Non-commutativity and effective actions in curved space-time

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+ work in progress: with S. Guttenberg, M. Herbst, R. Rashkov

Contents:

Deformation quantization

- Phase space $\mathcal{M}(p_i, q^j)$: Poisson bracket $\{p_i, q^j\}_{PB} = \delta_i^j$
  - general coordinates $M(x^\mu)$: $\{f, g\}_{PB} = \Theta^{\mu\nu}(x)\partial_\mu f \partial_\nu g$
  - Jacobi id. $\sum\!\!\!\!\!\!\circlearrowleft \{\{f, g\}_{PB}, h\}_{PB} = 0 \;\Leftrightarrow\; \sum\!\!\!\!\!\!\circlearrowleft \theta^{\mu\alpha}\partial_\alpha\theta^{\nu\rho} = 0$
  - Polyvector fields $\Lambda^\bullet TM$: vector fields $\xi \in \Lambda^1 T$, bi-vector $\Theta \in \Lambda^2\, TM$, ...
  - Schouten–Nijenhuis bracket: extends Lie bracket to graded bi-derivation on polyvector fields, i.e. $[T, .]$ and $[., T]$ are left/right (anti)derivations

    $[\xi, T] = \mathcal{L}_\xi T, \quad [\theta, \theta]^{\mu\nu\rho} = \frac{2}{3} \sum\!\!\!\!\!\!\circlearrowleft \theta^{\mu\alpha}\partial_\alpha\theta^{\nu\rho} \;\in \Lambda^3\, TM$
- **Deformation Quantization** $\sim$ Associative "operator product"

  $f \cdot g \longrightarrow (f \star g)(x) = fg + \frac{i}{2}\hbar\{f, g\}_{PB} + \mathcal{O}(\hbar^2) \;\in C^\infty[[\hbar]]$
- **Moyal** product: $\quad = \exp\left(\frac{i}{2}\hbar\, \Theta^{\mu\nu}\partial_{y^\mu}\partial_{z^\nu}\right) f(y)g(z)_{\,|\, y\,=\,z\,=\,x}$
  - for constant Θ ! ... what about change of coordinates?

- Symplectic case: $\det\Theta \neq 0 \quad \Rightarrow \quad [\Theta,\Theta] = 0 \;\Leftrightarrow\; d\omega = 0$ with $\omega = \Theta^{-1}$
  existence: De Wilde + Lecompte '83; constructive: Fedosov '85

- Poisson case: Kontsevich [q-alg/9709040]
  - **Formality map** U: Polyvector $T \longrightarrow$ Poly-DifferentialOperator D
  - $U(T_1,\ldots,T_n) = \sum_\Gamma w_\Gamma D_\Gamma \qquad w_\Gamma =$ convergent integrals (string-inspired)
  - Cattaneo–Felder: = topological σ-model perturbative expansion
  - Quasi-isomorphism of $\mathcal{L}_\infty$ algebras
    Hochschild complex
    Schouten–Nijenhuis $\mapsto$ Gerstenhaber bracket (="commutator" of PDOs)

Associativity, gauge equivalence & Hochschild complex

- Let $f \star g = fg + \hbar B_1(f,g) + \mathcal{O}(\hbar^2)$ with $B_1(f,g) = B^{\mu\nu} f_\mu g_\nu$, $f_\mu = \partial_{x^\mu} f$

- "associator" $\sim$ Hochschild co-cycle:

$$f\star(g\star h) - (f\star g)\star h = \hbar\Big(f B_1(g,h) - B_1(fg,h) + B_1(f,gh) - B_1(f,g)h\Big) + \hbar^2 \ldots$$

- gauge equivalence: $f \to Df$ with $D(1) = 1 \;\Rightarrow$

  $D = 1 + \hbar(D_1^\mu \partial_\mu + D_1^{\mu\nu}\partial_\mu\partial_\nu + \ldots) + \hbar^2 \ldots,$

