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Nonanticommutative SUSY field theories

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Outline

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  -SUSY transformations
- Deformed (noncommutative):
  -SUSY transformations
  -superspace
  -chiral fields
- Deformed Wess-Zumino Lagrangian
  -construction
  -equations of motion
- Comments, conclusions

Undeformed superspace

Generated by (anti)commuting coordinates

$$[x^m, x^n] = 0, \quad \{\theta^\alpha, \theta^\beta\} = 0, \quad \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = 0, \tag{1}$$

with $m = 0, \dots 3$ and $\alpha, \beta, \dot\alpha, \dot\beta = 1, 2$.
Derivatives consistent with this algebra are given by

$$[\partial_m, x^n] = \delta_m^n, \quad \{\partial_\alpha, \theta^\beta\} = \delta_\alpha^\beta, \quad \{\bar\partial^{\dot\alpha}, \bar\theta_{\dot\beta}\} = \delta_{\dot\alpha}^{\dot\beta} \tag{2}$$

and

$$[\partial_m, \partial_n] = 0, \quad \{\partial_\alpha, \partial_\beta\} = \{\bar\partial^{\dot\alpha}, \bar\partial^{\dot\beta}\} = 0, \dots. \tag{3}$$

Superfiled $F(x, \theta, \bar\theta)$ can be expanded in powers of θ and $\bar\theta$

$$\begin{aligned} F(x,\theta,\bar\theta) \; &= \; f(x) + \theta\phi(x) + \bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta n(x) \\ &+\theta\sigma^m\bar\theta v_m + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\varphi(x) + \theta\theta\bar\theta\bar\theta d(x). \end{aligned} \tag{4}$$

Under infinitesimal SUSY transformations

$$\delta_\xi F \;=\; \big(\xi Q + \bar\xi\bar Q\big)F, \tag{5}$$

$$Q_\alpha = \partial_\alpha - i\sigma^m{}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m\,, \quad \bar Q^{\dot\alpha} = \bar\partial^{\dot\alpha} - i\theta^\alpha\sigma^m{}_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_m. \tag{6}$$

Also $\xi^a, \bar\xi_{\dot\alpha} = const.$ and $\{\xi^\alpha,\xi^\beta\} = \{\xi^\alpha,\bar\xi_{\dot\alpha}\} = \{\bar\xi_{\dot\alpha},\bar\xi_{\dot\beta}\} = 0$.

Transformations (5) close in the algebra

$$[\delta_\xi,\delta_\eta] = -2i\big(\eta\sigma^m\bar\xi - \xi\sigma^m\bar\eta\big)\partial_m. \tag{7}$$

Leibniz rule is undeformed

$$\begin{aligned} \delta_\xi\big(F\cdot G\big) \;&=\; (\delta_\xi F)\cdot G + F\cdot(\delta_\xi G) \\ &=\; \big(\xi Q + \bar\xi\bar Q\big)\big(F\cdot G\big). \end{aligned} \tag{8}$$

Hopf algebra of undeformed SUSY transformations

- algebra

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \quad \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2i\sigma^m{}_{\alpha\dot{\beta}}\partial_m,$$
$$[\partial_m, \partial_n] = [\partial_m, Q_\alpha] = [\partial_m, \bar{Q}_{\dot{\alpha}}] = 0. \qquad (9)$$

- coproduct

$$\Delta Q_\alpha = Q_\alpha \otimes 1 + 1 \otimes Q_\alpha, \quad \Delta\bar{Q}_{\dot{\alpha}} = \bar{Q}_{\dot{\alpha}} \otimes 1 + 1 \otimes \bar{Q}_{\dot{\alpha}},$$
$$\Delta\partial_m = \partial_m \otimes 1 + 1 \otimes \partial_m. \qquad (10)$$

- counit

$$\varepsilon(Q_\alpha) = \varepsilon(\bar{Q}_{\dot{\alpha}}) = \varepsilon(\partial_m) = 0. \qquad (11)$$

- antipod

$$S(Q_\alpha) = -Q_\alpha, \quad S(\bar{Q}_{\dot{\alpha}}) = -\bar{Q}_{\dot{\alpha}}, \quad S(\partial_m) = -\partial_m. \qquad (12)$$

Deformation by twist

Abelian twist $\mathcal{F} = e^{-\frac{i}{2}\theta^{mn}\partial_m\otimes\partial_n}, \quad \theta^{mn} = -\theta^{nm} \in \mathbb{R}$ leads to

