Theoretical Tools for Heavy Quark Physics

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Contents

Why Heavy Quark Physics?

- Flavour Mixing and CP-Violations are two of the most important topics of contemporary Particle Physics
- It is all encoded in the UNITARY CKM MATRIX appearing in the charged current interaction:

$$\mathcal{L} = \frac{g}{\sqrt{2}} (\bar{u} \quad \bar{c} \quad \bar{t}) \gamma^\mu (1 - \gamma_5) W_\mu \; V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Entries in the CKM matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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CKM Matrix: Basics

- Tree dimensional (real) Rotation: Three angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- Single phase δ: $$U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}.$$
- PDG CKM Parametrization:

$$V_{\rm CKM} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12}$$

- Large Phases in $V_{ub} = |V_{ub}| e^{-i\gamma} = s_{13} e^{-i\delta_{13}}$ and $V_{td} = |V_{td}| e^{i\beta}$

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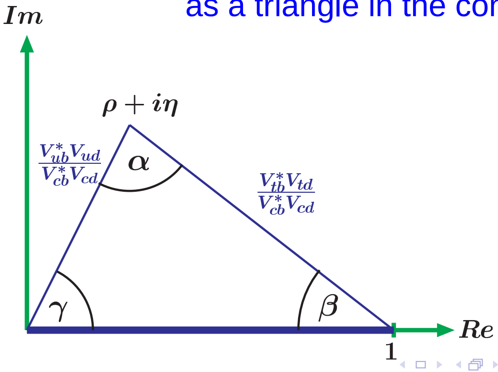
CKM Unitarity: Unitarity Triangle

- Out of six Unitarity Triangles only two have sides of comparable lengths:
- Depict the relation $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ as a triangle in the complex plane



- Heavy Quarks: $m_Q \gg \Lambda_{\rm QCD}$
  - Top Quark: $m_t \sim 175$ GeV (too heavy)
  - Bottom Quark: $m_b \sim 4.5$ GeV (just o.k.)
  - Charm Quark: $m_c \sim 1.5$ GeV (boarderline case)
  - Strange Quarks: $m_s \sim 0.1$ GeV (too light, but ...)
- Almost all CKM matrix elements describe transitions involving one or even two heavy quarks.
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Effective Field Theories

- Weak decays:
  Very different mass scales are involved:
  - $\Lambda_{QCD} \sim 200$ MeV: Scale of strong interactions
  - $m_c \sim 1.5$ GeV: Charm Quark Mass
  - $m_b \sim 4.5$ GeV: Bottom Quark Mass
  - $m_t \sim 175$ GeV and $M_W \sim 81$ GeV:
    Top Quark Mass and Weak Boson Mass
  - Λ_{NP} Scale of "new physics"
- At low scales the high mass particles / high energy degrees of freedom are irrelevant.
- Construct an "effective field theory" where the massive / energetic degrees of freedom are removed ("integrated out")

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Integrating out heavy degrees of freedom

- ϕ: light fields, Φ: heavy fields with mass Λ
- Generating functional as a functional integral
  Integration over the heavy degrees of freedom

$$Z[j] = \int [d\phi][d\Phi] \exp\left(\int d^4x \, [\mathcal{L}(\phi, \Phi) + j\phi]\right)$$

$$= \int [d\phi] \exp\left(\int d^4x \, [\mathcal{L}_{\rm eff}(\phi) + j\phi]\right) \text{ with}$$

$$\exp\left(\int d^4x \, \mathcal{L}_{\rm eff}(\phi)\right) = \int [d\Phi] \exp\left(\int d^4x \, \mathcal{L}(\phi, \Phi)\right)$$

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- For length scales $x \gg 1/\Lambda$: local effective Lagrangian
- Technically: (Operator Product) Expansion in inverse powers of Λ

$$\mathcal{L}_{\rm eff}(\phi) = \mathcal{L}_{\rm eff}^{(4)}(\phi) + \frac{1}{\Lambda}\mathcal{L}_{\rm eff}^{(5)}(\phi) + \frac{1}{\Lambda^2}\mathcal{L}_{\rm eff}^{(6)}(\phi) + \cdots$$

