SUSY Dark Matter and Colliders

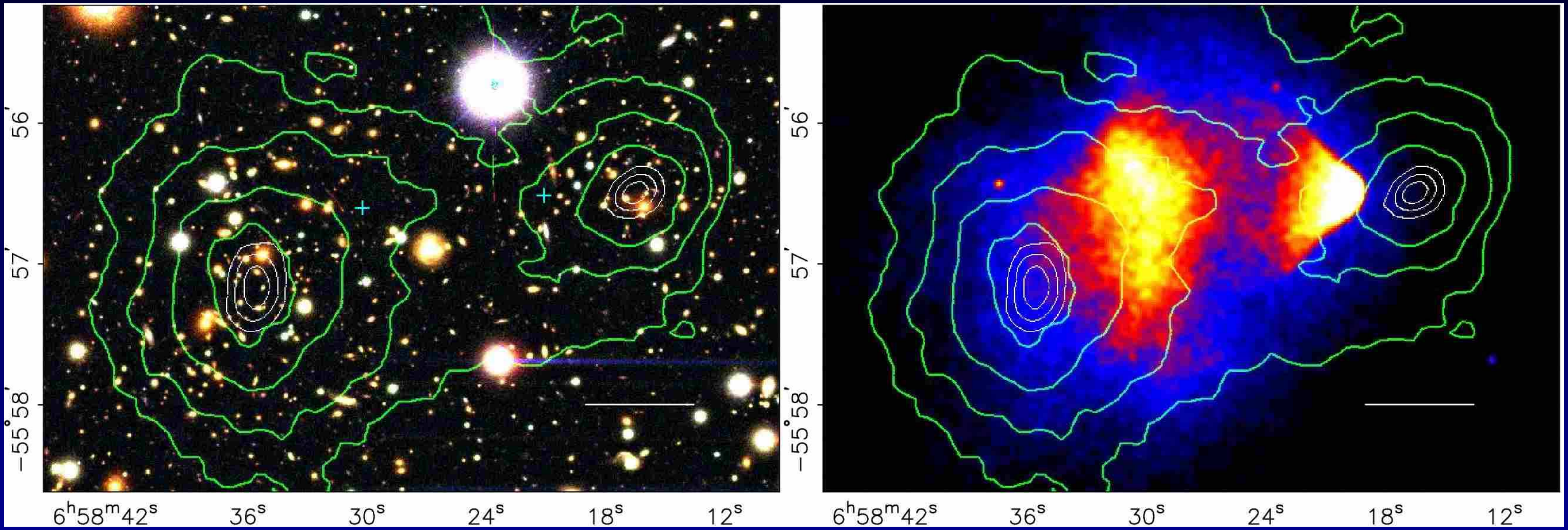
by

Ben Allanach (University of Cambridge)

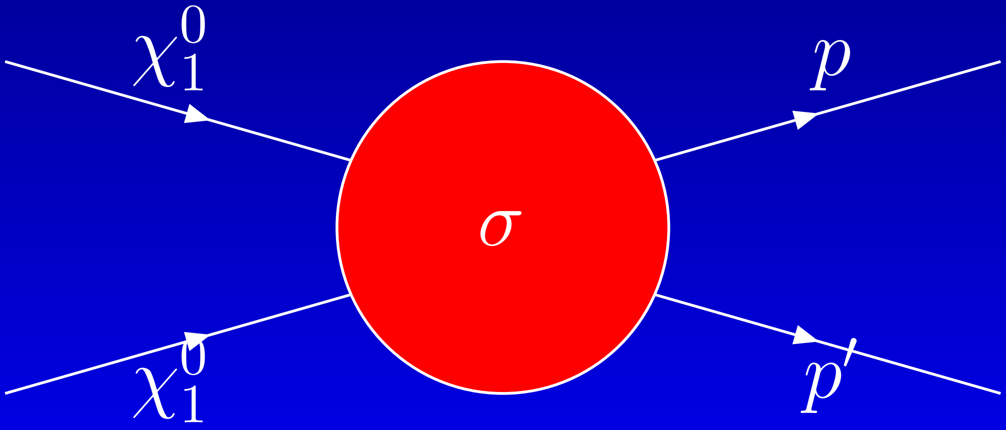
Talk outline

- SUSY Dark Matter
- Global Fits
- LHC measurements

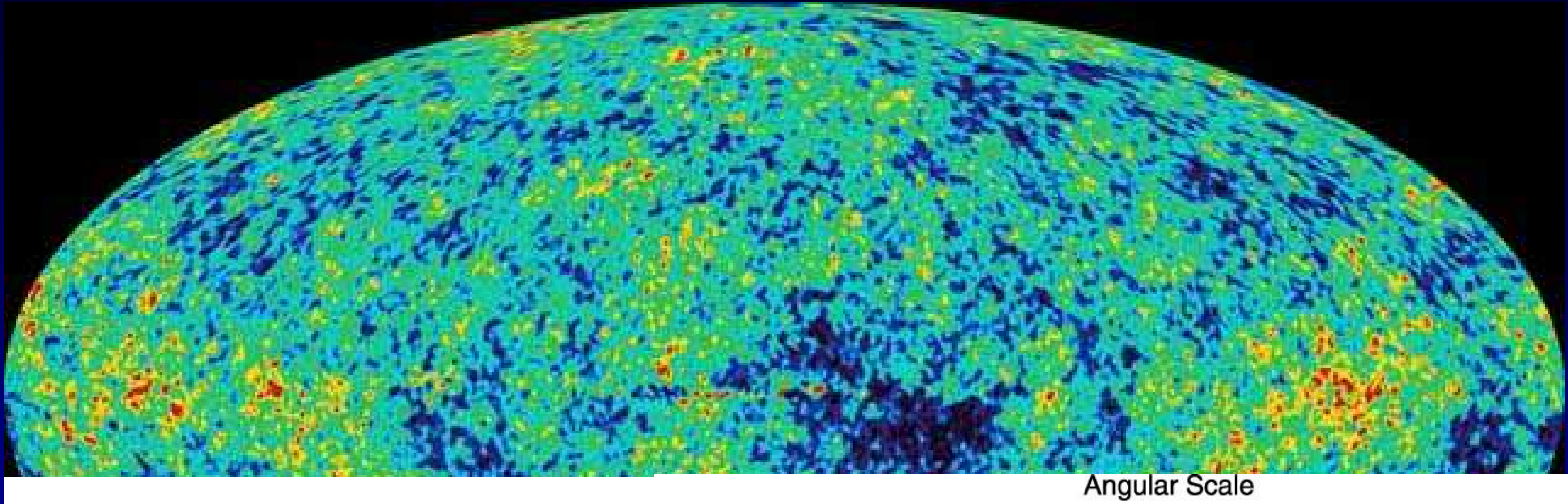
SUSY Dark Matter

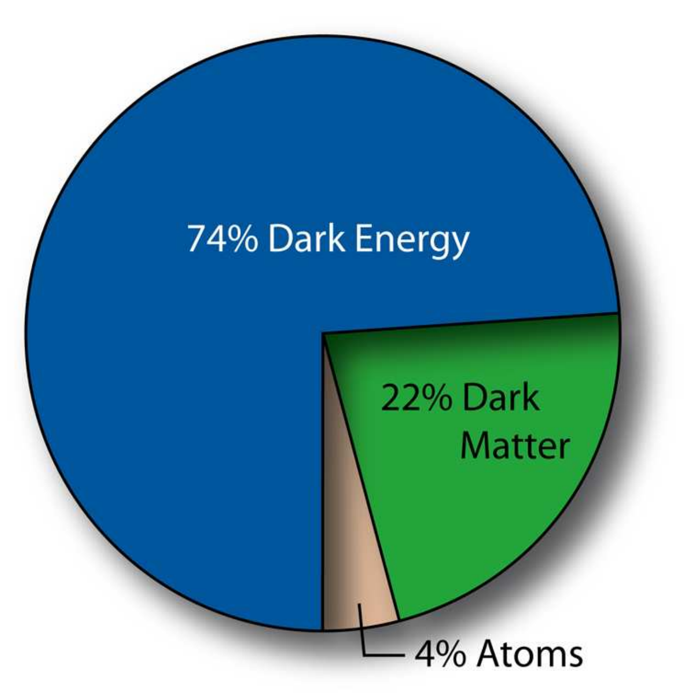


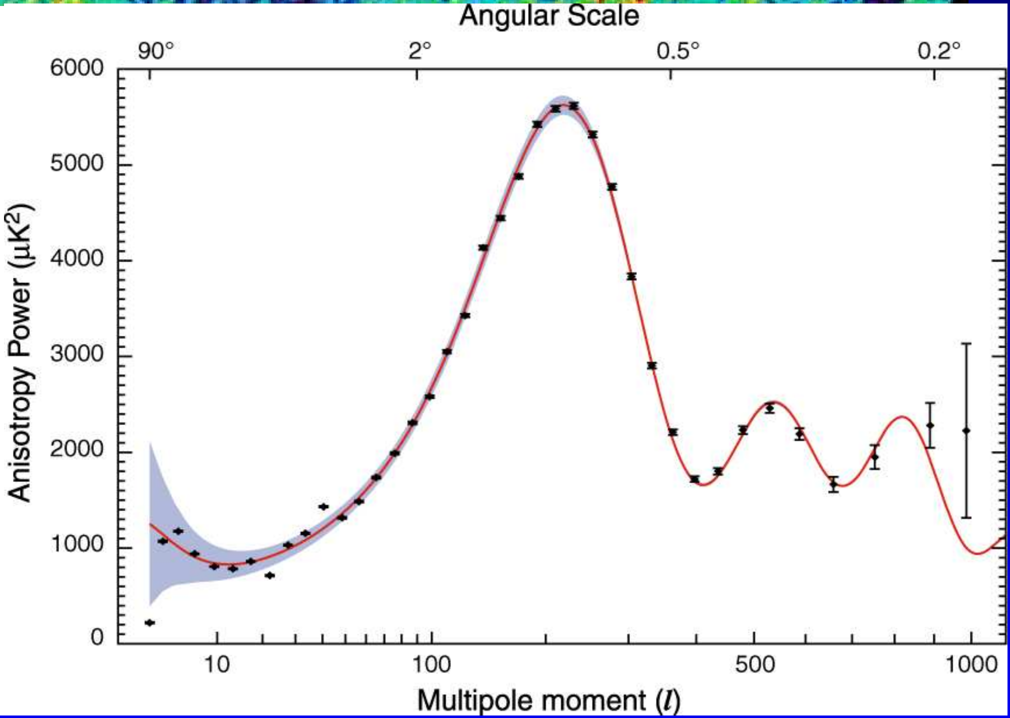
astro-ph/0608407



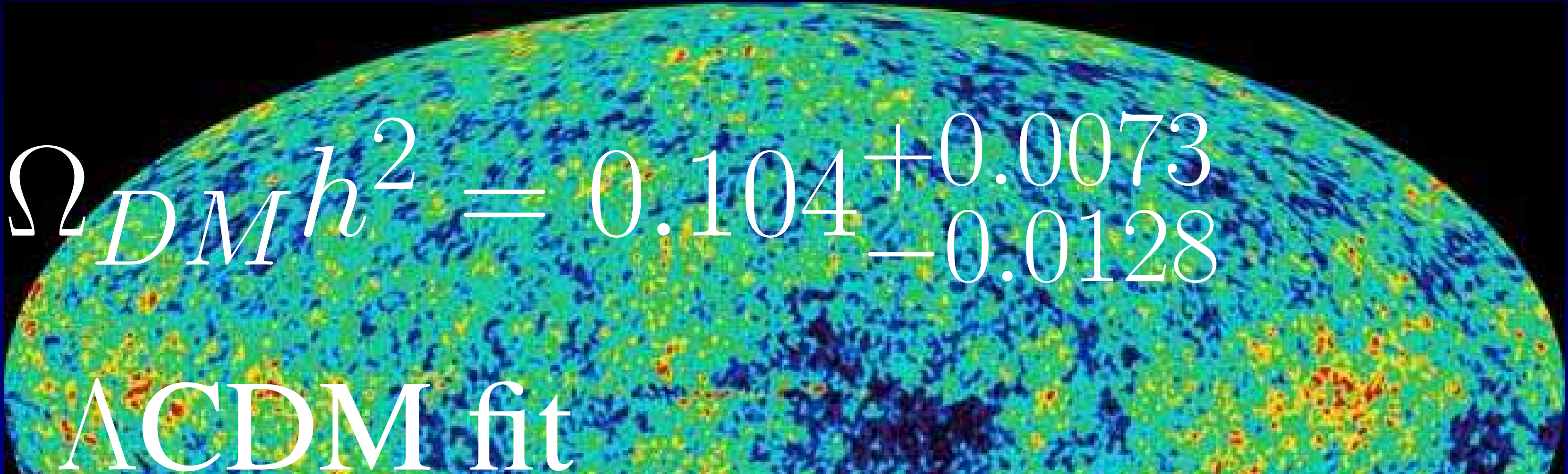
WMAP3+2dFRGS Fits





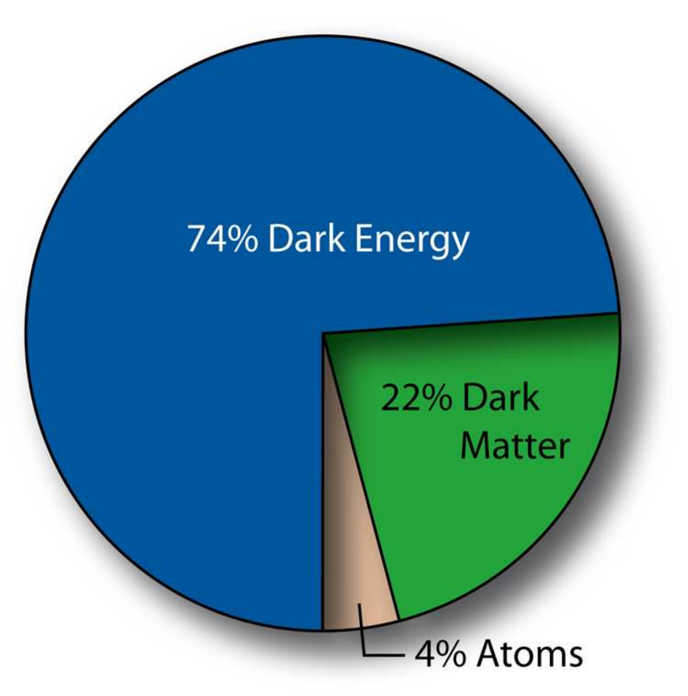


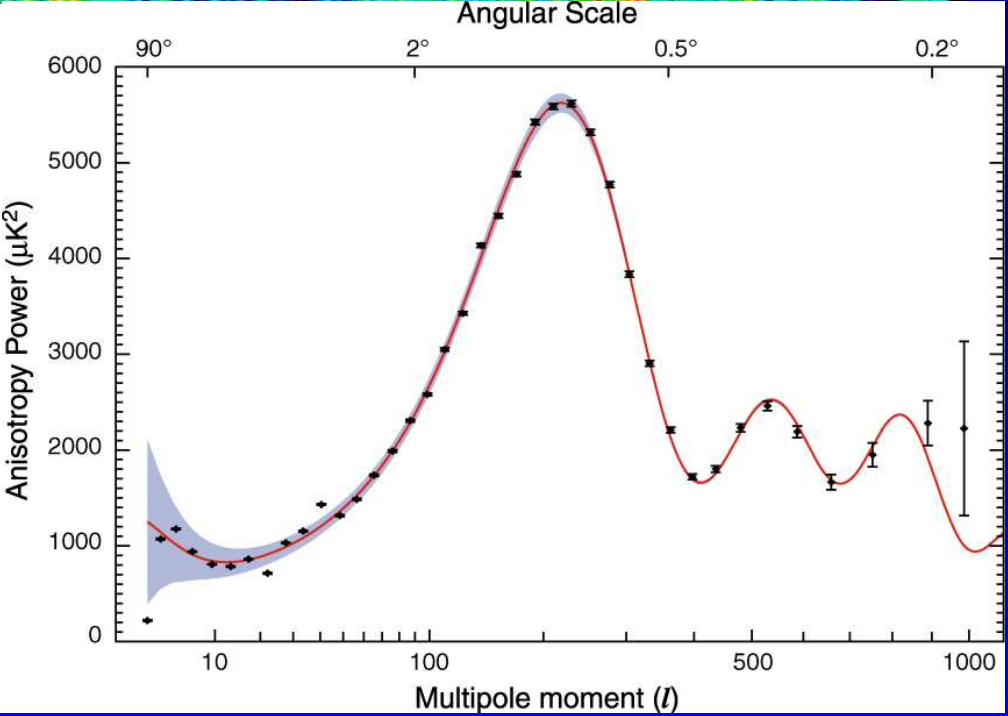
WMAP3+2dFRGS Fits



$$\Omega_{DM}h^2 = 0.104^{+0.0073}_{-0.0128}$$

ΛCDM fit





SUSY Prediction of Ωh^2

- Assume relic in thermal equilibrium with $n_{eq} \propto (MT)^{3/2} exp(-M/T)$.
- Freeze-out with $T_f \sim M_f/25$ once interaction rate $<$ expansion rate (t_{eq} critical)
- We use `microMEGAs` : $\Omega h^2 \propto 1/ < \sigma v >$ to solve coupled Boltzmann equations
- Generate SUSY spectrum with `SOFTSUSY` linked with SLHA

Belanger *et al*, CPC 149 (2002) 103
BCA, CPC 143 (2002) 305
BCA et al, JHEP0407 (2004) 036

Universality

Reduces number of SUSY breaking parameters from 100 to 3:

- $\tan\beta \equiv v_2/v_1$
- m_0, the common scalar mass (flavour).
- $M_{1/2}$, the common gaugino mass (GUT/string).
- A_0, the common trilinear coupling (flavour).

These conditions should be imposed at $M_X \sim O(10^{16-18})$ GeV and receive radiative corrections

$$\propto \; 1/(16\pi^2)\ln(M_X/M_Z).$$

Also, Higgs potential parameter sgn(μ)=±1.

Global CMSSM Fits

Q: Why perform global fits to SUSY using DM+indirect

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Implementation

Input parameters are: m_0, A_0, $M_{1/2}$, $\tan\beta$,

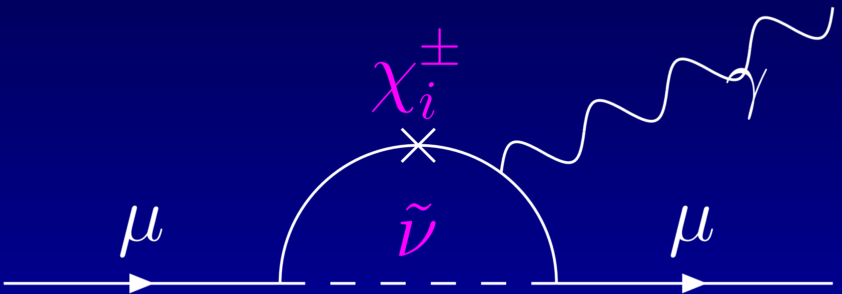
- $m_t = 171.4 \pm 2.9$, $m_b(m_b) = 4.24 \pm 0.11$ GeV,
- $\alpha_s(M_Z)^{\overline{MS}} = 0.1176 \pm 0.002$,
  $\alpha^{-1}(M_Z)^{\overline{MS}} = 127.918 \pm 0.018$

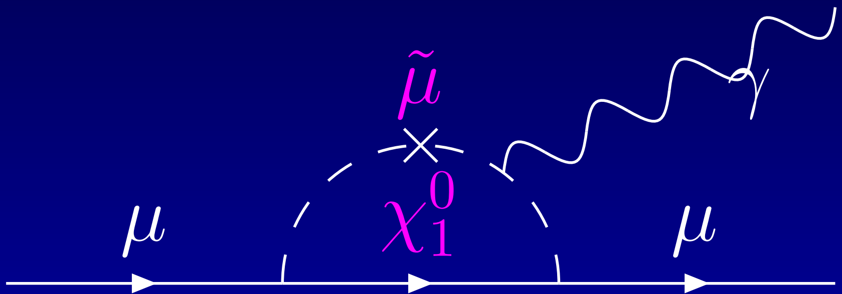
For the likelihood, we also use

- $\Omega_{DM}h^2 = 0.104^{+0.0073}_{-0.0128}$
- $\delta(g-2)_\mu/2 = (22 \pm 10) \times 10^{-10}$ Stöckinger *et al*
- $BR[b \to s\gamma] = (3.55 \pm 0.38) \times 10^{-5}$
- $\sin^2\theta^l_w(\text{eff}) = 0.23153 \pm 0.000175$
- $M_W = 80.392 \pm 0.031$ GeV W Hollik, A Weber *et al*

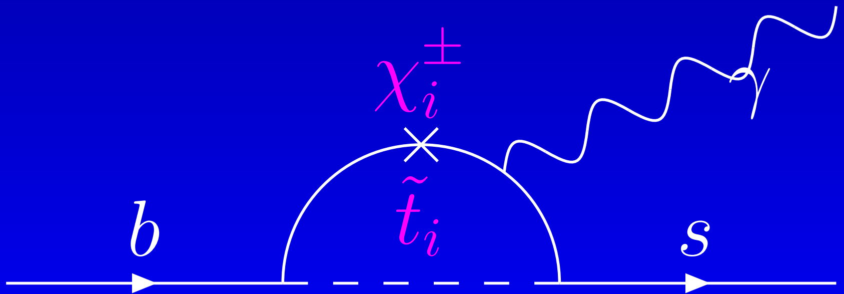
Additional observables

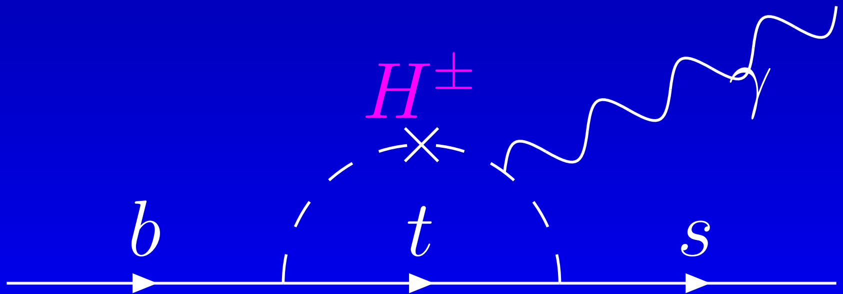
$$\delta \frac{(g-2)_\mu}{2} \sim 13 \times 10^{-10} \left(\frac{100 \text{ GeV}}{M_{SUSY}} \right)^2 \tan\beta$$





$$BR[b \to s\gamma] \propto \tan\beta (M_W / M_{SUSY})^2$$





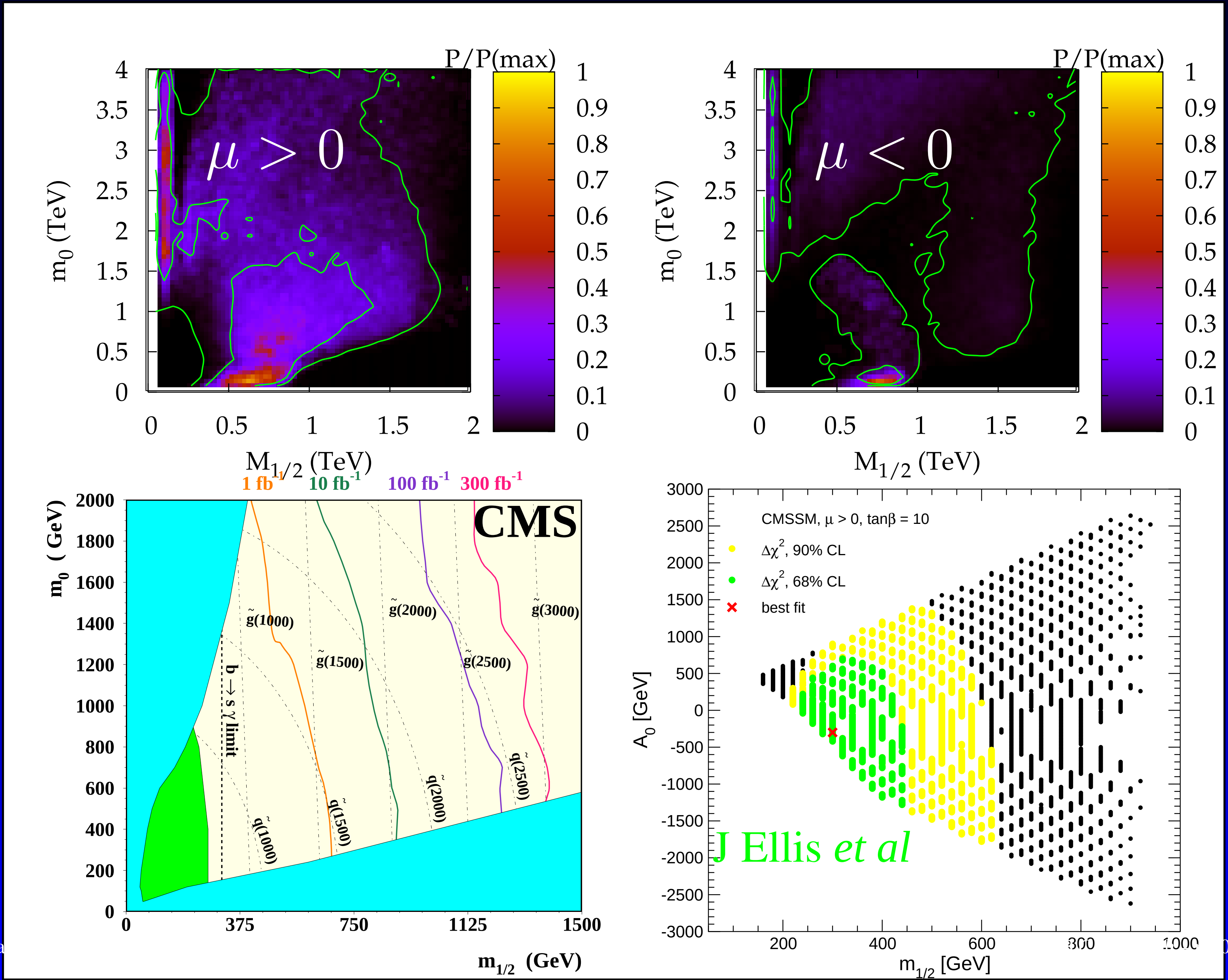
CMSSM Regions

After WMAP+LEP2, bulk region diminished. Need specific mechanism to reduce overabundance:

- $\tilde{\tau}$ coannihilation: small m_0, $m_{\tilde{\tau}_1} \approx m_{\chi_1^0}$. Boltzmann factor $\exp(-\Delta M/T_f)$ controls ratio of species. $\tilde{\tau}_1 \chi_1^0 \rightarrow \tau\gamma$, $\tilde{\tau}_1 \tilde{\tau}_1 \rightarrow \tau\bar{\tau}$.
- Higgs Funnel: $\chi_1^0 \chi_1^0 \rightarrow A \rightarrow b\bar{b}/\tau\bar{\tau}$ at large $\tan\beta$. Also via h at large m_0 small $M_{1/2}$.
- Focus region: Higgsino LSP at large m_0: $\chi_1^0 \chi_1^0 \rightarrow WW/ZZ/Zh/t\bar{t}$.
- $\tilde{t}$ coannihilation: high $-A_0$, $m_{\tilde{t}_1} \approx m_{\chi_1^0}$. $\tilde{t}_1 \chi_1^0 \rightarrow gt$, $\tilde{t}\tilde{t} \rightarrow tt$

Datta, Djouadi, Drees, hep-ph/0504090

Global Fits


P/P(max)
μ > 0
μ < 0
m_0 (TeV)
$M_{1/2}$ (TeV)
1 fb^{-1}
10 fb^{-1}
100 fb^{-1}
300 fb^{-1}
CMS
m_0 (GeV)
$m_{1/2}$ (GeV)
$\tilde{g}$(1000)
$\tilde{g}$(1500)
$\tilde{g}$(2000)
$\tilde{g}$(2500)
$\tilde{g}$(3000)
$\tilde{q}$(1000)
$\tilde{q}$(1500)
$\tilde{q}$(2000)
$\tilde{q}$(2500)
b → s γ limit
CMSSM, μ > 0, tanβ = 10
$\Delta\chi^2$, 90% CL
$\Delta\chi^2$, 68% CL
best fit
A_0 [GeV]
$m_{1/2}$ [GeV]
J Ellis *et al*

LHC vs LC in SUSY Measurement

- LHC (start date 2007) produces strongly interacting particles up to a few TeV. Precision measurements of mass *differences* possible if the decay chains exist: possibly per mille for leptons, several percent for jets.
- ILC has several energy options: 500-1000 GeV, CLIC up to 3 TeV. Linear colliders produce less strong particles but much easier to make precision measurements of masses/couplings.

