Spacetime and Noncommutative Geometry

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I am very honored and pleased that I have been invited to give this public lecture.

Let me first thank Helmuth Hüffel and the organisers of this meeting for this kind invitation.

Giving this talk has another significance for me.

My first trip abroad from India was to Vienna.

I arrived here on 1 May 1962 as Professor Walter Thirring's post-doc.

My stay in Vienna for six months, interrupted by participation in the 1962 Trieste meeting, has been very important in my scientific formation.

This public lecture is a final talk on what happened here last week.

A sweet, after a wonderful dinner.

It is also a taste of the future, what is to come, its intimations, with a personal focus.

A VIEW FROM VIENNA

It is well known that *sacher-torte* does not exist outside Vienna, just as there is no pizza outside Napoli.

My talk cannot compare with sacher-torte, but I hope that at least it will not disappoint you.

ON SPACETIME IN THE SMALL

There are good reasons to imagine that spacetime at the smallest scales will display some sort of discreteness.

They come from *quantum physics* and *the nature of black holes*.

Let me outline them.

Let us look first at some length scales.

The size of an atom is 10^{-8} centimetres. It is roughly the wavelength of visible light.

An atom is composed of a nucleus of size 10^{-13} centimetres with electrons orbiting it at 10^{-8} centimetres. The nucleus is composed of protons and neutrons of size 10^{-13} centimetres. The size of an electron is just 10^{-15} centimeters.

Thus atom is mostly empty space.

The next small length scale occurs when we deal with the new elementary particles of weak interactions like radioactive decay.

There the typical length scale is smaller by a factor of one hundred, being 10^{-15} centimetres, the same as the size of an electron.

The length scales in gravity are far smaller than all these scales, being 10^{-32} centimetres. This is 10^{19} times smaller than the size of an atom.

That is because gravity is so much weaker.

Newton's constant is very small. We feel it so vividly only because it is universal between bodies and is enhanced by large masses.

At such small length scales, there is no reason to suppose that the nature of spacetime would be anything like what we are familiar with.

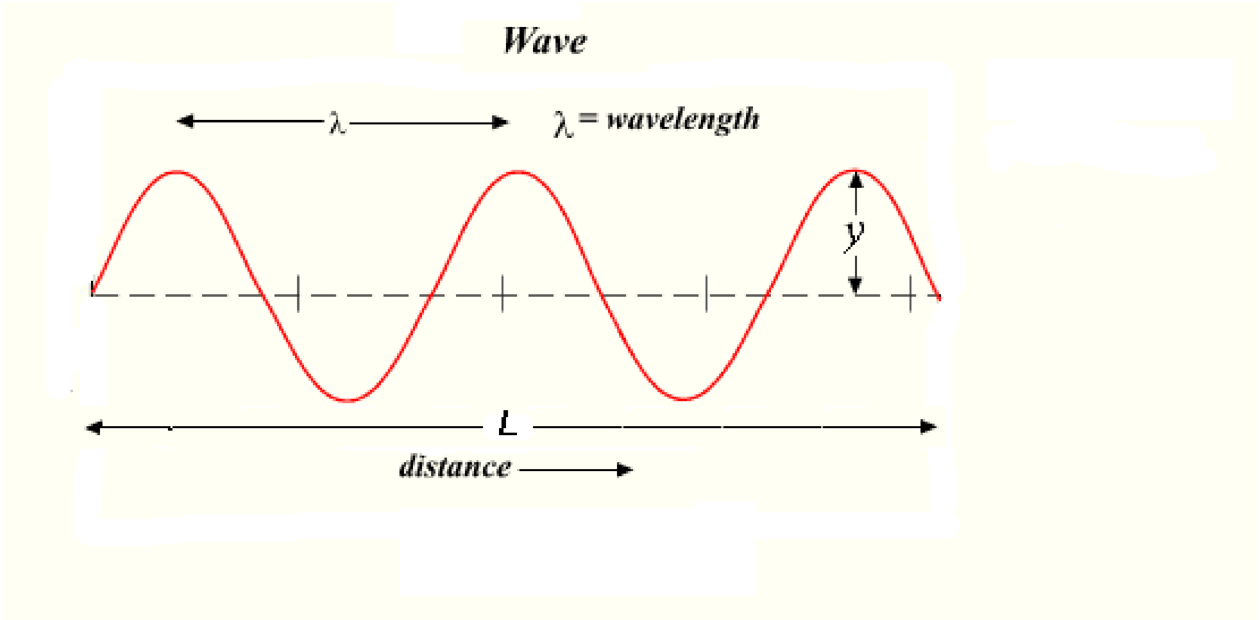
In fact, there are good reasons to suppose that it would be radically different.