- $\mathcal{L}_{\rm eff}$ is in general non-renormalizable, but ...
- $\mathcal{L}_{\rm eff}^{(4)}$ is the renormalizable piece
- For a fixed order in $1/\Lambda$: Only a finite number of insertions of $\mathcal{L}_{\rm eff}^{(4)}$ is needed!
- $\rightarrow$ can be renormalized
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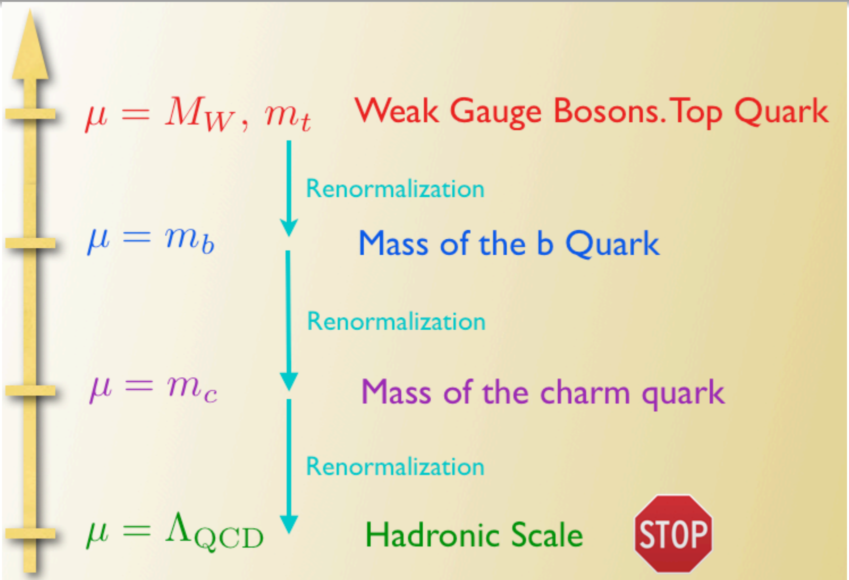
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Weak Gauge Bosons. Top Quark
Renormalization
$\mu = m_b$
Mass of the b Quark
Renormalization
$\mu = m_c$
Mass of the charm quark
Renormalization
$\mu = \Lambda_{QCD}$
Hadronic Scale
STOP

Heavy Quark Limit

Isgur, Wise, Voloshin, Shifman, Georgi, Grinstein, ...

- $1/m_Q$ Expansion: Substantial Theoretical Progress!
- Static Limit: $m_b,\, m_c \to \infty$ with fixed (four)velocity

$$v_Q = \frac{p_Q}{m_Q}, \qquad Q = b, c$$

- In this limit we have

$$\left. \begin{array}{l} m_{Hadron} = m_Q \\ p_{Hadron} = p_Q \end{array} \right\} v_{Hadron} = v_Q$$

- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics I!

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- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics !!

Heavy Quark Limit

Isgur, Wise, Voloshin, Shifman, Georgi, Grinstein, ...

- $1/m_Q$ Expansion: Substantial Theoretical Progress!
- Static Limit: $m_b,\, m_c \to \infty$ with fixed (four)velocity

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Heavy Quark Symmetries

- The interaction of gluons is identical for all quarks
- Flavour enters QCD only through the mass terms
  - $m \to 0$: (Chiral) Flavour Symmetry (Isospin)
  - $m \to \infty$ Heavy Flavour Symmetry
  - Consider b and c heavy: Heavy Flavour SU(2)
- Coupling of the heavy quark spin to gluons:

$$H_{int} = \frac{g}{2m_Q} \bar{Q}(\vec{\sigma} \cdot \vec{B})Q \quad \xrightarrow{m_Q \to \infty} \quad 0$$

  - Spin Rotations become a symmetry
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Baryonic Ground States

$$\left|[(ud)_0 Q]_{1/2}\right\rangle = |\Lambda_Q\rangle$$

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Wigner Eckart Theorem for HQS

- HQS imply a "Wigner Eckart Theorem"

$$\langle H^{(*)}(v) | \, Q_v \Gamma Q_{v'} \, | H^{(*)}(v') \rangle = C_\Gamma(v, v') \xi(v \cdot v')$$

with $H^{(*)}(v) = D^{(*)}(v)$ or $B^{(*)}(v)$

- $C_\Gamma(v, v')$: Computable Clebsh Gordan Coefficient
- $\xi(v \cdot v')$: Reduced Matrix Element
- $\xi(v \cdot v')$: universal non-perturbative Form Faktor: Isgur Wise Funktion
- Normalization of ξ at $v = v'$:

$$\xi(v \cdot v' = 1) = 1$$

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  - $\xi(v\cdot v')$: universal non-perturbative Form Faktor: Isgur Wise Funktion
  - Normalization of ξ at $v = v'$:

$$\xi(v\cdot v' = 1) = 1$$

Heavy Quark Effective Theory

- The heavy mass limit can be formulated as an effective field theory
- Expansion in inverse powers of m_Q
- Define the static field h_v for the velocity v

$$h_v(x) = e^{im_Q v\cdot x}\frac{1}{2}(1+\not{v})b(x) \qquad p_Q = m_Q v + k$$

- HQET Lagrangian

$$\mathcal{L} = \bar{h}_v(iv\cdot D)h_v + \frac{1}{2m_Q}\bar{h}_v(i\not{D})^2 h_v + \cdots$$

- Dim-4 Term: Feynman rules, loops, renormalization...

