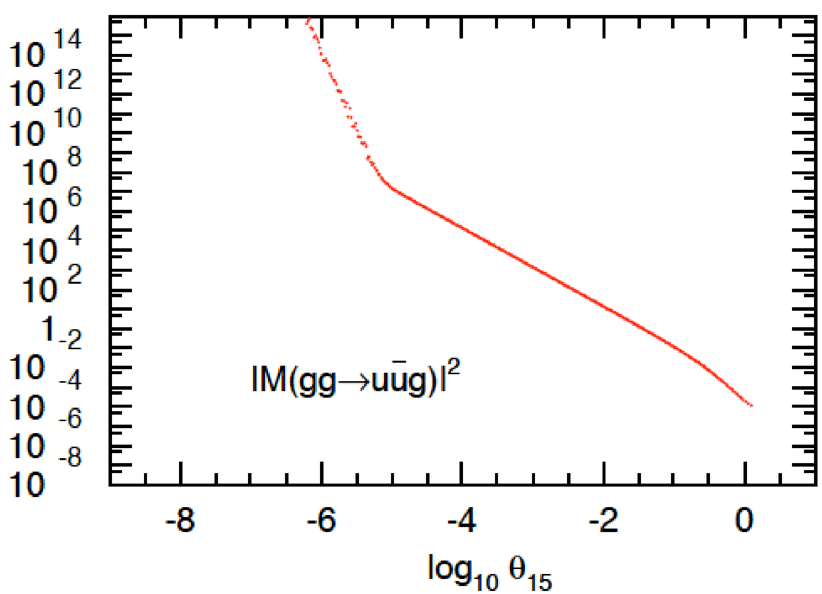
$$\sum D_i = D_2 + D_5 + D_6$$

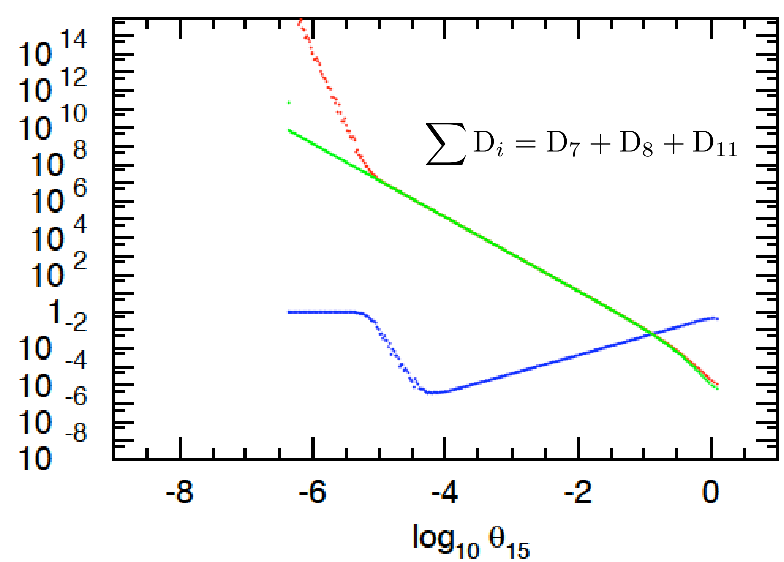




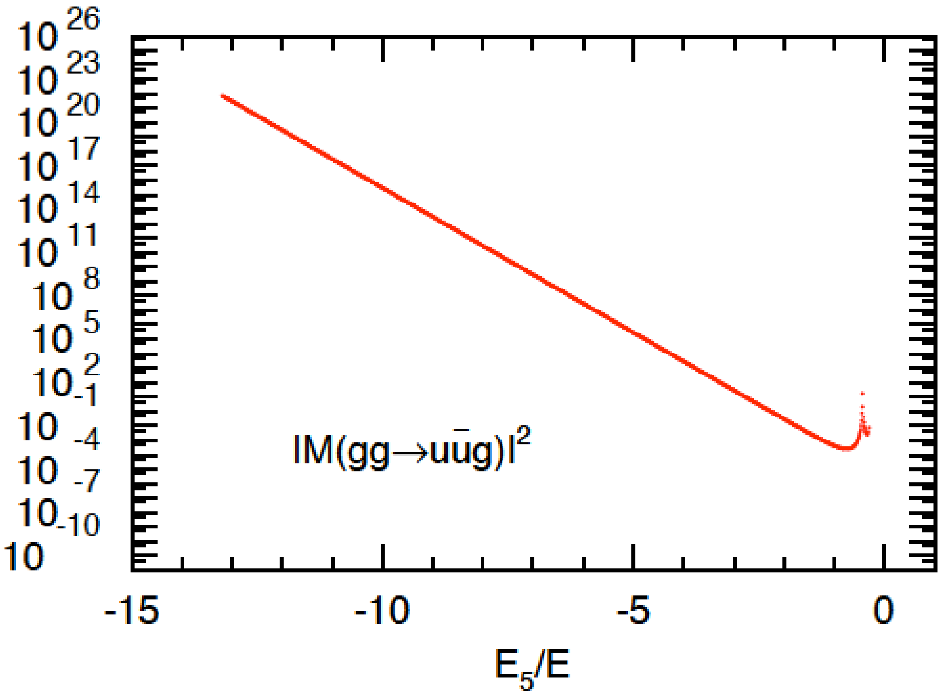
$$(|M|^2 - \sum D_i)/|M|^2 \quad \text{(Blue)}$$