$\mathcal{Q}$: What energy for LC?
$\mathcal{Q}$: What do we get from LHC ?

LHC/ILC Working Group Report: `hep-ph/0410364`

95% CL Upper Limits on Masses

particle	flat prior	natural prior
h^0	0.123	0.120
A^0	1.45	1.50
χ_1^0	0.65	0.45
$\chi_1^\pm$	1.20	0.85
$\tilde{g}$	3.25	2.30
$\tilde{e}_R$	1.90	1.90
$\tilde{q}_L$	3.20	2.45
$\tilde{t}_1$	2.45	1.80

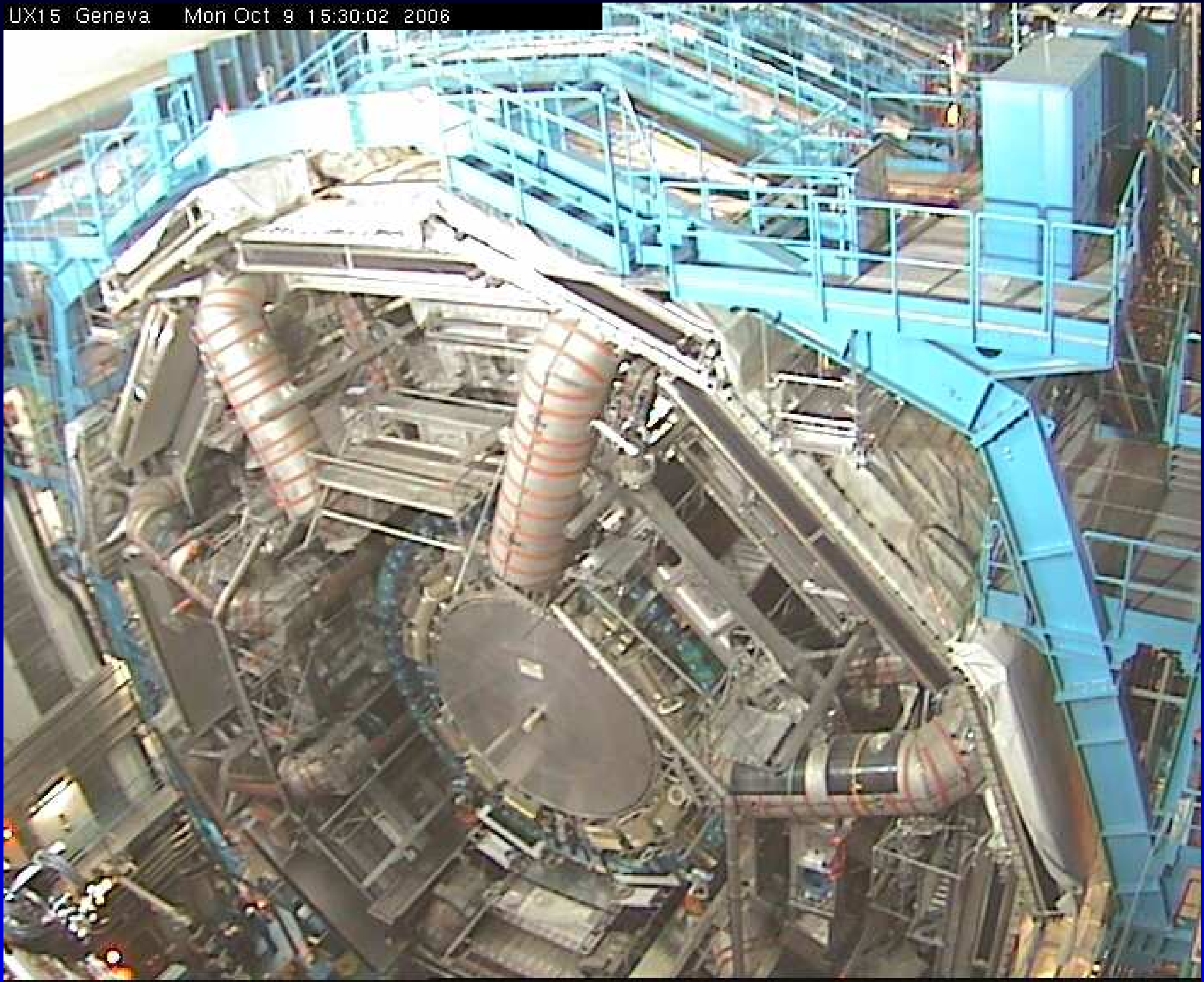
P(500 GeV ILC>$\chi_1^0\chi_1^0$, $\chi_1^\pm\chi_1^\pm$) ILC=0.7,0.33

P(800 GeV ILC>$\chi_1^0\chi_1^0$, $\chi_1^\pm\chi_1^\pm$) ILC=0.93,0.58

Caveats

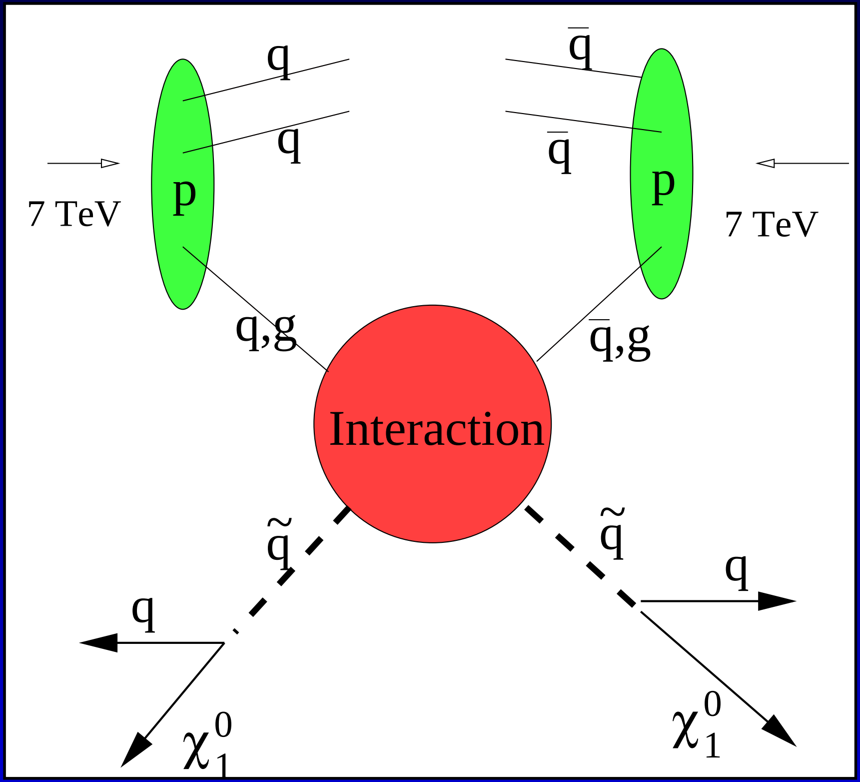
- Implicitly assumed that LSP constitutes *all* of dark matter
- Assumed radiation domination in post-inflation era. No clear evidence between freeze-out+BBN that this is the case (t_{eq} changes).
- Examples of non-standard cosmology that would change the prediction:
  - Extra degrees of freedom
  - Low reheating temperature
  - Extra dimensional models
  - Anisotropic cosmologies
  - Non-thermal production of neutralinos (late decays?)

LHC (ATLAS)


UX15 Geneva Mon Oct 9 15:30:02 2006

Collider SUSY Dark Matter Production

Strong sparticle production and decay to dark matter particles.



$\mathcal{Q}$: Can we measure enough to predict σ?

Collider SUSY Dark Matter Production

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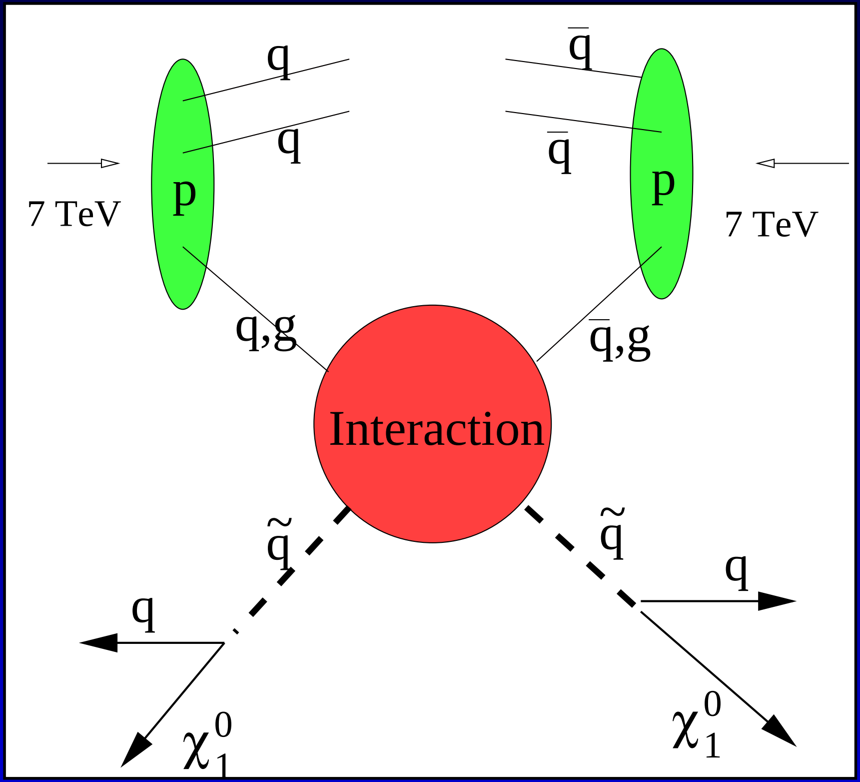


Any dark matter candidate that couples to hadrons can be produced at the LHC

Collider Check

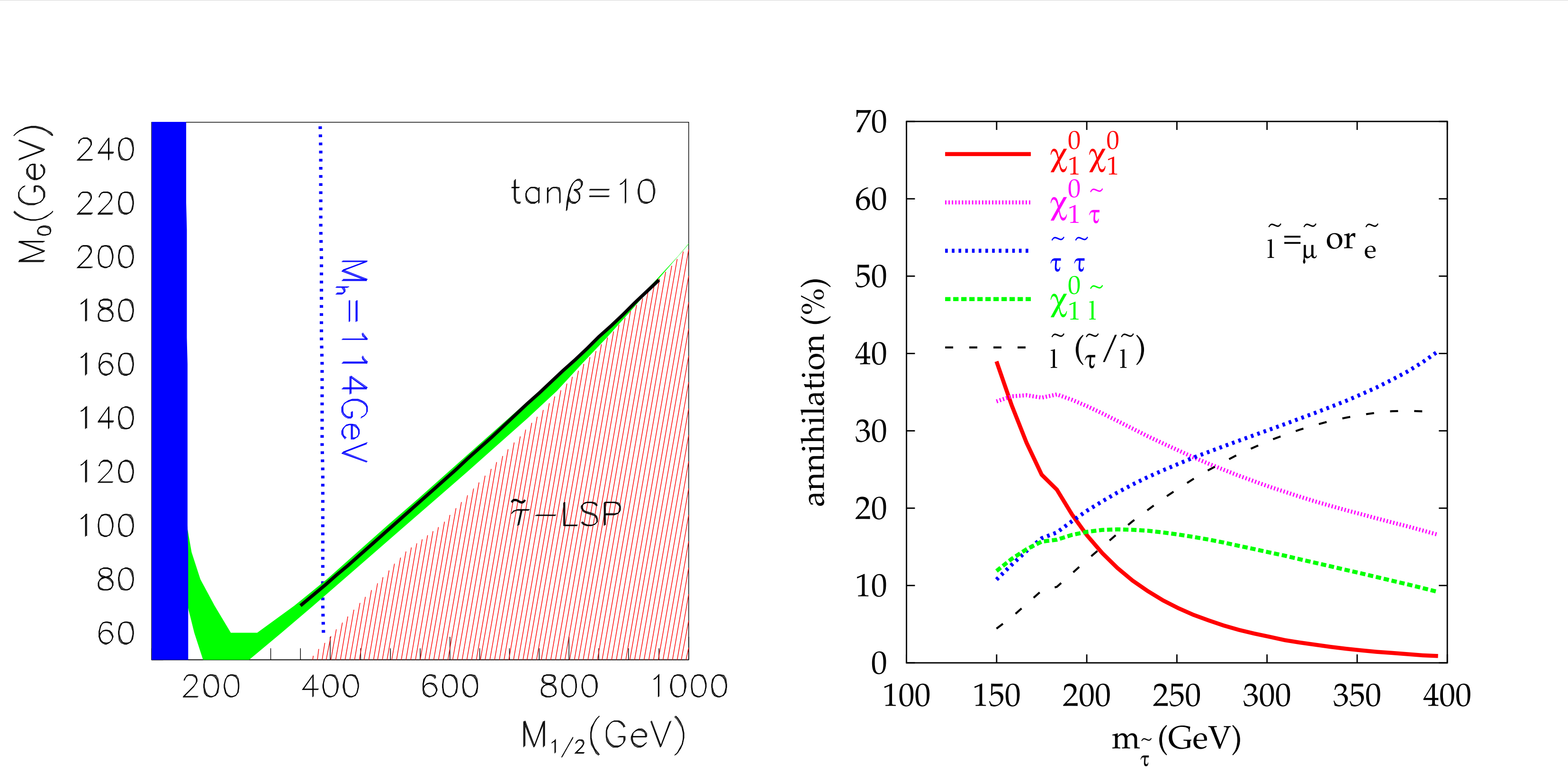
Need corroboration with *direct detection*.

If we can pin particle physics down, a comparison between the predicted relic density and that observed is a test of the cosmological assumptions used in the prediction.

Thus, if it doesn't fit, you change the cosmology until it does.

BCA, G. Belanger, F. Boudjema, A. Pukhov, JHEP 0412 (2004) 020.; M. Nojiri, D. Tovey, JHEP 0603 (2006) 063

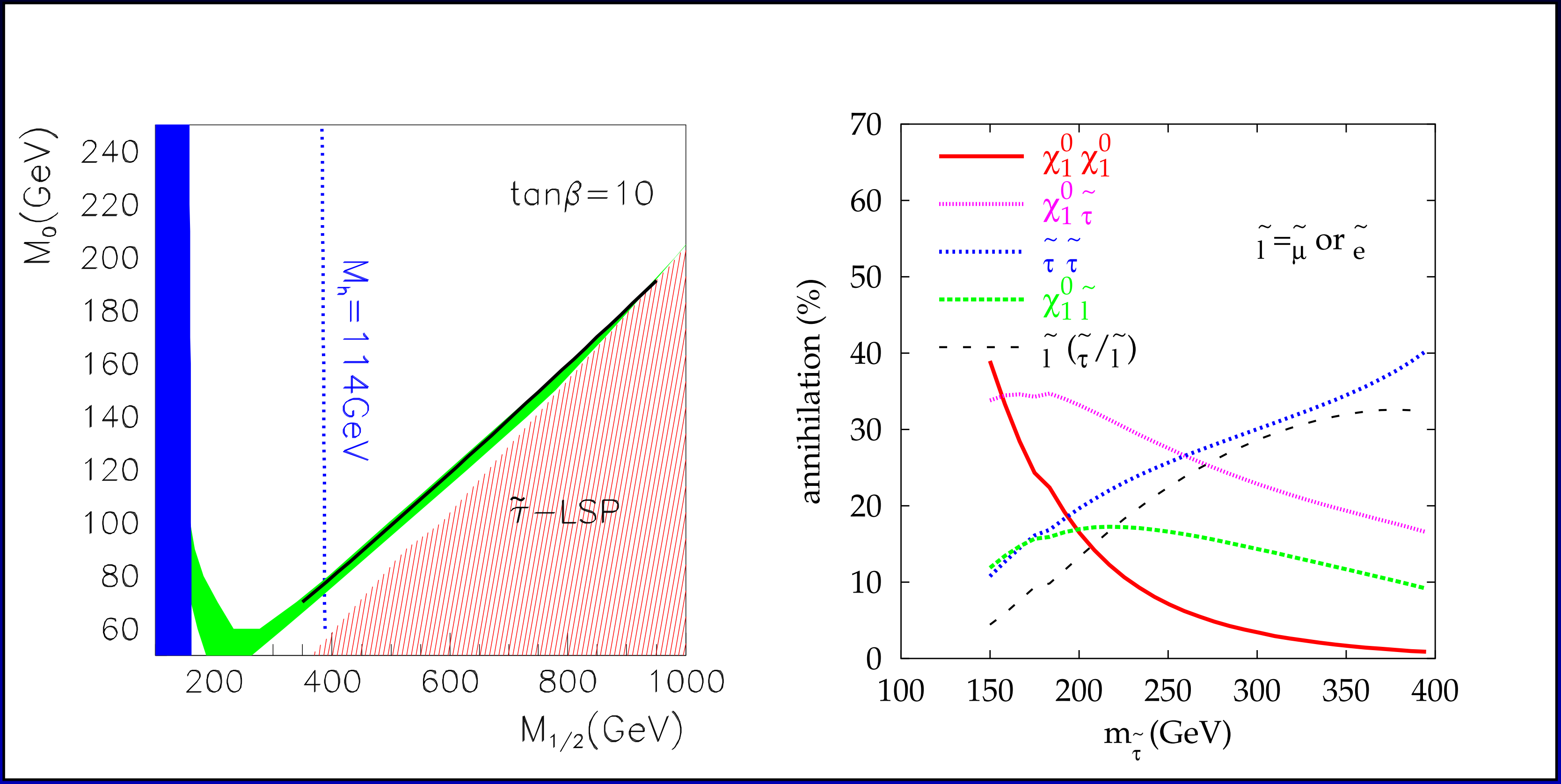
Coannihilation Slope



$$m^2_{\tilde l_R} \approx m_0^2 + 0.15 M^2_{1/2}, \qquad M_{\chi_1^0} \approx 0.4 M_{1/2}$$

Low enough $M_{1/2} \Rightarrow$ quasi-degenerate $\tilde\tau$, $M_{\chi_1^0}$

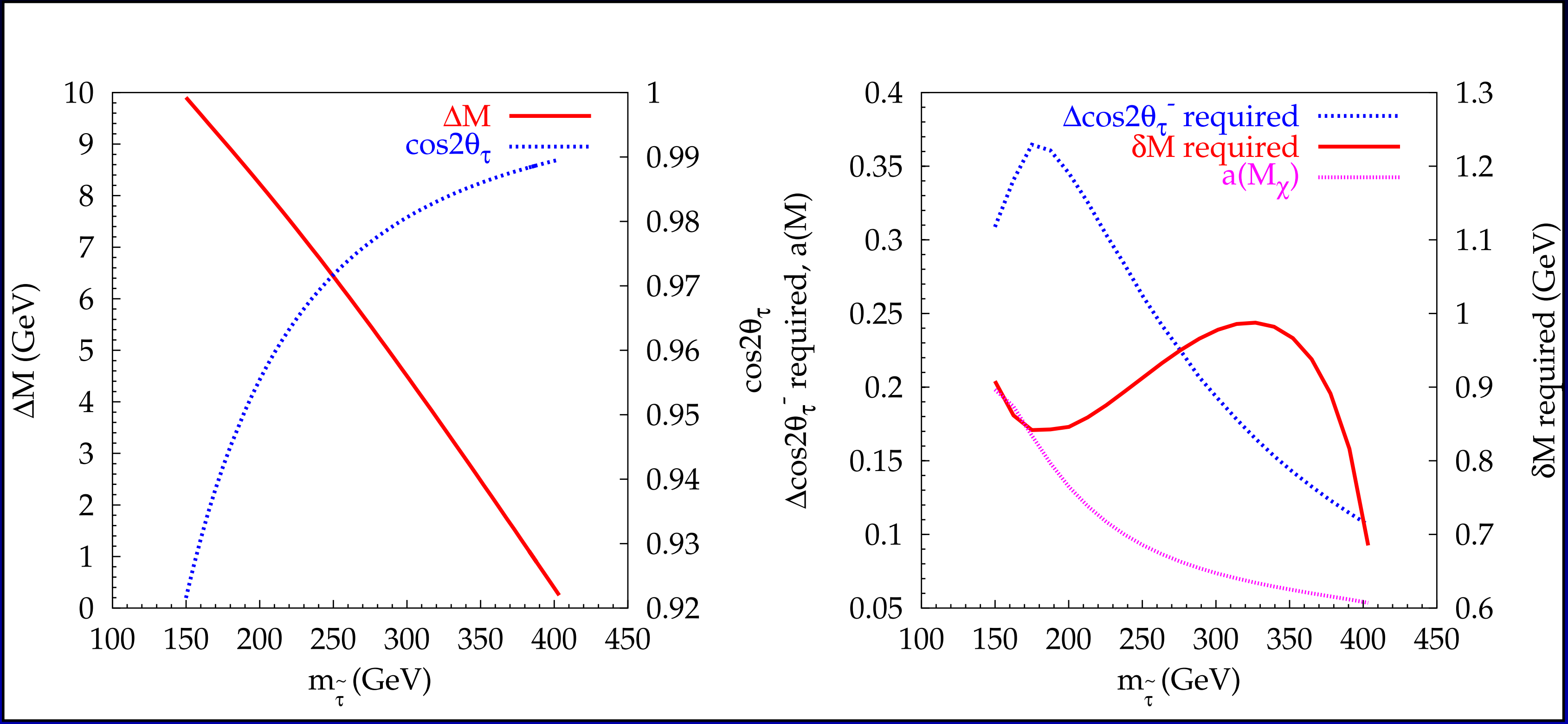
Coannihilation Slope



$$m^2_{\tilde l_R} \approx m_0^2 + 0.15 M^2_{1/2}, \qquad M_{\chi^0_1} \approx 0.4 M_{1/2}$$

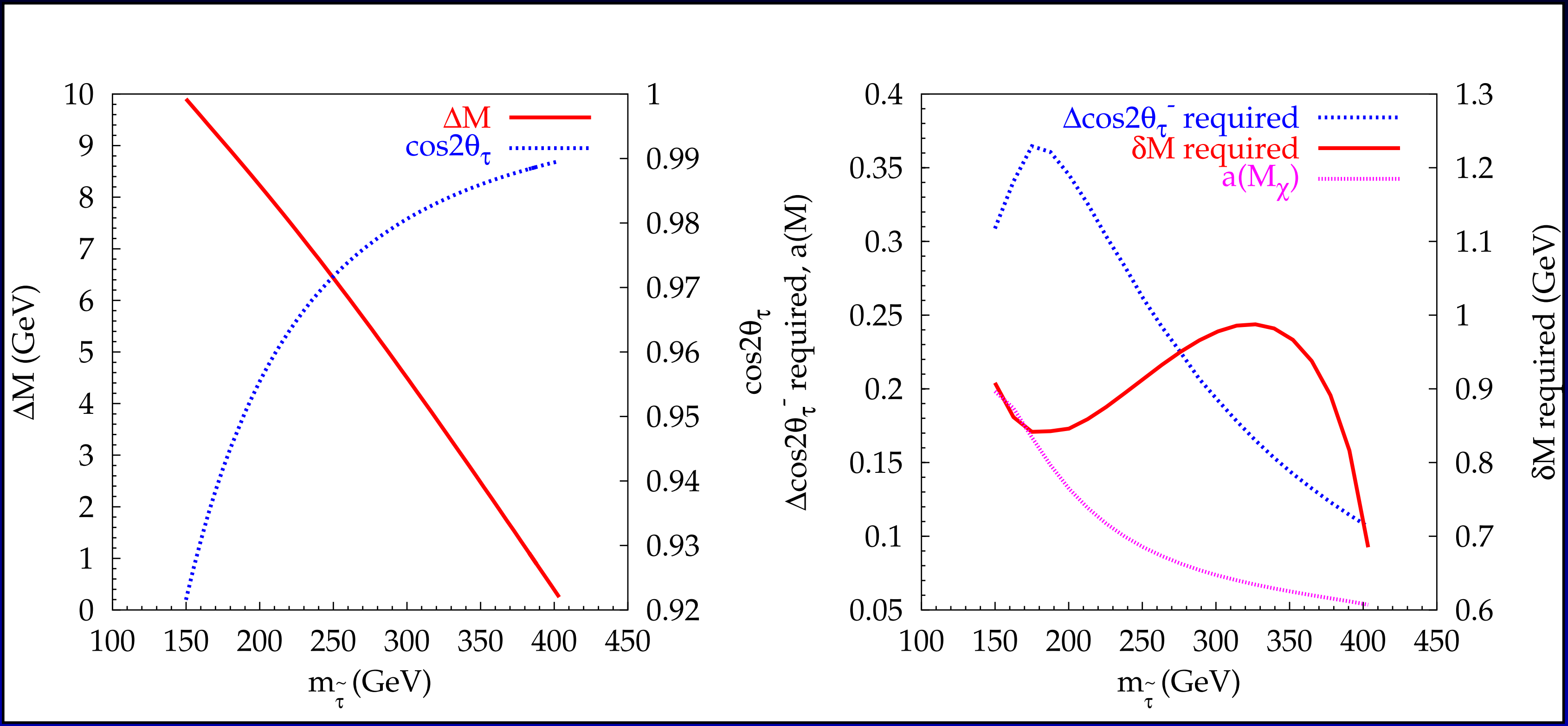
If we do not assume CMSSM, we will also have to measure selectron and smuon properties.