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Heavy to Heavy: $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega)(\mathcal{F}(\omega))^2$$
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- Normalization of the Form Factors is known at $vv' = 1$ from Heavy Quark Symmetries:
- Corrections can be calculated / estimated in HQET

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- Unquenched Calculations become available!
- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

$$\mathcal{F}(1) = 0.91^{+0.03}_{-0.04}$$

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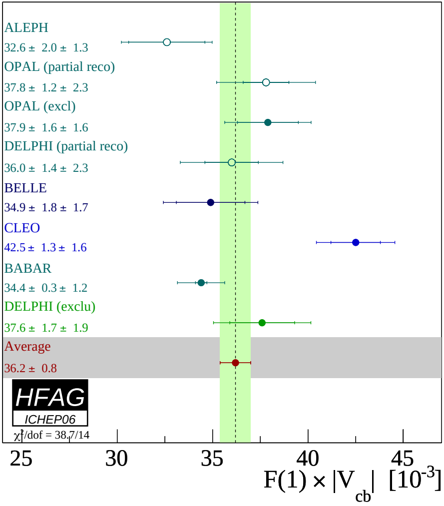

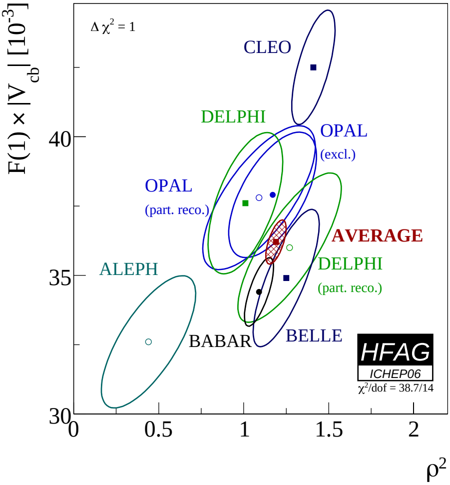

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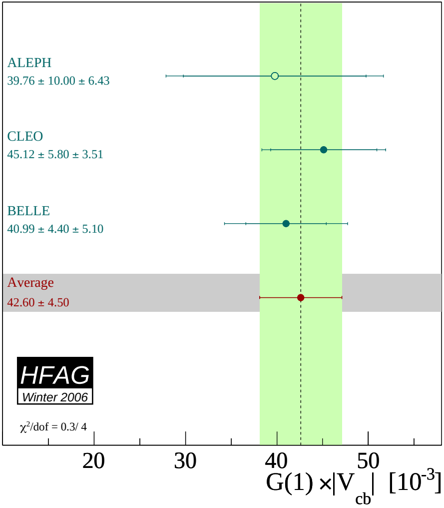

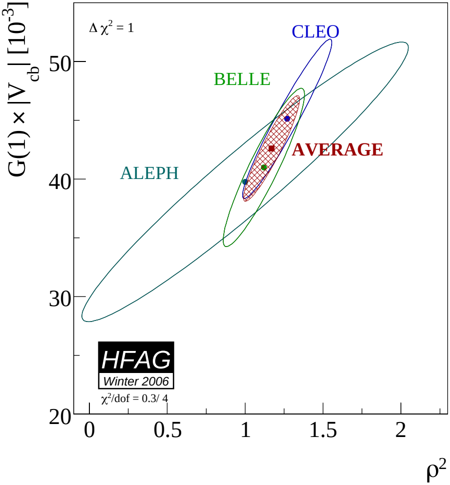

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Inclusive Decays: Using OPE

Operator Product Expansion = Heavy Quark Expansion

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$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v)\rangle|^2$$

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$\rightarrow$ The rate for $B \rightarrow X_c \ell \bar{\nu}_\ell$ can be written as

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- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

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μ_π: Kinetic energy and μ_G: Chromomagnetic moment

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New: $1/m_b^4$ Contribution Γ_4 (Dassinger, Turczyk, M.)