  - Initial-Final splitting

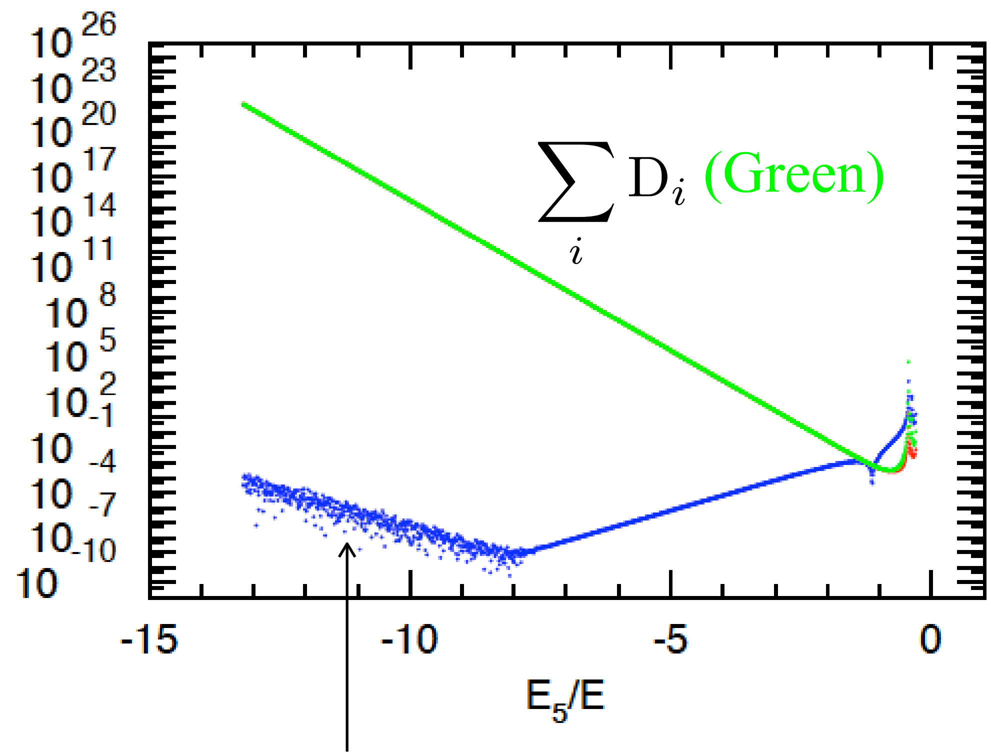




- Soft limit



$$\sum D_i = D_1 + \cdots + D_{12}$$



$\left(|M|^2 - \sum D_i\right)/|M|^2$ (Blue)

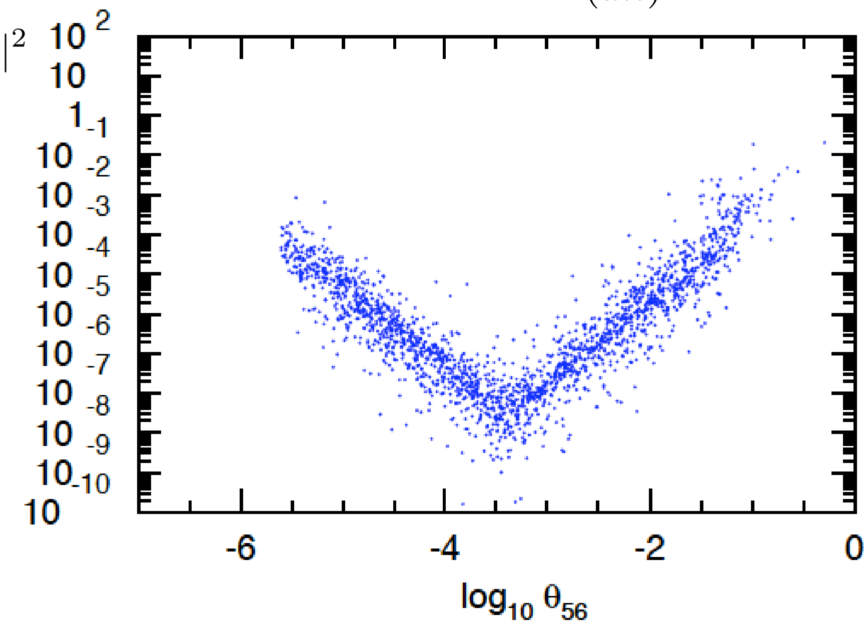
■ $2 \rightarrow 4$ process

$$gg \rightarrow t\bar{t}gg$$

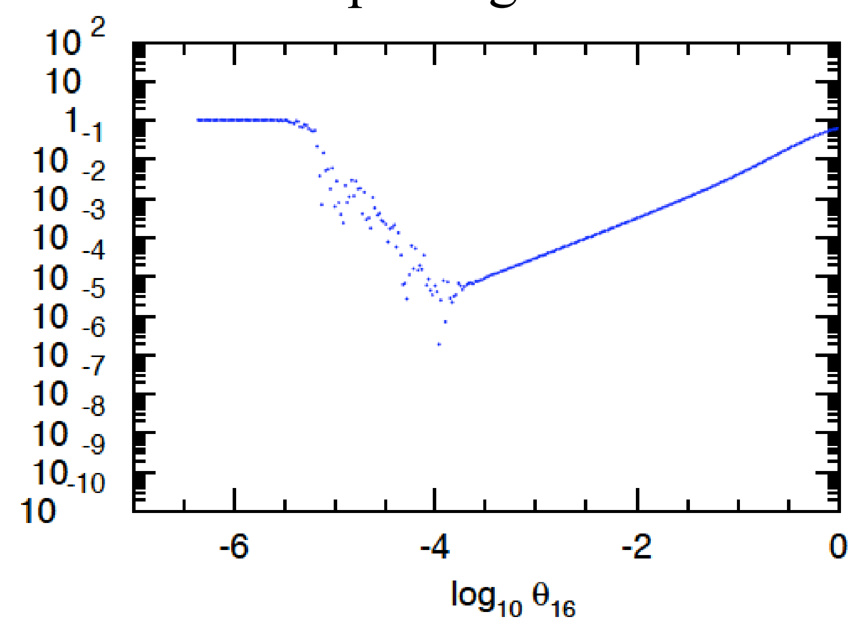
```
[Dipole1] : 36

 B1 : 36

{Splitting (1):(i,j)=(f,g)}:  16 (16)

                [1.(ij,k)=(fg,k): Dij,k]   8 (8)

                [2.(ij,a)=(fg,a): Dij^a]   8 (8)

{Splitting (2):(i,j)=(g,g)}:  4 (2)

                [3.(ij,k)=(gg,k): Dij,k]   2 (2)

                [4.(ij,a)=(gg,a): Dij^a]   2 (0)

{Splitting (3):(a,i)=(f,g)}:  0 (0)

                [5.(ai,k)=(fg,k): D^ai,k]   0 (0)

                [6.(ai,b)=(fg,b): D^ai,b]   0 (0)

{Splitting (4):(a,i)=(g,g)}:  16 (8)

                [7.(ai,k)=(gg,k): D^ai,k]   12 (8)

                [8.(ai,b)=(gg,b): D^ai,b]   4 (0)

[Total] : 36

 (Massive dipoles : 26)

--------------------
```