In particular we expect fundamental limitations on probing such small length scales. This comes about from considerations involving gravity and quantum theory.

The following arguments were described by Doplicher, Fredenhagen and Roberts.

WHAT QUANTUM PHYSICS TELLS US

We know that to see the details of an object of size L, we need to use light of wavelength λ less than L.



Now quantum physics teaches us that there is “wave-particle duality”.

Light can sometimes behave like a particle as in photo-electric effect.

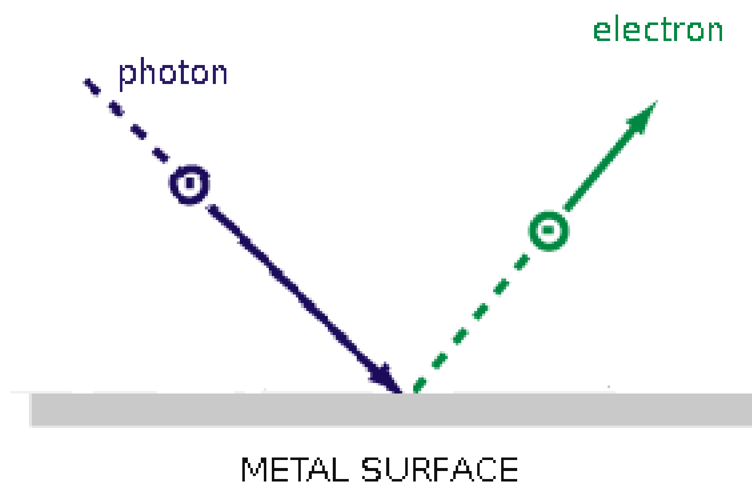


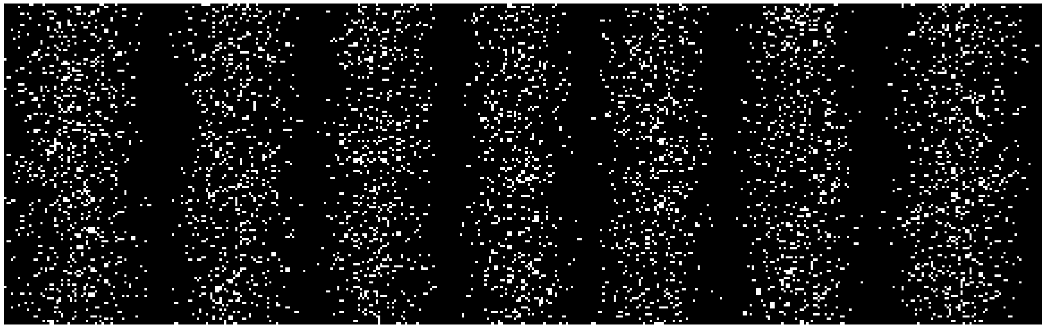
PHOTO-ELECTRIC EFFECT

Electron of mass m can behave like a wave of wavelength

$$\lambda = \frac{\hbar}{mc} = \text{Compton wavelength}$$

$\hbar = h/2\pi$, h = Planck's constant $=6.626068 \times 10^{-34} m^2 kg/s$, c = speed of light

Interference is a property of waves. Particles can also show interference. The picture below shows the interference pattern of electrons.