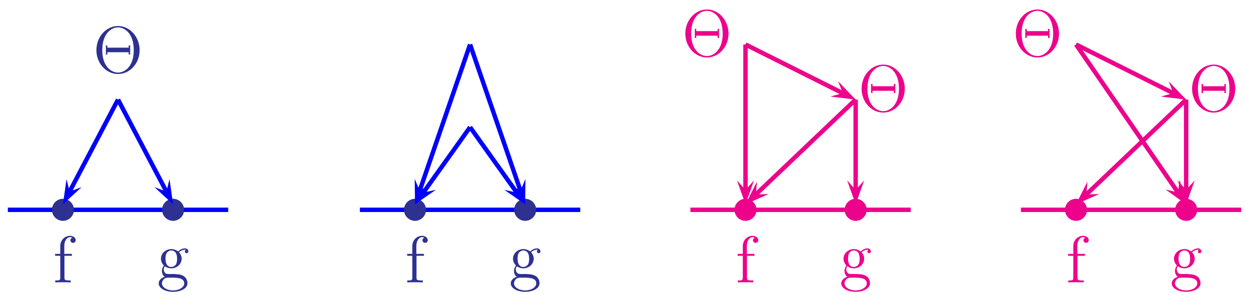
  $f \star' g = D(D^{-1} f \star D^{-1} g) = f \star g \;\Rightarrow$

  $B_1'(f,g) - B_1(f,g) = -f D_1(g) + D_1(fg) - D_1(f)g \;\ldots$ coboundary

- $D_1 = D^{\mu\nu}\partial_\mu\partial_\nu$ removes symmetric $B^{(\mu\nu)}$ $\longrightarrow$ choose $B^{\mu\nu} = \Theta^{\mu\nu} \in \Lambda^2$

define $[f\star g] \equiv fg + i\frac{\hbar}{2}\Theta^{\mu\nu} f_\mu g_\nu - \frac{\hbar^2}{8}\Theta^{\mu\alpha}\Theta^{\nu\beta} f_{\mu\alpha} g_{\nu\beta} \ldots$ "Moyal part"

$$\Rightarrow \quad f\star g = [f\star g] - \hbar^2\, \Theta^{\mu\rho}\partial_\rho \Theta^{\alpha\beta}\big(a\,[f_{\alpha\mu}\star g_\beta] + b\,[f_\beta \star g_{\alpha\mu}]\big) + \mathcal{O}\big((\partial\Theta)^2\big)$$



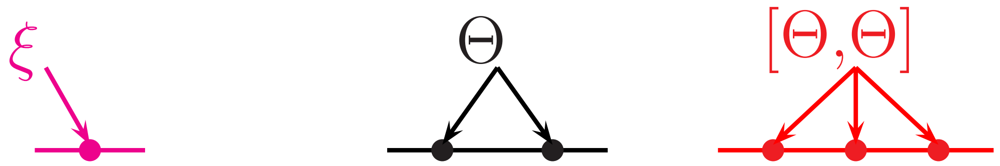
– f, g boundary insertions (open strings)

– integration over Θ's in upper half plane (vertices)

- $f\star(g\star h) - (f\star g)\star h = 0$ at order $\hbar^2$ (only need to keep $\Theta\partial\Theta$ terms!)
  – if and only if $a = b = \frac{1}{12}$ and $[\Theta, \Theta] = 0$

  $\Rightarrow$ associativity requires Poisson structure (mod gauge equivalence)

Formality, associativity and diffeomorphisms

- Formality: 

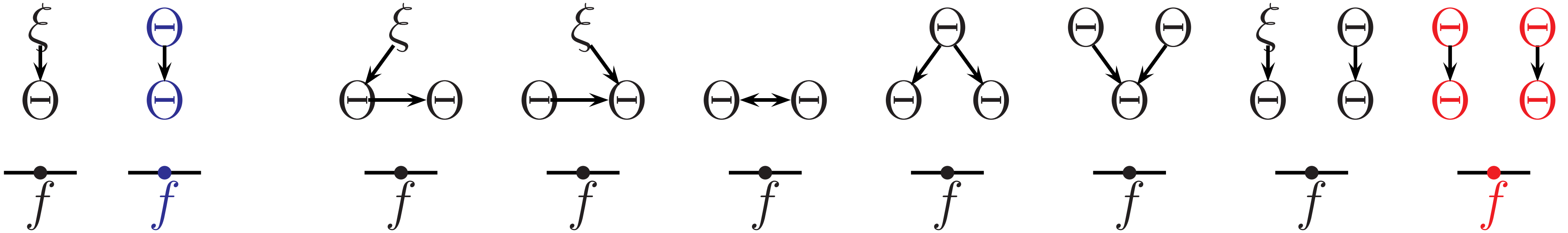
- $[\Theta,\Theta] \;\mapsto$ Gerstenhaber bracket $[\star,\star]$ of $\star(f,g) \equiv$ associator
  $\Rightarrow$ this proves associativity for Poisson structures Θ

