Emergent Gravity from Noncommutative Gauge Theory

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Introduction

- Classical space-time meaningless at Planck scale

  due to gravity $\leftrightarrow$ Quantum Mechanics

  $\Rightarrow$ "quantized" (noncommutative?) spaces

- What about gravity on/for quantized spaces ??

  should be simple & naturally related to NC

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- ∃ simple models for dynamical NC space:

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Matrix Models and dynamical space(time)

Consider Matrix Model:

$$S_{YM} = -Tr[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'}, \qquad a = 0, 1, 2, 3$$

(toy candidate for fundamental theory)

dynamical objects: $X^a \in L(\mathcal{H})$ ... hermitian matrices

equation of motion: $[X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

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- $[X^a, X^b] = 0$ ...classical objects; ignore here
- $[X^a, X^b] = \bar{\theta}^{ab}$, "Moyal-Weyl quantum plane"
  where $\bar{\theta}^{ab}$ ... antisymmetric tensor, nondegenerate
- many more, of type $[X^a, X^b] = \theta^{ab}(x)$

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fluctuating quantum spaces and gauge fields

consider fluctuations

$$X^a = \overline{Y}^a + A^a$$ ("covariant coordinates")

around solution

$$[\overline{Y}^a, \overline{Y}^b] = i\overline{\theta}^{ab}$$ "Moyal-Weyl plane"

note

$$[\overline{Y}^a, f(\overline{Y})] \sim i\theta^{ab}\partial_b f(\overline{y})$$

obtain

$$\begin{aligned}[X^a, X^b] - i\overline{\theta}^{ab} &= \overline{\theta}^{aa'}\overline{\theta}^{bb'}(\partial_{a'}A_{b'} - \partial_{b'}A_{a'} + [A_{a'}, A_{b'}]) \\ &= \overline{\theta}^{aa'}\overline{\theta}^{bb'} F_{a'b'}\end{aligned}$$

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$U(1)$ Yang-Mills on quantum plane

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nonabelian $U(n)$ case: similar, $Y^a \to \overline{Y}^a \otimes 1_n$

however:

- $U(1)$ sector cannot be disentangled
- space itself obtained as "vacuum", is dynamical;

  fluctuations of covariant coords $X^a \leftrightarrow$ gravity ?!

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Geometry from $u(1)$ sector:

consider general quantum space determined by full $\mathfrak{u}(1)$ sector:

$$\begin{aligned} X^a &= \overline{Y}^a + A^a \\ [X^a, X^b] &= i\theta^{ab}(x) \qquad (= i\overline{\theta}^{ab} + iF^{ab}(x)\,) \end{aligned}$$

... general quantized Poisson manifold $(\mathcal{M}, \theta^{ab}(x))$

$$[X^a, \Phi(x)] \sim i\theta^{ab}(x)\partial_b\Phi(x)$$

couple to scalar matter Φ

$$\begin{aligned} S[\Phi] &= Tr\,\eta_{aa'}[X^a, \Phi][X^{a'}, \Phi] \\ &\sim \int d^4x\; G^{ab}(x)\,(\partial_a + [A_a, .])\Phi(\partial_b + [A_b, .])\Phi \end{aligned}$$

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- $\theta^{ac}(x)$ ... vielbein ("gauge-fixed"!)

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generalization to $su(n)$ gauge fields

separate $\mathfrak{u}(1)$ and $\mathfrak{su}(n)$ components

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\begin{aligned}
X^a &= (\overline{Y}^a + \overline{\theta}^{ab} A_b^0) \otimes \mathbf{1}_n + (\overline{\theta}^{ab} A_b^\alpha \otimes \lambda_\alpha) \\
&=: Y^a \otimes \mathbf{1}_n + \theta^{ab} A_b^\alpha \otimes \lambda_\alpha
\end{aligned}
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will see:

$\mathfrak{u}(1)$ component Y^a ... dynamical geometry, gravity

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effective action in semi-classical limit:

express NC action in terms of ordinary gauge fields on $(\mathcal{M}, G_{ab}(x))$:
(Seiberg-Witten map)

$$S_{YM} = \int d^4y\, \rho(y) tr\left(4\eta(y) - G^{cc'} G^{dd'} F_{cd} F_{c'd'}\right) + 2\int \eta(y)\, tr\, F \wedge F$$

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linearized NC gravity:

effective metric for Moyal-Weyl $\overline{\theta}^{ab} = \text{const:}$

$$\overline{\eta}^{ab} := \overline{\theta}^{ac}\,\overline{\theta}^{bd}\,\eta_{cd}$$ ... flat Minkowski

metric fluctuations over flat (Moyal-Weyl) space:

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vacuum e.o.m.:

$$G^{ac}\partial_c\,\theta^{-1}_{ab}(y) = 0 \qquad (\Leftrightarrow D^a F_{ab} = 0)$$

implies vacuum equations of motion (linearized)

$$R_{ab} = 0 \; + O(\theta^2)$$

while $R_{abcd} = O(\bar\theta) \neq 0$ ... nonvanishing curvature

$\Rightarrow$ on-shell d.o.f. of gravitational waves on Minkowski space

note

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matter, Newtonian limit

Question: sufficient d. o. f. in G^{ab} for gemetries with matter?

Answer: o.k. at least for Newtonian limit

$$ds^2 = -c^2 dt^2 \Big(1 + \frac{2U}{c^2}\Big) + d\vec{x}^2 \Big(1 + O(\frac{1}{c^2})\Big)$$

where $\Delta_{(3)} U(y) = 4\pi G \rho(y)$ and ρ ...static mass density

can show: $\exists$ sufficient d.o.f. in G^{ab} for arbitrary $\rho(y)$

moreover, vacuum e.o.m. imply

$$ds^2 = -c^2 dt^2 \Big(1 + \frac{2U}{c^2}\Big) + d\vec{x}^2 \Big(1 - \frac{2U}{c^2}\Big)$$

as in G.R.

Question: what about the Einstein-Hilbert action?

Answer:

- tree level: e.o.m. for gravity follow from $\mathfrak{u}(1)$ sector:

$$R_{ab} \sim 0,$$

at least for linearized gravity.

- one-loop: gauge or matter fields couple to G_{ab}
$\Rightarrow$ (Sakharov) induced E-H action:

$$S_{1-loop} \sim \int d^4y \sqrt{G}\left(c_1 \Lambda^4 + c_2 \Lambda^2 R[G] + O(\log(\Lambda))\right)$$

(modifications due to different role of density factors)

- E-H action arises from UV/IR mixing

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- simple, intrinsically NC mechanism to generate gravity

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  - vacuum equation $R_{ab} \sim 0$ at least in linearized case
  - Newtonian limit, some post-newtonian corrections o.k.

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