PROBING PLANCK LENGTH-SCALES

In order to probe physics at the Planck scale $L \simeq 10^{-32} cms$, the Compton wave-length λ_C of the probe of mass M must therefore fulfill

$$\lambda_C = \frac{\hbar}{Mc} \leq L \;\; or \;\; M \geq \frac{\hbar}{Lc} \simeq \text{Planck mass} \simeq 10^{19}\text{GeV} \simeq 10^{-5}\text{grams}.$$

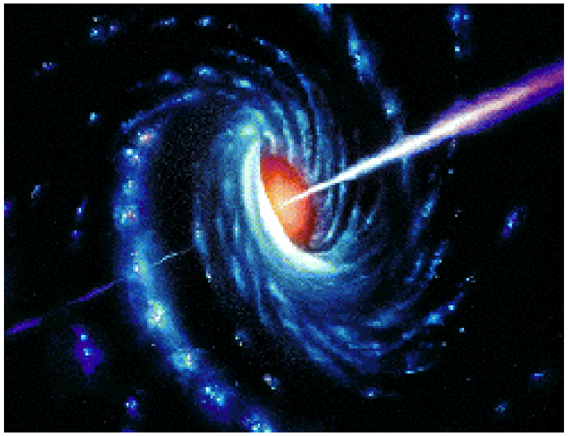
Such huge mass in the small volume $L^3 \simeq (10^{-32} cm)^3$ will strongly affect gravity and can cause black holes and their horizons to form: gravity can become so large that no signal can escape the volume.

ON BLACK HOLES

These are objects with such strong gravity on their surfaces that no signal can escape from them.

It is now beleived that supermassive black holes exist in the centres of all massive galaxies.

Picture below shows an artist’s concept of an active galactic nucleus [Courtesy of NASA].



If an object is thrown up on earth, gravity pulls it back.

We must throw it with sufficient speed v for it to escape earth.

As the gravity of an object gets stronger, v gets larger and eventually it becomes the speed of light c.

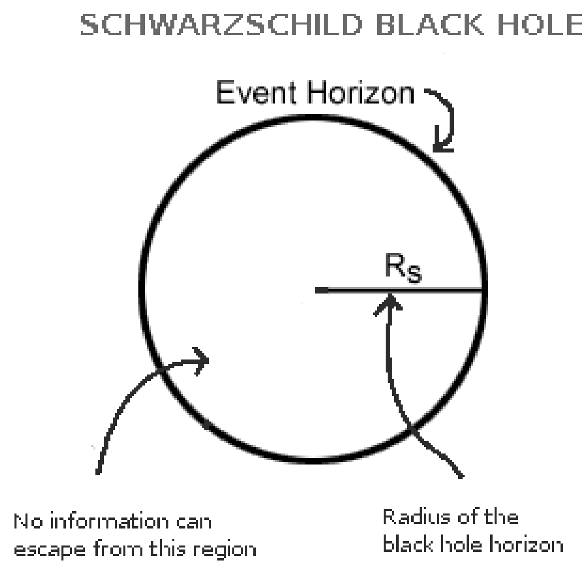
But no object can travel faster than light.

At this strong limit of gravity this object becomes a black hole.

It traps even light in its interior, within its “event horizon”.

All black holes have event horizons.

Behind this horizon, gravity is so strong that no information about the black hole's interior can escape to the region outside.



Black holes were first predicted by the English geologist John Michell in 1783 and by Karl Schwarzschild in 1916.

LESSON FROM QUANTUM MECHANICS

Quantum theory too puts fundamental limitations on measurements of position x and momentum p:

$$\Delta x \Delta p \geq \hbar/2$$

Δx, Δp = uncertaintities in position and momentum measurements.

There is a precise mathematical manner to achieve this [Schrödinger (from Vienna), Heisenberg, Dirac, ...]:

Make x and p “noncommuting”:

$$xp - px = i\hbar.$$

This suggests we can incorporate limitations on spatial resolution by making position noncommuting as in

$$xy - yx = i\theta$$

where θ is a new Planck-like fundamental constant with dimension $(\text{length})^2$.

When spacetime is noncommuting, we say that it obeys “noncommutative” geometry.

Now

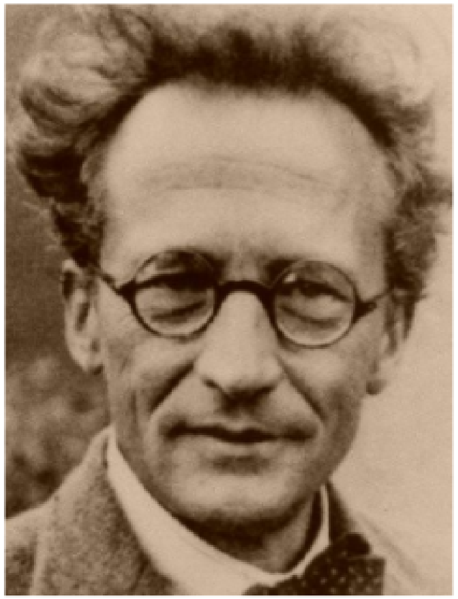
$$\Delta x \Delta y \geq \theta/2$$

where Δx, Δy = intrinsic uncertaintities in x and y coordinate measurements.