- $\Theta^{\mu\nu} f_\mu g_\nu \;\mapsto f \star g$ … this defines the Kontsevich product

- $\xi \rightarrow D_\xi = \mathcal{L}_\xi + \mathcal{O}(\hbar)$ deformed Lie derivative !
  Formality $\Rightarrow \;\partial_t(f \star_t g) = D_\xi f \star g + f \star D_\xi g - D_\xi(f \star g)$ with $\dot{x}^\mu = \xi^m$
  – coordinate transformation entails gauge transformation !
  – used by Cattaneo–Felder to "globalize" the $\star$ product

- $\xi \longrightarrow D_\xi = \mathcal{L}_\xi + \mathcal{O}(\hbar)$ deformed Lie derivative

Formality $\Rightarrow$ $\partial_t(f \star_t g) = D_\xi f \star g + f \star D_\xi g - D_\xi(f \star g)$ with $\dot{x}^\mu = \xi^m$



... plus Moyal-type series on top of this

$\mapsto$ Gauge transformations: $f \longrightarrow f + D_\xi f + \mathcal{O}(\xi^2)$

$$\mathcal{D}_\xi = \xi^\mu \partial_\mu + \frac{1}{24} \xi^\alpha_{\mu\rho} \Theta^{\mu\nu} \partial_\nu \Theta^{\rho\beta} \partial_\alpha \partial_\beta \quad (1)$$

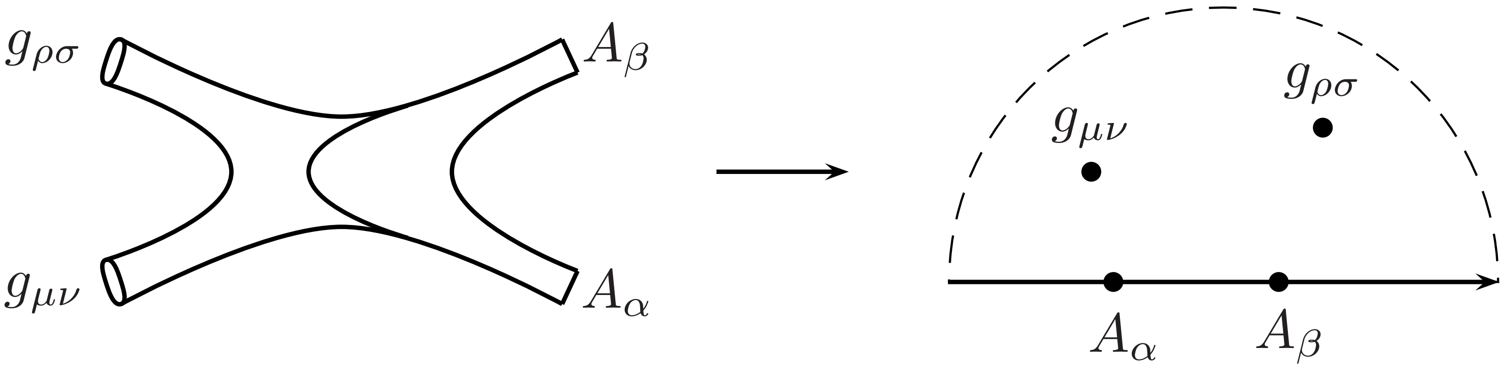
$$+ \ldots \quad (2)$$

$$+ P\, \xi^\alpha_{\rho\sigma} \partial_\nu \Theta^{\beta\mu} \partial_\mu \Theta^{\rho\gamma} \Theta^{\delta\nu} \partial_\nu \Theta^{\sigma\varepsilon} \partial_\alpha \partial_\beta \partial_\gamma \partial_\delta \partial_\varepsilon \quad (3)$$