PCMSSM Dependencies



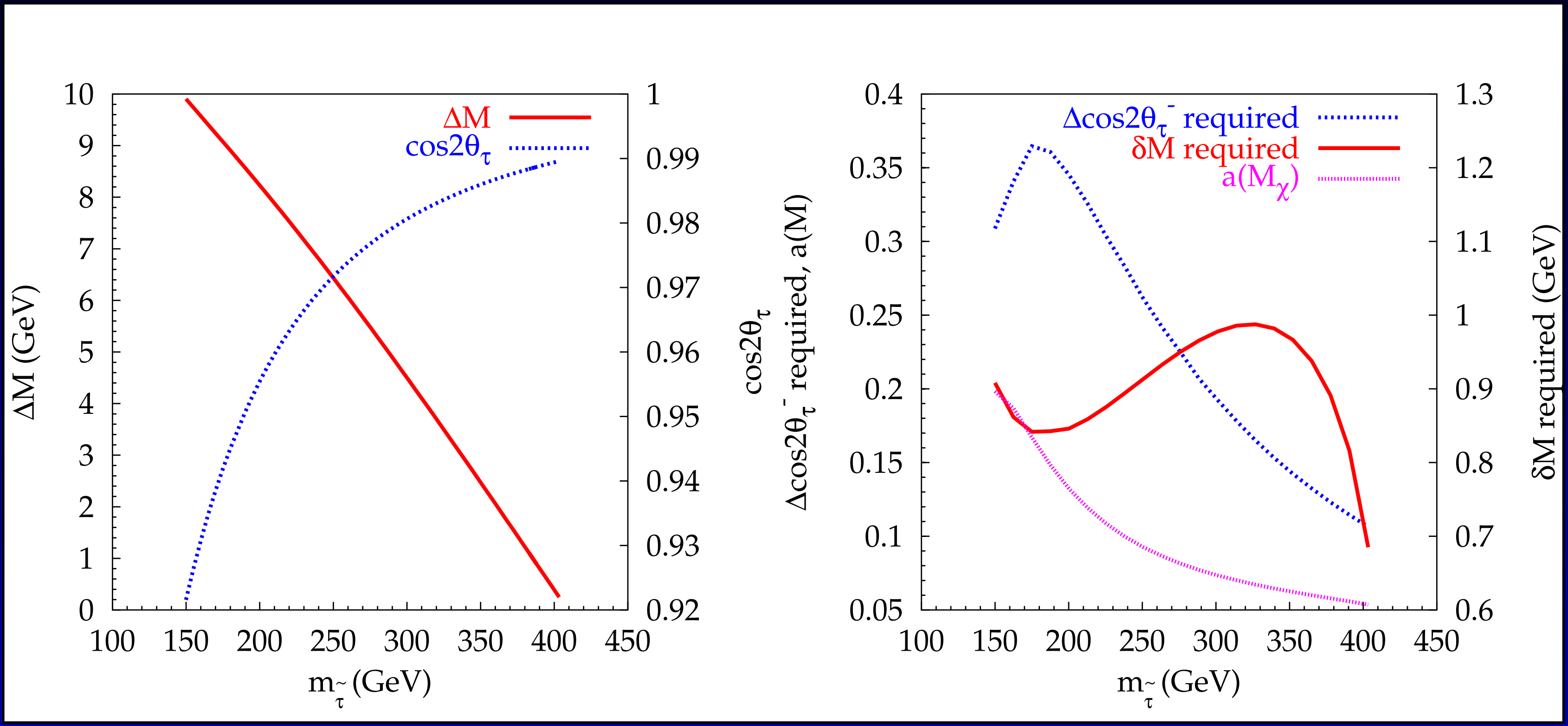
LHS: plots of quantities along CMSSM slope. Below $\Delta M = 1.78$ GeV, no two-body stau decay. LC studies indicate $\Delta M > 5$ GeV is OK.

PCMSSM Dependencies



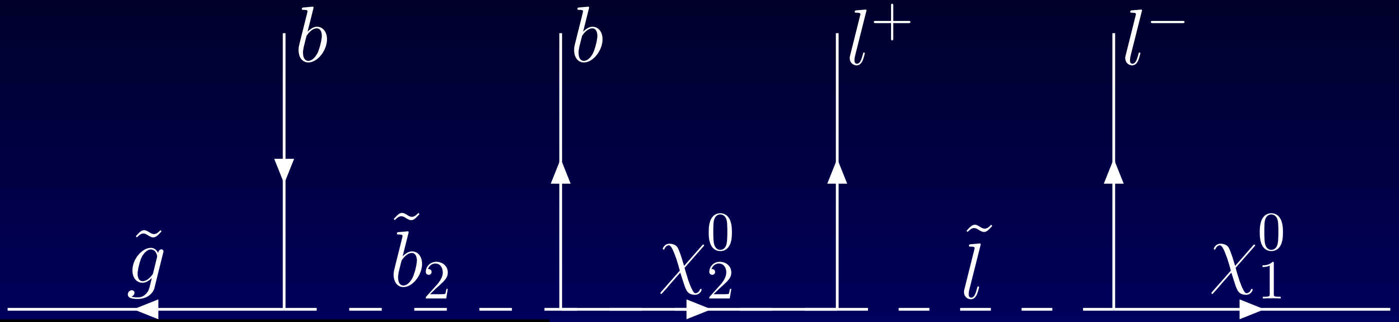
$\tilde{\tau}_1\chi_1^0 \to \tau\gamma \propto 3\cos 2\theta_\tau + 5$ from coupling of neutralino to $\tilde{\tau}_{L/R}$.

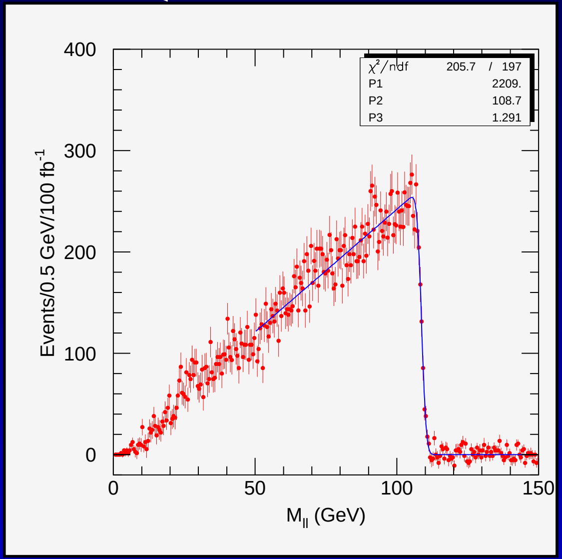
PCMSSM Dependencies



$a(M_\chi)$ found by keeping ΔM constant, δM by just varying stau mass. $m_{\tilde{e}}, m_{\tilde{\mu}}$ *needed to about* 1.5%

LHC SUSY Measurements





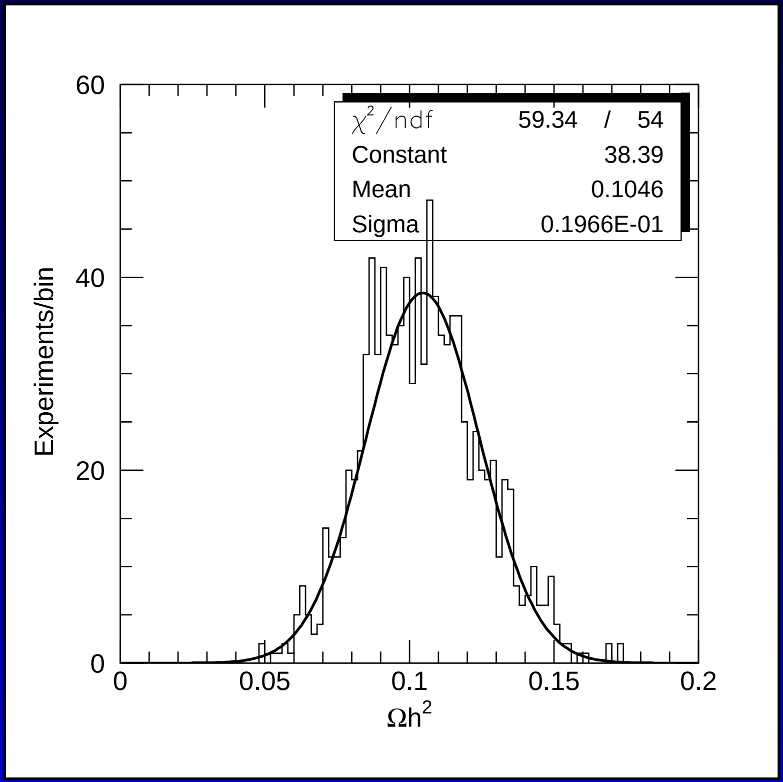
$$m_{ll}^2 = \frac{(m^2_{\chi_2^0} - m^2_{\tilde l})(m^2_{\tilde l} - m^2_{\chi_1^0})}{m^2_{\tilde l}}$$

Q: Can we measure enough of these to pin SUSY dow

BCA, Lester, Parker, Webber, JHEP 0009 (2000) 004

Predicting Ωh^2

Not much left that's allowed but edge measurements allow reasonable Ωh^2 error for 300 fb^{-1}.



Q: What about other bits of parameter space?

M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`.

Bulk Region

M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`. for 300 fb^{-1}. SPA point $m_0 = 70$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -300$ GeV, $\tan\beta = 10$, $\mu > 0$: $\Omega h^2 = 0.108$. Put in m_{ll}^{max}, m_{llq}^{max}, m_{lq}^{low}, m_{lq}^{high}, m_{llq}^{min}, $m_{l_L} - m_{\chi_1^0}$, $m_{ll}^{max}(\chi_4^0)$, $m_{\tau\tau}^{max}$, m_h.

$\tilde\chi_1^0\tilde\chi_1^0 \rightarrow \ell^+\ell^-$	40%
$\tilde\chi_1^0\tilde\chi_1^0 \rightarrow \tau^+\tau^-$	28%
$\tilde\chi_1^0\tilde\chi_1^0 \rightarrow \nu\bar\nu$	3%
$\tilde\chi_1^0\tilde\tau_1 \rightarrow Z\tau$	4%
$\tilde\chi_1^0\tilde\tau_1 \rightarrow A\tau$	18%
$\tilde\tau_1\tilde\tau_1 \rightarrow \tau\tau$	2%

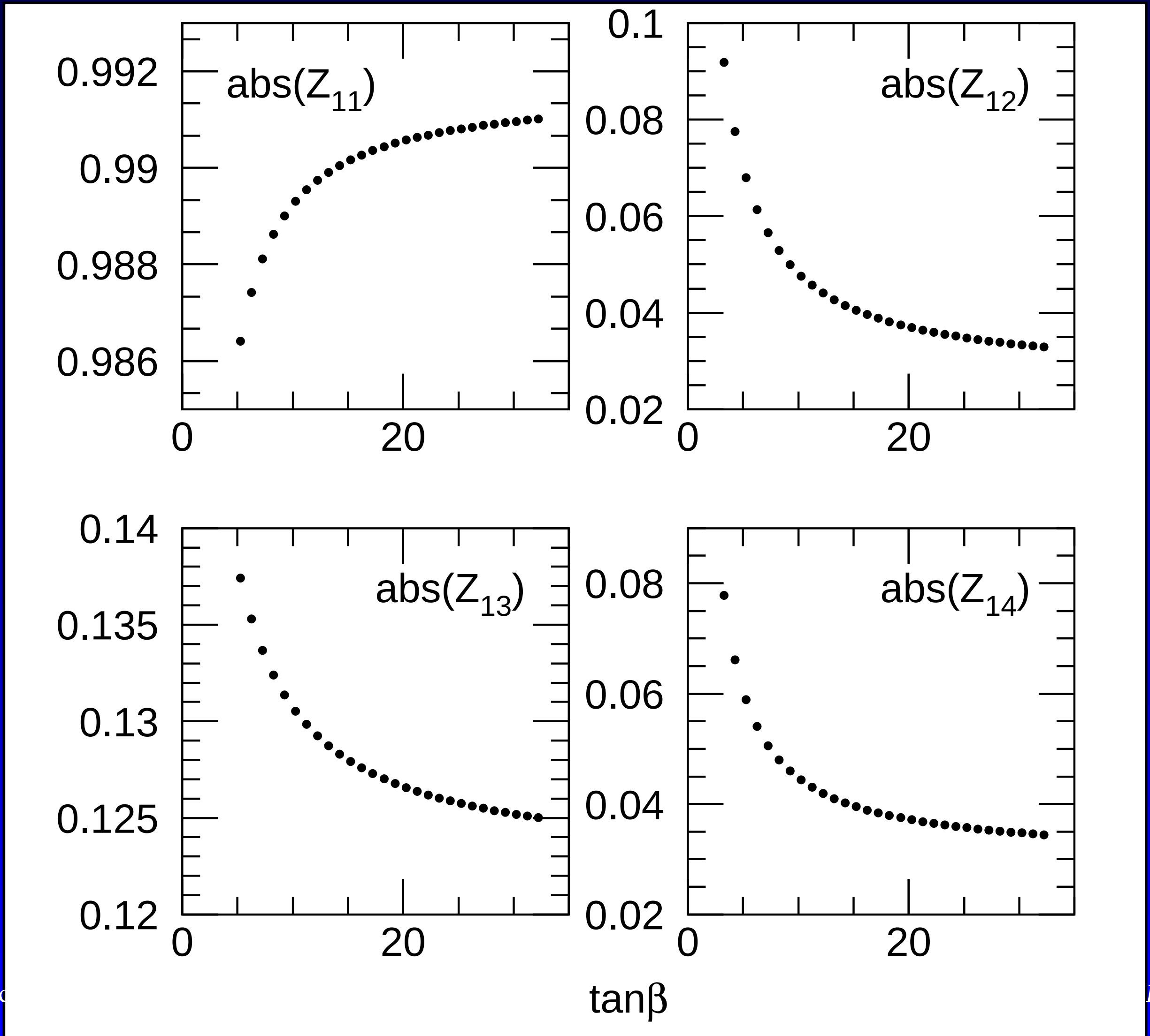
Neutralino mass matrix

Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.

$$\left[\begin{array}{cccc} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{array}\right] \tag{1}$$

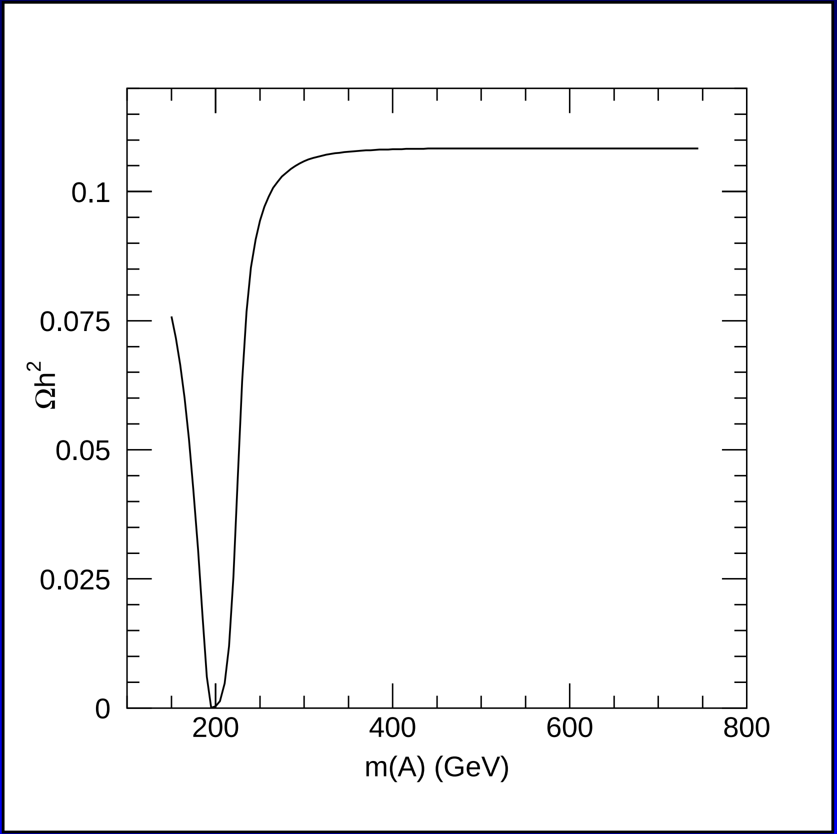
Neutralino mass matrix

Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.



Slepton/A^0 Higgs

$\Gamma(\chi_2^0 \rightarrow \tilde{l}_R l)/\Gamma(\chi_2^0 \rightarrow \tilde{\tau}_1 \tau)$ then helps determine θ_τ for a given $\tan\beta$. Exclusion of A^0 helps you to exclude A^0 appearing in cascade decays. Meaurement of m_h provides constraints in $m_A - \tan\beta$ plane.

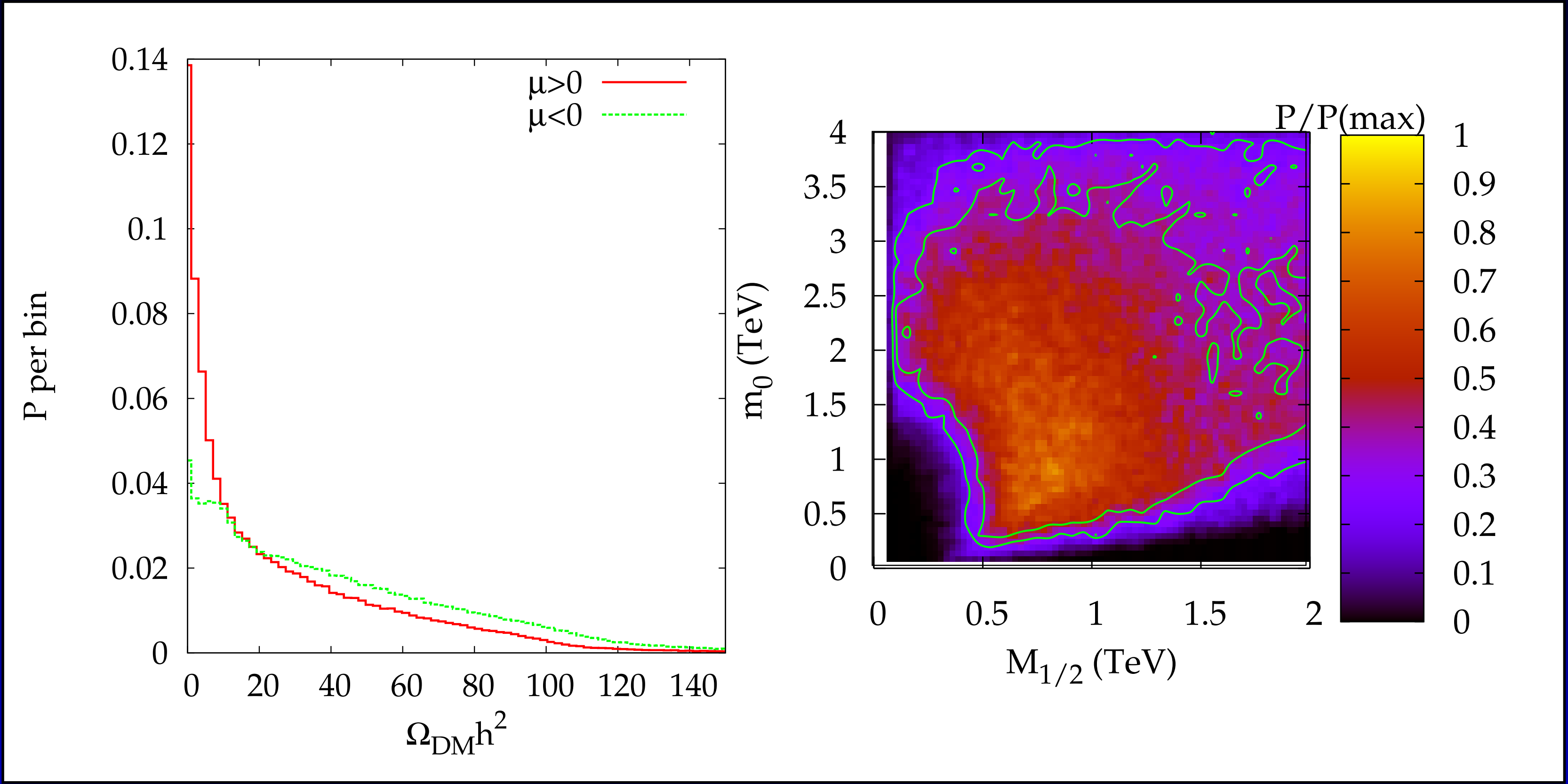


Summary

- CMSSM fits indicate LHC should be able to find SUSY.
- Light gauginos from fits encouraging for ILC.
- LHC is likely to *not* have enough precision to provide an accurate prediction of Ωh^2: you really need an ILC for that.
- We want to predict relic density to improve our understanding of cosmology.