- Five new parameters:

$\langle \vec{E}^2 \rangle$: Chromoelectric Field squared
$\langle \vec{B}^2 \rangle$: Chromomagnetic Field squared
$\langle (\vec{p}^2)^2 \rangle$: Fourth power of the residual b quark momentum
$\langle (\vec{p}^2)(\vec{\sigma} \cdot \vec{B}) \rangle$: Mixed Chromomag. Mom. and res. Momentu
$\langle (\vec{p} \cdot \vec{B})(\vec{\sigma} \cdot \vec{p}) \rangle$: Mixed Chromomag. field and res. helicity

- Some of these can be estimated in naive factorization

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Heavy to Heavy: $B \to X_c \ell \bar{\nu}_\ell$

- Determine the HQE parameters from
  - Charged lepton energy spectrum
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- From the theoretical side:
  Calculation of moments of the spectra

$$\langle M_X^n \rangle = \frac{1}{\Gamma} \int dM_X \, M_X^n \int_{E_{\rm cut}} dE_\ell \, \frac{d^2\Gamma}{dM_X \, dE_\ell}$$

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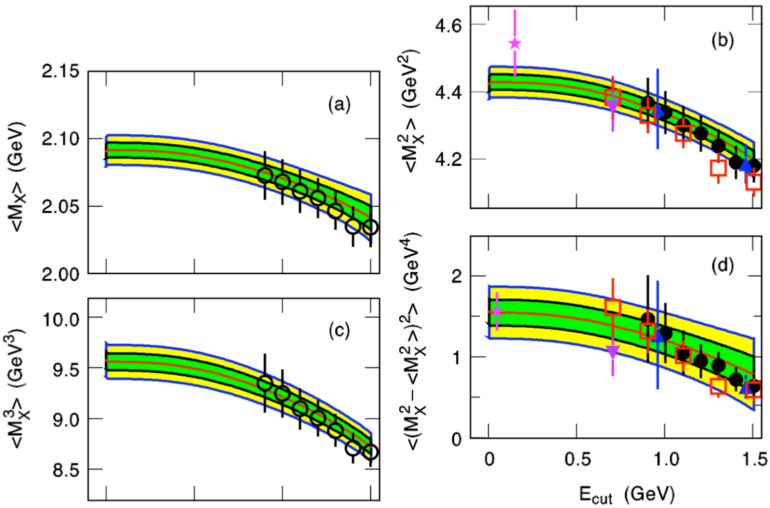
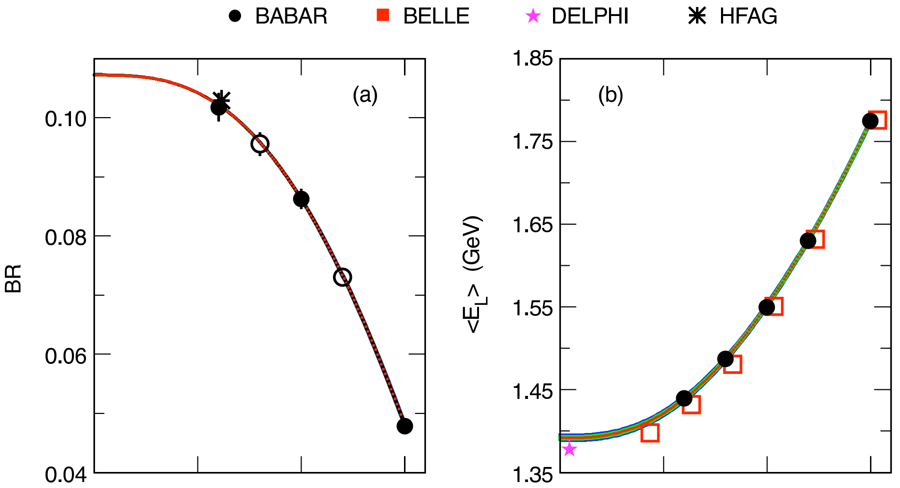
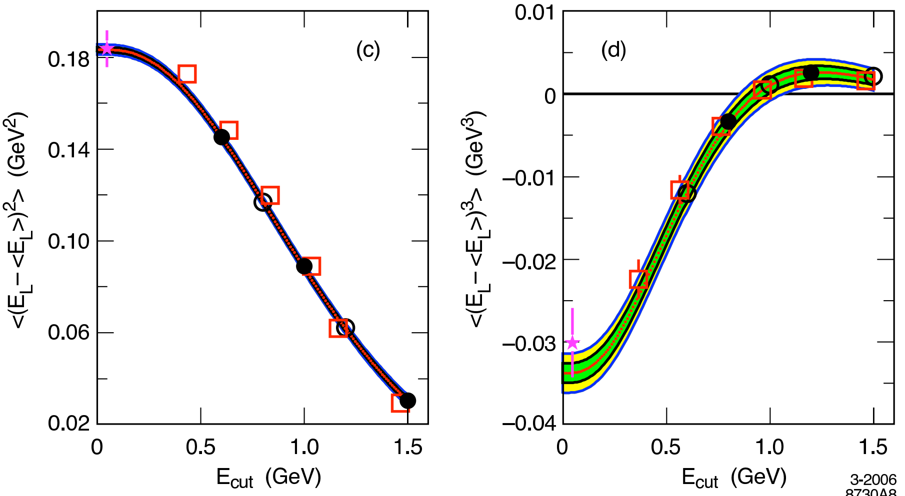
Hadronic Invariant Mass Moments (Buchmüller, Flächer)



