Consistent Dimensional Reduction over Coset Spaces

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Outlook

- Introduction
- Reductions and Consistency
- Consistency check for CSDR
- Conclusions

Introduction

Higher dimensions provide a framework for unification of interactions.
Coset Space Dimensional Reduction (CSDR) provides:

- Gauge-Higgs-Yukawa unification
- Interesting GUT models/Chiral fermions in 4-dims
- N=1 Softly broken SUSY Langrangians

Important issue for the effective theory: Consistency of the reduction$\longrightarrow$ CSDR consistency has to be examined.

Reduction on Group Manifold

Reduction of gauge field A on group manifold S (Scherk-Schwarz '79).
Ansatz:

$$A = A_\mu dx^\mu + A_I(x) e^I(y),$$

$I = 1, \ldots, \mathrm{dim} S$,
$e^I \longrightarrow$ left-invariant 1-forms.

$\hookrightarrow$ Keep only S_L singlets under the full isometry group $S_L \times S_R$.

Invariance Condition:

$$\mathcal{L}_{X^I} A = 0,$$

$X^I \longrightarrow$ Killing vectors dual to right-invariant 1-forms.

Reduction on Coset Space

Reduction of gauge field A on coset space S/R (Forgacs-Manton '79).
Generalized Invariance Condition:

$$\mathcal{L}_{X^I} A = D W_I,$$

$W_I \longrightarrow$ gauge transformation parameter.

$\hookrightarrow$ Constraints on the gauge field.

Analysis of constraints provides:

- 4-dim fields
- gauge group of effective 4-dim theory

Consistency

Definition. *(Pons-Talavera '04) A truncation is said to be consistent if its implementation at the level of the variational principle agrees with its implementation at the level of the equations of motion.*

The following diagram has to be commutative:

$$
\begin{array}{ccc}
\mathcal{L} & \xrightarrow{\text{Red.}} & \mathcal{L}_R \\
\Big\downarrow \text{e.o.m.} & & \Big\downarrow \text{e.o.m.} \\
\frac{\delta \mathcal{L}}{\delta \Phi} = 0 & \xrightarrow{\text{Red.}} & \left(\frac{\delta \mathcal{L}}{\delta \Phi}\right)_R = 0 \Leftrightarrow \frac{\delta \mathcal{L}_R}{\delta \Phi} = 0
\end{array}
$$

In general this diagram is not commutative.

- For reductions on Group Manifolds consistency is guaranteed (Scherk-Schwarz '79, Cvetic et.al. '03)

- For coset spaces consistency has to be examined explicitly

Check the consistency of CSDR (A.C., P.Manousselis, N.Prezas, G.Zoupanos, PLB 656 (2007))

Set up

D-dim Einstein-Yang-Mills Langrangian:

$$\mathcal{L} = \widehat{R} *_D \mathbf{1} - \frac{1}{2}\mathrm{Tr}\widehat{F}_{(2)} \wedge *_D \widehat{F}_{(2)} - \lambda_{(D)} *_D \mathbf{1},$$

with Lie(G)-valued field strength:

$$\widehat{F}_{(2)} = d\widehat{A}_{(1)} + \widehat{A}_{(1)} \wedge \widehat{A}_{(1)},$$

and $\lambda_{(D)}$ $\longrightarrow$ D-dim cosmological constant.
Background metric on $M^4 \times S/R$:

$$g_{(D)} = \eta_{mn} e^m e^n + \delta_{ab} e^a e^b,$$

η_{mn} $\longrightarrow$ Minkowski metric
δ_{ab} $\longrightarrow$ S-invariant metric on the internal space.

Reduction of the Action

Action (YM part):

$$S = -\frac{1}{2}\int Tr\widehat{F} \wedge *_D \widehat{F},$$

Ansatz:

$$\widehat{A}^{\tilde{I}}(x,y) = A^{\tilde{I}}(x) + \chi^{\tilde{I}}_{\alpha}(x,y)dy^{\alpha},$$

where

$$\chi^{\tilde{I}}_{\alpha}(x,y) = \phi^{\tilde{I}}_{A}(x)e^{A}_{\alpha}(y)$$

and $\tilde{I}$ G-index, A S-index, α (S/R)-index.