There are similar equations in any pair of coordinates.

A BIT OF HISTORY

The idea that spacetime geometry may be noncommutative is old. It goes back to Schrödinger and Heisenberg.



Erwin Schrodinger

Werner Heisenberg

Heisenberg raises this possibilty in a letter to Rudolph Peierls in the 30's.

Heisenberg also complains that he does not know enough mathematics to explore the physical consequences of this possibilty.

Peierls mentions Heisenberg's ideas to Wolfgang Pauli.

He in turn explains it to Hartland Snyder.

And it is Snyder who publishes the first paper on the subject in Physical Review in 1941/42.

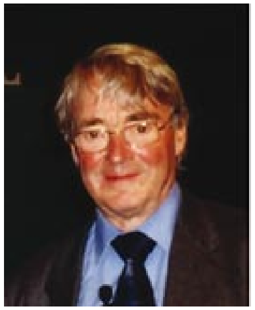
I should also mention the role of Joe Weinberg in these developments. Joe was a student of Oppenheimer and was a close associate of Pauli and a classmate of Schwinger. He was the person accused of passing nuclear secrets to the Soviets and who lost his job in 1952 at Minnesota for that reason. His wife supported the family for several years. Eventually he got a faculty position at Case in 1958 and from there, he came to Syracuse.

Joe was remarkable. He seemed to know everything, from Sanskrit to noncommutative geometry, and published very little. He had done extensive research on this new vision of spacetime. He had showed me his manuscripts. They are now in Syracuse University archives.

QUANTUM FIELDS ON THE MOYAL PLANE

Pioneered by

- Doplicher, Fredenhagen, Roberts
- Julius Wess and his group

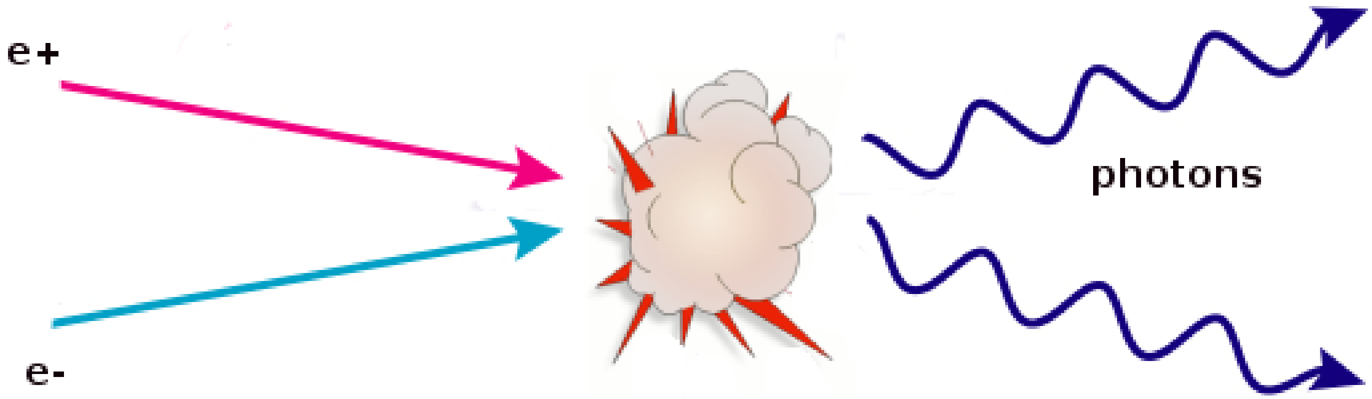


Wess was a student of Hans Thirring, father of Walter Thirring.

All of us in this field miss him for his kindness and wisdom.