Open strings, gauge invariance & electrodynamics

Closed string (conformal gauge): $\mathcal{L} = \frac{1}{2\pi\alpha'}\partial X^\mu \bar{\partial} X^\nu \Big(g_{\mu\nu}(X) + B_{\mu\nu}(X) \Big)$

String diagram for photon–graviton scattering:



- Gauge invariance: $\delta_\Lambda \left(\int_\Sigma X^* B \right) = 0$ for $\delta_\Lambda B = d\Lambda$ (Stockes' theorem)

  But: open strings $\rightarrow$ boundary term $\delta_\Lambda \left(\int_\Sigma B \right) = \int_{\partial\Sigma} \Lambda$

- Introduce compensator field ($\sim$ surface charge in E-dyn.):

$$S_A = \int_{\partial\Sigma} A = \int_{\partial\Sigma} ds \partial_s X^\mu A_\mu(X) = \int_\Sigma F \qquad \text{with} \qquad F = dA$$

Gauge invariance: $\delta B = d\Lambda, \qquad \delta A = -\frac{1}{2\pi\alpha'}\Lambda + d\lambda \quad \Rightarrow$

Invariant field strengths: $H = dB = d\mathcal{F}$, $\mathcal{F} = B + 2\pi\alpha' F$.

– D-brane effective action . . . $\mathcal{L}_{BI} \sim \sqrt{|\, g_{\mu\nu} + \mathcal{F}_{\mu\nu} \,|}$ . . . gauge invariant

– Superstrings: $A \ \rightarrow$ superpartner gaugino λ

Boundary conditions and noncommutativity

- **strings $\rightarrow$ operators $X^\mu(\sigma, \tau) \rightarrow$ NC operator product**

$$S = \frac{1}{2\pi\alpha'} \int_{\mathbb{H}} \mathrm{d}^2 z \, \partial X^\mu \bar{\partial} X^\nu \Big(g_{\mu\nu}(X) + \mathcal{F}_{\mu\nu}(X) \Big), \qquad \mathcal{F} = B + 2\pi\alpha' \, dA$$

- **Surface terms . . . fluctuations about background value $X \rightarrow X + \xi$:**

$$\begin{aligned} &\partial(\delta X^\mu \bar{\partial} X^\nu (g_{\mu\nu} + \mathcal{F}_{\mu\nu})) + \bar{\partial}(\partial X^\mu \delta X^\nu (g_{\mu\nu} + \mathcal{F}_{\mu\nu})) \\ &= \partial(\delta X^\mu \bar{\partial} X^\nu (g_{\mu\nu} + \mathcal{F}_{\mu\nu})) + \bar{\partial}(\partial X^\nu \delta X^\mu (g_{\mu\nu} - \mathcal{F}_{\mu\nu}) \end{aligned}$$

- **Boundary conditions:** $\int dy \partial() = - \int dy \bar{\partial}()$

$$g_{\mu\nu}(\partial - \bar{\partial}) X^\nu - \mathcal{F}_{\mu\nu}(\partial + \bar{\partial}) X^\nu = g_{\mu\nu} \partial_{\textbf{tang.}} X^\nu - \mathcal{F}_{\mu\nu} \partial_{\textbf{normal}} X^\nu = 0$$

Propagator:

$$\langle \zeta^\mu(u,\bar{u})\,\zeta^\nu(w,\bar{w})\rangle_{\mathcal{F}} = -\alpha'\Big\{ g^{\mu\nu}(\ln|u-w| - \ln|u-\bar{w}|) + G^{\mu\nu}\ln|u-\bar{w}|^2 - \Theta^{\mu\nu}\ln\Big(\frac{\bar{w}-u}{\bar{u}-w}\Big)\Big\}$$