1) θ-deformed Poincaré symmetry,

Chaichian et al. (Phys. Lett. B604, 98 (2004)), Wess (hep-th/0408080), Koch et al.(Nucl. Phys. B717, 387 (2005))

$$\begin{aligned}
[\partial_m, \partial_n] &= 0, \quad [\delta^\star_\omega, \partial_r] = \omega_r{}^m\partial_m, \quad [\delta^\star_\omega, \delta^\star_{\omega'}] = \delta^\star_{[\omega,\omega']}, \\
\Delta(\delta^\star_\omega) &= \delta^\star_\omega \otimes 1 + 1 \otimes \delta^\star_\omega + \frac{i}{2}\theta^{rs}\Big(\omega^l{}_r\partial_l \otimes \partial_s + \partial_r \otimes \omega^l{}_s\partial_l\Big).
\end{aligned}$$

2) θ-deformed gravity,

Aschieri et al. (Class. Quant. Grav. 22, 3511 (2005) and 23, 1883 (2006))

$$[\delta^\star_\xi, \delta^\star_\eta] = \delta^\star_{[\xi,\eta]},\ \Delta(\delta^\star_\xi) = \delta^\star_\xi \otimes 1 + 1 \otimes \delta^\star_\xi - \frac{i}{2}\theta^{rs}\Big(\delta^\star_{(\partial_r\xi)} \otimes \partial_s + \partial_r \otimes \delta^\star_{(\partial_s\xi)}\Big) + \dots .$$

3) θ-deformed gauge theory,

Aschieri et al. (Lett. Math. Phys. 78 (2006) 61), Vassilevich (Mod. Phys. Lett. A 21 (2006) 1279), Giller et al. (Phys. Lett. B655, 80 (2007))

$$[\delta^\star_\alpha, \delta^\star_\beta] = \delta^\star_{-i[\alpha,\beta]},\ \Delta(\delta^\star_\alpha) = \delta^\star_\alpha \otimes 1 + 1 \otimes \delta^\star_\alpha - \frac{i}{2}\theta^{rs}\Big(\delta^\star_{(\partial_r\alpha)} \otimes \partial_s + \partial_r \otimes \delta^\star_{(\partial_s\alpha)}\Big) + \dots .$$

Deformed SUSY transformations

We choose twist $\mathcal{F}$

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}\partial_\alpha\otimes\partial_\beta+\frac{1}{2}\bar{C}_{\dot{\alpha}\dot{\beta}}\bar{\partial}^{\dot{\alpha}}\otimes\bar{\partial}^{\dot{\beta}}}, \quad C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}. \qquad (13)$$

Hopf algebra of deformed SUSY transformations

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \quad \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2i\sigma^m_{\ \alpha\dot{\beta}}\partial_m, \ \dots, \qquad (14)$$

$$\begin{aligned}
\Delta_{\mathcal{F}}(Q_\alpha) &= \mathcal{F}\Big(Q_\alpha\otimes 1 + 1\otimes Q_\alpha\Big)\mathcal{F}^{-1} \qquad (15)\\
&= Q_\alpha\otimes 1 + 1\otimes Q_\alpha \\
&\quad -\frac{i}{2}\bar{C}_{\dot{\alpha}\dot{\beta}}\Big(\sigma^m_{\alpha\dot{\gamma}}\varepsilon^{\dot{\gamma}\dot{\alpha}}\partial_m\otimes\bar{\partial}^{\dot{\beta}} + \bar{\partial}^{\dot{\alpha}}\otimes\sigma^m_{\ \alpha\dot{\gamma}}\varepsilon^{\dot{\gamma}\dot{\beta}}\partial_m\Big), \\
\Delta_{\mathcal{F}}(\bar{Q}_\alpha) &= \bar{Q}_{\dot{\alpha}}\otimes 1 + 1\otimes\bar{Q}_{\dot{\alpha}} \\
&\quad +\frac{i}{2}C^{\alpha\beta}\Big(\sigma^m_{\alpha\dot{\alpha}}\partial_m\otimes\partial_\beta + \partial_\alpha\otimes\sigma^m_{\ \beta\dot{\alpha}}\partial_m\Big), \\
\varepsilon(Q_\alpha) &= \varepsilon(\bar{Q}_{\dot{\alpha}}) = 0, \qquad S(Q_\alpha) = -Q_\alpha, \quad S(\bar{Q}_{\dot{\alpha}}) = -\bar{Q}_{\dot{\alpha}}.
\end{aligned}$$