Supplementary Material

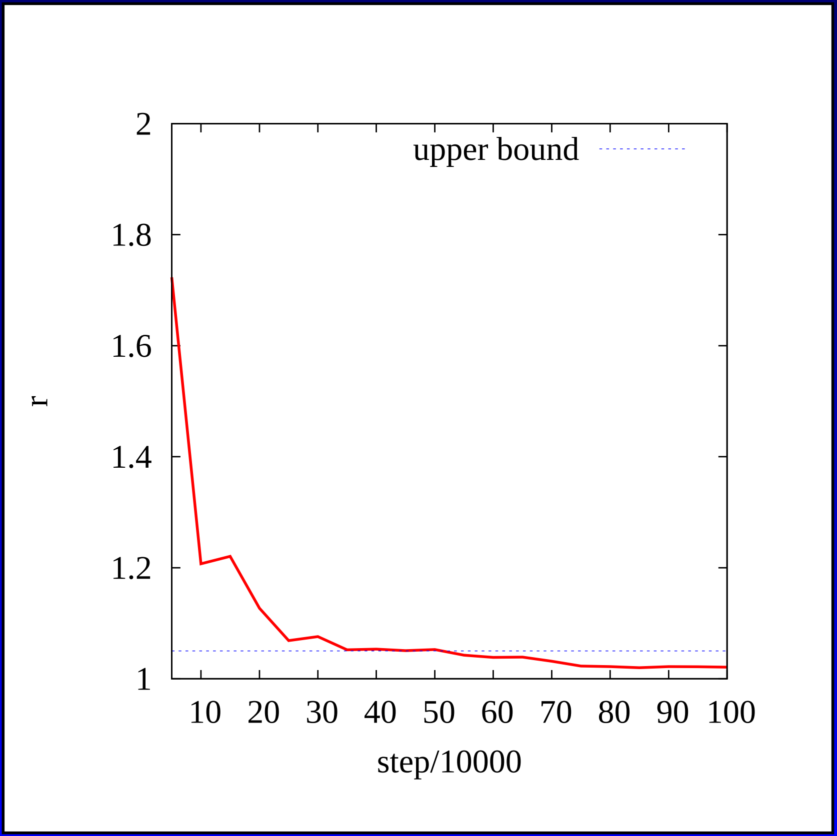
No Dark Matter Fits



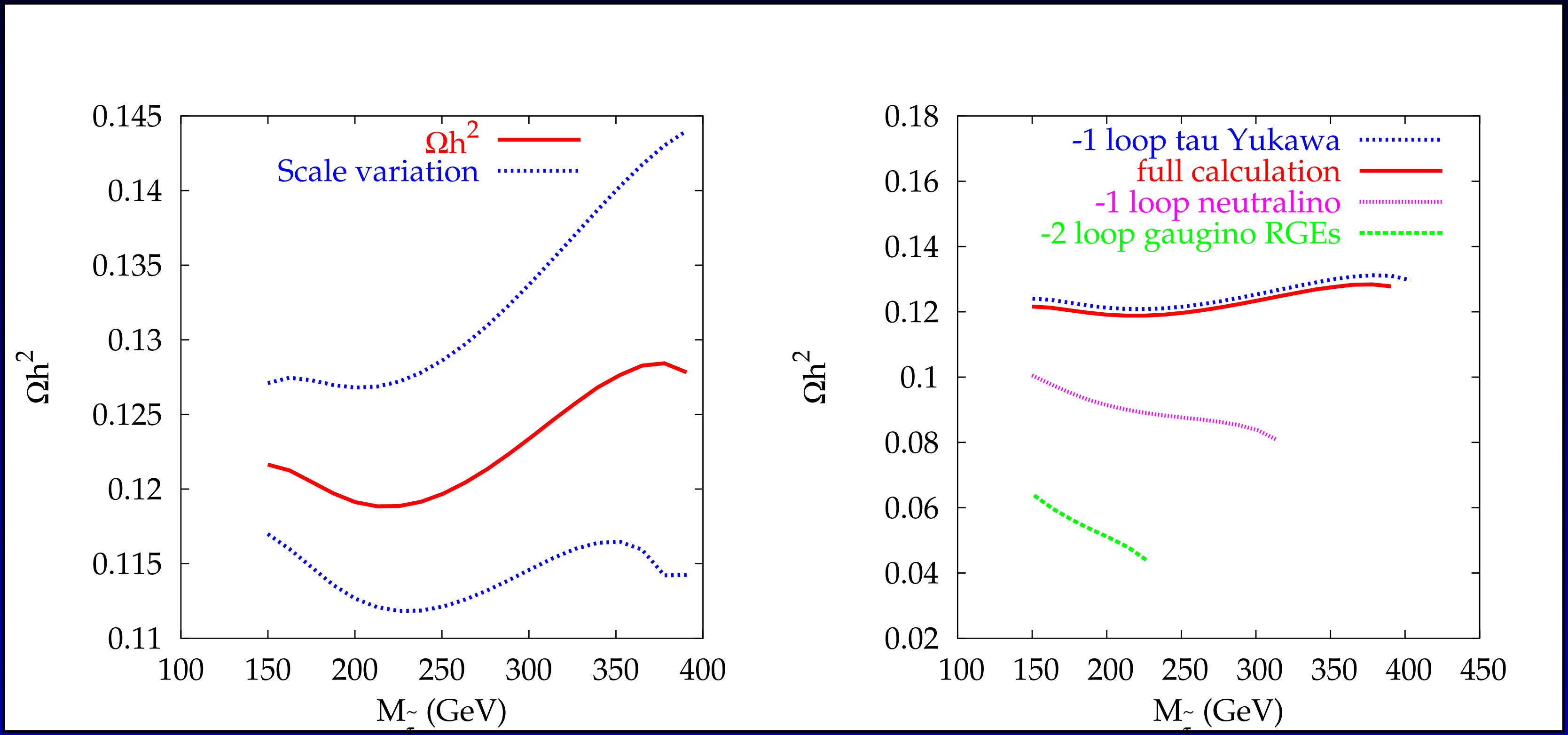
Huge χ^2 from the dark matter relic density.

Convergence

We run 9×1 000 000 points. By comparing the 9 independent chains with random starting points, we can provide a statistical measure of convergence: an upper bound r on the excepted variance decrease for infinite statistics.

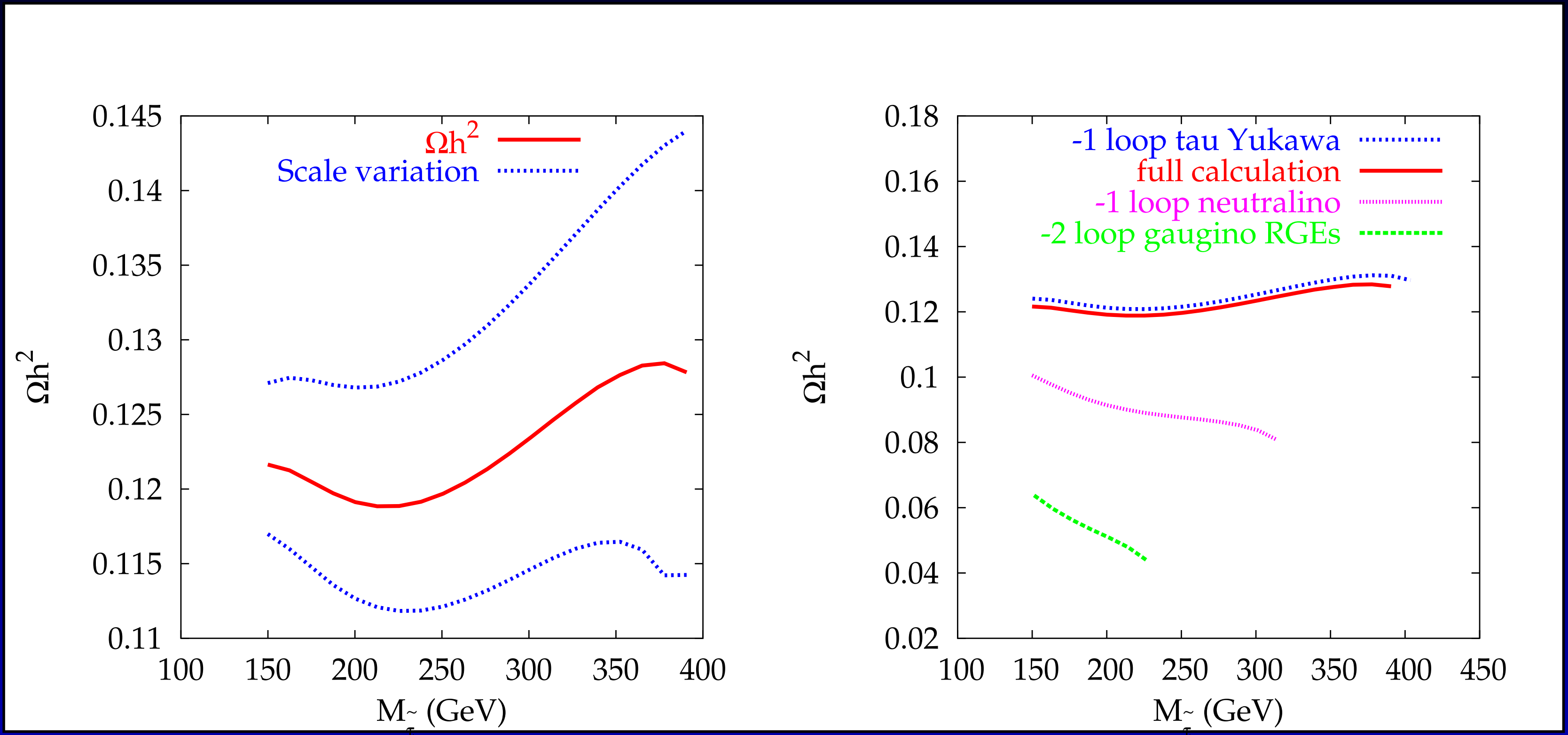


Coannihilation Theory Uncertainties

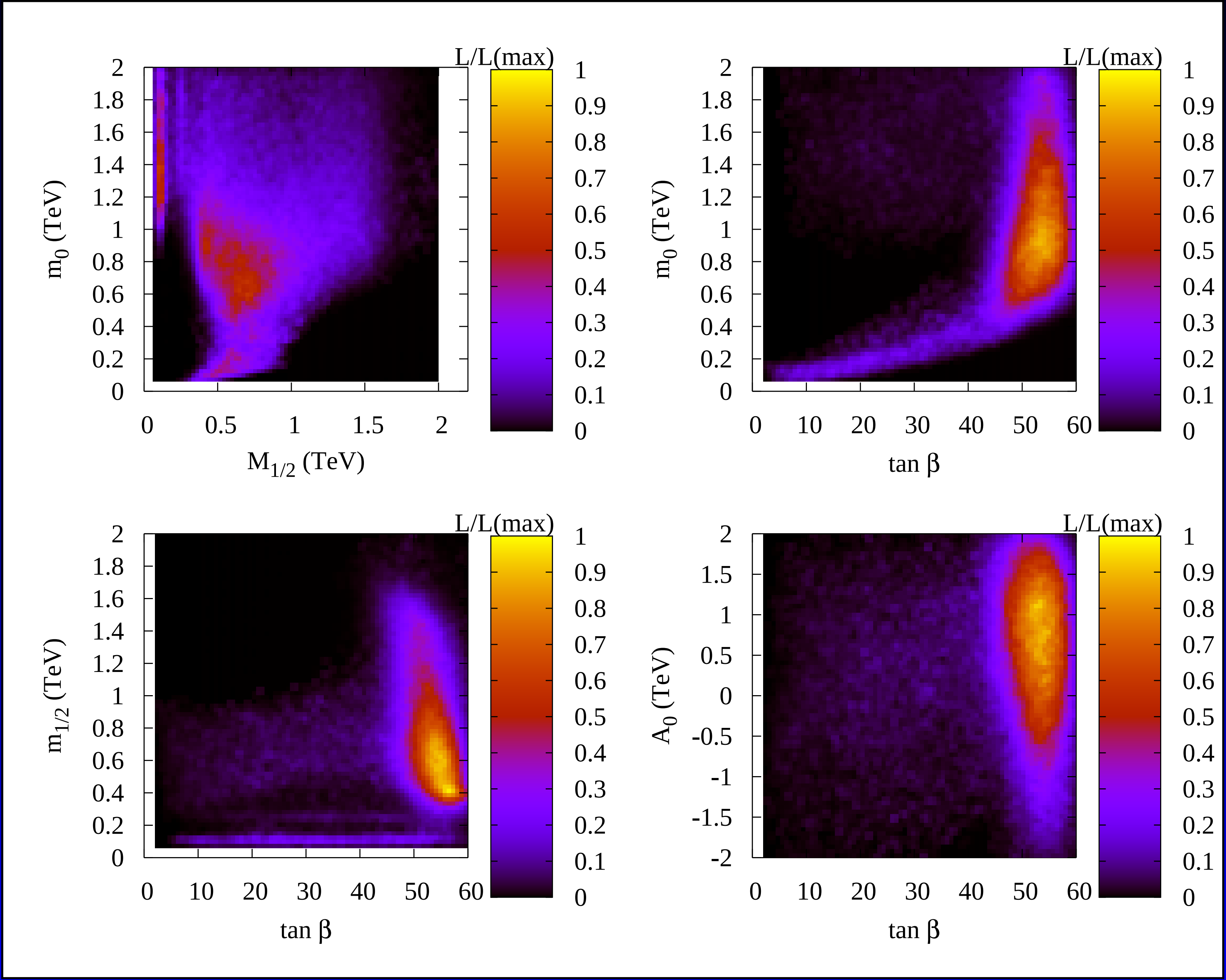


Expect higher orders to be 100 times smaller than these differences: 3-loop terms could possibly be important!

Coannihilation Theory Uncertainties



Effect of 2-loop RGE terms suggest a possible effect from 3 loops. Jack and Jones find that it's not significant for the neutralino.



m_0 (TeV)
M_1/2 (TeV)
L/L(max)
tan β
m_1/2 (TeV)
A_0 (TeV)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.5
1.5
10
20
30
40
50
60
0.1
0.3
0.5
0.7
0.9
-0.5
-1
-1.5
-2

Iterative Procedure

What change in a parameter produces a 10% change in Ωh^2?

Take a parameter point with $\omega_{-1} \equiv \Omega h^2$. Change *one parameter at a time* by fraction a_0. Result is ω_0, then iterate

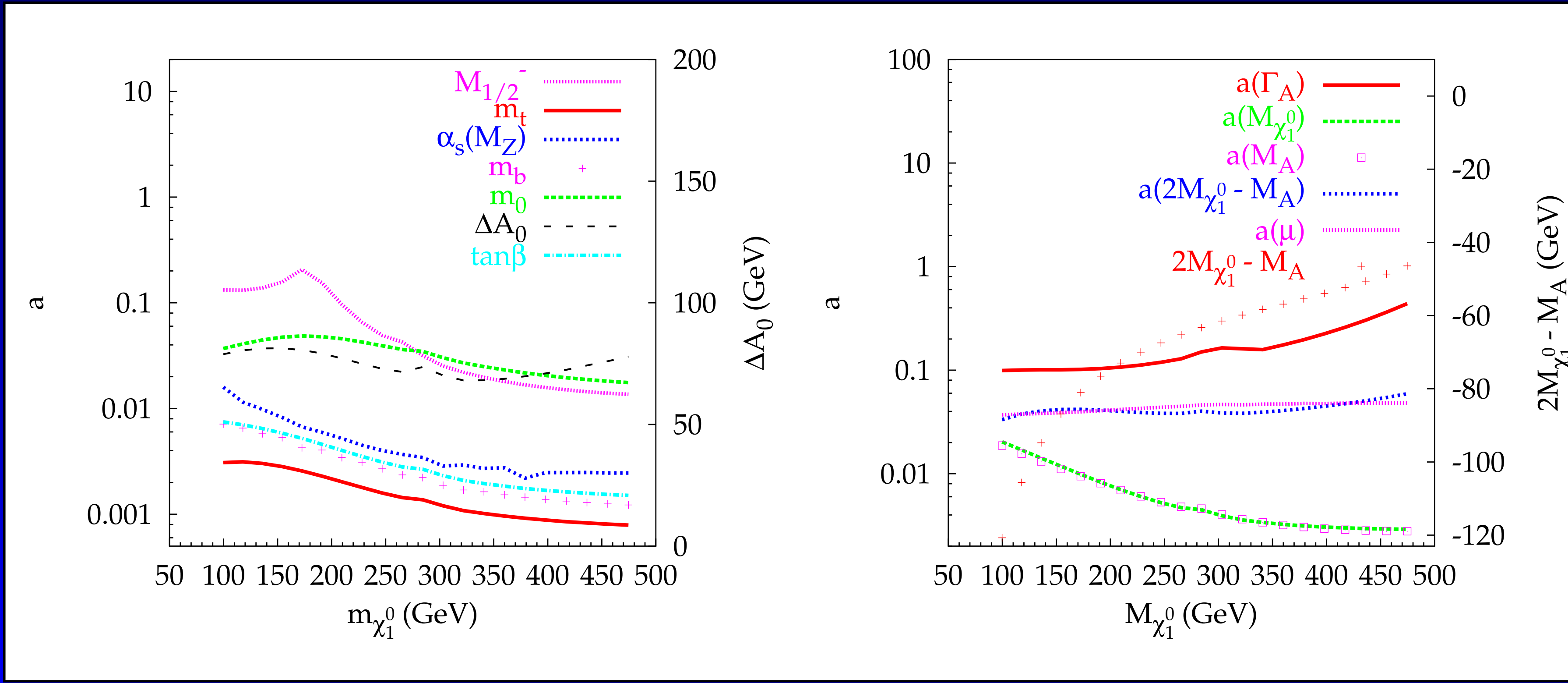
$$a_{i+1} = a_i \omega_{-1} \frac{10\%}{w_i - \omega_{-1}}.$$

Small accuracy $a \equiv a_\infty$ means the parameter has to be known very accurately in order to predict Ωh^2 to 10%.

For parameters that are zero, we take the absolute value as a rather than the fractional value.

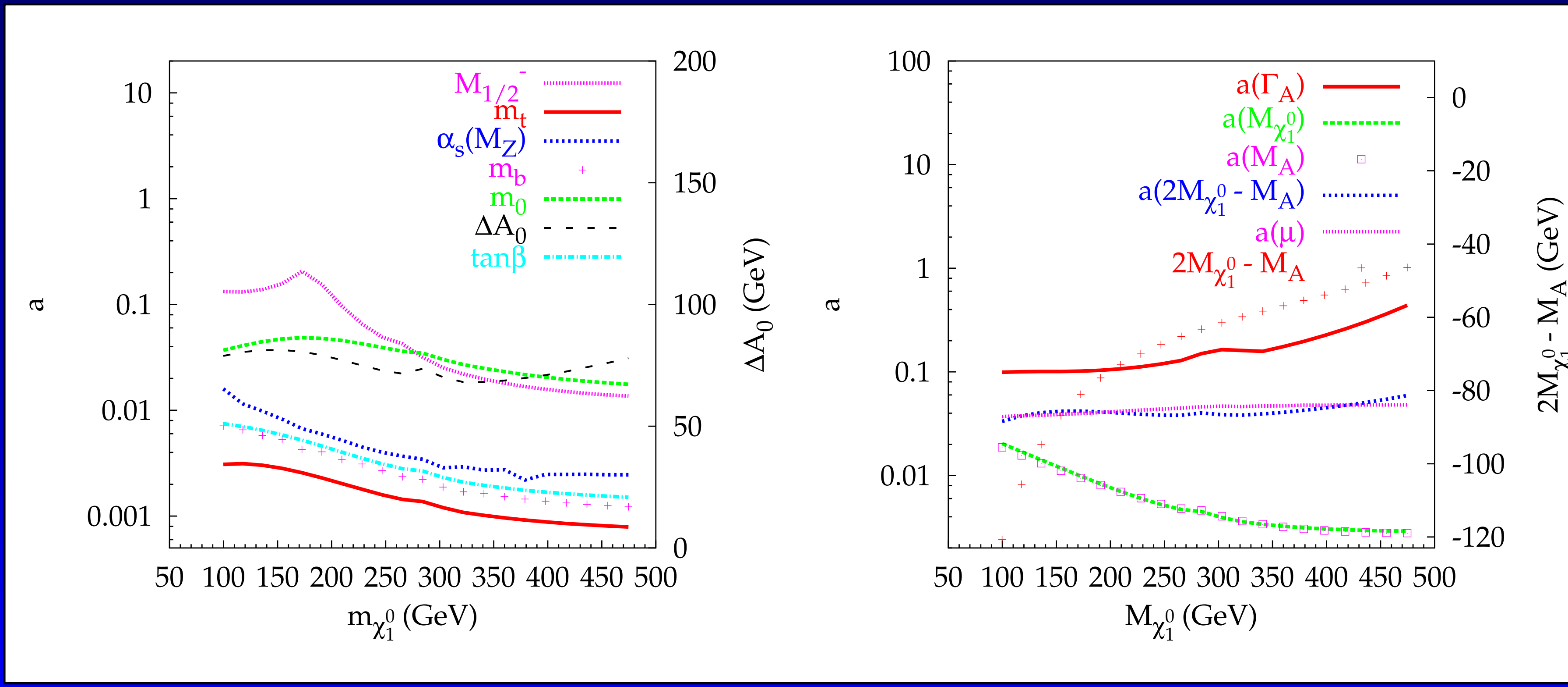
Funnel Accuracies

LHS: CMSSM. $a(m_b)$ *worrying*. $\alpha_s(M_Z)$ dependence comes about through its effect on $m_b(m_b)$. m_0, $M_{1/2}$ might be feasible at LHC, m_t possible at ILC. $\tan\beta$ looks impossible.



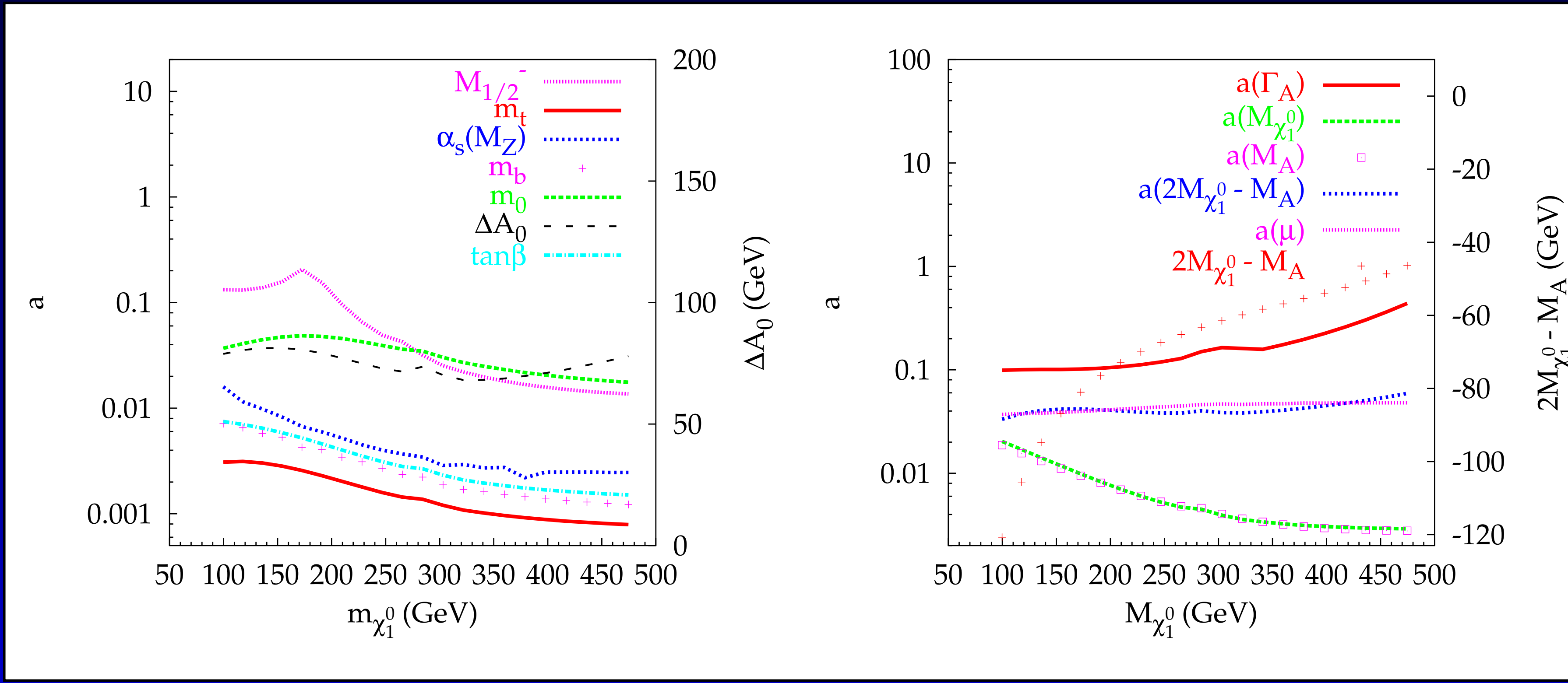
Funnel Accuracies

SM inputs and $\tan\beta$ uncertainties can be controlled by measuring M_A, Γ_A. $A\chi_1^0\chi_1^0$ coupling $\sim 1/\mu$. $\Gamma_A \propto M_A \tan^2\beta(m_b^2 + m_\tau^2)$ ($\gamma\gamma$ option of LC, $A \to \mu\mu$ at LHC).

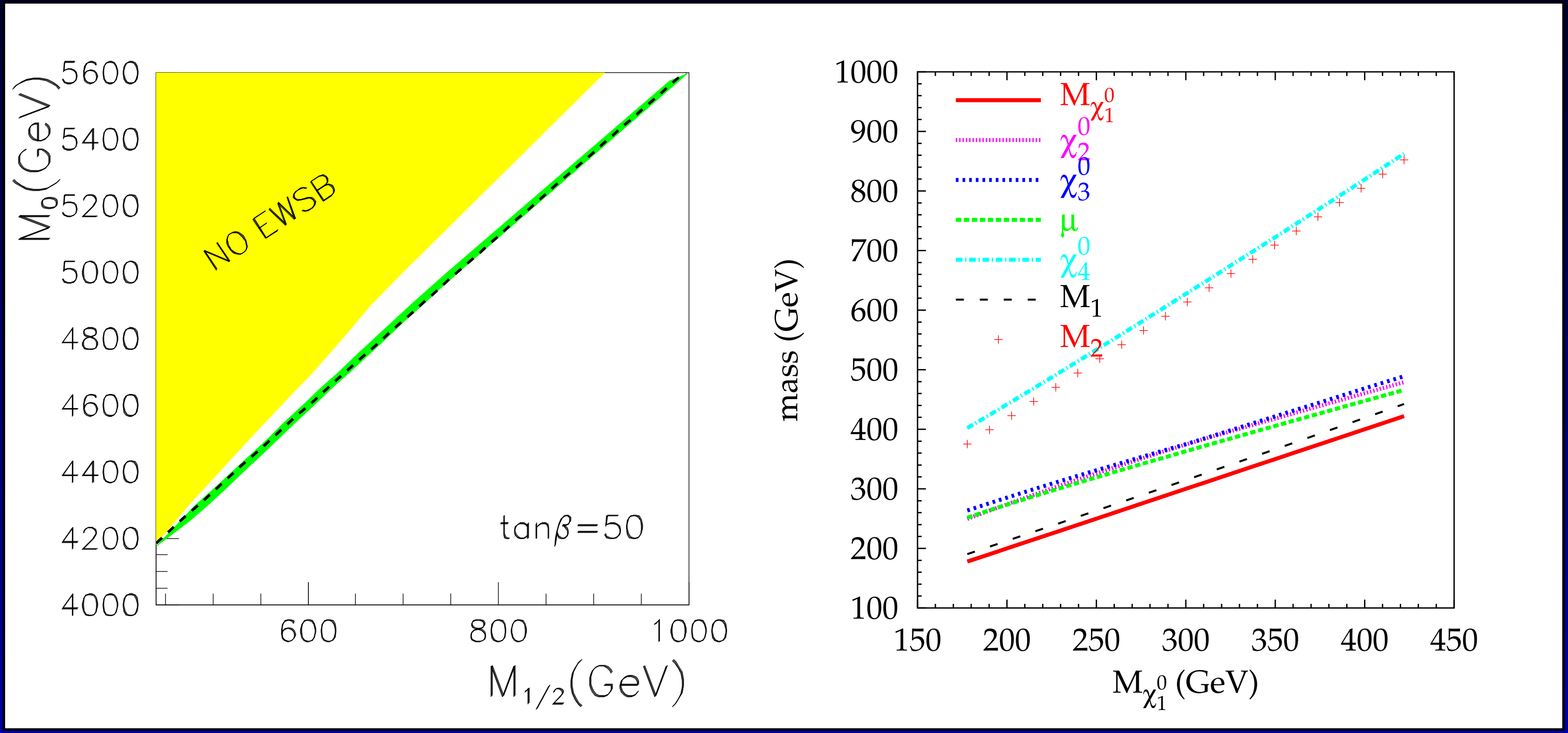


Funnel Accuracies

Mass of χ_1^0 is important, but not mixing (see $a(\mu)$).