- Final-Final splitting : $\sum_{(all)} \mathrm{D}_i$

$(|\mathrm{M}|^2 - \sum \mathrm{D}_i)/|\mathrm{M}|^2$



- Initial-Final splitting: $\sum \mathrm{D}_i = \mathrm{D}_{27} + \mathrm{D}_{28} + \mathrm{D}_{29} + \mathrm{D}_{35}$

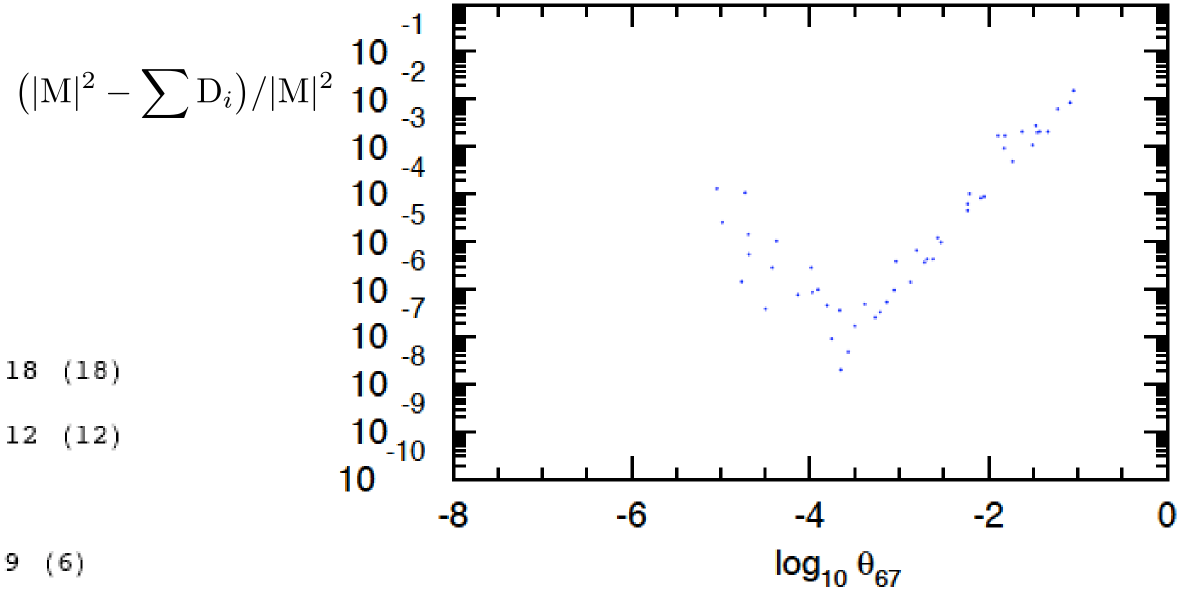


■ $2 \to 5$ process $\Leftarrow$ NLO of $2 \to 4$ LO process: Cutting edge !

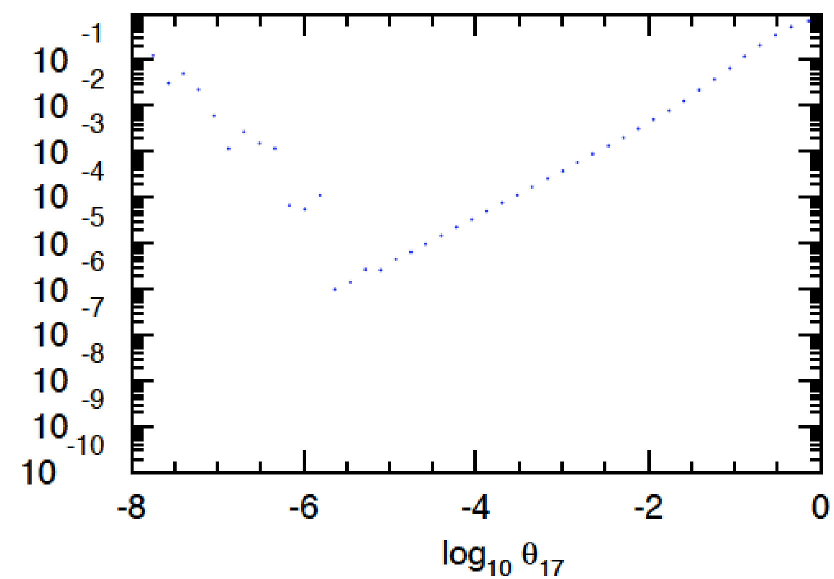
$gg \to t\bar{t}ggg$

```
[Dipole1] : 75

 B1 : 75

{Splitting (1):(i,j)=(f,g)}:  30 (30)

               [1.(ij,k)=(fg,k): Dij,k]  18 (18)

               [2.(ij,a)=(fg,a): Dij^a]  12 (12)

{Splitting (2):(i,j)=(g,g)}:  15 (6)

               [3.(ij,k)=(gg,k): Dij,k]  9 (6)

               [4.(ij,a)=(gg,a): Dij^a]  6 (0)

{Splitting (3):(a,i)=(f,g)}:  0 (0)

               [5.(ai,k)=(fg,k): D^ai,k]  0 (0)

               [6.(ai,b)=(fg,b): D^ai,b]  0 (0)

{Splitting (4):(a,i)=(g,g)}:  30 (12)

               [7.(ai,k)=(gg,k): D^ai,k]  24 (12)

               [8.(ai,b)=(gg,b): D^ai,b]  6 (0)

------------------

[Total] : 75

 (Massive dipoles : 48)

--------------------
```