$\star$-product

Inverse of $\mathcal{F}$ introduces a $\star$-product on the superspace as

$$
\begin{aligned}
F\star G &= \mu\{\mathcal{F}^{-1}\,F\otimes G\} \\
&= F\cdot G-\frac{1}{2}(-1)^{|F|}C^{\alpha\beta}(\partial_\alpha F)\cdot(\partial_\beta G)-\frac{1}{2}(-1)^{|F|}\bar{C}_{\dot\alpha\dot\beta}(\bar\partial^{\dot\alpha}F)(\bar\partial^{\dot\beta}G) \\
&\quad -\frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}(\partial_\alpha\partial_\gamma F)\cdot(\partial_\beta\partial_\delta G)-\frac{1}{8}\bar{C}_{\dot\alpha\dot\beta}\bar{C}_{\dot\gamma\dot\delta}(\bar\partial^{\dot\alpha}\bar\partial^{\dot\gamma}F)(\bar\partial^{\dot\beta}\bar\partial^{\dot\delta}G) \\
&\quad -\frac{1}{4}C^{\alpha\beta}\bar{C}_{\dot\alpha\dot\beta}(\partial_\alpha\bar\partial^{\dot\alpha}F)(\partial_\beta\bar\partial^{\dot\beta}G) \\
&\quad +\frac{1}{16}(-1)^{|F|}C^{\alpha\beta}C^{\gamma\delta}\bar{C}_{\dot\alpha\dot\beta}(\partial_\alpha\partial_\gamma\bar\partial^{\dot\alpha}F)(\partial_\beta\partial_\delta\bar\partial^{\dot\beta}G) \\
&\quad +\frac{1}{16}(-1)^{|F|}C^{\alpha\beta}\bar{C}_{\dot\alpha\dot\beta}\bar{C}_{\dot\gamma\dot\delta}(\partial_\alpha\bar\partial^{\dot\alpha}\bar\partial^{\dot\gamma}F)(\partial_\beta\bar\partial^{\dot\beta}\bar\partial^{\dot\delta}G) \\
&\quad +\frac{1}{64}C^{\alpha\beta}C^{\gamma\delta}\bar{C}_{\dot\alpha\dot\beta}\bar{C}_{\dot\gamma\dot\delta}(\partial_\alpha\partial_\gamma\bar\partial^{\dot\alpha}\bar\partial^{\dot\gamma}F)(\partial_\beta\partial_\delta\bar\partial^{\dot\beta}\bar\partial^{\dot\delta}G), \qquad (16)
\end{aligned}
$$

where $|F|=1$ if F is odd and $|F|=0$ if F is even.

Under complex conjugation

$$(F \star G)^* = G^* \star F^*. \tag{17}$$

Special examples

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \bar C_{\dot\alpha\dot\beta}, \quad [x^m \stackrel{\star}{,} x^n] = 0. \tag{18}$$

Nonanticommutative space; nonanticommutativity is encoded in terms of the $\star$-product (16).
Derivatives consistent with (18) are just the usual derivatives (2).

Usual integral is the "good" integral.

Deformed SUSY transformation is defined as

$$\delta^\star_\xi F(x,\theta,\bar\theta) = \big(\xi Q + \bar\xi\bar Q\big) F(x,\theta,\bar\theta). \tag{19}$$

As a consequence of (24) the $\star$-product of two superfields is again a superfield

$$\begin{aligned} \delta^\star_\xi (F\star G) &= \big(\xi Q + \bar\xi\bar Q\big)(F\star G), \\ &\neq (\delta_\xi F)\star G + \underbrace{F\star(\delta_\xi G)}_{F\star\big((\xi Q+\bar\xi\bar Q)G\big)} . \end{aligned} \tag{20}$$

Leibniz rule is deformed

$$\begin{aligned} \delta^\star_\xi (F\star G) &= (\delta_\xi F)\star G + F\star(\delta_\xi G) \\ &+ \frac{i}{2} C^{\alpha\beta}\Big(\bar\xi^{\dot\gamma}\sigma^m_{\alpha\dot\gamma}(\partial_m F)\star(\partial_\beta G) + (\partial_\alpha F)\star\bar\xi^{\dot\gamma}\sigma^m_{\beta\dot\gamma}(\partial_m G)\Big) \\ &- \frac{i}{2}\bar C_{\dot\alpha\dot\beta}\Big(\xi^\alpha\sigma^m_{\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\alpha}(\partial_m F)\star(\bar\partial^{\dot\beta}G) + (\bar\partial^{\dot\alpha}F)\star\xi^\alpha\sigma^m_{\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\beta}(\partial_m G)\Big). \end{aligned} \tag{21}$$