Lepton Energy Moments I (Buchmüller, Flächer)



Lepton Energy Moments II (Buchmüller, Flächer)



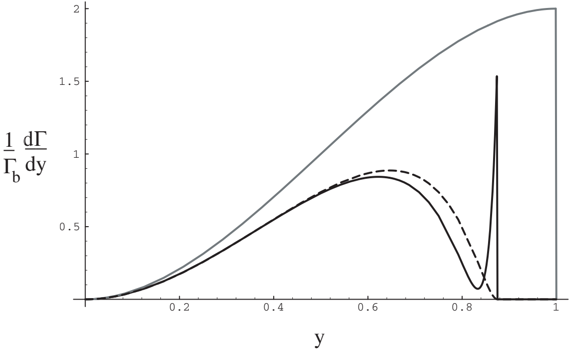
$$V_{cb,incl} = (41.96 \pm 0.23_{exp} \pm 0.35_{HQE} \pm 0.59_{\Gamma_{sl}}) \times 10^{-3}$$

O. Buchmüller, HQL2006

Twist Expansion

- Calculation of spectra within the OPE

$$\frac{d\Gamma}{dy} \propto \Theta(1-y-\rho)\left[2 - \frac{\mu_\pi^2}{(m_Q(1-y))^2}\left(\frac{\rho}{1-\rho}\right)^2\left\{3 - 4\left(\frac{\rho}{1-\rho}\right)\right\}\right]$$



- $y = \frac{2E_\ell}{m_b}$
- $\rho = \frac{m_c^2}{m_b^2}$

- In the massless case this becomes for $B \to X_u \ell \bar{\nu}_\ell$

$$\frac{d\Gamma}{dy} \stackrel{y\to 1}{=} \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \left[\Theta(1-y) + \frac{\mu_\pi^2 - \mu_G^2}{6m_b^2} \delta(1-y) + \frac{\mu_\pi^2}{6m_b^2} \delta'(1-y) + \cdots \right]$$

- Likewise for $B \to X_s \gamma$ ($x = \frac{2E_\gamma}{m_b}$)

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb*}|^2 |C_7|^2 \left(\delta(1-x) - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \delta'(1-x) + \frac{\mu_\pi^2}{6m_b^2} \delta''(1-x) + \cdots \right)$$

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Shape- or Light-Cone Distribution Functions

- Resummation into a shape function or light cone distribution function (Bigi, Shifman, Uraltsev, Neubert, M., ...)

$$2M_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D))|B(v)\rangle$$

such that

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- Coefficients a_i are the moments of the spectrum:
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- Twist Expansion in complete analogy to deep inelastic scattering

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- Problem: How to calculate corrections to the shape functions?
- More than two scales involved!
- Inclusive Rates in the Endpoint become (Korchemski, Sterman)

$$d\Gamma = H * J * S$$

with * = Convolution

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
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Basics of Soft Collinear Effective Theory

- Heavy-to-light decays:
  Kinematic Situations with energetic light quarks
  hadronizing into jets or energetic light mesons
  p_{fin}: Momentum of a light final state meson

$$p_{\rm fin}^2 \sim \mathcal{O}(\Lambda_{\rm QCD} m_b) \quad v \cdot p_{\rm fin} \sim \mathcal{O}(m_b)$$

- Use light-cone vectors $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$:

$$p_{\rm fin} = \frac{1}{2}(n \cdot p_{\rm fin})\bar{n} \quad \text{and} \quad v = \frac{1}{2}(n + \bar{n})$$

- Momentum of a light quark in such a meson:

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- Extremely important for the determination of V_{ub}
- Shape functions are modelled or taken from $B \to X_s \gamma$