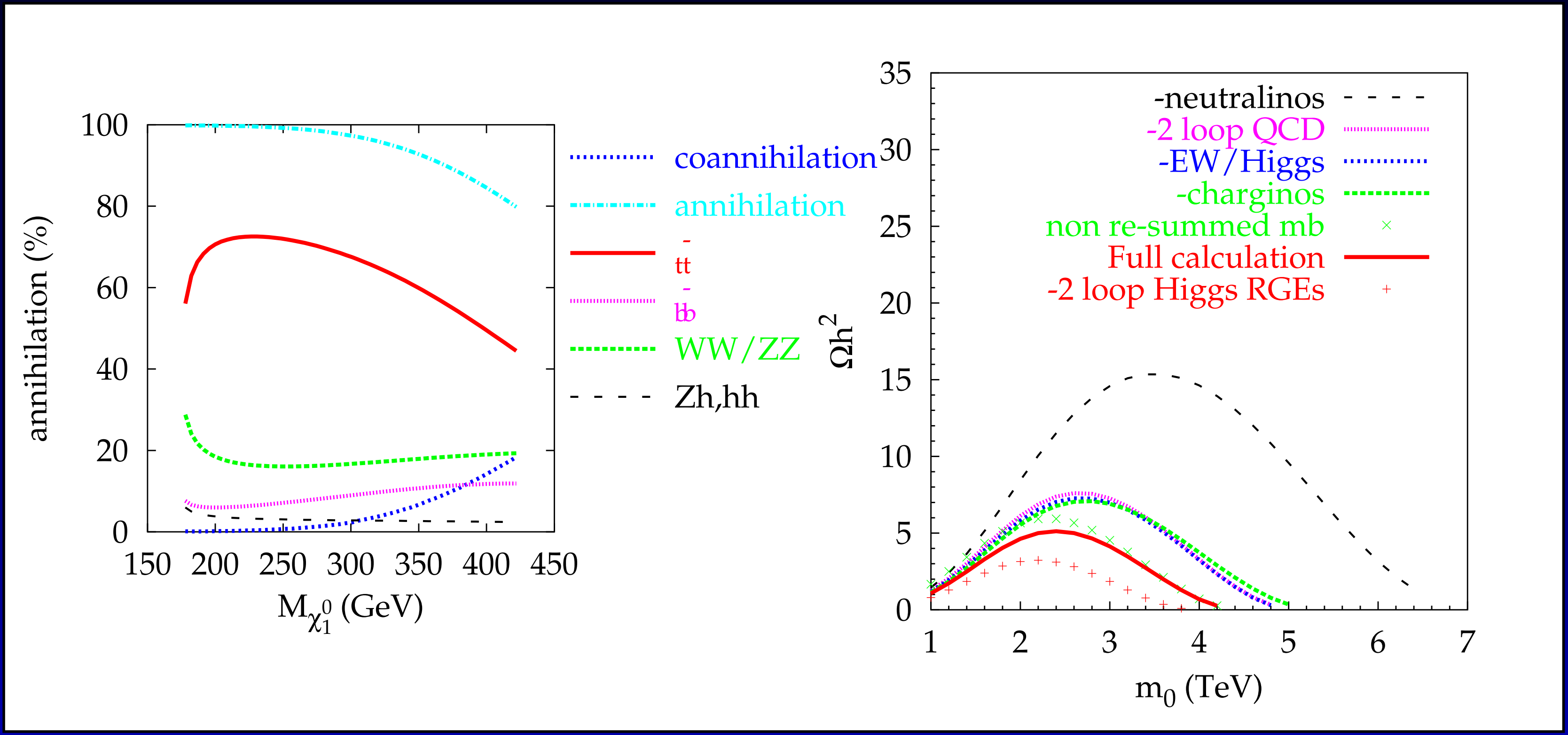


Focus Point Slope



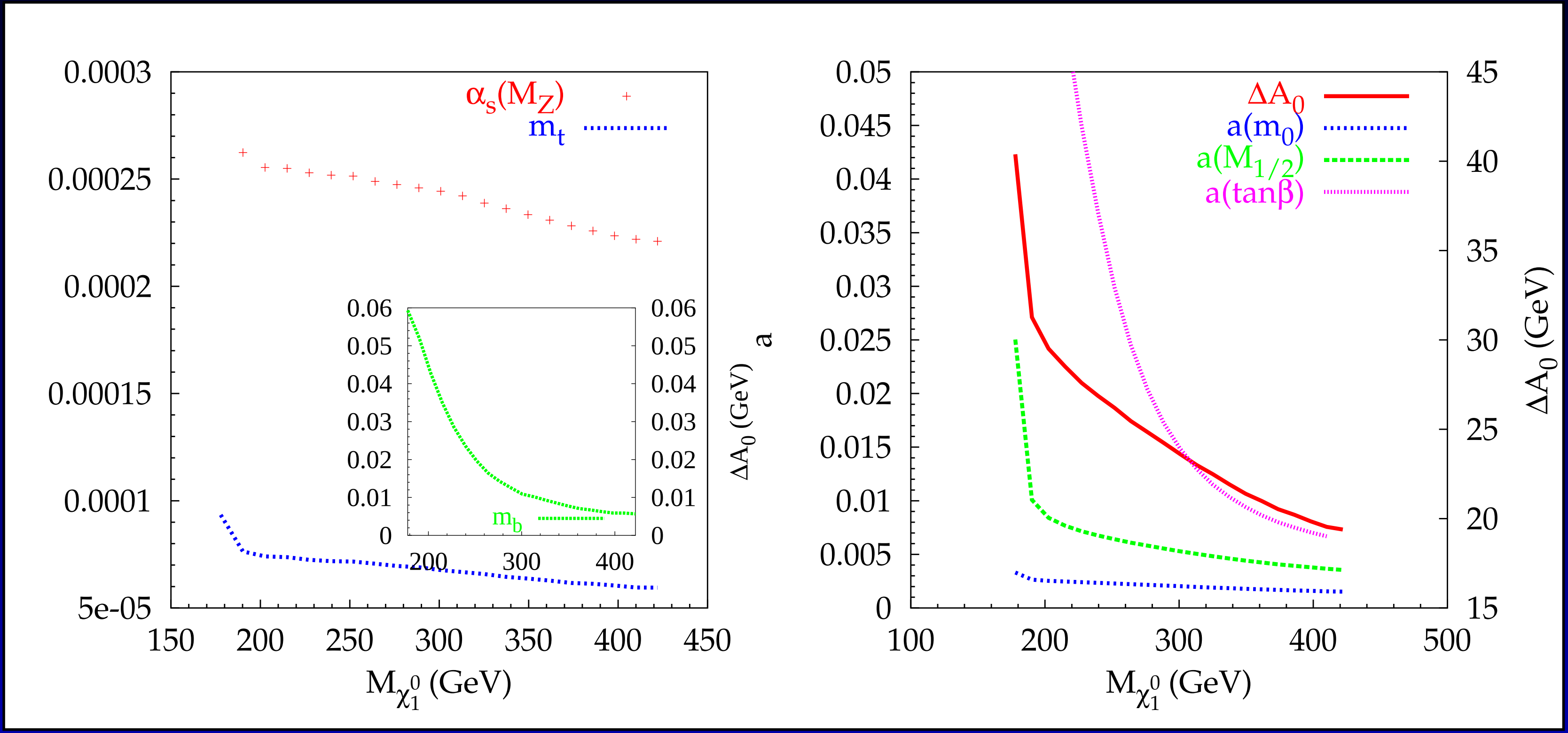
Heavy sfermions and A^0. $M_1 < \mu < M_2$, ie significant *Higgsino* component $\sim 25\%$.

Focus Channels and Theory



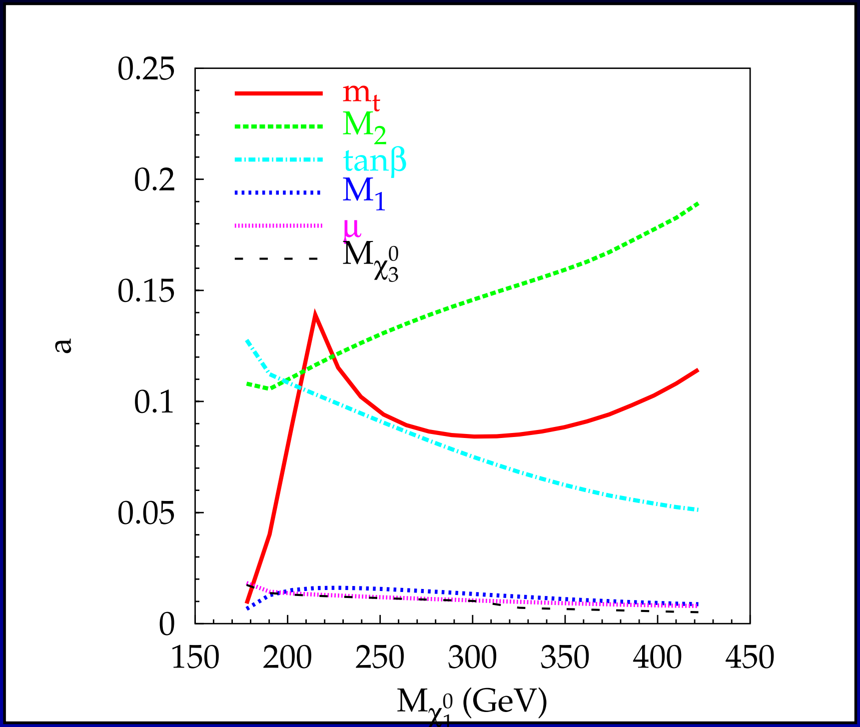
$t\bar{t}$ annihilation predominantly through Z. Coannihilation $\equiv \chi_1^0\chi_i^0$ or $\chi_1^0\chi_1^\pm$. Several competing channels.

Focus CMSSM Accuracies



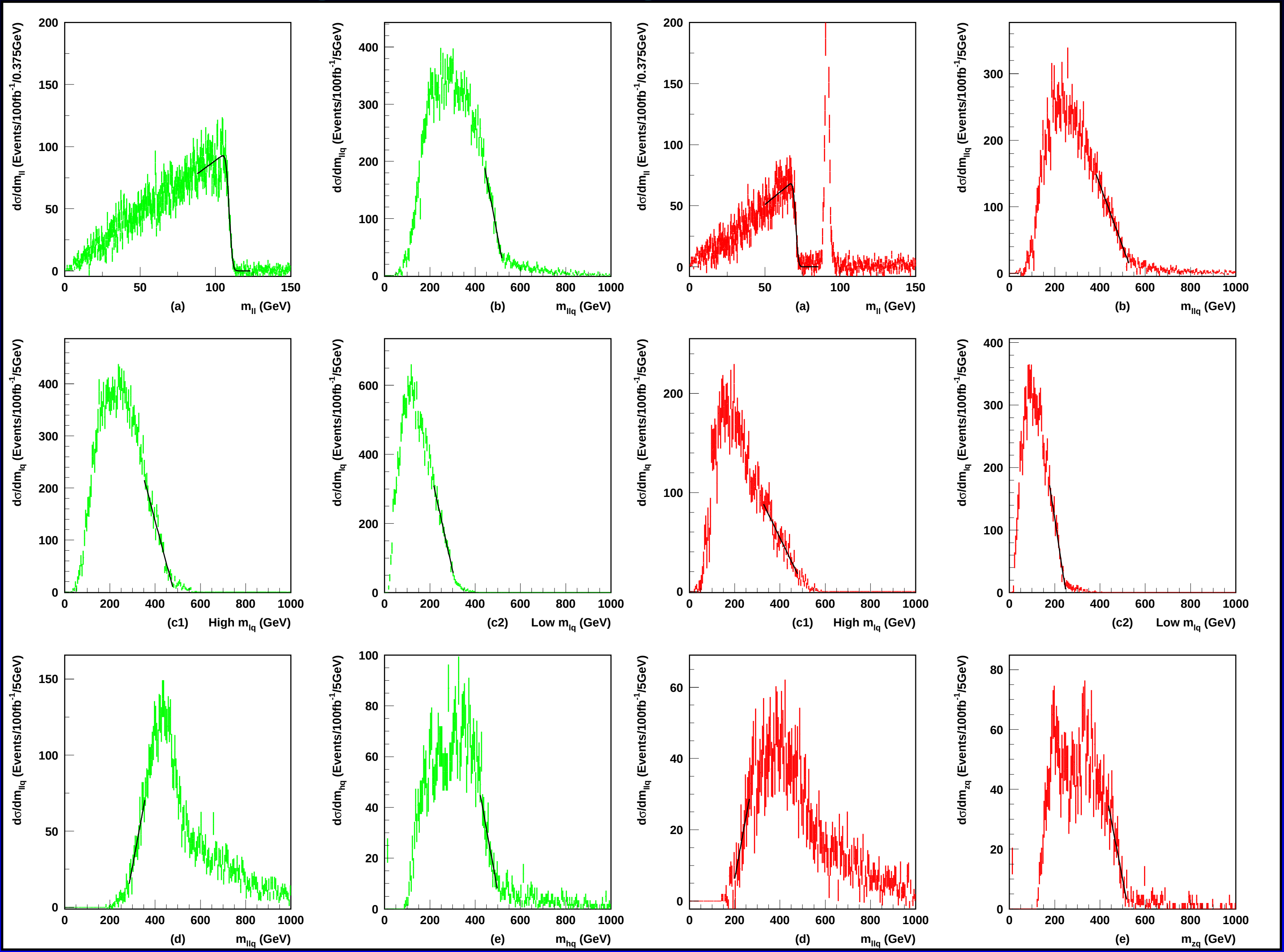
$\delta m_t = 30$ MeV might be possible at future ILC but $a(m_0) < 0.5\%$ looks completely unfeasible.

Focus PCMSSM Accuracies



Easier outside of CMSSM, eg μ no longer sensitive to m_t ($\propto$ coupling to neutral goldstone).

Edge Fitting at S5 and O1

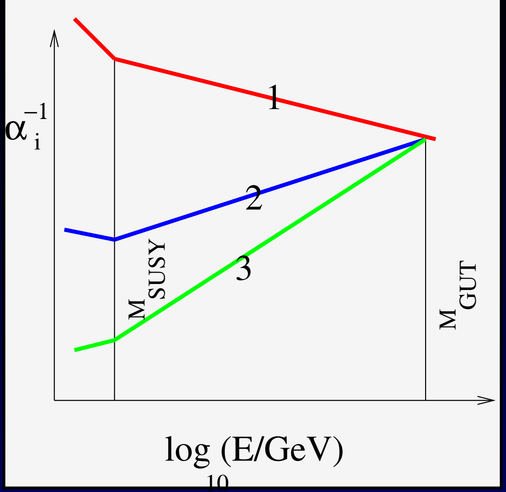


Edge Positions

endpoint	S5 fit	O1 fit
m_{ll}	109.10±0.13	70.47±0.15
m_{llq} edge	532.1±3.2	544.1±4.0
lq high	483.5±1.8	515.8±7.0
lq low	321.5±2.3	249.8±1.5
llq thresh	266.0±6.4	182.2±13.5

Best case lepton mass measurements can be as accurate as 1 per mille, but jets are a few percent

SOFTSUSY



Get $g_i(M_Z)$, $h_{t,b,\tau}(M_Z)$. ←

↓

Run to M_S.

↓

REWSB, iterative solution of μ

↓

M_X. Soft SUSY breaking BC.

↓

Run to M_S. Calculate sparticle pole masses.

↓

Run to M_Z →

BCA, Comp. Phys. Comm. 143 (2002) 305.

Other Observables

Often more complicated, eg m_{llq} edge:

$$\max\left[\frac{(m_{\tilde{q}}^2 - m_{\chi_2^0}^2)(m_{\chi_2^0}^2 - m_{\chi_1^0}^2)}{m_{\chi_2^0}^2}, \frac{(m_{\tilde{q}}^2 - m_{\tilde{l}}^2)(m_{\tilde{l}}^2 - m_{\chi}^2}{m_{\tilde{l}}^2}\right.$$

$$\left.\frac{(m_{\tilde{q}} m_{\tilde{l}} - m_{\chi_2^0} m_{\chi_1^0})(m_{\chi_2^0}^2 - m_{\tilde{l}}^2)}{m_{\chi_2^0} m_{\tilde{l}}}\right]$$

Also m_{lq}^{high}, m_{lq}^{low}, llq *threshold* , $M_{T_2}^2(m) =$

$$\min_{\not{p}_1 + \not{p}_2 = \not{p}_T}\left[\max\left\{m_T^2(p_T^{l_1}, \not{p}_1, m), m_T^2(p_T^{l_2}, \not{p}_2, m)\right\}\right],$$

$\max[M_{T_2}(m_{\chi_1^0})] = m_{\tilde{l}}]$ for dislepton production.

Statistics Study

- Choose two model-points: S5 ($m_0 = 100, m_{1/2} = 300, A_0 = 300, \tan\beta = 2.1, \mu > 0$) and O1 ($m_{\tilde{l}} = 177, m_{1/2} = 306, A_{\tilde{q}} = 137, m_{\tilde{q}} = 0, A_{\tilde{l}} = 306, \tan\beta = 10, \mu > 0$)
- Find cuts to measure “signal” endpoints
- Estimate expected accuracy of ATLAS measurement: 100 fb^{-1}
- Perform χ^2 fits of sparticle masses to expected positions of edges expected from an ensemble of experiments
- Interpret results as statistics of measurement on sparticle masses

Important points: there is no one m

Cuts Example

We use `ATLFAST2.16`, `HERWIG6.0`, `ISAWIG` and `ISAJET7.42`. Assume 100 fb^{-1} of LHC data.

- $|\eta_j| \leq 5$, $p_T^j \geq 15$ GeV
- $p_T^e \geq 5$, $p_T^\mu \geq 6$, $|\eta_l| \leq 2.5$
- l isolation: 10 GeV in $\Delta R = 0.2$, $\Delta R(lj) \geq 0.4$.

eg for m_{ll}:

- 2 OSSF leptons, $p_T^{l_1} \geq p_T^{l_2} \geq 10$ GeV.
- $n_{jets} \geq 2$, $p_T^{j_1} \geq p_T^{j_2} \geq 150$ GeV, $\not{p}_T > 300$ GeV

OSSF-OSDF subtracts well the Standard Model background.

Uncertainties in Relic Density

Bulk region: $\tilde{B}\tilde{B} \rightarrow Z, h \rightarrow l\bar{l}$. Coannihilation: $\tilde{\tau}\chi_1^0 \rightarrow \tau + X$

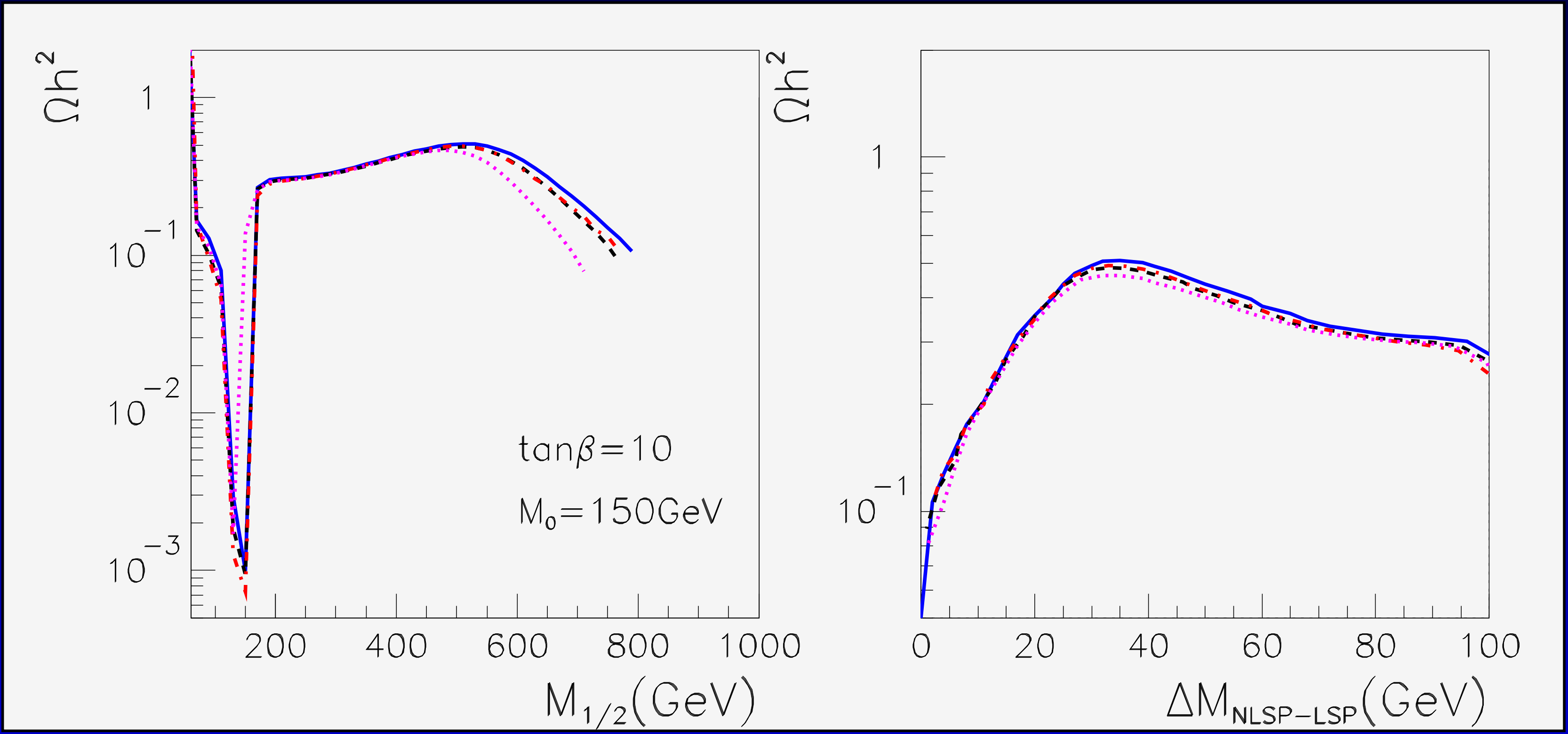


Figure 0: Bulk/coannihilation region. Full: `SoftSusy`, dotted: `SPheno`.