- define $(g_{\mu\nu} + \mathcal{F}_{\mu\nu})^{-1} = G^{(\mu\nu)} + \Theta^{[\mu\nu]}$
- $g^{\mu\nu}$-term $\to 0$ at boundary $\partial\Sigma \Rightarrow G^{\mu\nu}$ = "open string metric"
- for $G^{\mu\nu} \to 0$ propagator $\to$ antisym. step function at $\partial\Sigma$

  $\Rightarrow$ non-commutative non-singular operator product $\sim$ Moyal
- Proposal: define non-commutative product

$$f(x) \circ g(x) = \frac{1}{\sqrt{|g+\mathcal{F}|}} \int \mathcal{D}\xi \; e^{-S[X=x+\xi]} \; f(X(0))\; g(X(1))$$

- includes quantum corrections; regularized by putting $u = 0$, $w = 1$
- off-shell A_∞ administrating non-associativity
- BI measure comes up in sum over graphs

Philosophy:

- Non-commutative product $\equiv$ summing up leading strong B/F field effects
- Beyond topological limit (Seiberg-Witten; Kontsevich) non-associative
- Cyclicity = Connes-Flato-Sternheimer = (strongly) closed product w.r.t. measure Ω

$$\int \Omega \; f \circ g = \int \Omega \; f \cdot g \qquad \text{and} \qquad \int \Omega \; (f \circ g) \circ h = \int \Omega \; f \circ (g \circ h)$$

- Variation of effective action: cyclicity $\Rightarrow$ associativity not necessary

$$\delta \int \Omega \; (\phi \circ ((\phi \circ \phi) \circ \phi) = \int \Omega \; \delta\phi \cdot (\ldots)$$

  and: associativity not sufficient; cyclicity (partly) fixes gauge equivalence

- Felder Shoikhet '00: Kontsevich product cyclic if $\partial_\mu(\Omega\Theta^{\mu\nu}) = 0$
  – but not canonical: $\nexists$ Liouville measure for Poisson structure
- Maxwell equation: $D - branes$ suggest to treat bulk fields as background and to impose generalized Maxwell equation:

$$\partial_\mu(\sqrt{\det(g + \mathcal{F})}\Theta^{\mu\nu}) = 0 \quad \Leftrightarrow \quad G^{\rho\sigma} D_\rho \mathcal{F}_{\sigma\mu} - \frac{1}{2}\Theta^{\rho\sigma} H_{\rho\sigma}{}^\lambda \mathcal{F}_{\lambda\mu} = 0$$

How does it work?

- Derivative expansion: Exact to all order in θ, expand in $\partial\Theta$
- Tachyon effecitve action
  - 2-point function $\to f \circ g$; structure of 3-point functions & ghosts $\Rightarrow$

$$\Box f_i - (-2\pi) f_i = \frac{1}{\sqrt{\det(g-\mathcal{F})}}\, \partial_\mu \left(\sqrt{\det(g-\mathcal{F})} G^{\mu\nu} \partial_\nu f_i \right) - (-2\pi) f_i = 0.$$

$=$ on-shell condition $\Rightarrow$

$$S \;=\; -\frac{1}{2g_o^2} \int d^D x \sqrt{g-\mathcal{F}} \left\{ G^{\mu\nu} \cdot \partial_\mu T \cdot \partial_\nu T - \frac{1}{\alpha'}\, T \cdot T - \sqrt{\frac{8}{9\alpha'}}\, T \cdot (T \circ T) \right\}.$$

- 2nd derivative order and dilaton background: Kontsevich + gauge term:

$$f \circ g = f \star g - \frac{1}{24} \Theta^{\mu\rho} \Theta^{\nu\sigma} \partial_\rho \partial_\sigma (\log \sqrt{\det(g+\mathcal{F})\; e^{-\phi}}) f_\mu g_\nu$$

Green Schwarz String

- Type II target-superspace $x^M = (x^m, \theta^\mu, \hat\theta^{\hat\mu})$ with supersymmetry-transformation

$$\begin{aligned} \delta\theta^\mu &= \varepsilon^\mu, \qquad \delta\hat\theta^{\hat\mu} = \hat\varepsilon^{\hat\mu} \\ \delta x^m &= \varepsilon\gamma^m\theta + \hat\varepsilon\gamma^m\theta \end{aligned}$$