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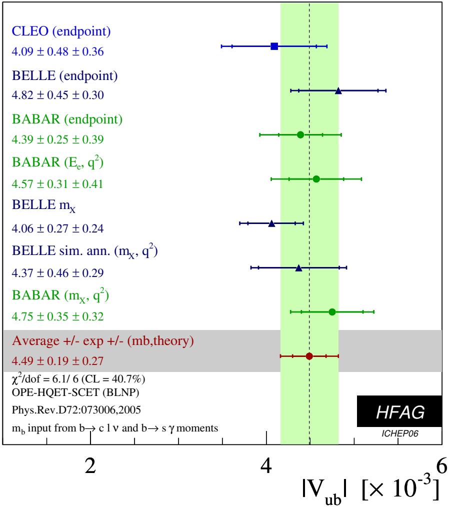
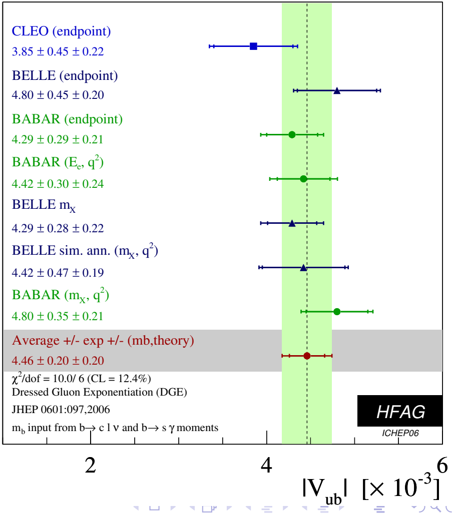


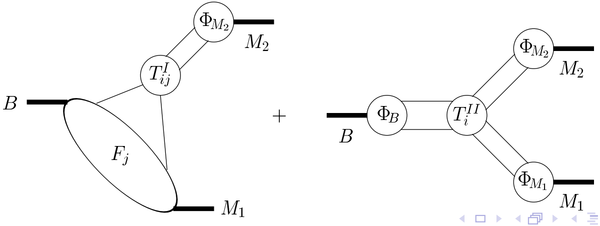


Towards Understanding Non-leptonic Decays

- Non-leptonic decays require the calculation of hadronic matrix elements of four-quark operators, e.g. for a decay like $B \to \pi\pi$

$$\mathcal{M} = \langle B|(\bar{b}\gamma_\mu(1-\gamma_5)q)(\bar{q}'\gamma_\mu(1-\gamma_5)q'')|\pi\pi\rangle$$

- In the large m_b limit a factorization theorem has been proven (QCD-Factorization, similar to SCET)
(Beneke, Buchalla, Neubert, Sachrajda)



General Properties of QCD-Factorization

- Leading term is (contains) naive factorization
- Non-perturbative qunatities are the soft form factor and the light cone distributions of the light hadrons and of the B meson
- The strong phases of the matrix elements are either perturbative ($\mathcal{O}(\alpha_s(m_b))$)
  or power suppressed ($\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b)$)
- $\longrightarrow$ The strong phases are predicted to be small(ish)
- $\longrightarrow$ Important for the calculation of CP Asymmetries
- Does it work?