Chiral field Φ fulfils $\bar{D}_{\dot\alpha}\Phi = 0$, with $\bar{D}_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\theta^\alpha\sigma^m_{\ \alpha\dot\alpha}\partial_m$

$$\begin{aligned}\Phi(x) \ &= \ A(x) + \sqrt{2}\theta^\alpha\psi_\alpha(x) + \theta\theta\mathsf{F}(x) + i\theta\sigma^l\bar\theta(\partial_l A(x))\\ &\quad - \frac{i}{\sqrt{2}}\theta\theta(\partial_m\psi^\alpha(x))\sigma^m_{\ \alpha\dot\alpha}\bar\theta^{\dot\alpha} + \frac{1}{4}\theta\theta\bar\theta\bar\theta(\Box A(x)). \qquad (22)\end{aligned}$$

The $\star$-product of two, three,... chiral superfields is not chiral

$$\begin{aligned}\Phi\star\Phi \ &= \ A^2 - \frac{C^2}{2}\mathsf{F}^2 + \frac{1}{4}C^{\alpha\beta}\bar C^{\dot\alpha\dot\beta}\sigma^m_{\ \alpha\dot\alpha}\sigma^l_{\ \beta\dot\beta}(\partial_m A)(\partial_l A) + \frac{1}{64}C^2\bar C^2(\Box A)^2\\ &\quad +\theta^\alpha\Big(2\sqrt{2}\psi_\alpha A - \frac{1}{\sqrt{2}}C^{\gamma\beta}\bar C^{\dot\alpha\dot\beta}\varepsilon_{\gamma\alpha}(\partial_m\psi^\rho)\sigma^m_{\ \rho\dot\beta}\sigma^l_{\ \beta\dot\alpha}(\partial_l A)\Big)\\ &\quad -\frac{i}{\sqrt{2}}C^2\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\psi_\alpha)\mathsf{F} + \theta\theta\Big(2A\mathsf{F} - \psi\psi\Big)\\ &\quad +\bar\theta\bar\theta\Big(-\frac{C^2}{4}\big(\mathsf{F}\Box A - \frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\big)\Big)\\ &\quad +\theta\sigma^m\bar\theta\Big(i(\partial_m A^2) + \frac{i}{4}C^{\alpha\beta}\bar C^{\dot\alpha\dot\beta}\sigma_{m\alpha\dot\alpha}\sigma^l_{\ \beta\dot\beta}(\Box A)(\partial_l A)\Big)\\ &\quad +i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}\big(\partial_m(\psi_\alpha A)\big) + \frac{1}{4}\theta\theta\bar\theta\bar\theta(\Box A^2), \qquad (23)\end{aligned}$$

where $\quad C^2 = C^{\alpha\beta}C^{\gamma\delta}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}$ and $\bar C^2 = \bar C_{\dot\alpha\dot\beta}\bar C_{\dot\gamma\dot\delta}\varepsilon^{\dot\alpha\dot\gamma}\varepsilon^{\dot\beta\dot\delta}$.

We project out chiral, antichiral and transverse component of $\Phi\star\Phi$ and $\Phi\star\Phi\star\Phi$ by using projectors P_1, P_2 and P_T

$$P_1=\frac{1}{16}\frac{D^2\bar{D}^2}{\Box},\quad P_2=\frac{1}{16}\frac{\bar{D}^2D^2}{\Box},\quad P_T=-\frac{1}{8}\frac{D\bar{D}^2D}{\Box}, \tag{24}$$