$\hookrightarrow$

$$\widehat{F}^{\tilde{I}} = F^{\tilde{I}} + D\phi^{\tilde{I}}_{A} \wedge e^{A} - \frac{1}{2}F^{\tilde{I}}_{AB}e^{A} \wedge e^{B},$$

where

$$\begin{aligned} F^{\tilde{I}} &= dA^{\tilde{I}} + \frac{1}{2} f^{\tilde{I}}_{\tilde{J}\tilde{K}} A^{\tilde{J}} \wedge A^{\tilde{K}}, \\ D\phi^{\tilde{I}}_A &= d\phi^{\tilde{I}}_A + f^{\tilde{I}}_{\tilde{J}\tilde{K}} A^{\tilde{J}} \phi^{\tilde{K}}_A, \\ F^{\tilde{I}}_{AB} &= f^C_{AB} \phi^{\tilde{I}}_C - [\phi_A, \phi_B]^{\tilde{I}}. \end{aligned}$$

Dualize:

$$*_D \widehat{F}^{\tilde{I}} = *_4 F^{\tilde{I}} \wedge vol_d + *_4 D\phi^{\tilde{I}}_A \wedge *_d e^A - \frac{1}{2} F_{AB} vol_4 \wedge *_d (e^A \wedge e^B).$$

$\hookrightarrow$

$$\mathcal{L} = -\frac{1}{2} Tr F \wedge *_4 F + \frac{1}{2} Tr D\phi_a \wedge *_4 D\phi_a - \frac{1}{4} F_{ab} F^{ab} vol_4,$$

by imposing constraints:

$$D\phi^{\tilde{I}}_i = F^{\tilde{I}}_{ai} = F^{\tilde{I}}_{ij} = 0.$$

where i R-index.

or

$$\begin{aligned} F_{ib} &= f_{ib}{}^{c}\phi_c - [\phi_i, \phi_b] = 0, \\ F_{ij} &= f_{ij}{}^{k}\phi_k - [\phi_i, \phi_j] = 0, \end{aligned}$$

$$[A_\mu, \phi_i] = 0.$$

$\looparrowright$ Exactly the CSDR constraints.

- Residual gauge group: $H = C_G(R)$

- 4-dim scalars ϕ_a:

$$S \supset R, \quad adjS = adjR + \sum s_i,$$

$$G \supset R, \quad adjG = (adjR, 1) + (1, adjH) + \sum (r_i, h_i).$$

$\rightarrow \forall r_i \equiv s_i \rightsquigarrow$ multiplet $h_i \in H$

Reduction of e.o.m.

Higher-dim Yang-Mills:

$$\hat{D} *_D \hat{F}^{\tilde{I}} = \hat{d} *_D \hat{F}^{\tilde{I}} + f^{\tilde{I}}_{\tilde{J}\tilde{K}} \hat{A}^{\tilde{J}} \wedge *_D \hat{F}^{\tilde{K}} = 0.$$

Substituting the ansatz we obtain:

$$D *_4 F^{\tilde{I}} \wedge vol_d - f^{\tilde{I}}_{\tilde{J}\tilde{K}} \phi^{\tilde{J}}_A *_4 D\phi^{\tilde{K}}_B \wedge e^A \wedge *_d e^B - *_4 D\phi^{\tilde{I}}_A \wedge d *_d e^A = 0$$

and

$$D *_4 D\phi^{\tilde{I}}_A \wedge *_d e^A - \frac{1}{2} f^{\tilde{I}}{}_{\tilde{J}\tilde{K}} \phi^{\tilde{J}}_A F^{\tilde{K}}_{BC} vol_4 \wedge e^A \wedge *_d (e^B \wedge e^C) - \frac{1}{2} F^{\tilde{I}}_{BC} vol_4 \wedge d *_d (e^B \wedge e^C)$$

By imposing the same constraints as before, namely

$$D\phi^{\tilde{I}}_i = F^{\tilde{I}}_{ai} = F^{\tilde{I}}_{ij} = 0.$$

$\hookrightarrow$

$$
\begin{aligned}
D *_4 F^{\tilde{I}} &= f^{\tilde{I}}_{\tilde{J}\tilde{K}} \phi^{\tilde{J}}_a *_4 D\phi^{\tilde{K}}_a , \\
D *_4 D\phi^{\tilde{I}}_a &= -(f^{\tilde{I}}_{\tilde{J}\tilde{K}} \phi^{\tilde{J}}_c F^{\tilde{K}}_{ca} + \frac{1}{2} F^{\tilde{I}}_{bc} f_{bca}) vol_4 .
\end{aligned}
$$

These are the equations of motion for a Yang-Mills theory coupled to charged scalars with non-trivial potential, namely the e.o.m. coming from the variation of the reduced Langrangian.

$\rightsquigarrow$ The truncation is consistent.

Conclusions and current work

There is no standard recipe for consistent coset space reductions.

Utilizing a particular ansatz and a set of constraints (CSDR constraints) we proved the consistency of coset reduction.

- Including Fermions is straightforward

- Include Kaluza-Klein fluctuations

$$d\hat{s}^2_{(D)} = ds^2_{(4)} + h_{\alpha\beta}(x,y)(dy^\alpha - \mathcal{A}^\alpha(x,y))(dy^\beta - \mathcal{A}^\beta(x,y))$$

  For general S-invariant metric the reduction can remain consistent

$\rightsquigarrow$Modified constraints, 4-dim potential modified from new scalar fields and new rules for the 4-dim gauge theory

- Application to reduction of Heterotic SuGra on suitable coset spaces