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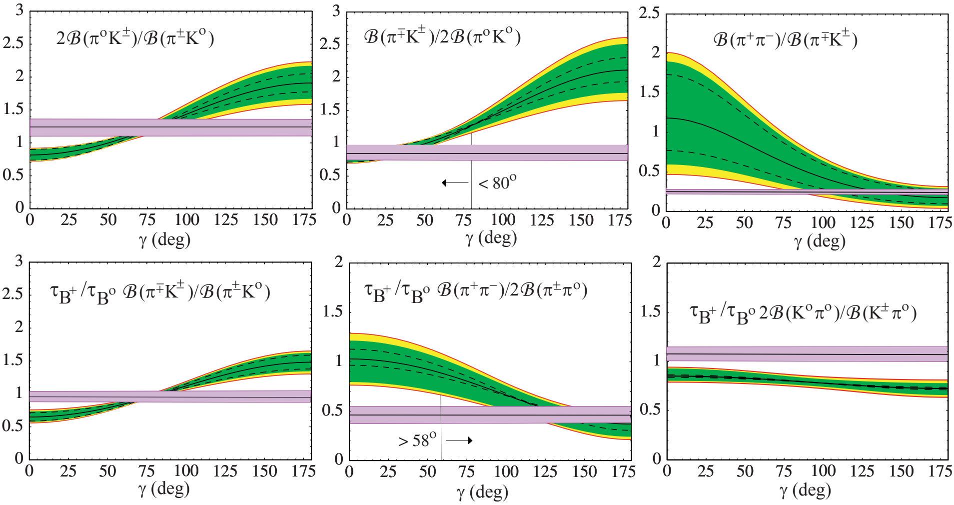
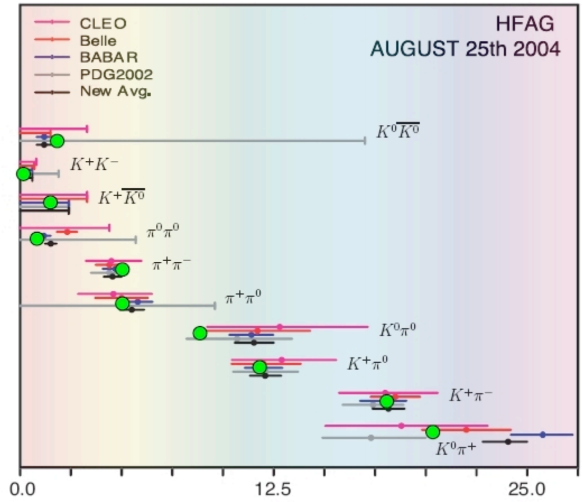


(Beneke, Buchalla, Neubert, Sachrajda, 2001)

Update on the BR’s by Neubert (CKM 2005, San Diego)



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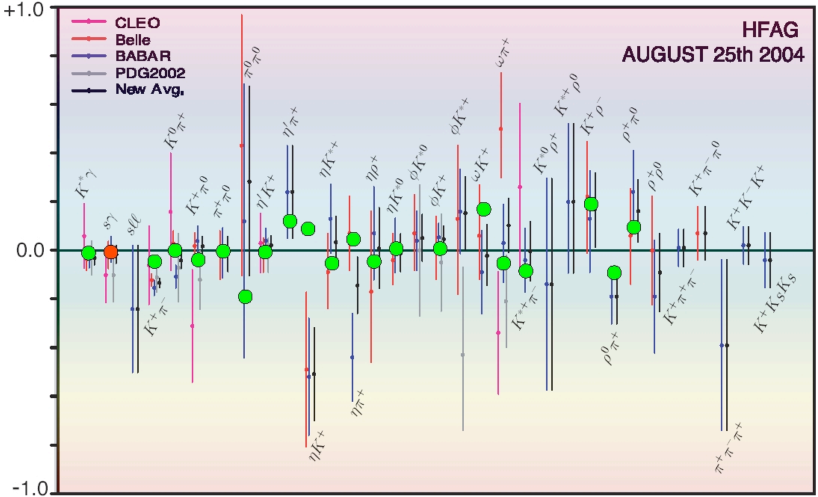

Summary I: The achievements

- Effective field theory methods made precise calculations in heavy quark physics possible
- Starting about 1989 HQET and HQE put heavy quark physics on a model independent basis
- $\longrightarrow$ Model dependence often appears only at subleading orders
- SCET is an ansatz to undestand also exclusive non-leptonic decays systematically
- I did not talk about Lattice QCD calculations: Enormous progress due to better algorithms and to stronger computers
- Heavy Flavour Physics has become (in some corners) a precision field.

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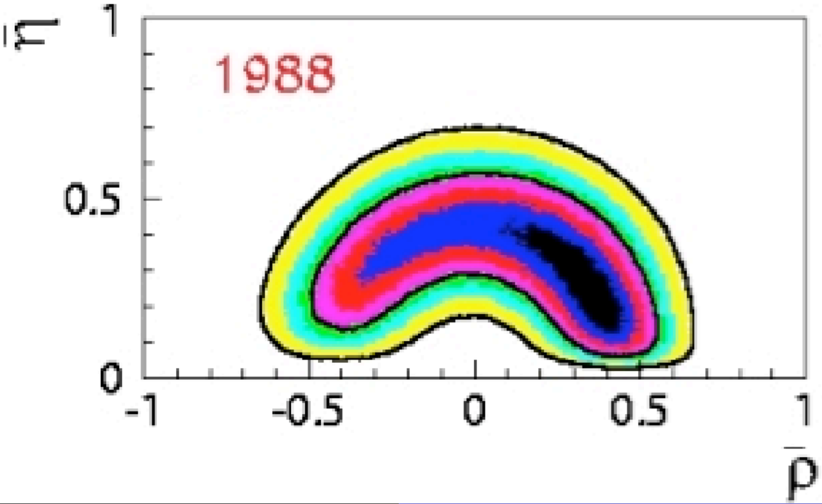
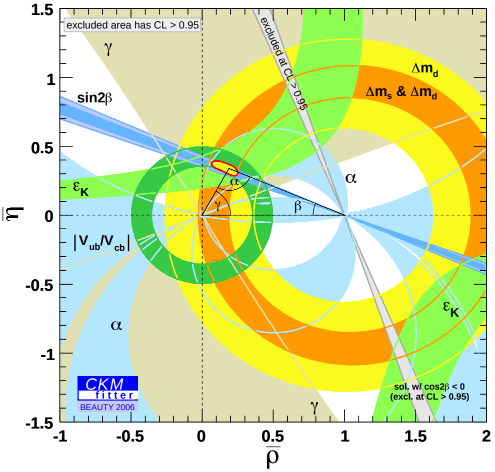
1988: Pre-historic Unitarity Triangle