SUSY-invariant one-forms (supervielbeins) in flat superspace

$$E^A \equiv \mathrm{d}x^M E_M{}^A = (\underbrace{\mathrm{d}x^a + \mathrm{d}\theta\gamma^a\theta + \mathrm{d}\hat\theta\gamma^a\hat\theta}_{\Pi^a}\ , \quad \mathrm{d}\theta^\alpha \quad , \quad \mathrm{d}\hat\theta^{\hat\alpha})$$

- GS-action (in conformal gauge)

$$\begin{aligned} S_{GS} &= \int \quad \tfrac{1}{2}\Pi^a_z\eta_{ab}\Pi^b_{\bar z} + \mathcal{L}_{WZ} \\ \mathcal{L}_{WZ} &= -\tfrac{1}{2}\Pi^a_z\left(\theta\gamma_a\bar\partial\theta - \hat\theta\gamma_a\bar\partial\hat\theta\right) + \tfrac{1}{2}\left(\partial\theta\gamma^a\theta\right)\left(\hat\theta\gamma_a\bar\partial\hat\theta\right) - (z \leftrightarrow \bar z) \end{aligned}$$

- Fermionic momenta are constrained (constraints $d_\alpha = p_\alpha - f\alpha$)

$$p_{z\alpha} = (\gamma_a\theta)_\alpha\left(\partial x^a - \tfrac{1}{2}\theta\gamma^a\partial\theta - \tfrac{1}{2}\hat\theta\gamma^a\partial\hat\theta\right) = f_\alpha(\theta^\mu, \partial_1 x^m, \partial_1\theta^\mu, p_a)$$

- Constraints are mixed first (κ-symmetry)/ second class

$$\{d_{z\alpha}(\sigma), d_{z\beta}(\sigma')\} \quad \propto \quad 2\gamma^a_{\alpha\beta}\Pi_{za}\delta(\sigma-\sigma')$$

Siegel's idea (NPB'93): complete to a (centrally extended) closed algebra

$$\begin{aligned}
\{d_{z\alpha}, \Pi_{za}\} &\propto 2\gamma_{a\,\alpha\beta}\partial\theta^\beta\delta(\sigma-\sigma') \\
\{\Pi_{za}, \Pi_{zb}\} &\propto \eta_{ab}\delta'(\sigma-\sigma') \\
\{d_{z\alpha}, \partial\theta^\beta\} &\propto \delta^\beta_\alpha\delta'(\sigma-\sigma')
\end{aligned}$$

- Same chiral algebra from the following free Lagrangian

$$\begin{aligned}
S_{free} &= \int \quad \tfrac{1}{2}\partial x^m\eta_{mn}\bar\partial x^n + \bar\partial\theta^\alpha p_{z\alpha} + \partial\hat\theta^{\hat\alpha}\hat p_{\bar z\hat\alpha} = \\
&= \int \quad \underbrace{\frac{1}{2}\Pi^a_z\eta_{ab}\Pi^b_{\bar z} + \mathcal{L}_{WZ}}_{\mathcal{L}_{GS}} + \bar\partial\theta^\alpha d_{z\alpha} + \partial\hat\theta^{\hat\alpha}\hat d_{\bar z\hat\alpha}
\end{aligned}$$

Classically coincides with GS for $d_\alpha = \hat d_{\hat\alpha} = 0$ (still mixed first/second class).