- Final-Final splitting: $\sum \mathrm{D}_i = (37, 38, 39, 44, 45)$



- Initial-Final splitting: $\sum \mathrm{D}_i = (62, 63, 64, 65, 74)$



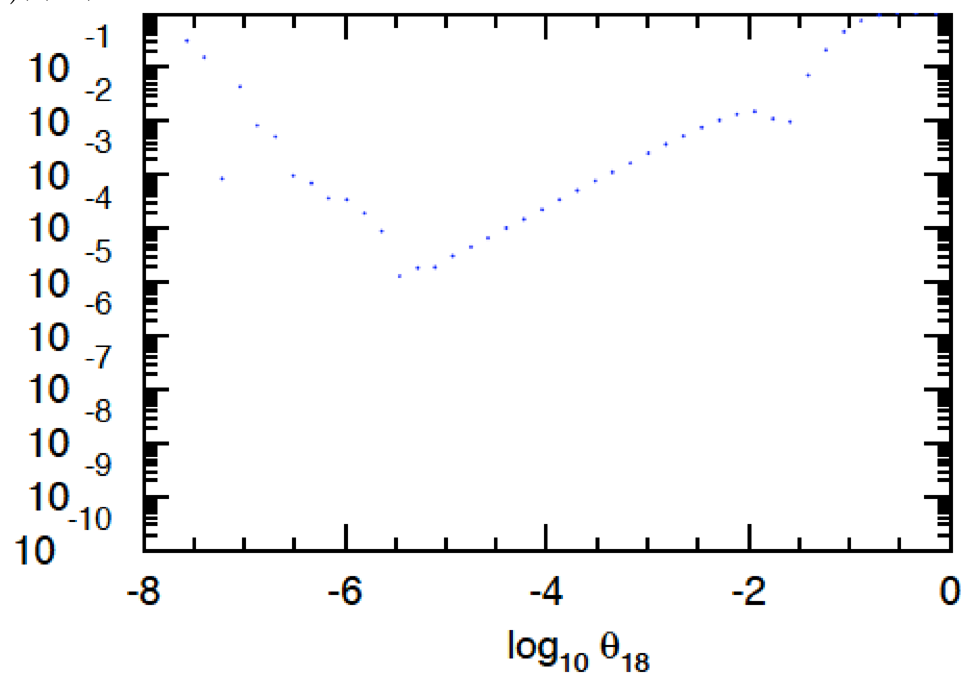
- $2 \to 6$ process $\Longleftarrow$ NLO of $2 \to 5$ LO process: Cutting edge !

$$u\bar{u} \to t\bar{t}b\bar{b}gg$$

- Initial-Final splitting: $\sum \mathrm{D}_i = (65, 66, 67, 68, 69, 77)$

```
[Dipole1] : 78

 B1 : 78

{Splitting (1):(i,j)=(f,g)}:  48 (48)

                  [1.(ij,k)=(fg,k): Dij,k]  32 (32)

                  [2.(ij,a)=(fg,a): Dij^a]  16 (16)

{Splitting (2):(i,j)=(g,g)}:  6 (4)

                  [3.(ij,k)=(gg,k): Dij,k]  4 (4)

                  [4.(ij,a)=(gg,a): Dij^a]  2 (0)

{Splitting (3):(a,i)=(f,g)}:  24 (16)

                  [5.(ai,k)=(fg,k): D^ai,k]  20 (16)

                  [6.(ai,b)=(fg,b): D^ai,b]  4 (0)

{Splitting (4):(a,i)=(g,g)}:  0 (0)

                  [7.(ai,k)=(gg,k): D^ai,k]  0 (0)

                  [8.(ai,b)=(gg,b): D^ai,b]  0 (0)

 -------------------

[Total] : 78

 (Massive dipoles : 68)

--------------------
```

$(|\mathrm{M}|^2 - \sum \mathrm{D}_i)/|\mathrm{M}|^2$



4. Outlook

■ Achivements

- Full automatization of dipole subtraction
  by Mathematica code and interface with MadGraph

- Checking at regular phase space point
  Complete agreement with real corrections to $t\bar{t} + 1\mathrm{jet}$
  (S. Dittmaier, P. Uwer and S. Weinzierl, arXiv:0810.0452)

-Presented examples: $gg \to t\bar{t}gg$ $gg \to t\bar{t}ggg$ . . .

■ Plan

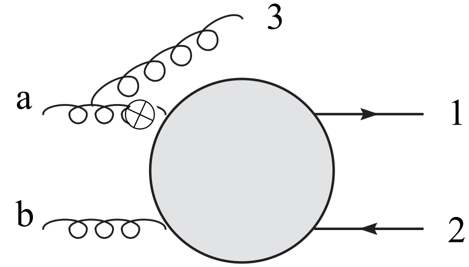
- Phase space integral by Monte Carlo method with optimization

- Combine with Virtual correction

- The complete code will be publicly available

Extra slides

- Emitter = gluon case



$$\mathrm{D}^{a3}_{1}(p_1,p_2,p_3,p_a,p_b) = -\frac{1}{2p_a\cdot p_3}\frac{1}{x_{31,a}}\underset{(\mu)}{\langle \tilde{g}g} \to \tilde{u}\bar{u}\Big|\frac{\mathrm{T}_u\cdot\mathrm{T}_{gg}}{\mathrm{T}^2_{gg}}\mathrm{V}^{a3}\underline{(\mu,\nu)}\Big|\underset{(\nu)}{\tilde{g}g} \to \tilde{u}\bar{u}\rangle_2$$