ON STANDARD QUANTUM FIELD THEORIES

– They are devised to account for processes involving particle creation and annihilation, and particle transmutation:



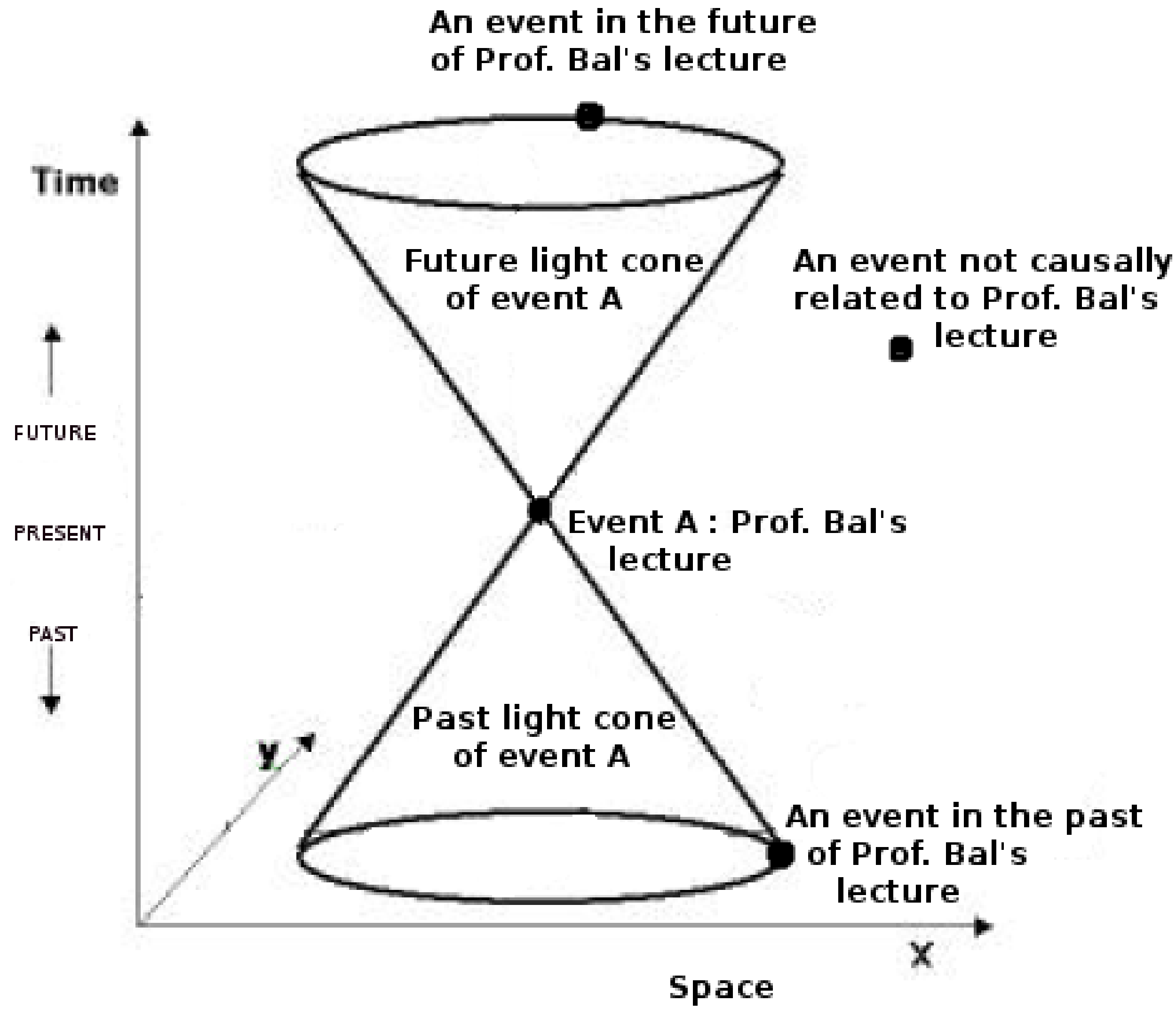
Pair annihilation of electron (e-) and positron (e+)

– They treat particles as point particles, with no extension.

– They obey “causality”.

Thus relativity teaches us that no signal can travel faster than light.

Signals can travel only within the “light cone”.