Berkovits Pure Spinor String

- Berkovits (hep-th/0001035): impement $d_\alpha = 0$ in cohomology

$$Q = \oint \quad \lambda^\alpha d_{z\alpha}, \qquad \hat{Q} = \oint \mathrm{d}\bar{z} \quad \hat{\lambda}^{\hat{\alpha}} \hat{d}_{\bar{z}\hat{\alpha}}$$

d_α not pure first class $\Rightarrow$ $Q^2 = 0$ requires pure spinor constraint $\lambda\gamma^a\lambda = 0$

- Berkovits pure spinor string action in flat background (add only $\mathcal{L}_{gh}$)

$$S_{ps} = \int \quad \underbrace{\tfrac{1}{2}\Pi^a_z \eta_{ab} \Pi^b_{\bar{z}} + \mathcal{L}_{WZ}}_{\mathcal{L}_{GS}} + \bar{\partial}\theta^\alpha d_{z\alpha} + \partial\hat{\theta}^{\hat{\alpha}} \hat{d}_{\bar{z}\hat{\alpha}} + \mathcal{L}_{gh}$$

$$\mathcal{L}_{WZ} = -\tfrac{1}{2}\Pi^a_z \left(\theta\gamma_a\bar{\partial}\theta - \hat{\theta}\gamma_a\bar{\partial}\hat{\theta}\right) + \tfrac{1}{2}\left(\partial\theta\gamma^a\theta\right)\left(\hat{\theta}\gamma_a\bar{\partial}\hat{\theta}\right) - (z \leftrightarrow \bar{z})$$

$$\Pi^a_z = \partial x^a + \partial\theta\gamma^a\theta + \partial\hat{\theta}\gamma^a\hat{\theta}$$

$$\mathcal{L}_{gh} = \bar{\partial}\lambda^\alpha\omega_{z\alpha} + \partial\hat{\lambda}^{\hat{\alpha}}\omega_{\hat{\alpha}} + L_{z\bar{z}a}(\lambda\gamma^a\lambda) + \hat{L}_{z\bar{z}a}(\hat{\lambda}\gamma^a\hat{\lambda})$$

- Lagrange multiplier L good enought at classical level; quantization of (λ, ω) is tricky
- PS constraint (first class) generates antighost gauge symmetry $\delta_{(\mu)}\omega_{z\alpha} = \mu_{za}(\gamma^a\lambda)_\alpha$

Type II PS String in General Background

Berkovits&Howe [hep-th/0112160]: deform by vertex; Bedoya&Chandía [hep-th/0609161]: 1-loop; Guttenberg'06: type II BRST-transformations

- Curved background: up to field redefinitions most general renormalizable action with ghostnumber 0 is (with $G_{MN} = E_M{}^a e^{2\Phi}\eta_{ab}E_N{}^b$):

$$S = \int \frac{1}{2}\partial x^M \big(G_{MN}(x) + B_{MN}(x)\big)\bar\partial x^N + \bar\partial x^M E_M{}^{\boldsymbol\alpha}(x)\, d_{\boldsymbol\alpha} + \partial x^M E_M{}^{\hat{\boldsymbol\alpha}}(x)\, \hat d_{\hat{\boldsymbol\alpha}} + \mathcal{T}(x)$$

$$+ d_{\boldsymbol\alpha}\mathcal{P}^{\boldsymbol\alpha\hat{\boldsymbol\beta}}(x)\, \hat d_{\bar z\hat{\boldsymbol\beta}} + \boldsymbol\lambda^{\boldsymbol\alpha} C_{\boldsymbol\alpha}{}^{\boldsymbol\beta\hat{\boldsymbol\gamma}}(x)\, \boldsymbol\omega_{\boldsymbol\beta}\hat d_{\hat{\boldsymbol\gamma}} + \hat{\boldsymbol\lambda}^{\hat{\boldsymbol\alpha}} \hat C_{\hat{\boldsymbol\alpha}}{}^{\hat{\boldsymbol\beta}\boldsymbol\gamma}(x)\, \hat{\boldsymbol\omega}_{\hat{\boldsymbol\beta}} d_{\boldsymbol\gamma} + \boldsymbol\lambda^{\boldsymbol\alpha}\hat{\boldsymbol\lambda}^{\hat{\boldsymbol\alpha}} S_{\boldsymbol\alpha\hat{\boldsymbol\alpha}}{}^{\boldsymbol\beta\hat{\boldsymbol\beta}}(x)\, \boldsymbol\omega_{\boldsymbol\beta}\hat{\boldsymbol\omega}_{\hat{\boldsymbol\beta}}$$