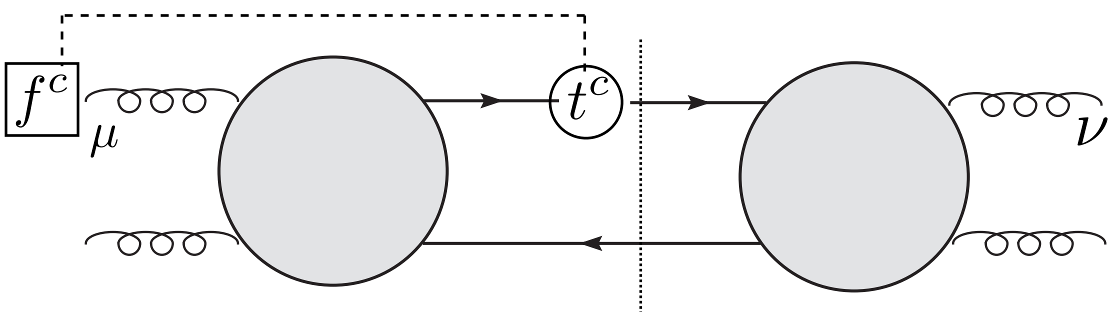
splitting function:

$$\mathrm{V}^{ai}_{k}(x,u)\underline{^{\mu\nu}} = 16\pi\alpha_s C_A\left[-g^{\mu\nu}\left(\frac{1}{1-x_{ik,a}+u_i}-1+x_{ik,a}(1-x_{ik,a})\right)+\frac{1-x_{ik,a}}{x_{ik,a}}\frac{u_i(1-u_i)}{p_i\cdot p_k}\left(\frac{p^\mu_i}{u_i}-\frac{p^\mu_k}{1-u_i}\right)\left(\frac{p^\nu_i}{u_i}-\frac{p^\nu_k}{1-u_i}\right)\right]$$

$$\times$$

Color linked Born squared (CLBS)

$\epsilon^{\lambda}_{\mu}\Big)$



$\Big(\epsilon^{\lambda'\star}_{\nu}$ (Amputate polarization vector)

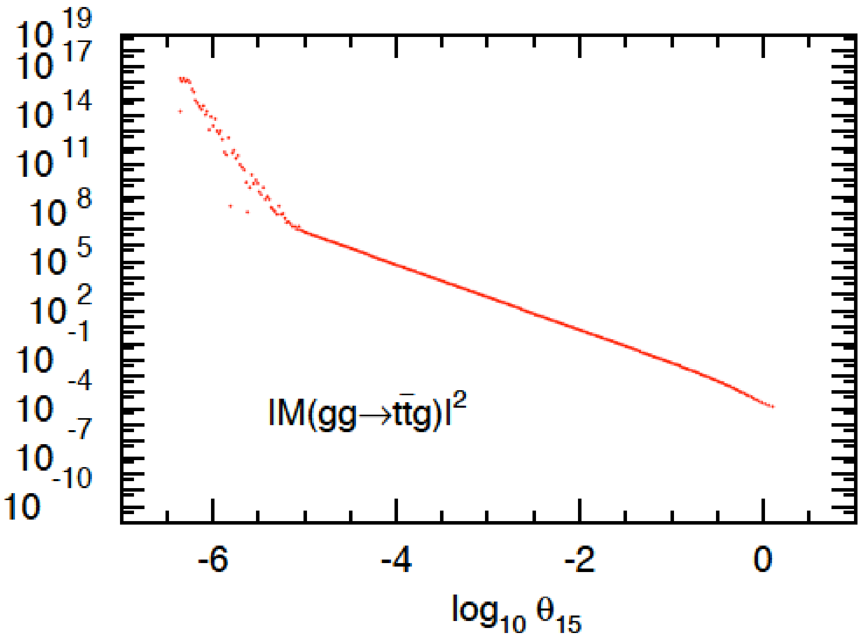
Different helicity squared (DHS)