2006: Todays Unitarity Triangle



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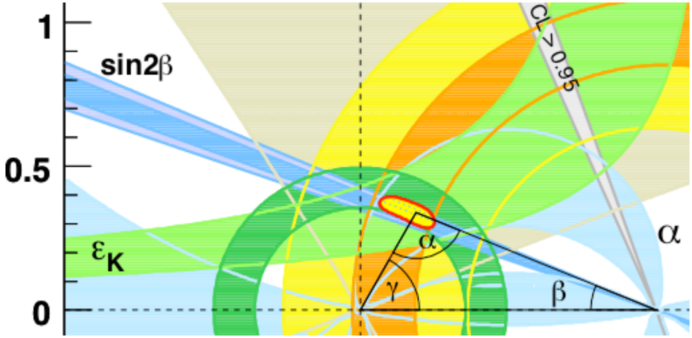
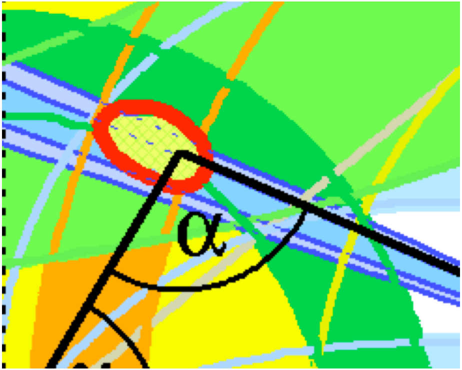
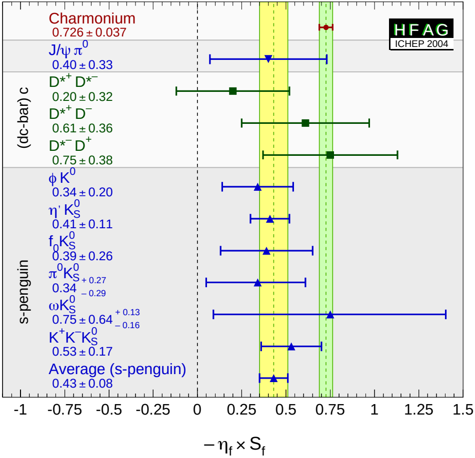


There are interesting tensions ...





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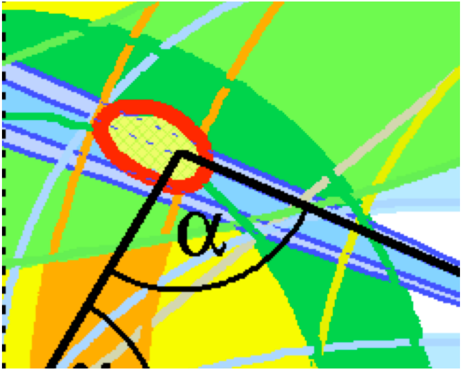

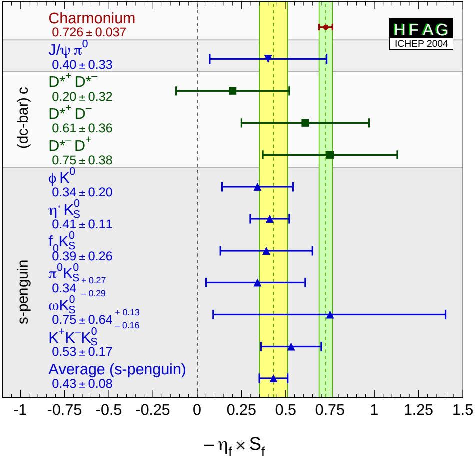

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