Focus Point

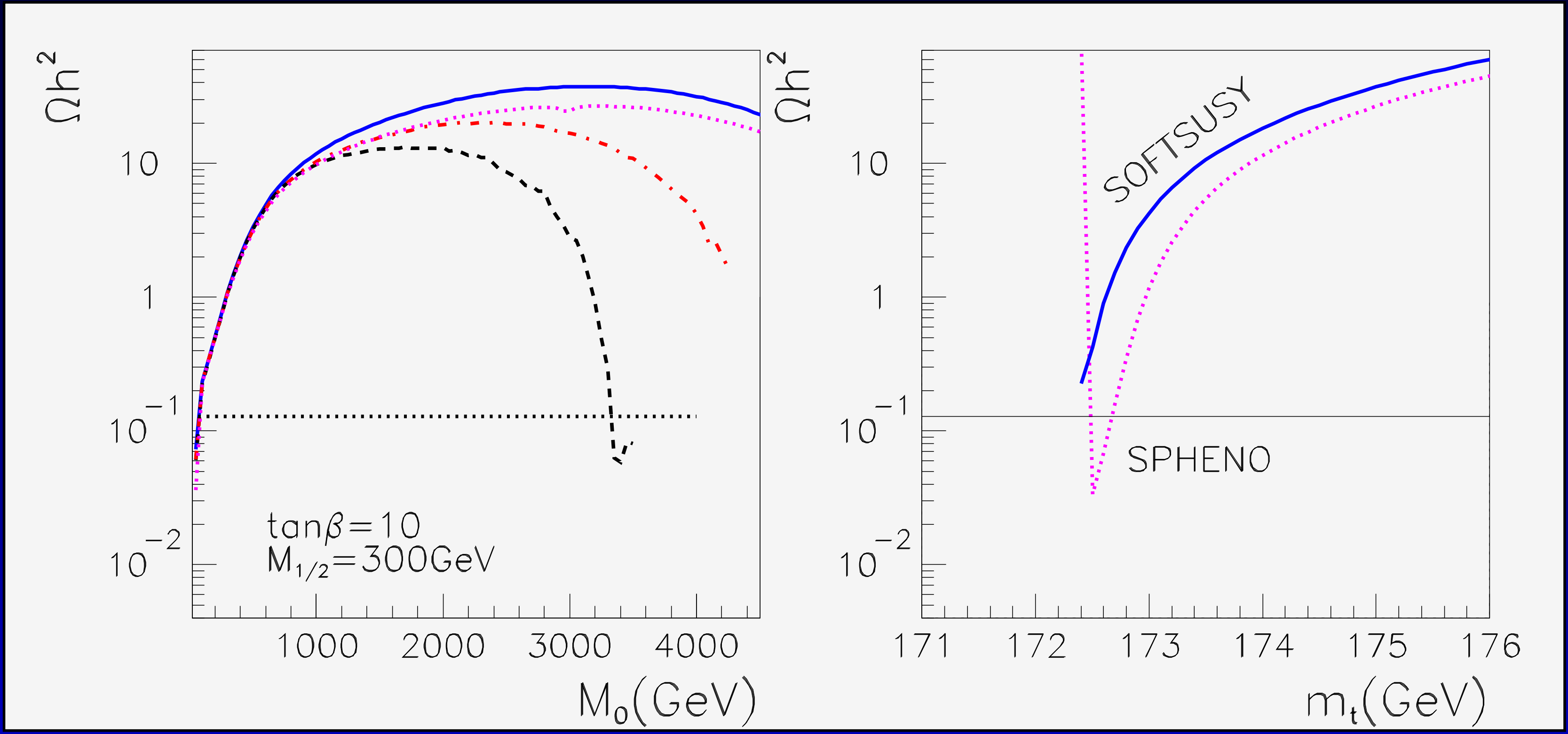


Figure 0: Focus point region. Full: `SoftSusy`, dotted: `SPheno`, dashed: `SuSpect`. Higgsino LSP annihilates into ZZ/WW

High $\tan\beta$

BCA, Belanger, Boudjema, Pukhov, Porod, hep-ph/0402161. Baer *et al*

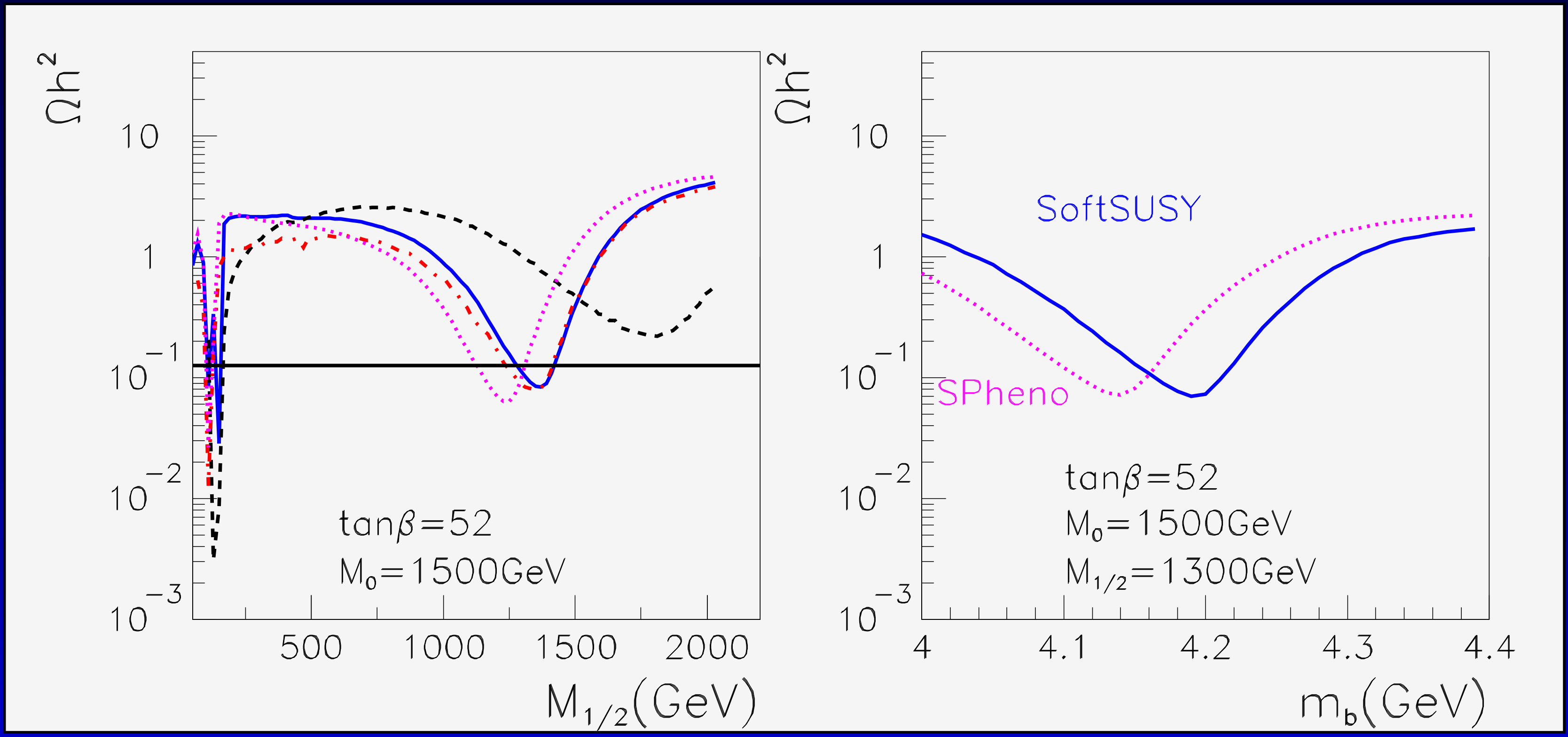
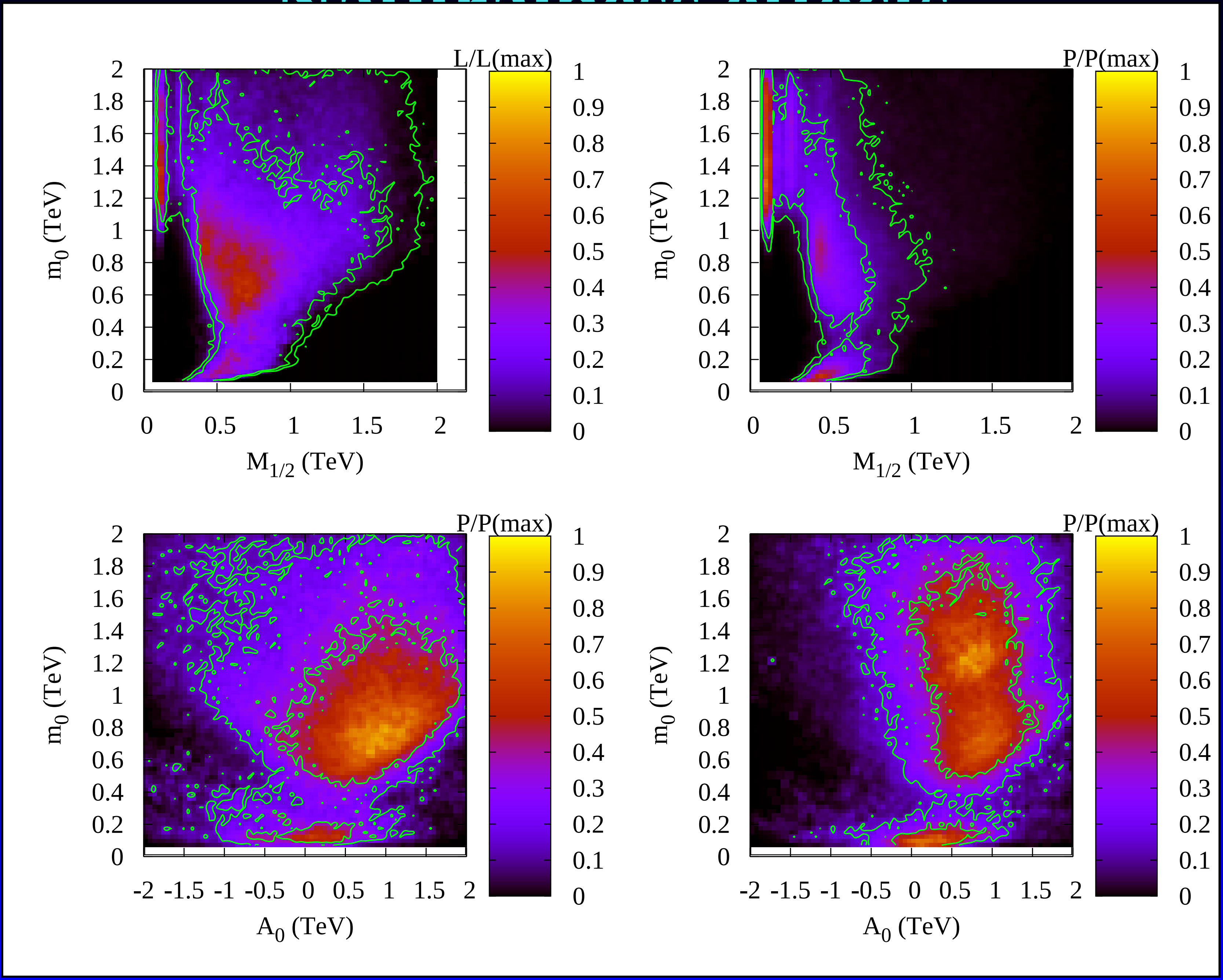
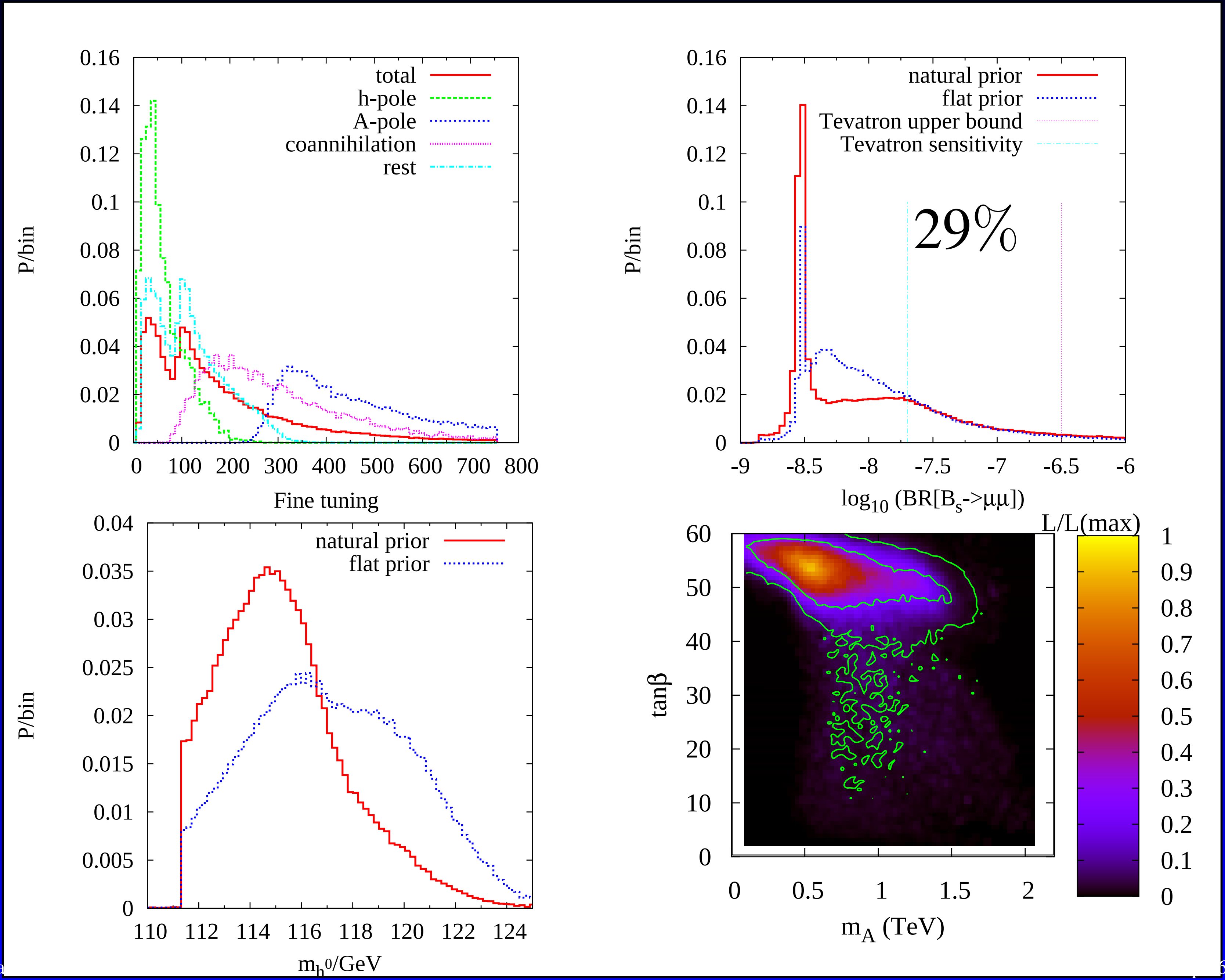


Figure 0: High $\tan\beta$ region. Full: `SoftSusy`, dotted: `SPheno`, dashed: `SuSpect`. Get annihilation into A.

Naturalness effects

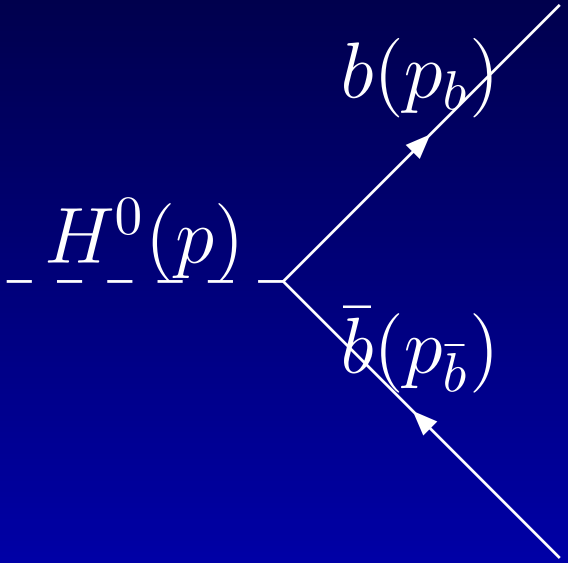


Prior Dependence



SUSY Kinematics: a Reminder

Take a particle decaying into 2 particles, eg $H^0 \rightarrow b\bar{b}$. We define the invariant mass of the $b\bar{b}$ pair such that:



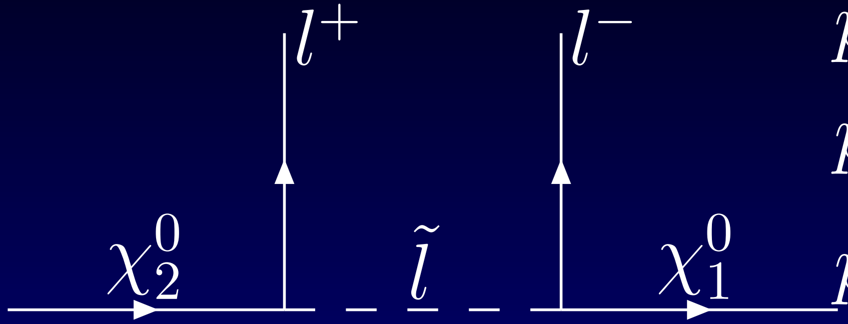
$$p^\mu = (\sqrt{m_H^2 + p^2}, \underline{p}) = p_b^\mu + p_{\bar{b}}^\mu$$

$$\Rightarrow p^2 = m_H^2 = (p_b + p_{\bar{b}})^2$$

Is *invariant* in boosted frames

Question: What happens to invariant mass in SUSY cascade decays, where we miss the final particle?

Cascade Decay



$$p^\mu_{\tilde{l}} = (m_{\tilde{l}}, \underline{0})$$

$$p^\mu_{l^\pm} = (|\underline{p}_{l^\pm}|, \underline{p}_{l^\pm})$$

$$p^\mu_{\chi^0_{1,2}} = (\sqrt{{m_{\chi^0_{1,2}}}^2 + |\underline{p}_{\chi^0_{1,2}}|^2},$$

The invariant mass of the l^+l^- pair is

$$m^2_{ll} = (p_{l^+} + p_{l^-})^\mu (p_{l^+} + p_{l^-})_\mu = p^2_{l^+} + p^2_{l^-} + 2p_{l^+} \cdot p_{l^-}$$

$$= 2|\underline{p}_{l^+}||\underline{p}_{l^-}|(1 - \cos\theta) \leq 4|\underline{p}_{l^+}||\underline{p}_{l^-}|.$$

Momentum conservation:

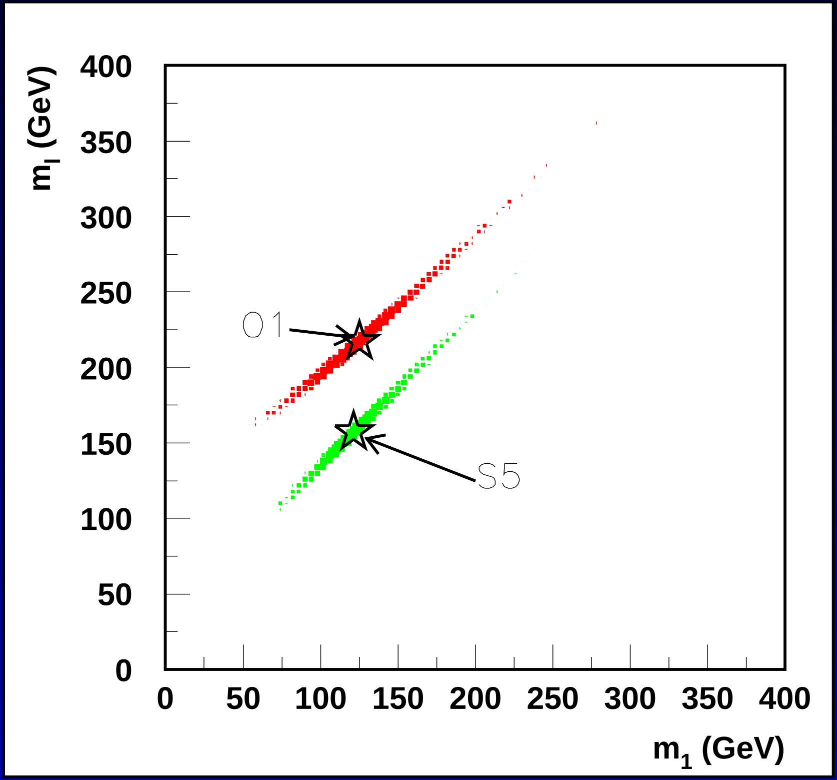
$$\Rightarrow \underline{p}_{\chi^0_2} + \underline{p}_{l^+} = \underline{0}, \qquad \underline{p}_{l^-} + \underline{p}_{\chi^0_1} = \underline{0}.$$

Energy conservation: $\sqrt{{m_{\chi^0_2}}^2 + |\underline{p}_{l^+}|^2} = m_{\tilde{l}} + |\underline{p}_{l^+}|,$

$$\Rightarrow |\underline{p}_{l^+}| = \frac{m^2_{\chi^0_2} - m^2_{\tilde{l}}}{2m_{\tilde{l}}}.$$ Similarly $$|\underline{p}_{l^-}| = \frac{m^2_{\tilde{l}} - m^2_{\chi^0_1}}{2m_{\tilde{l}}}.$$

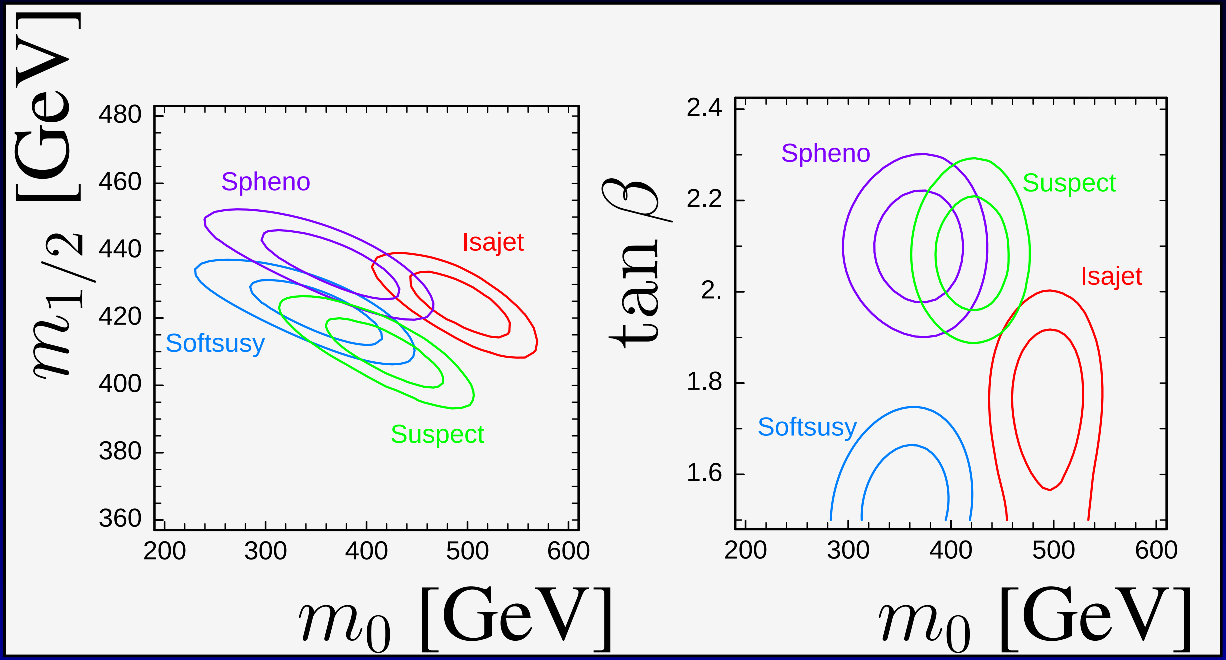
Edge to Mass Measurements

	width S5	width O1
χ_1^0	17	22
$\tilde{l}_R$	17	20
χ_2^0	17	20
$\tilde{q}$	22	20



Mass differences well constrained, but overall mass scale not so well constrained by LHC

Fitting to SUSY Breaking Model



- Experimenters pick a SUSY breaking point
- They derive observables and errors after detector simulation
- We fit this “data” with our codes

BCA, S Kraml, W Porod, JHEP 0303 (2003) 016

Uncertainties

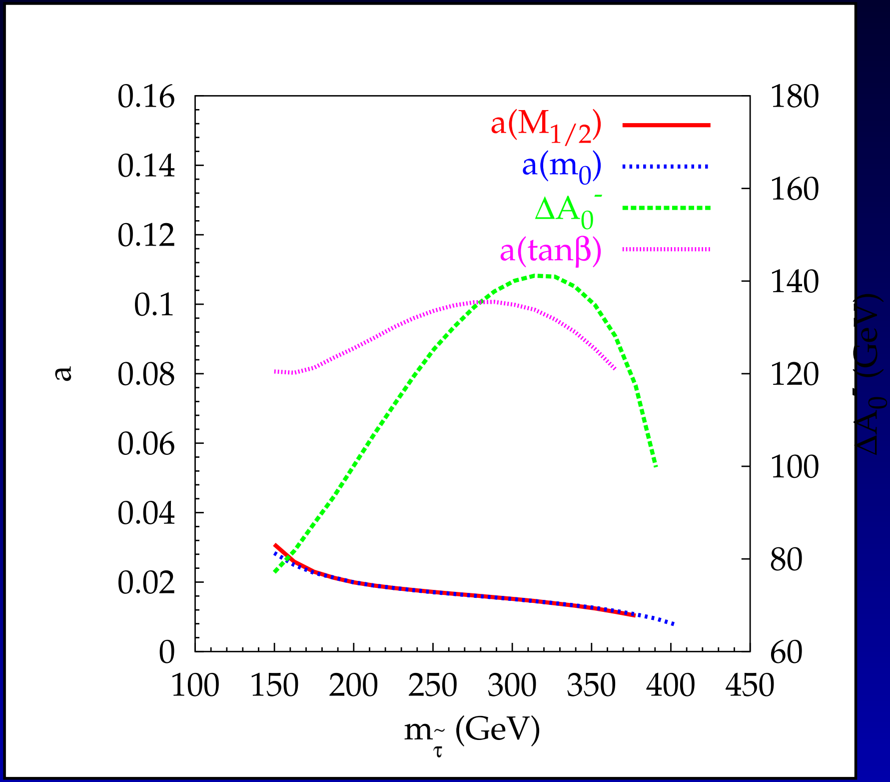
We use two approaches to determine what variation of parameters produce a 10% variation in Ωh^2:

- **PCMSSM** - variation of weak scale parameters (*not* on CMSSM trajectory): $m_{\chi_1^0}, M_A, m_b$ etc.
- **CMSSM** - simple variation of mSUGRA parameters and experimental inputs: $m_0, M_{1/2}, \alpha_s(M_Z), m_t$ etc.

mSUGRA theory uncertainties estimated by varying scale at which radiative corrections added to sparticle masses:

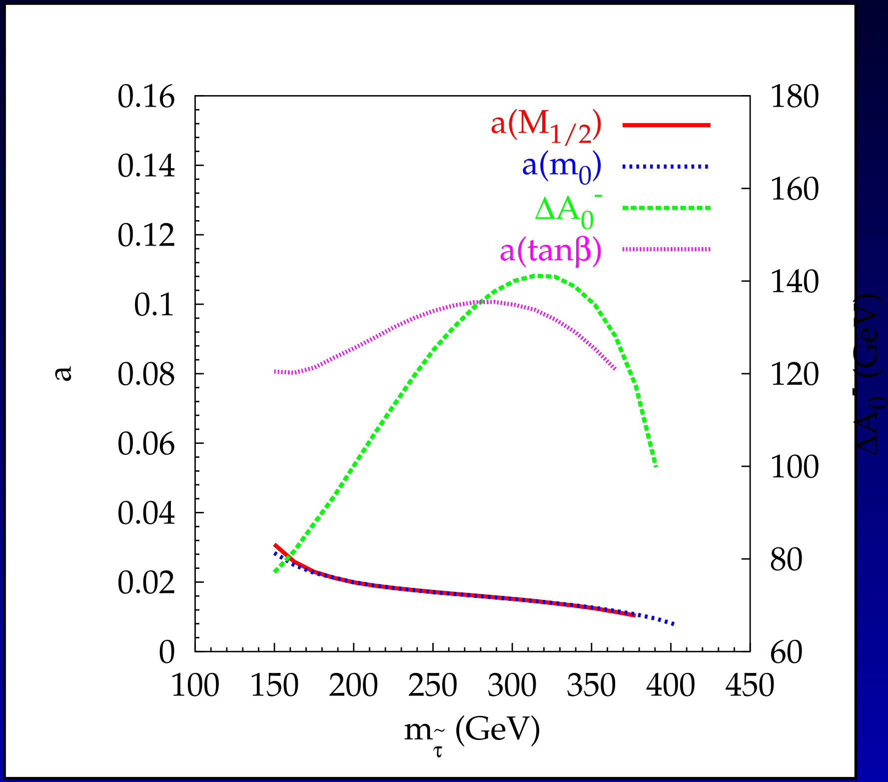
$$0.5 < x \equiv \frac{M_{SUSY}}{\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}} < 2, \qquad M_{SUSY} > M_Z$$

CMSSM Coannihilation Uncertainties



$a(m_0) \approx a(M_{1/2})$ comes from the sensitivity to $\exp[-(m_{\tilde{\tau}} - M_{\chi_1^0})]$

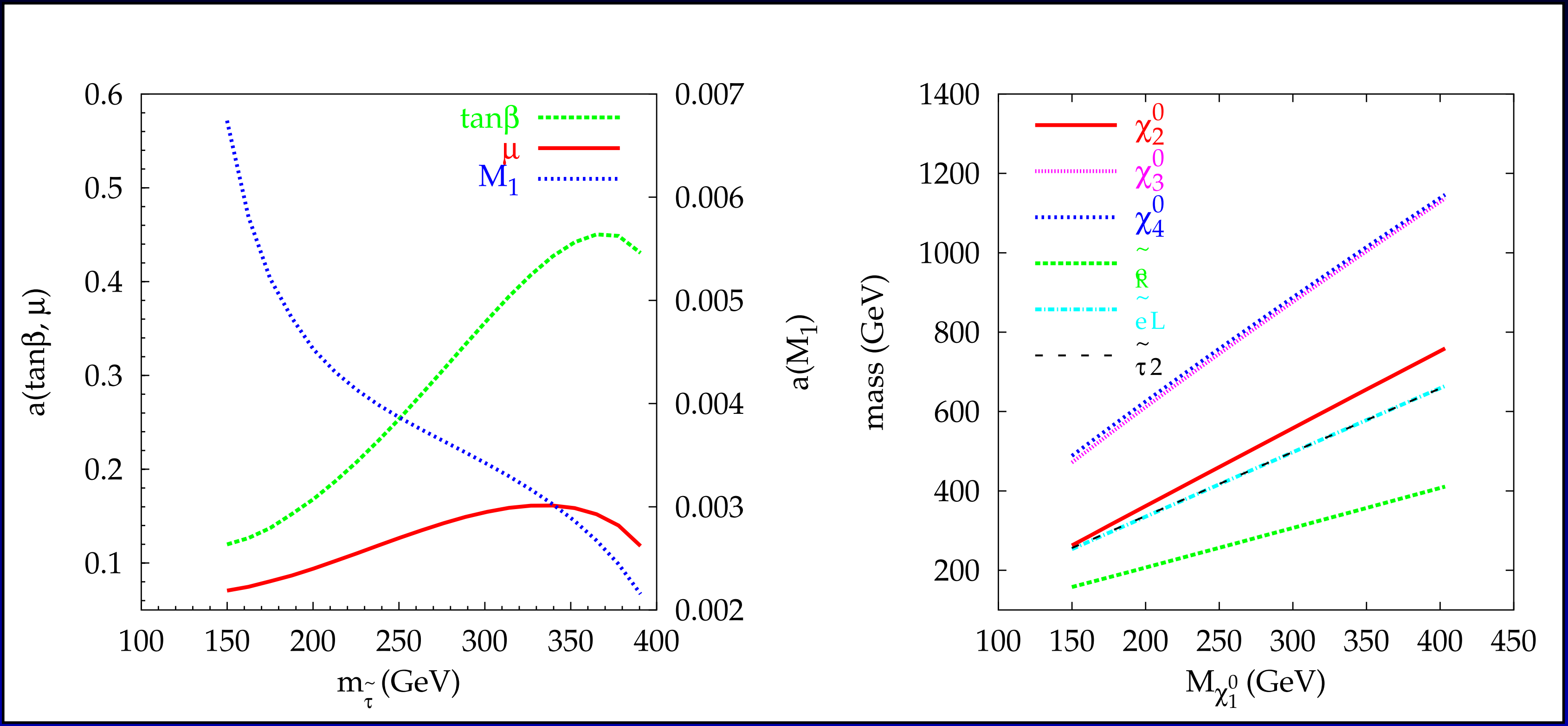
CMSSM Coannihilation Uncertainties



Unknown whether accuracies can be reached - but it looks difficult : $\Delta\Omega h^2 \sim .03$ in diminished bulk region.

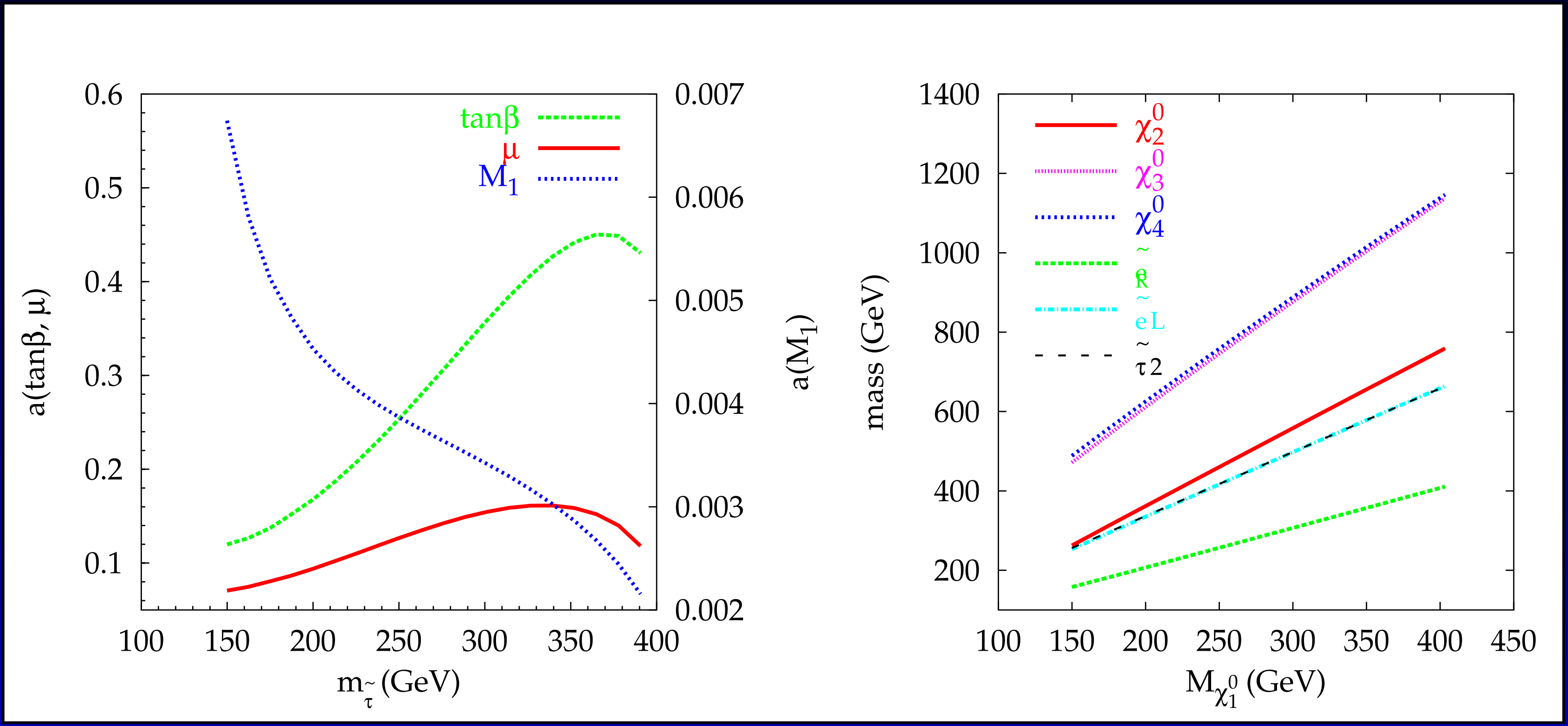
Polesello, Tovey, JHEP05 (2004) 071

PCMSSM Coannihilation



RHS: Spectrum useful for optimal energy of linear collider. $\tilde{e}_R, \tilde{\mu}_R$ also possible. Cascade $\tilde{q}_L \longrightarrow \chi_2^0 \longrightarrow \tilde{e}_R \longrightarrow \chi_1^0$ available.

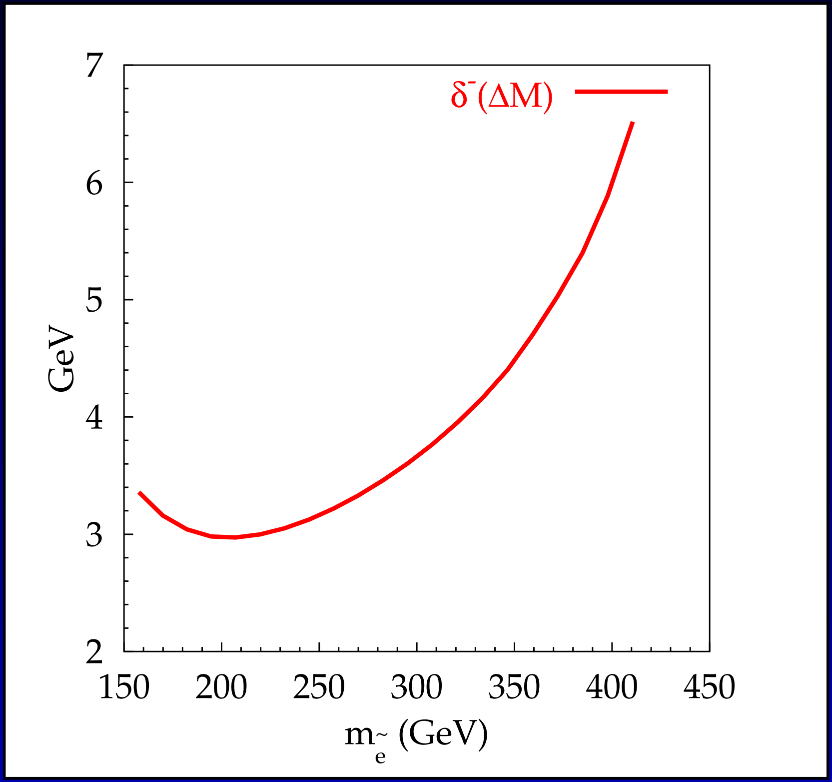
PCMSSM Coannihilation



LHS: Dependences in left-hand plot all come from the effect on LSP mass.

Need to know $M_{\chi_1^0}$ very accurately.

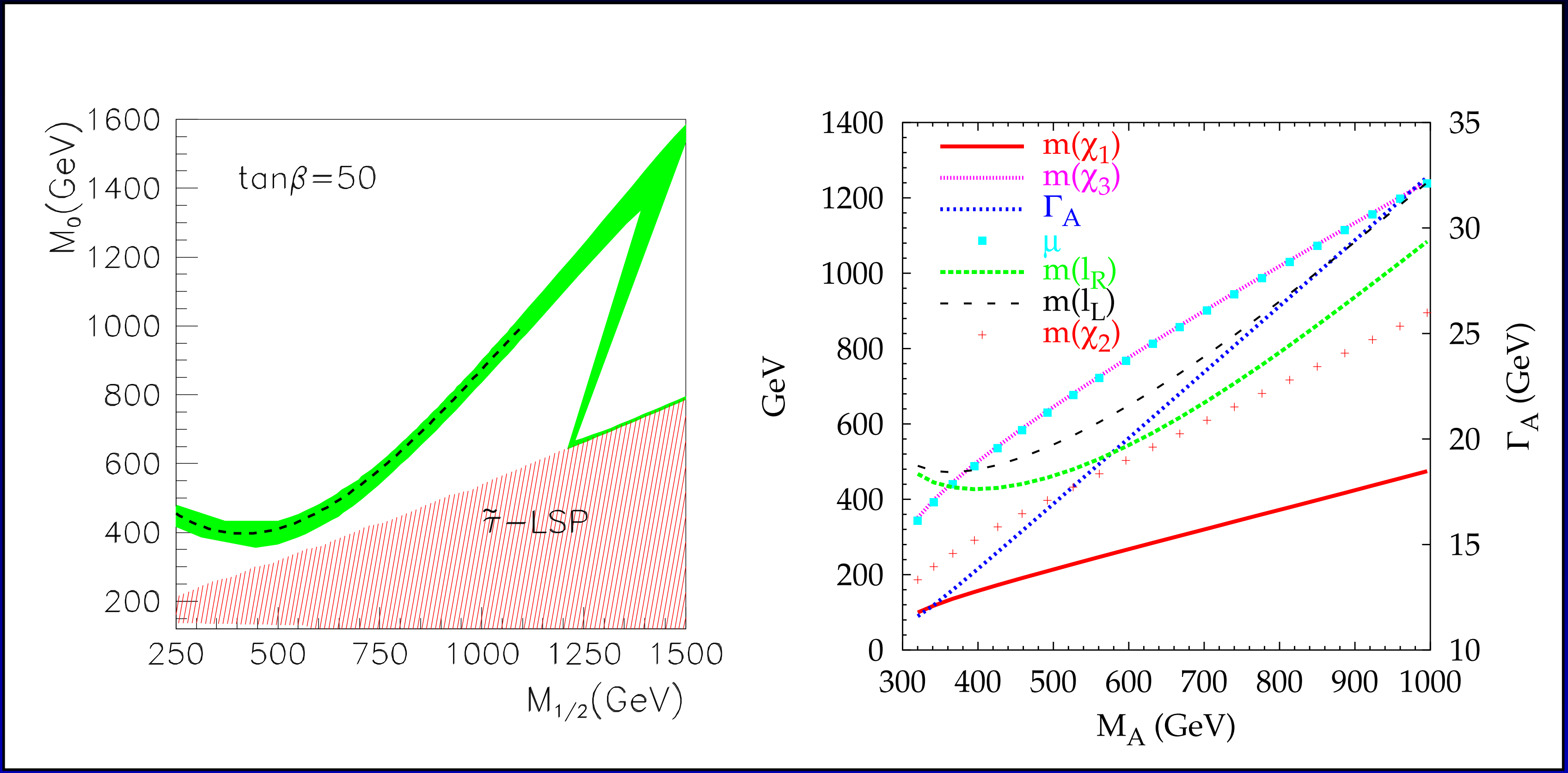
Slepton Dependence



Accuracy required on $m_{\tilde{l}} - m_{\chi_1^0}$ for WMAP precision.

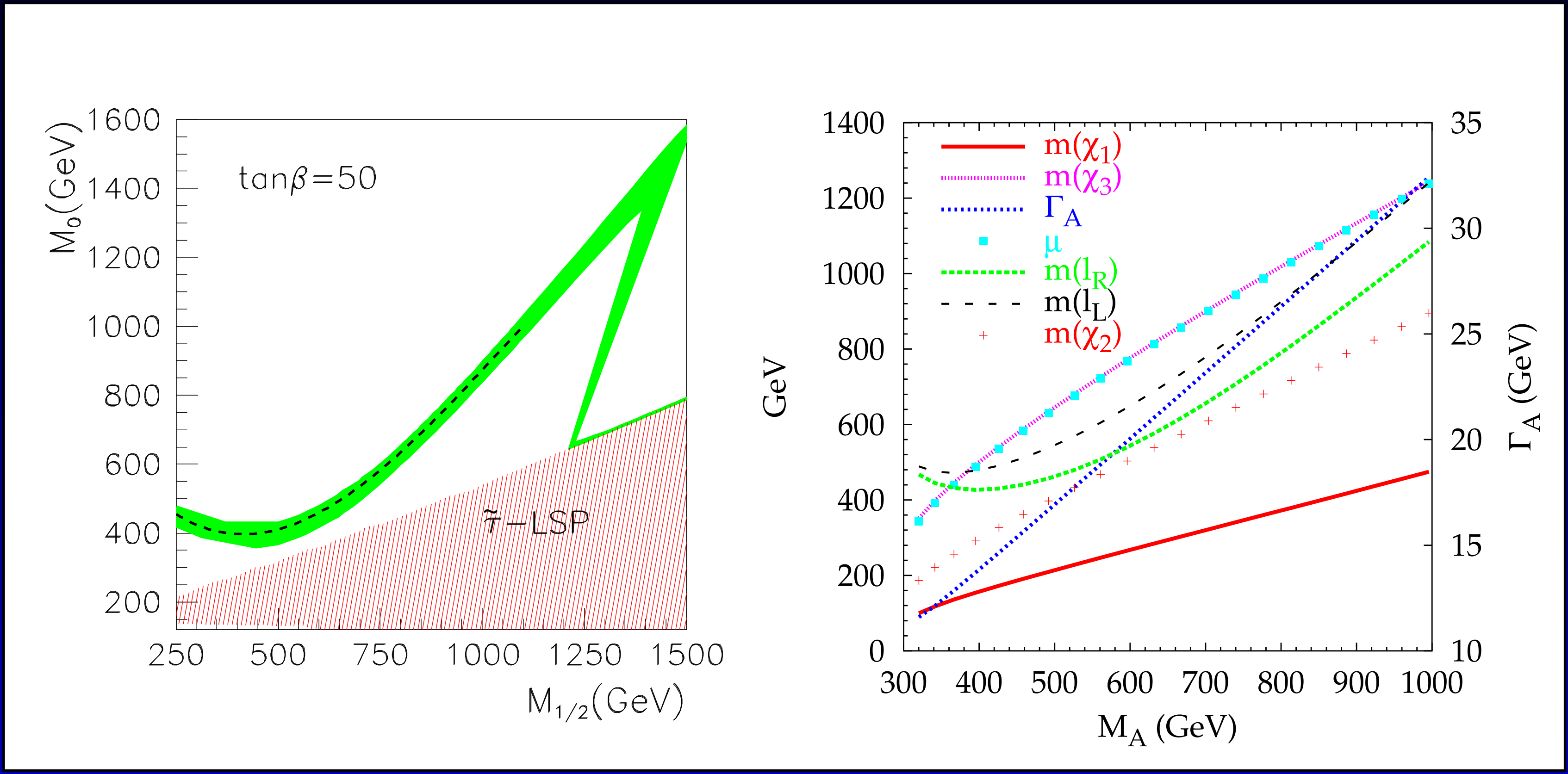
LC studies say this is acheivable, but need more work for $\cos\theta_\tau$ (=0.987±0.06 at lower end of slope).

Funnel Slope



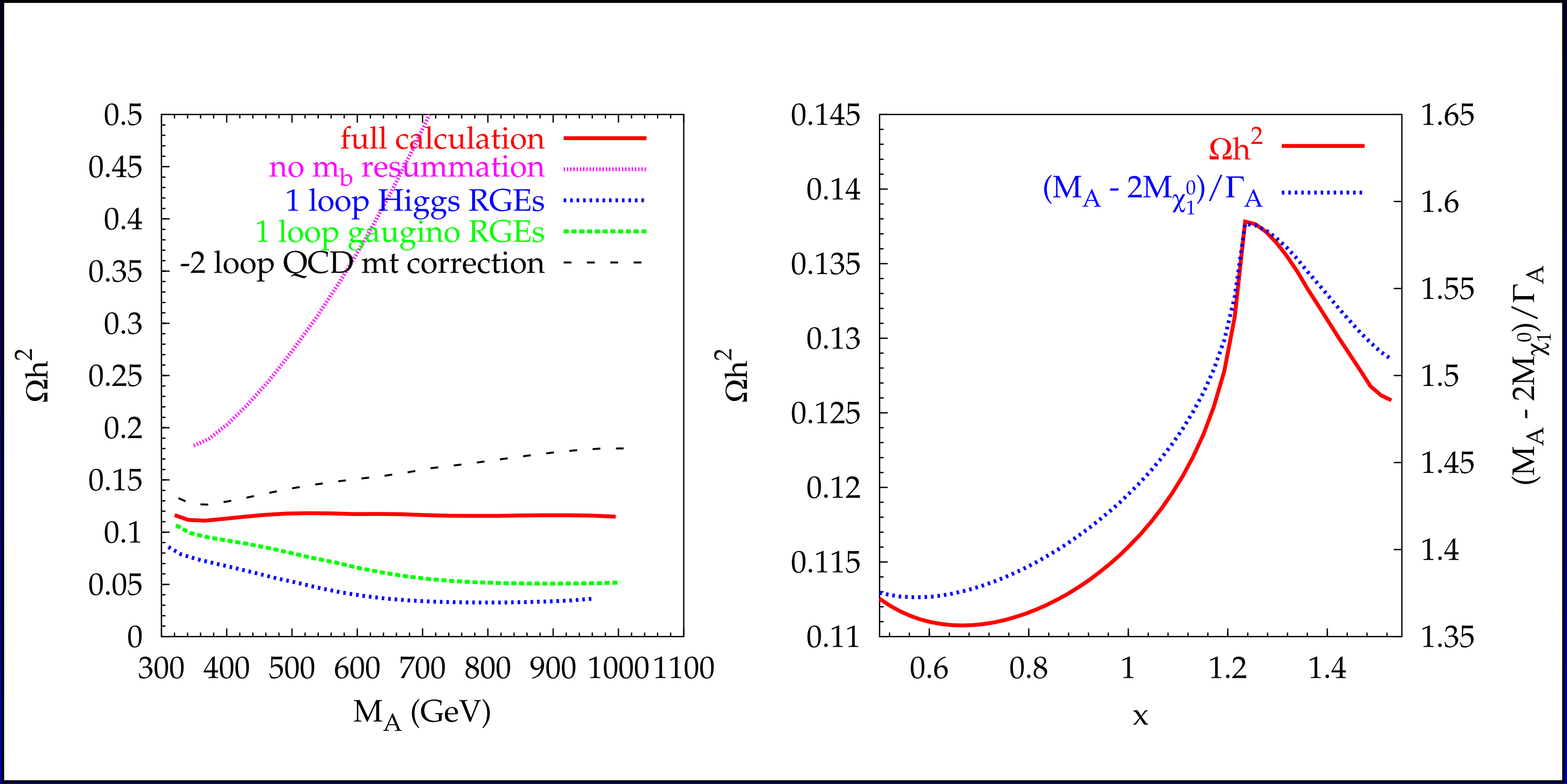
$$< \sigma v >^{-1} \sim \frac{4 m_{\chi_1^0} \Gamma_A}{g^2_{m_{\chi_1^0}\tilde{\chi}_1^0 A}} \left(4 \left(\frac{M_A - 2 m_{\chi_1^0}}{\Gamma_A} \right)^2 + 1 \right).$$

Funnel Slope



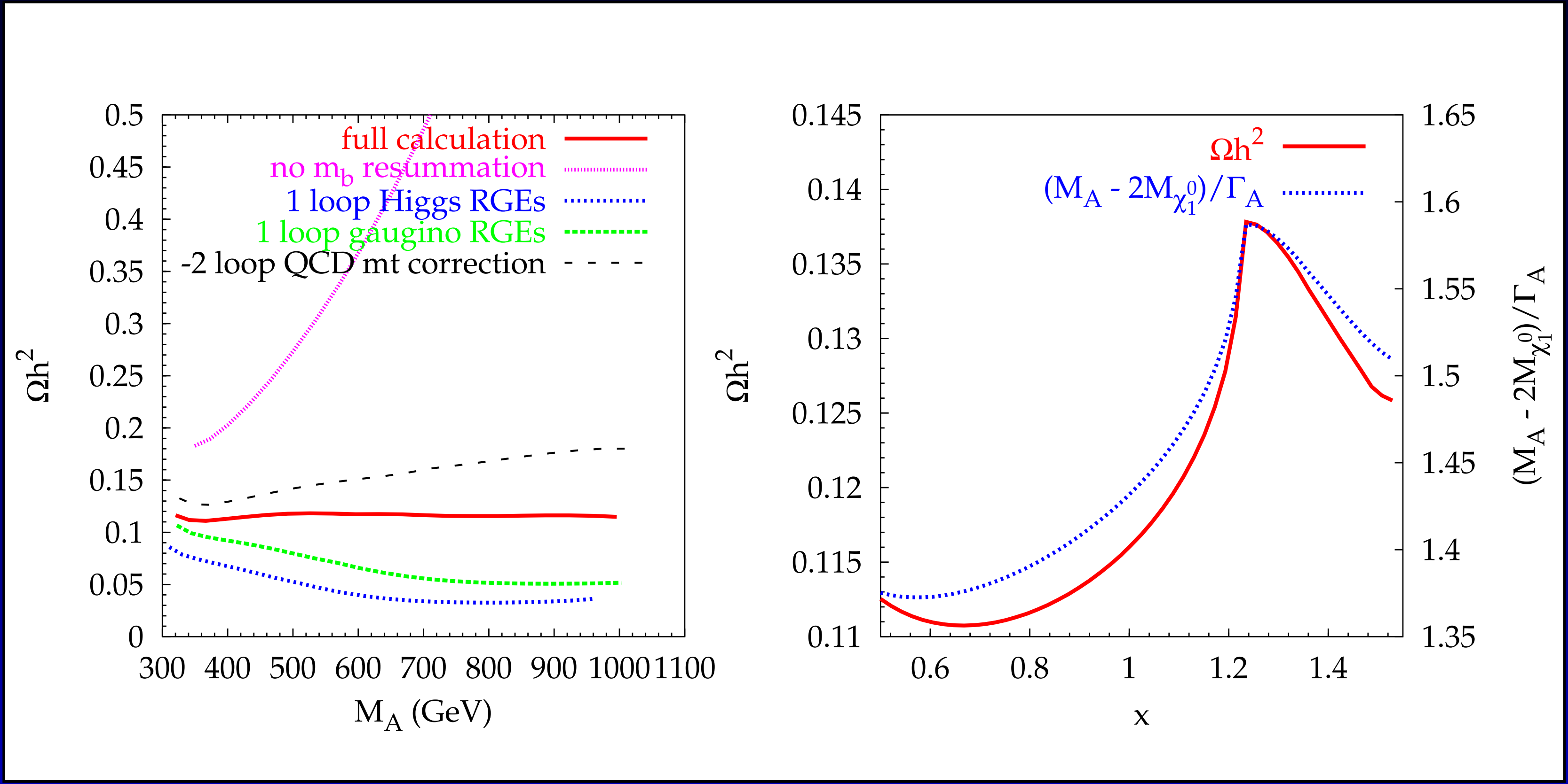
Notice that spectrum is quite *heavy*: need a high energy ILC! Γ_A will be important.