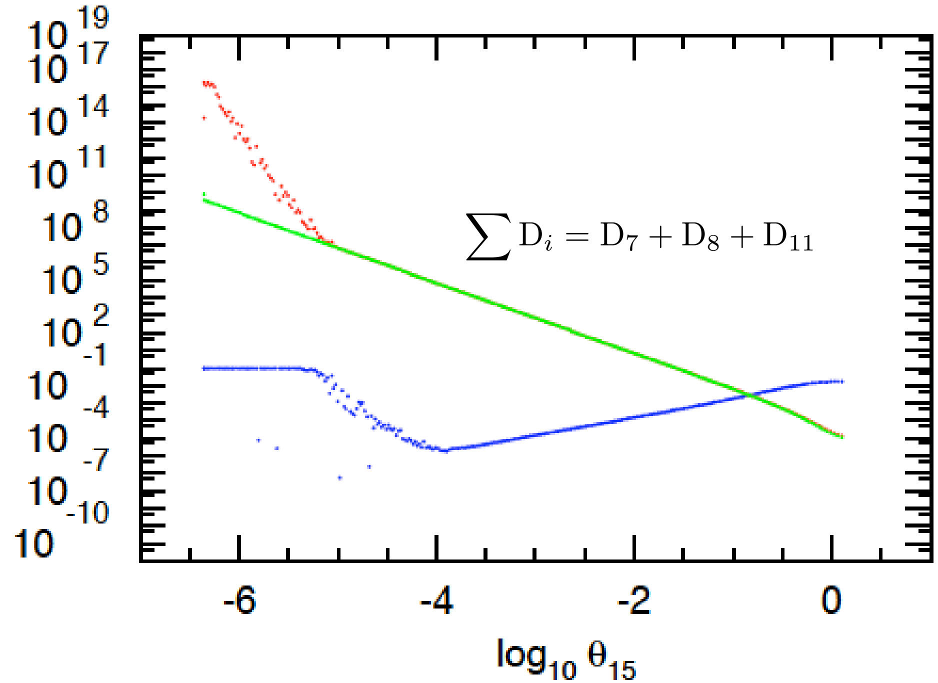
■ $2 \to 3$ process

$gg \to t\bar{t}g$

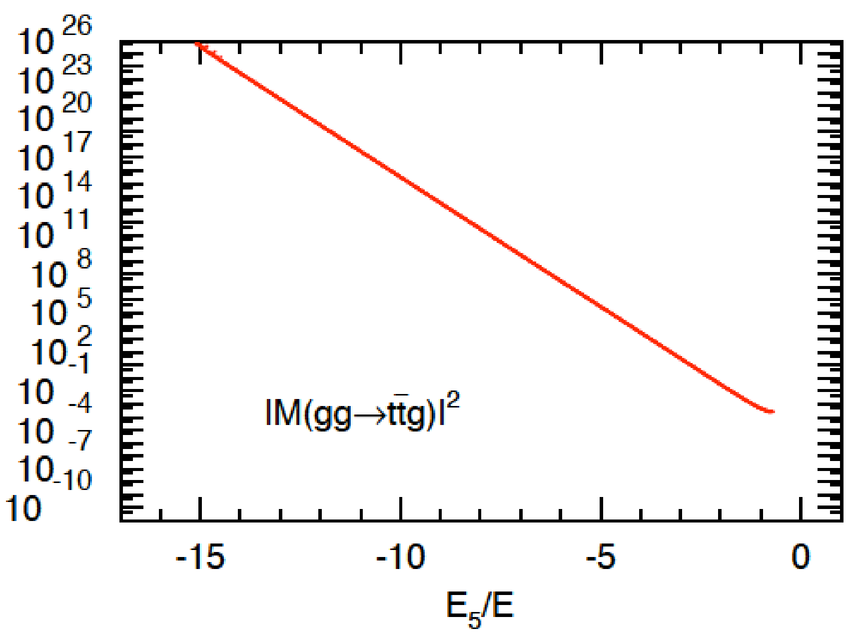
```
Number of dipoles
[Dipole1] : 12
 B1 : 12
{Splitting (1):(i,j)=(f,g)}:  6 (6)
               [1.(ij,k)=(fg,k): Dij,k]  2 (2)
               [2.(ij,a)=(fg,a): Dij^a]  4 (4)
{Splitting (2):(i,j)=(g,g)}:  0 (0)
               [3.(ij,k)=(gg,k): Dij,k]  0 (0)
               [4.(ij,a)=(gg,a): Dij^a]  0 (0)
{Splitting (3):(a,i)=(f,g)}:  0 (0)
               [5.(ai,k)=(fg,k): D^ai,k]  0 (0)
               [6.(ai,b)=(fg,b): D^ai,b]  0 (0)
{Splitting (4):(a,i)=(g,g)}:  6 (4)
               [7.(ai,k)=(gg,k): D^ai,k]  4 (4)
               [8.(ai,b)=(gg,b): D^ai,b]  2 (0)
-------------------
[Dipole2] : 0
{Splitting (5):(i,j)=(f,fbar)}
  (u-ubar splitting) B2u : 0
               [9-u.(ij,k)=(u ubar,k): Dij,k]  0 (0)
               [10-u.(ij,a)=(u ubar,a): Dij^a]  0 (0)
  (d-dbar splitting) B2d : 0
               [9-d.(ij,k)=(u ubar,k): Dij,k]  0 (0)
               [10-d.(ij,a)=(u ubar,a): Dij^a]  0 (0)
  (b-bbar splitting) B2b : 0
               [9-b.(ij,k)=(u ubar,k): Dij,k]  0 (0)
               [10-b.(ij,a)=(u ubar,a): Dij^a]  0 (0)
  (t-tbar splitting) B2t : 0
               [9-t.(ij,k)=(u ubar,k): Dij,k]  0 (0)
               [10-t.(ij,a)=(u ubar,a): Dij^a]  0 (0)
-------------------
[Total] : 12
 (Massive dipoles : 10)
-------------------
END
```

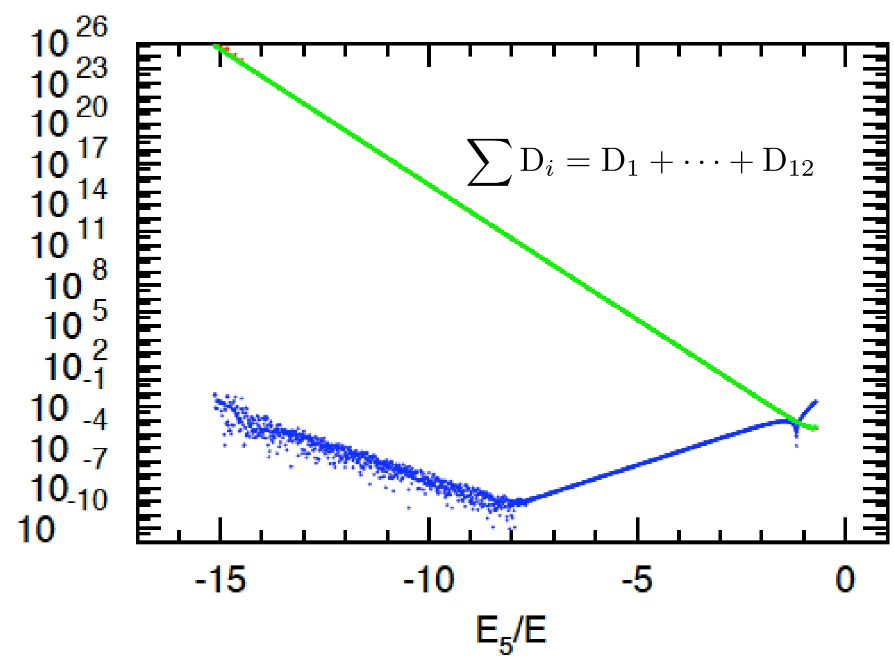
- Collinear limit - Initial-Final splitting