$$f(x)\frac{1}{\Box}g(x)=f(x)\int \mathrm{d}^4y\; G(x-y)g(y).$$

$$\begin{aligned}
P_2\big(\Phi\star\Phi\big) \;=\; & A^2-\frac{C^2}{8}\mathsf{F}^2\\
&+\frac{1}{16}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma^m{}_{\alpha\dot\alpha}\sigma^l{}_{\beta\dot\beta}\Big((\partial_mA)(\partial_lA)+\frac{2}{\Box}\partial_m\big((\Box A)(\partial_lA)\big)\Big)\\
&+\sqrt{2}\theta^\alpha\Big(2\psi_\alpha A-\frac{1}{4}C^{\gamma\beta}\bar{C}^{\dot\alpha\dot\beta}\varepsilon_{\gamma\alpha}(\partial_m\psi^\rho)\sigma^m{}_{\rho\dot\beta}\sigma^l{}_{\beta\dot\alpha}(\partial_lA)\Big)\\
&+\theta\theta\Big(2A\mathsf{F}-\psi\psi\Big)+i\theta\sigma^k\bar\theta\partial_k\Big(A^2-\frac{C^2}{8}\mathsf{F}^2\\
&+\frac{1}{16}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma^m{}_{\alpha\dot\alpha}\sigma^l{}_{\beta\dot\beta}\Big((\partial_mA)(\partial_lA)+\frac{2}{\Box}\partial_m\big((\Box A)(\partial_lA)\big)\Big)\Big)\\
&+i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{k\dot\alpha\alpha}\partial_k\Big(\psi_\alpha A-\frac{1}{8}C^{\gamma\beta}\bar{C}^{\dot\alpha\dot\beta}\varepsilon_{\gamma\alpha}(\partial_m\psi^\rho)\sigma^m{}_{\rho\dot\beta}\sigma^l{}_{\beta\dot\alpha}(\partial_lA)\Big)\\
&+\frac{1}{4}\theta\theta\bar\theta\bar\theta\Box\Big(A^2-\frac{C^2}{8}\mathsf{F}^2+\frac{1}{16}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma^m{}_{\alpha\dot\alpha}\sigma^l{}_{\beta\dot\beta}\\
&\qquad\Big((\partial_mA)(\partial_lA)+\frac{2}{\Box}\partial_m\big((\Box A)(\partial_lA)\big)\Big)\Big)+\mathcal{O}(C^3).
\end{aligned} \tag{25}$$

Wess-Zumino Lagrangian

Undeformed Wess-Zumino Lagrangian

$$\mathcal{L} = \Phi^+ \cdot \Phi\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \Big(\frac{m}{2}\Phi \cdot \Phi\Big|_{\theta\theta} + \frac{\lambda}{3}\Phi \cdot \Phi \cdot \Phi\Big|_{\theta\theta} + \text{ c.c. }\Big), \quad (26)$$

with m and λ real constants.

Deformation

$$\Phi^+ \cdot \Phi \rightarrow \Phi^+ \star \Phi,$$

$$\Phi \cdot \Phi \rightarrow P_2\big(\Phi \star \Phi\big),$$

$$\Phi \cdot \Phi \cdot \Phi \rightarrow \begin{cases} P_2\Big(\Phi \star P_2\big(\Phi \star \Phi\big)\Big), \\ P_2\Big(P_2\big(\Phi \star \Phi\big) \star \Phi\Big), \\ P_2\Big(\Phi \star \Phi \star \Phi\Big). \end{cases}$$

Comments I

- kinetic term

$$\Phi^+ \star \Phi\Big|_{\theta\theta\bar\theta\bar\theta} = \mathsf{F}^*\mathsf{F} + \frac{1}{4}A^*\Box A + \frac{1}{4}A\Box A^* \qquad (27)$$
$$-\frac{1}{2}(\partial_m A^*)(\partial^m A) + \frac{i}{2}(\partial_m\bar\psi)\bar\sigma^m\psi - \frac{i}{2}\bar\psi\bar\sigma^m(\partial_m\psi)$$

remains undeformed.

- mass term

$$P_2\big(\Phi \star \Phi\big)\Big|_{\theta\theta} + \text{c.c.} \; = 2A\mathsf{F} - \psi\psi + 2A^*\mathsf{F}^* - \bar\psi\bar\psi$$

remains undeformed.