Funnel Theory Uncertainties



LHS: Γ_A affected by large $m_b^{SM}/(1+\Delta_{SUSY})$ corrections since $A \rightarrow b\bar{b} \propto Ab\bar{b}$ coupling $\propto m_b \tan\beta$, and $\tan\beta = 50$.

Funnel Theory Uncertainties



RHS: $x > 1.5$ yielded $M_A^2 < 0$ ie no EWSB. Strong correlation of theory error with its effect on $(M_A - 2M_{\chi_1^0})/\Gamma_A$ - could measure it!

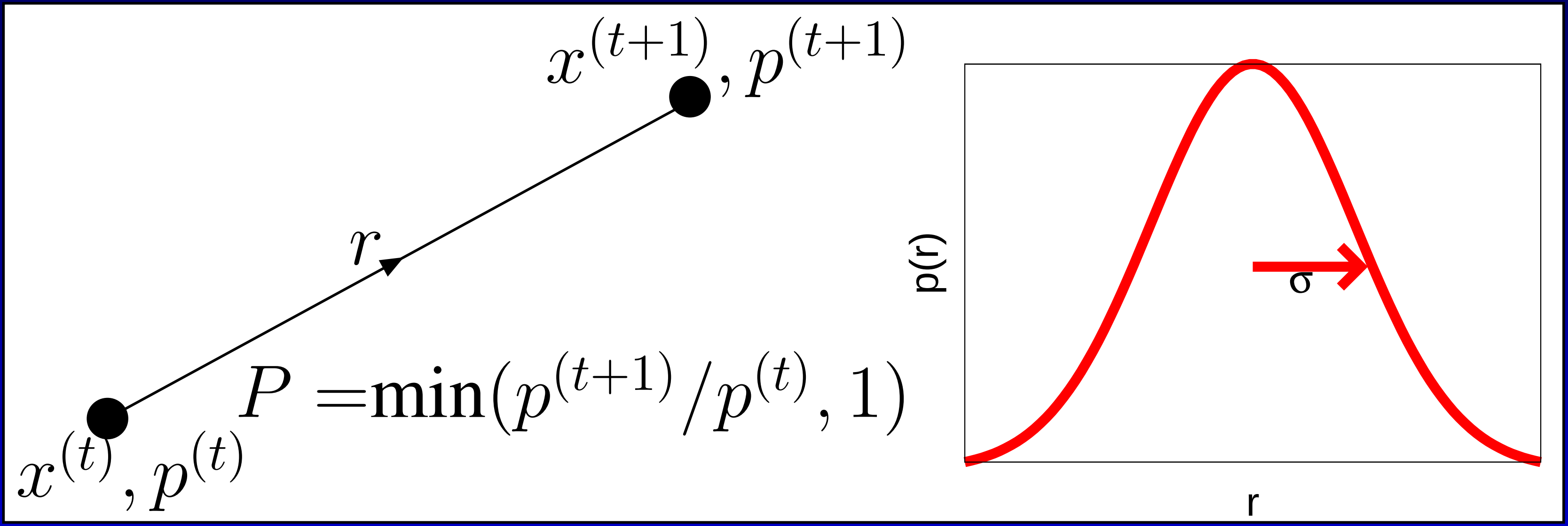
Fit Development

- Really, would like to combine likelihoods from different measurements
- Typically only 2d scans, but in general we have $\alpha(M_Z)$, $\alpha_s(M_Z)$, m_t, m_b, m_0, $M_{1/2}$, A_0, $\tan\beta$ to vary
- Effective 3d type scan done which parameterises a 2d surface of correct Ωh^2
- Baltz *et al* managed to perform a 4d scan, but lost the likelihood interpretation. They used the impressive *Markov Chain Monte Carlo technique*.

Done in 2d in Ellis *et al*, `hep-ph/0310356`
Ellis *et al*, `hep-ph/0411218`

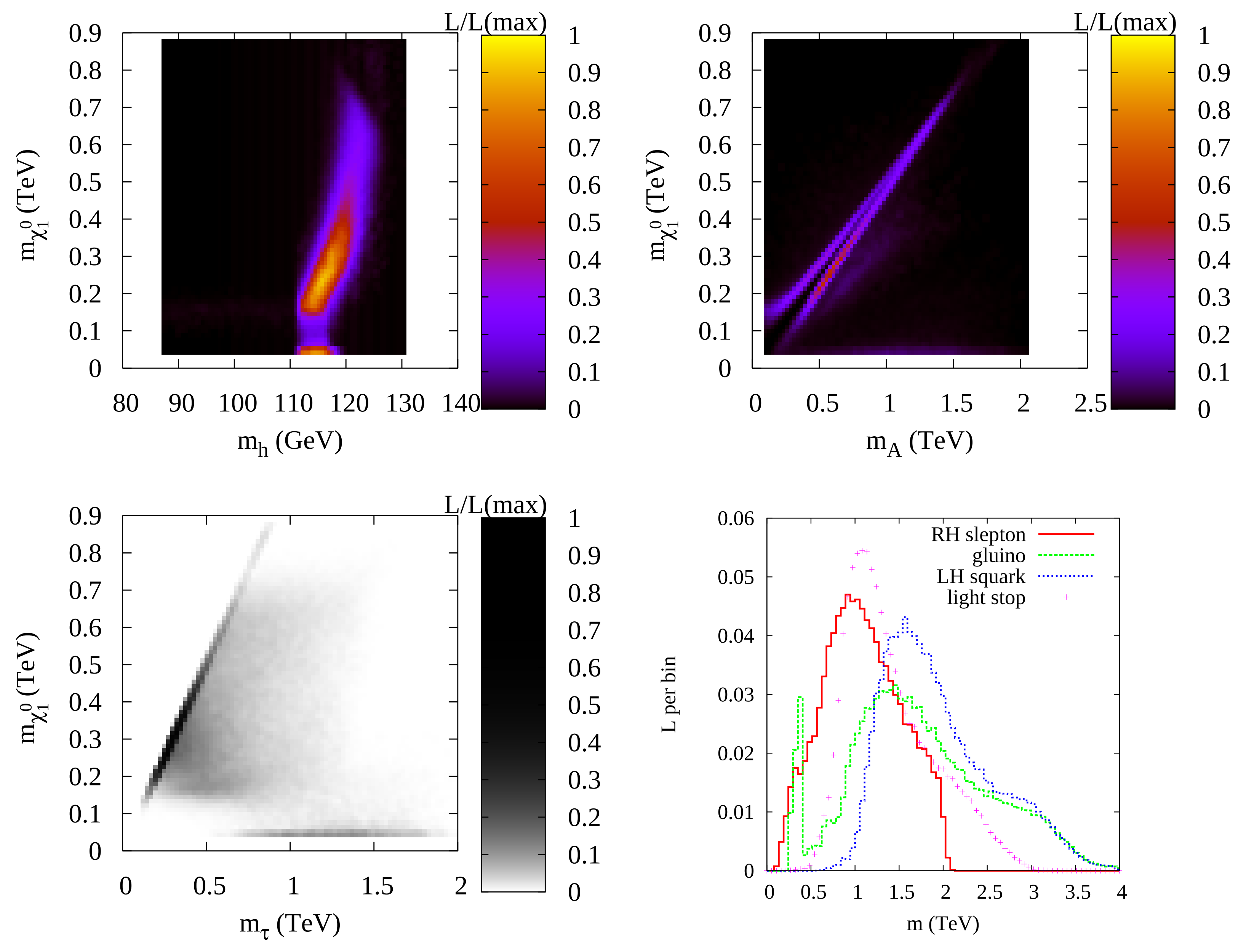
Markov-Chain Monte Carlo

Metropolis-Hastings Markov chain sampling consists of list of parameter points $x^{(t)}$ and associated posterior probabilities $p^{(t)}$.

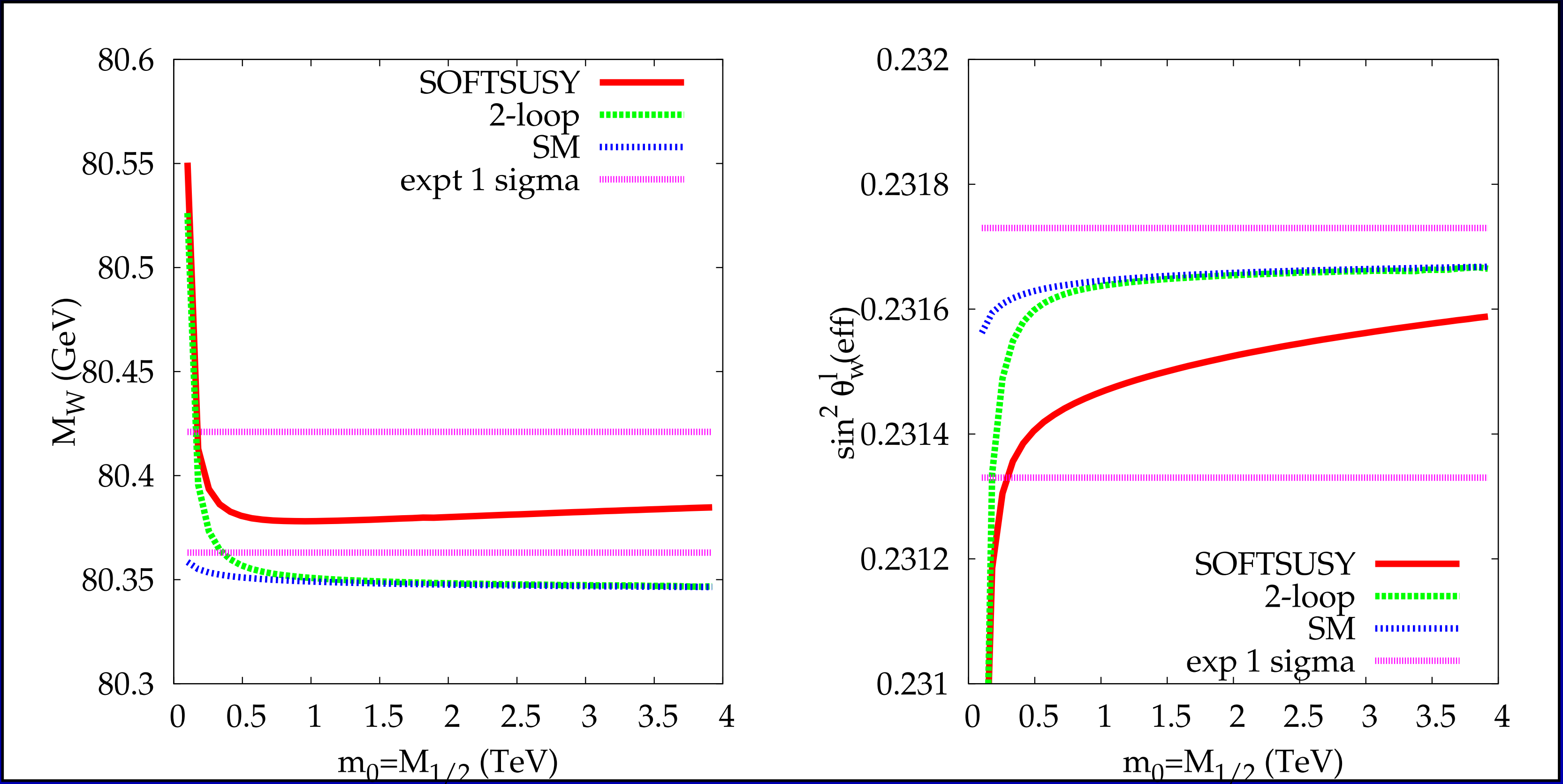


Final density of x points $\propto p$. Required number of points goes *linearly* with number of dimensions.

Sanity Check



Electroweak Observables



They prefer light SUSY . Be careful of 1-loop approx.

Ellis *et al*, hep-ph/0411216; hep-ph/0602220.

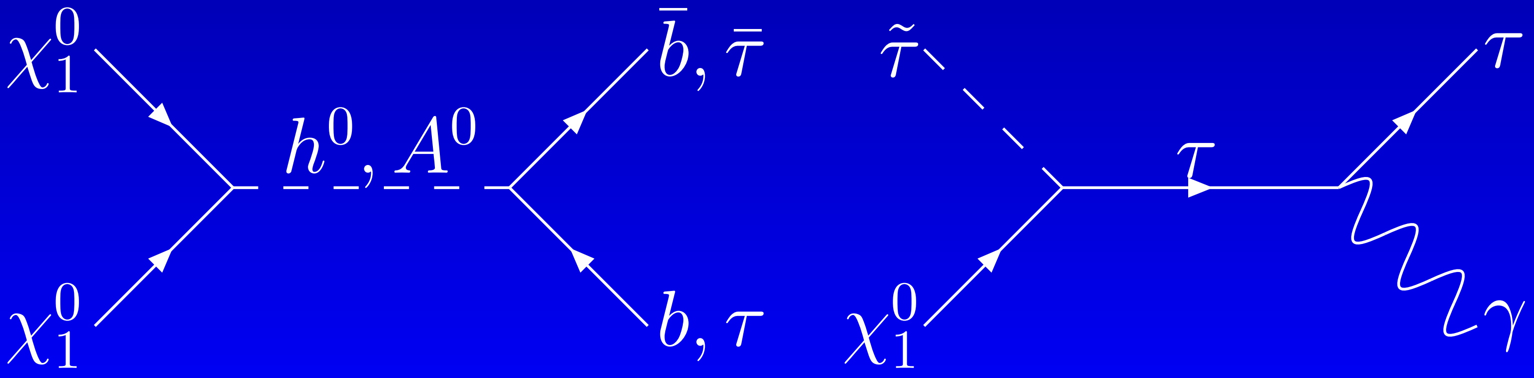
Volume Effects

Can't rely on a good χ^2 *in non-Gaussian situation*

Annihilation Mechanism

Define stau co-annihilation when $m_{\tilde{\tau}}$ is within 10% of $m_{\chi_1^0}$ and Higgs pole when $m_{h,A}$ is within 10% of $2m_{\chi_1^0}$.

mechanism	flat prior	natural prior
h^0-pole	0.025	0.07
A^0-pole	0.41	0.14
$\tilde{\tau}-$co-annihilation	0.26	0.18
rest	0.31	0.61



Naturalness

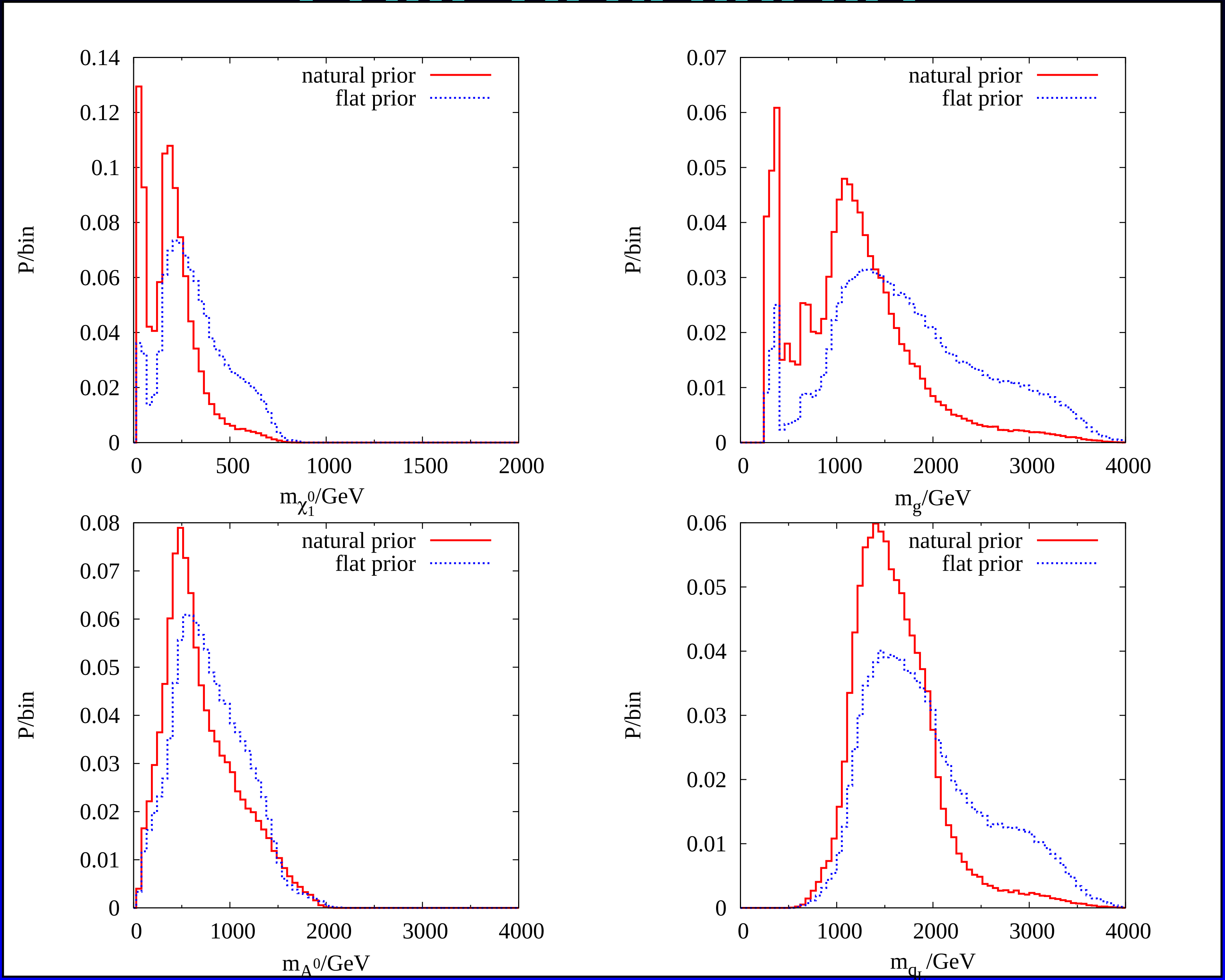
$$M_Z^2 = \tan 2\beta \left[m_{H_2}^2 \tan\beta - m_{H_1}^2 \cot\beta\right] - 2\mu^2$$

Cancellation implied by sparticle mass bounds. Quantify by

$$f = \text{max}_x\{||\frac{d\ln M_Z^2}{d\ln x}||\}$$

where $x \in \{M_{1/2}, m_0, A_0, \mu, B\}$. We will choose the prior to be $1/f$.

Prior Dependence



Likelihood and Posterior

Q: What's the chance of observing someone to be pregnant, given that they are female?

Likelihood

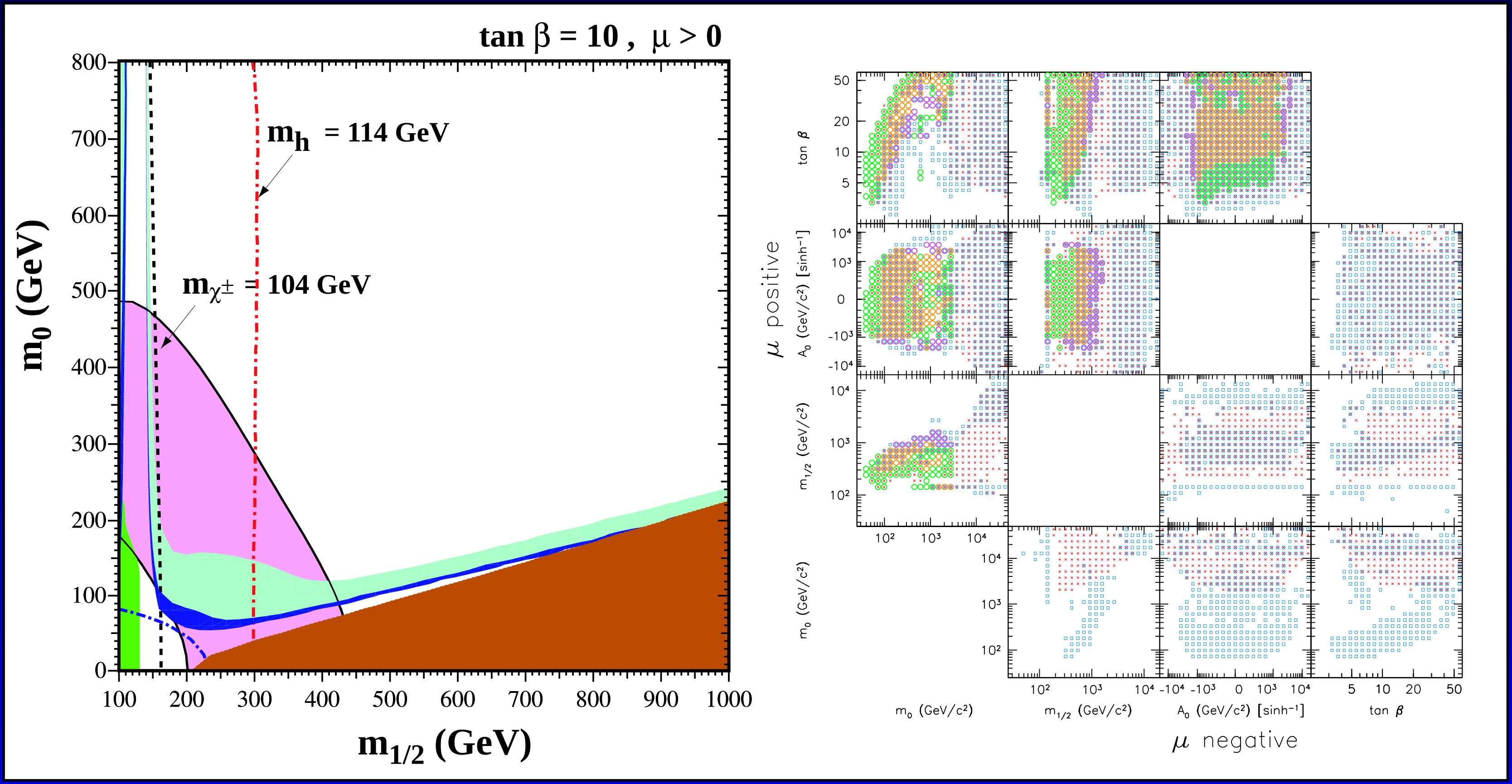
p(pregnant | female, human) = 0.01

Posterior

p(female | pregnant, human) = 1.00

Constraints on SUSY Models

CMSSM well-studied in literature: eg Ellis, Olive *et al* PLB565 (2003) 176; Roszkowski *et al* JHEP 0108 (2001) 024; Baltz, Gondolo, JHEP 0410 (2004) 052;…



Application of Bayes'

$\mathcal{L} \equiv p(d|m)$ is pdf of reproducing data d assuming CMSSM hypothesis H and parameter point m

$$\begin{aligned} p(m|d,H) &= p(d|m,H)\frac{p(m,H)}{p(d,H)} \\ \frac{p(m_1|d)}{p(m_2|d)} &= \frac{p(d|m_1)p(m_1)}{p(d|m_2)p(m_2)} \end{aligned}$$

$p(m|d)$ is called the posterior pdf. We will compare $p(m_i) = 1$ with a *naturalness* prior: 1/(fine tuning).