- Soft limit





■ $2 \to 4$ process

$gg \to u\bar{u}gg$

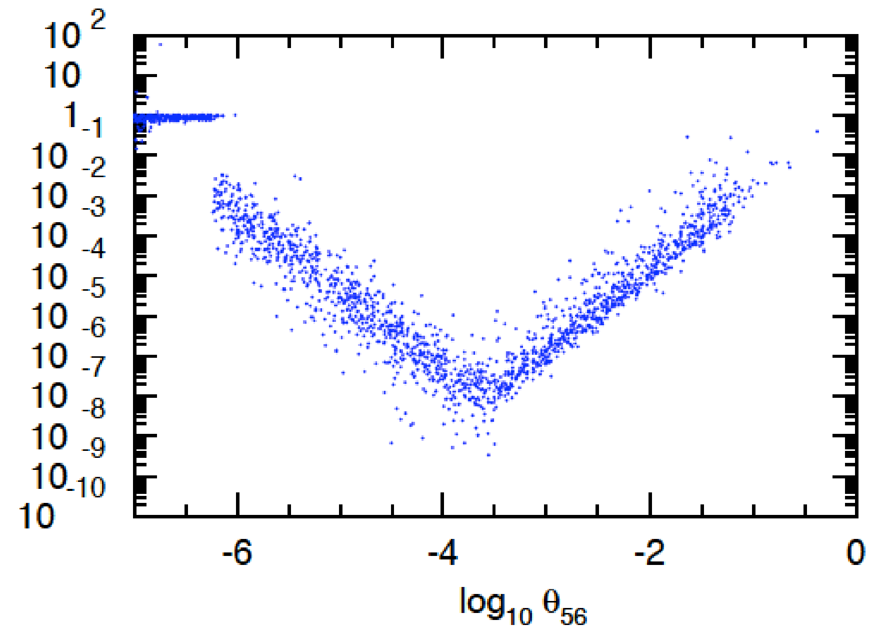
```
[Dipole1] : 36
 B1 : 36
{Splitting (1):(i,j)=(f,g)}:  16 (0)
               [1.(ij,k)=(fg,k): Dij,k]  8 (0)
               [2.(ij,a)=(fg,a): Dij^a]  8 (0)
{Splitting (2):(i,j)=(g,g)}:  4 (0)
               [3.(ij,k)=(gg,k): Dij,k]  2 (0)
               [4.(ij,a)=(gg,a): Dij^a]  2 (0)
{Splitting (3):(a,i)=(f,g)}:  0 (0)
               [5.(ai,k)=(fg,k): D^ai,k]  0 (0)
               [6.(ai,b)=(fg,b): D^ai,b]  0 (0)
{Splitting (4):(a,i)=(g,g)}:  16 (0)
               [7.(ai,k)=(gg,k): D^ai,k]  12 (0)
               [8.(ai,b)=(gg,b): D^ai,b]  4 (0)
-------------------
[Dipole2] : 4
{Splitting (5):(i,j)=(f,fbar)}
  (u-ubar splitting) B2u : 4
               [9-u.(ij,k)=(u ubar,k): Dij,k]  2 (0)
               [10-u.(ij,a)=(u ubar,a): Dij^a]  2 (0)
  (d-dbar splitting) B2d : 0
               [9-d.(ij,k)=(u ubar,k): Dij,k]  0 (0)
               [10-d.(ij,a)=(u ubar,a): Dij^a]  0 (0)
  (b-bbar splitting) B2b : 0
               [9-b.(ij,k)=(u ubar,k): Dij,k]  0 (0)
               [10-b.(ij,a)=(u ubar,a): Dij^a]  0 (0)
  (t-tbar splitting) B2t : 0
               [9-t.(ij,k)=(u ubar,k): Dij,k]  0 (0)
               [10-t.(ij,a)=(u ubar,a): Dij^a]  0 (0)
-------------------
[Dipole4] : 16
```

```
{Splitting (7):(a,i)=(g,f) or (g,fbar)}
 ((a,i)=(g,u) splitting)  B4u : 8
               [13-u.(ai,k)=(gu,k): D^ai,k]  6 (0)
               [14-u.(ai,b)=(gu,b): D^aib]  2 (0)
 ((a,i)=(g,ubar) splitting)   B4ubar : 8
               [13-ubar.(ai,k)=(g ubar,k): D^ai,k]  6 (0)
               [14-ubar.(ai,b)=(g ubar,b): D^aib]  2 (0)
 ((a,i)=(g,d) splitting) B4d : 0
               [13-d.(ai,k)=(gd,k): D^ai,k]  0 (0)
               [14-d.(ai,b)=(gd,b): D^aib]  0 (0)
 ((a,i)=(g,dbar) splitting) B4dbar : 0
               [13-dbar.(ai,k)=(g ubar,k): D^ai,k]  0 (0)
               [14-dbar.(ai,b)=(g ubar,b): D^aib]  0 (0)
-------------------
[Total] : 56
 (Massive dipoles : 0)
-------------------
```

- Collinear limit
  - Final-Final splitting:

$$\sum \mathrm{D}_i = \mathrm{D}_{17} + \mathrm{D}_{18} + \mathrm{D}_{19} + \mathrm{D}_{20}$$