An event in the future
of Prof. Bal's lecture
Time
Future light cone
of event A
An event not causally
related to Prof. Bal's
lecture
FUTURE
PRESENT
Event A : Prof. Bal's
lecture
PAST
Past light cone
of event A
y
An event in the past
of Prof. Bal's
lecture
x
Space

These features lead to profound consequenses:

PAULI PRINCIPLE

As I mentioned above, atoms and molecules are mostly empty. If that is the case, why is it that we cannot pass a finger right through another ? Or a professor cannot walk through a door ?

It is true that yogis in India routinely walk through mountains (and disappear from spot x only to reappear instantly at spot y) after aeons of meditation, but the physicists' question is : why can't we all do this?

The reason is a deep physical principle called Pauli principle which says that no two electrons or two protons can occuppy the same spot.

So when one pushes one electron or proton near the other, there is a strong repulsion, which is impossible to overcome.

It is this principle which gives stabilty to matter,and much of of their observed behaviour.

CPT THEOREM

This relates processes involving particles and anti-particles

Particle	Anti-particle
Electron	Positron
Neutron	Anti-neutron
K^0-meson	$\bar{K}^0$-meson

CPT theorem says that

Life time of a particle before it decays = Life time of its anti-particle before it decays

Magnetic moment of a particle = magnetic moment of its antiparticle

ON NONCOMMUTATIVE QUANTUM FIELD THEORIES

– Particles have “extensions”

– Causality is violated. Signals can travel faster than light.

That is because light cone itself is not sharp: we cannot localize spacetime points.

So “inside” and “outside” light cone loses precise meaning.

So Pauli principle and CPT theorem are violated.

Consequences

Normally no two electrons (protons) can occupy the same atomic (nuclear) level.

So consider an atom (nucleus) already occupied by an electron (proton).

Look for a “forbidden” transition where an electron say makes a transition to an occupied level.

It is allowed in noncommutative geometry.

From existing data from neutrino experiments at Gran Sasso (Italy), super-kamiokande (Japan), and also from Maryland experiments for forbidden electron transitions,

$$\text{(Noncommutativity parameter)}^{\frac{1}{2}} \leq 10^{-24} cms,$$

$$\text{Energy scale} \geq 10^{11} \text{ GeV.}$$

CPT

Life times and magnetic moments of particles and anti-particles can differ.

Electron magnetic moment μ_e the best measured number ever in science.

μ_e (in units of $\frac{e\hbar}{2m_e c}$) = 2.002319304 . It can differ from that of positron. Estimates of θ from such experiments are in progress.

COSMIC MICROWAVE BACKGROUND (CMB)

About 400, 000 years after the big bang, photons (radiation) began to fill the universe.

In 1948 George Gamow and Ralph Alpher predicted the existence of these photons.

It was Arno Penzias and Robert Wilson in 1964 who detected and measured the temperature of this radiation of cosmic origin. Their answer was about 3 K.

They were given Nobel prize for this work.