- interaction term

$$
\begin{aligned}
P_2\Big(\Phi\star P_2\big(\Phi\star\Phi\big)\Big)\Big|_{\theta\theta} + \text{c.c.} \; &= 3\big(A^2\mathsf{F} - (\psi\psi)A\big)\\
&-\frac{1}{4}K^{ab}K_{ab}\mathsf{F}^3\\
&+\frac{1}{2}K^m{}_aK^{*na}\mathsf{F}\Big((\partial_mA)(\partial_nA) + \frac{2}{\Box}\partial_m\big((\Box A)(\partial_nA)\big)\Big)\\
&-\Big(K^m{}_aK^{*na}\psi(\partial_n\psi) - 2K^m{}_aK^{*n}{}_c(\partial_n\psi)\sigma^{ca}\psi\Big)(\partial_mA) \qquad (28)\\
&+\frac{1}{2}K^*_{ab}(\bar\sigma^{ab}\bar\sigma^{lm})^{\dot\beta}{}_{\dot\beta}(\partial_mA)\partial_l\Big[A^2 - \frac{1}{4}K^{ab}K_{ab}\mathsf{F}^2\\
&+\frac{1}{2}K^m{}_aK^{*na}\mathsf{F}\Big((\partial_mA)(\partial_nA) + \frac{2}{\Box}\partial_m\big((\Box A)(\partial_nA)\big)\Big)\Big]\\
&+\text{ c.c. } + \mathcal{O}(K^4),
\end{aligned}
$$

where we introduced the following notation

$$
C_{\alpha\beta} = K_{ab}(\sigma^{ab}\varepsilon)_{\alpha\beta}, \quad \bar C_{\dot\alpha\dot\beta} = K^*_{ab}(\varepsilon\bar\sigma^{ab})_{\dot\alpha\dot\beta}. \qquad (29)
$$

Deformed Wess-Zumino Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \Phi^+ \star \Phi\Big|_{\theta\theta\bar{\theta}\bar{\theta}} \\
&\quad + \Big(\frac{m}{2} P_2\big(\Phi \star \Phi\big)\Big|_{\theta\theta} + \frac{\lambda}{3} P_2\Big(\Phi \star P_2\big(\Phi \star \Phi\big)\Big)\Big|_{\theta\theta} + \text{ c.c. } \Big) \\
&= A^* \Box A + i(\partial_m \bar{\psi})\bar{\sigma}^m \psi + \mathsf{F}^*\mathsf{F} \\
&\quad + \Big[\frac{m}{2}\Big(2A\mathsf{F} - \psi\psi\Big) + \lambda\Big(\mathsf{F}A^2 - A\psi\psi\Big) \\
&\quad - \frac{\lambda}{3}\Big(K^m{}_a K^{*na}\psi(\partial_n\psi) - 2K^m{}_a K^{*n}{}_b(\partial_n\psi)\sigma^{ba}\psi\Big)(\partial_m A) \\
&\quad + \frac{\lambda}{6} K^m{}_a K^{*na}\mathsf{F}\Big((\partial_m A)(\partial_n A) + \frac{2}{\Box}\partial_m\big((\partial_n A)\Box A\big)\Big) \qquad (30) \\
&\quad - \frac{\lambda}{12} K^{mn} K_{mn} \mathsf{F}^3 + \text{ c.c. } \Big] + \mathcal{O}(K^4).
\end{aligned}
$$

Equations of motion for fields F and F^*

$$\mathsf{F}^* + mA + \lambda A^2 - \frac{\lambda}{4}K^{ab}K_{ab}\mathsf{F}^2 + \frac{\lambda}{6}K^{m}{}_{a}K^{*na}(\partial_m A)(\partial_n A)$$
$$+\frac{\lambda}{3}K^{m}{}_{a}K^{*na}\frac{1}{\Box}\partial_m\big((\partial_n A)\Box A\big) + \mathcal{O}(K^4) = 0, \quad (31)$$
$$\mathsf{F} + mA^* + \lambda(A^*)^2 - \frac{\lambda}{4}K^{*ab}K^*_{ab}(\mathsf{F}^*)^2$$
$$+\frac{\lambda}{6}K^{m}{}_{a}K^{*na}(\partial_m A)^*(\partial_n A)^*$$
$$+\frac{\lambda}{3}K^{m}{}_{a}K^{*na}\frac{1}{\Box}\partial_m\big((\partial_n A)^*\Box A^*\big) + \mathcal{O}(K^4) = 0 \quad (32)$$

are solved perturbatively and solutions inserted in the Lagrangian (40). That gives