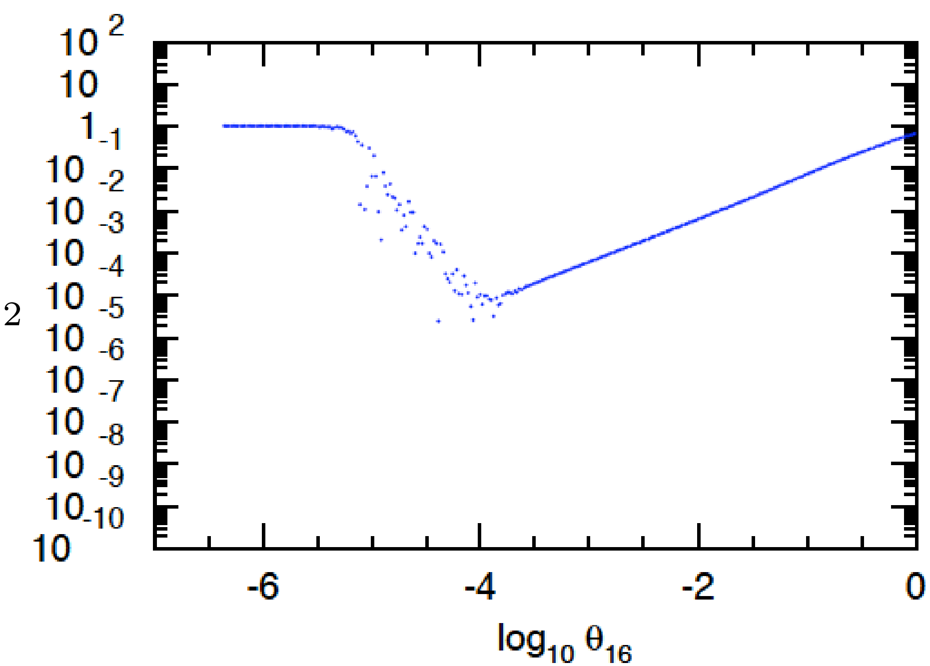
$$\left(|\mathrm{M}|^2 - \sum \mathrm{D}_i\right)/|\mathrm{M}|^2$$



  - Initial-Final splitting:

$$\sum \mathrm{D}_i = \mathrm{D}_{27} + \mathrm{D}_{28} + \mathrm{D}_{29} + \mathrm{D}_{35}$$

$$\left(|\mathrm{M}|^2 - \sum \mathrm{D}_i\right)/|\mathrm{M}|^2$$



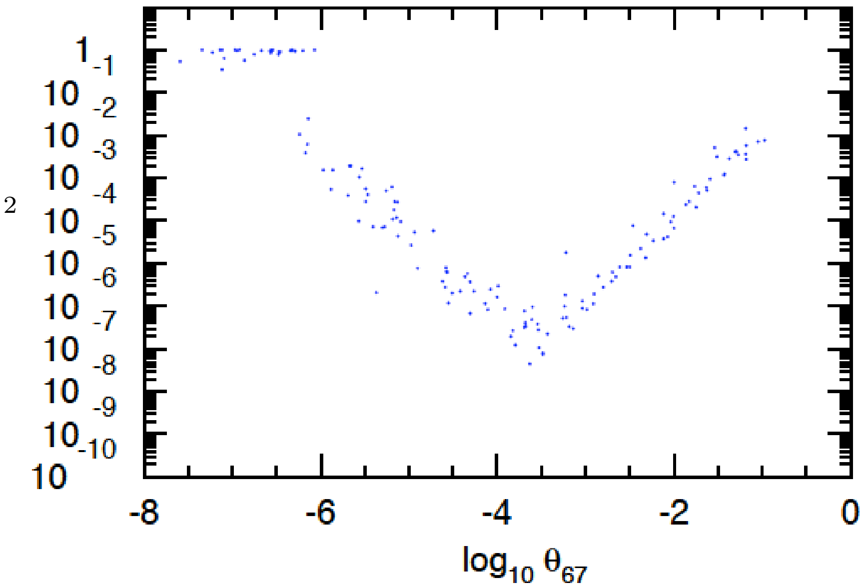
■ $2 \longrightarrow 5$ process

$$u\bar{u} \longrightarrow d\bar{d}ggg$$

```
[Dipole1] : 75
 B1 : 75
{Splitting (1):(i,j)=(f,g)}:  30 (0)
                [1.(ij,k)=(fg,k): Dij,k]  18 (0)
                [2.(ij,a)=(fg,a): Dij^a]  12 (0)
{Splitting (2):(i,j)=(g,g)}:  15 (0)
                [3.(ij,k)=(gg,k): Dij,k]  9 (0)
                [4.(ij,a)=(gg,a): Dij^a]  6 (0)
{Splitting (3):(a,i)=(f,g)}:  30 (0)
                [5.(ai,k)=(fg,k): D^ai,k]  24 (0)
                [6.(ai,b)=(fg,b): D^ai,b]  6 (0)
{Splitting (4):(a,i)=(g,g)}:  0 (0)
                [7.(ai,k)=(gg,k): D^ai,k]  0 (0)
                [8.(ai,b)=(gg,b): D^ai,b]  0 (0)
-------------------
[Dipole2] : 5
{Splitting (5):(i,j)=(f,fbar)}
  (u-ubar splitting) B2u : 0
                [9-u.(ij,k)=(u ubar,k): Dij,k]  0 (0)
                [10-u.(ij,a)=(u ubar,a): Dij^a]  0 (0)
  (d-dbar splitting) B2d : 5
                [9-d.(ij,k)=(u ubar,k): Dij,k]  3 (0)
                [10-d.(ij,a)=(u ubar,a): Dij^a]  2 (0)
  (b-bbar splitting) B2b : 0
                [9-b.(ij,k)=(u ubar,k): Dij,k]  0 (0)
                [10-b.(ij,a)=(u ubar,a): Dij^a]  0 (0)
  (t-tbar splitting) B2t : 0
                [9-t.(ij,k)=(u ubar,k): Dij,k]  0 (0)
                [10-t.(ij,a)=(u ubar,a): Dij^a]  0 (0)
-------------------
[Total] : 80
 (Massive dipoles : 0)
-------------------
```

- Final-Final splitting: $\sum \mathrm{D}_i = (37, 38, 39, 44, 45)$

$(|\mathrm{M}|^2 - \sum \mathrm{D}_i)/|\mathrm{M}|^2$



- Initial-Final splitting: $\sum \mathrm{D}_i = (62, 63, 64, 65, 74)$