$$+ \underbrace{\big(\bar\partial\boldsymbol\lambda^{\boldsymbol\beta} + \boldsymbol\lambda^{\boldsymbol\alpha}\bar\partial x^M \Omega_{M\boldsymbol\alpha}{}^{\boldsymbol\beta}(x)\big)}_{\equiv\nabla_{\bar z}\boldsymbol\lambda^{\boldsymbol\beta}}\boldsymbol\omega_{z\boldsymbol\beta} + \underbrace{\big(\partial\hat{\boldsymbol\lambda}^{\hat{\boldsymbol\beta}} + \hat{\boldsymbol\lambda}^{\hat{\boldsymbol\alpha}}\partial x^M \hat\Omega_{M\hat{\boldsymbol\alpha}}{}^{\hat{\boldsymbol\beta}}(x)\big)}_{\equiv\hat\nabla_z\boldsymbol\lambda^{\hat{\boldsymbol\beta}}}\hat{\boldsymbol\omega}_{\bar z\hat{\boldsymbol\beta}} + L_{z\bar z a}(\boldsymbol\lambda\gamma^a\boldsymbol\lambda) + \hat L_{z\bar z\hat a}(\hat{\boldsymbol\lambda}\gamma^{\hat a}\hat{\boldsymbol\lambda})$$

- A nonconstant tachyon background $\mathcal{T}$ will not be BRST-invariant (flat: $\mathbf{s}\mathcal{T} = \boldsymbol\lambda^{\boldsymbol\alpha}\nabla_{\boldsymbol\alpha}\mathcal{T} \overset{!}{=} 0 \quad \Rightarrow [\nabla_{\boldsymbol\alpha}, \nabla_{\boldsymbol\beta}]\mathcal{T} = -2\gamma^a_{\boldsymbol\alpha\boldsymbol\beta}\nabla_a\mathcal{T} \overset{!}{=} 0 \quad \Rightarrow \mathcal{T} \overset{!}{=} const$)

- The general ansatz for the BRST-currents (gh#1, conf weight 1) in the curved background can (reparametrizing d) be brought to

$$\boldsymbol{j}_z = \boldsymbol\lambda^{\boldsymbol\alpha} d_{z\boldsymbol\alpha} + \boldsymbol\lambda^{\boldsymbol\alpha} W_{\boldsymbol\alpha M}(x)\,\partial_z x^M, \quad \boldsymbol{j}_{\bar z} = 0, \qquad (Q = \oint \mathrm{d}z \;\; \boldsymbol{j}_z)$$

SUGRA and superspace deformation

- Ooguri, Vafa [hep-th/0303063]: Hybrid formalism, 4d superspace
  de Boer, Grassi, Nieuwenhuizen [hep-th/0302078]:
  10d, constant RR background

- Berkovits [hep-th/0205154]: (non-abelien) SUSY Born-Infeld
  from pure spinor (with boundary fermions)

- Most tedious part: solution of constraints, θ-expansion

Recollections & Outlook

- Einstein's unification attempts:
  - Torsion / non-symmetric metric: $g_{(\mu\nu)} + B_{[\mu\nu]}$
  - Kaluza–Klein / hidden dimensions
- Good ingredients, but different dynamics:
  - $g_{(\mu\nu)} + \mathcal{F}_{[\mu\nu]} = (G^{(\mu\nu)} + \Theta^{[\mu\nu]})^{-1} \quad \ldots \quad \mathcal{F} = dA + B$
    - **two dynamical fields determine noncommutativity**
    - $\partial(\mu_{BI}\theta^{\mu\nu}) = 0$ generalized BI-Maxwell $\rightarrow$ cyclicity (D-brane)
    - $dB = *da$ a=Axion ... Peccei–Quinn 1977: strong-CP problem
  - KK $\rightarrow$ D-branes: NC = additional aspect of quantum geometry
    - gravity = **bulk** physics / NC dynamics = **brane** physics
  - SUSY: $B_{\mu\nu}$ deforms x-space, $\not{C}$ deforms θ-space (RR gauge fields)
- $\exists$ non-topological cyclicity to all orders? $\rightarrow$ non-commutative BI ?
- $\exists$ non-topological (&SUSY) version of formality?