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{O}(K^4)\ , \quad (33)$$

with

$$\mathcal{L}_0 = A^*\Box A + i(\partial_m\bar{\psi})\bar{\sigma}^m\psi - \lambda A^*\bar{\psi}\bar{\psi} - \lambda A\psi\psi - \frac{m}{2}\big(\psi\psi + \bar{\psi}\bar{\psi}\big)$$
$$-m^2A^*A - m\lambda A(A^*)^2 - m\lambda A^*A^2 - \lambda^2A^2(A^*)^2, \quad (34)$$
$$\mathcal{L}_2 = \frac{\lambda}{3}K^m{}_aK^{*na}\Big(m(\partial_mA) + 2\lambda A(\partial_mA)\Big)\frac{1}{\Box}\big((\partial_nA^*)\Box A^*\big)$$
$$+\frac{\lambda}{3}K^m{}_aK^{*na}\Big(m(\partial_mA^*) + 2\lambda A^*(\partial_mA^*)\Big)\frac{1}{\Box}\big((\partial_nA)\Box A\big)$$
$$+\frac{\lambda}{12}K^{ab}K_{ab}\Big(mA^* + \lambda(A^*)^2\Big)^3 + \frac{\lambda}{12}K^{*ab}K^*_{ab}\Big(mA + \lambda A^2\Big)^3$$
$$-\frac{\lambda}{6}K^m{}_aK^{*na}\Big((mA + \lambda A^2)(\partial_mA^*)(\partial_nA^*)$$
$$+(mA^* + \lambda(A^*)^2)(\partial_mA)(\partial_nA)\Big)$$
$$-\frac{\lambda}{3}\Big(K^m{}_aK^{*na}\psi(\partial_n\psi) - 2K^m{}_aK^{*n}{}_b(\partial_n\psi)\sigma^{ba}\psi\Big)(\partial_mA) \quad (35)$$
$$-\frac{\lambda}{3}\Big(K^m{}_aK^{*na}\bar{\psi}(\partial_n\bar{\psi}) - 2K^m{}_aK^{*n}{}_b\bar{\psi}\bar{\sigma}^{ab}(\partial_n\bar{\psi})\Big)(\partial_mA^*).$$

Comments II

- we constructed a deformation of the Wess-Zumino Lagrangian with the good classicl limit
- SUSY invariant action, but nonlocal interaction terms
- work in progress:
  -renormalisability of the model
  -different choice of the twist (13)
- future work
  -gauge theories
  -. . .

Comments III

Different types of deformation of the superspace present in the literature

- $[x^m \stackrel{\star}{,} x^n] = i\theta^{mn}, \quad [x^m \stackrel{\star}{,} \theta] = 0$,
  $\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \{\theta^\alpha \stackrel{\star}{,} \bar\theta_{\dot\alpha}\} = 0, \dots$
  Chu et al. (hep-th/9912153), Ferrara et al. (hep-th/0002084)
  -follows from the twist $\mathcal{F} = e^{-\frac{i}{2}\theta^{mn}\partial_m\otimes\partial_n}$.
- $[x^0 \stackrel{\star}{,} x^j] = iax^j, \quad [x^j \stackrel{\star}{,} x^l] = 0$,
  $[x^0 \stackrel{\star}{,} \theta^\alpha] = \frac{ia}{2}\theta^\alpha, \quad [x^j \stackrel{\star}{,} \theta^\alpha] = \{\theta^\alpha, \theta^\beta\} = 0, \dots$
  Kosinski et al. (hep-th/9405076, hep-th/0011053).
- $[x^m \stackrel{\star}{,} x^n] = [y^m \stackrel{\star}{,} y^n] = 0, \quad [x^m \stackrel{\star}{,} \theta^\alpha] = iC^{m\alpha}$,
  $\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \{\theta^\alpha \stackrel{\star}{,} \bar\theta_{\dot\alpha}\} = 0, \dots$
  Kobayashi et al. (hep-th/0505011), Banerjee et al. (hep-th/0511205).

- non(anti)commutative space

  $\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = \{\theta^\alpha, \bar\theta_{\dot\alpha}\} = 0,$

  $[y^m \stackrel{\star}{,} y^n] = 0, \quad [x^m \stackrel{\star}{,} x^n] \neq 0, \ldots$

  Seiberg (hep-th/0305248), Zupnik (hep-th/0506043), Ihl et al. (hep-th/0506057), Ferrara et al. (hep-th/0307039),...;

  -follows from the twist $\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}Q_\alpha \otimes Q_\beta}$

  -chirality preserved, $\star$-product non-hermitian, deformed Wess-Zumino action invariant under $N = 1/2$ SUSY.

  -also follows from the twist $\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha \otimes D_\beta}$; chirality is not preserved, $\star$-product is not hermitian, deformed Wess-Zumino action is invariant under $N = 1$ SUSY.

- ...