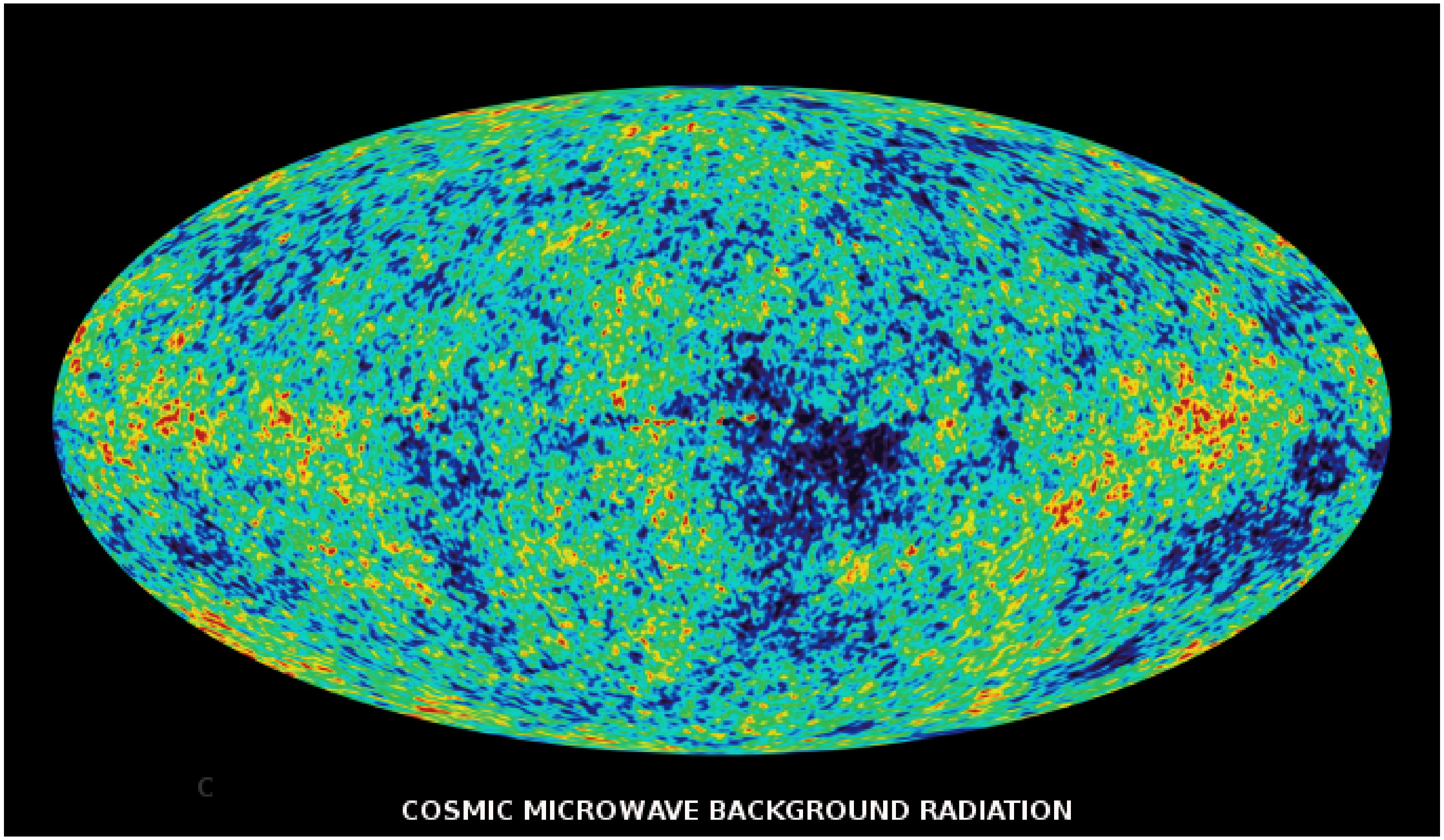
This temperature corresponds to a radiation in the microwave frequency - so it is called the cosmic microwave background (CMB) radiation.

In 1992, the COBE satellite detected temperature fluctuations (anisotropies) of the order of 10^{-5} in the CMB radiation.

It led to the conclusion that the early universe was not smooth: there were small perturbations in the radiation-matter fluid.

CMB radiation had its origin when the universe was much smaller in size than today.

So it can carry the signature of physics of the small scale - the noncommutative spacetime.



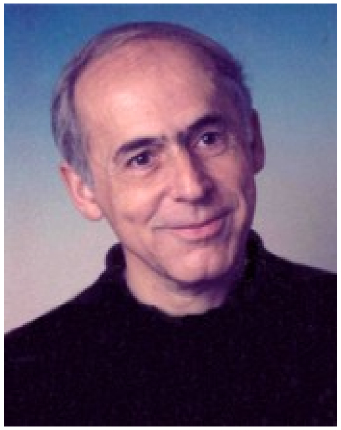
Estimates of the noncommutativity parameter from the CMB data are in progress.

FORMAL DEVELOPMENTS

Standard quantum field theories have many divengence problems because they treat particles as point-like.

In dramatic developments

Grosse and Wulkenhaar, Grosse and Wohlgenannt



Harald Grosse

and the Paris group:

Gurau, Magnen, Rivasseau, Vignes-Tournaret, ...

have shown that

Many problems of standard quantum field theories disappear in noncommutative field theories.

This work can have enduring impact on fundamental physics.

FINAL REMARKS

With noncommutativity, there are violations of Pauli principle, Lorentz and CPT invariance, and effects on the CMB radiation.

Perhaps one can isolate characteristic signals of such violations and confront them with experiments.

One can put limits on noncommutativity from the observed CMB data and data on Pauli-forbidden transitions.

Important avenues of research have also been opened up in foundations of quantum field theory.

There are many interesting possibilities!