# Theoretical Tools for Heavy Quark Physics

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#### **Recent Developments**

- Soft Collinear Effective Theory
- Towards Understanding Nonleptonic Decays

Why Heavy Quark Physics? Effective Field Theories

## Why Heavy Quark Physics?

- Flavour Mixing and CP-Violations are two of the most important topics of contemporary Particle Physics
- It is all encoded in the UNITARY CKM MATRIX appearing in the charged current interaction:

$$\mathcal{L} = \frac{g}{\sqrt{2}} \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \gamma^{\mu} (1 - \gamma_5) W_{\mu} \quad \bigvee_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

• Entries in the CKM matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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Why Heavy Quark Physics? Effective Field Theories

## CKM Matrix: Basics

• Tree dimensional (real) Rotation: Three angles  $\theta_{ij}$ 

 $U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix} , \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$ 

• Single phase  $\delta$ :  $U_{\delta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}$ .

• PDG CKM Parametrization:

$$V_{\mathrm{CKM}} = U_{23} U_{\delta}^{\dagger} U_{13} U_{\delta} U_{12}$$

• Large Phases in  $V_{ub} = |V_{ub}|e^{-i\gamma} = s_{13}e^{-i\delta_{13}}$  and  $V_{td} = |V_{td}|e^{i\beta}$ 

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Why Heavy Quark Physics? Effective Field Theories

## CKM Unitarity: Unitarity Triangle

- Out of six Unitarity Triangles only two have sides of comparable lengths:
- Depict the relation  $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$



Why Heavy Quark Physics? Effective Field Theories

#### • Heavy Quarks: $m_Q \gg \Lambda_{QCD}$

- Top Quark:  $m_t \sim 175$  GeV
- Bottom Quark: m<sub>b</sub> ~ 4.5 GeV
- Charm Quark:  $m_c \sim 1.5$  GeV
- Strange Quarks:  $m_s \sim 0.1$  GeV

(too heavy) (just o.k.) boarderline case) (too light, but ...)

- Almost all CKM matrix elements describe transitions involving one or even two heavy quarks.
- Determination of these matrix elements involve to deal with the strong interaction of heavy quarks.

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Why Heavy Quark Physics? Effective Field Theories

## **Effective Field Theories**

• Weak decays:

- Λ<sub>QCD</sub> ~ 200 MeV: Scale of strong interactions
- *m<sub>c</sub>* ~ 1.5 GeV: Charm Quark Mass
- *m<sub>b</sub>* ~ 4.5 GeV: Bottom Quark Mass
- *m<sub>t</sub>* ~ 175 GeV and *M<sub>W</sub>* ~ 81 GeV:
  - Top Quark Mass and Weak Boson Mass
- Λ<sub>NP</sub> Scale of "new physics"
- At low scales the high mass particles / high energy degrees of freedom are irrelevant.
- Construct an "effective field theory" where the massive / energetic degrees of freedom are removed ("integrated out")

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Why Heavy Quark Physics? Effective Field Theories

## Integrating out heavy degrees of freedom

- $\phi$ : light fields,  $\Phi$ : heavy fields with mass  $\Lambda$
- Generating functional as a functional integral Integration over the heavy degrees of freedom

$$Z[j] = \int [d\phi][d\Phi] \exp\left(\int d^4x \left[\mathcal{L}(\phi, \Phi) + j\phi\right]\right)$$
  
=  $\int [d\phi] \exp\left(\int d^4x \left[\mathcal{L}_{eff}(\phi) + j\phi\right]\right)$  with  
 $\exp\left(\int d^4x \mathcal{L}_{eff}(\phi)\right) = \int [d\Phi] \exp\left(\int d^4x \mathcal{L}(\phi, \Phi)\right)$ 

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Why Heavy Quark Physics? Effective Field Theories

- For length scales  $x \gg 1/\Lambda$ : local effective Lagrangian
- Technically: (Operator Product) Expansion in inverse powers of Λ

$$\mathcal{L}_{\rm eff}(\phi) = \mathcal{L}_{\rm eff}^{(4)}(\phi) + \frac{1}{\Lambda} \mathcal{L}_{\rm eff}^{(5)}(\phi) + \frac{1}{\Lambda^2} \mathcal{L}_{\rm eff}^{(6)}(\phi) + \cdots$$

- $\mathcal{L}_{eff}$  is in general non-renormalizable, but ...
- $\mathcal{L}_{eff}^{(4)}$  is the renormalizable piece
- For a fixed order in 1/Λ: Only a finite number of insertions of L<sup>(4)</sup><sub>eff</sub> is needed!
- $\bullet \rightarrow$  can be renormalized
- Renormalizability is not an issue here

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Why Heavy Quark Physics? Effective Field Theories

- For length scales  $x \gg 1/\Lambda$ : local effective Lagrangian
- Technically: (Operator Product) Expansion in inverse powers of Λ

$$\mathcal{L}_{\mathrm{eff}}(\phi) = \mathcal{L}_{\mathrm{eff}}^{(4)}(\phi) + rac{1}{\Lambda} \mathcal{L}_{\mathrm{eff}}^{(5)}(\phi) + rac{1}{\Lambda^2} \mathcal{L}_{\mathrm{eff}}^{(6)}(\phi) + \cdots$$

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Why Heavy Quark Physics? Effective Field Theories



Thomas Mannel, University of Siegen Theoretical Tools for Heavy Quark Physics

Heavy Quark Limit Heavy Quark Symmetries Heavy Quark Effective Theory

# Heavy Quark Limit

Isgur, Wise, Voloshin, Shifman, Georgi, Grinstein, ...

1/m<sub>Q</sub> Expansion: Substantial Theoretical Progress!
 Static Limit: m<sub>b</sub>, m<sub>c</sub> → ∞ with fixed (four)velocity

$$v_Q = rac{p_Q}{m_Q}, \qquad Q = b, c$$

$$\left. egin{array}{l} m_{Hadron} = m_{Q} \ p_{Hadron} = p_{Q} \end{array} 
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- For m<sub>Q</sub> → ∞ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics I!

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# Heavy Quark Symmetries

- The interaction of gluons is identical for all quarks
- Flavour enters QCD only through the mass terms
  - $m \rightarrow 0$ : (Chiral) Flavour Symmetry (Isospin)
  - $m \to \infty$  Heavy Flavour Symmetry
  - Consider b and c heavy: Heavy Flavour SU(2)
- Coupling of the heavy quark spin to gluons:

$$H_{int} = rac{g}{2m_o} ar{Q} (ec{\sigma} \cdot ec{B}) Q \stackrel{m_o o \infty}{\longrightarrow} 0$$

- Spin Rotations become a symmetry
- Heavy Quark Spin Symmetry: SU(2) Rotations
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### **Mesonic Ground States**

#### Bottom:

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angle &= |m{B}^{-}
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angle &= |m{B}^{0}
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angle &= |m{B}_{s}
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Charm:

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angle &= |m{B}_{s}
angle \end{aligned}$$

Charm:

$$\begin{array}{l} |(\boldsymbol{c}\bar{\boldsymbol{u}})_{J=0}\rangle = |\boldsymbol{D}^0\rangle \\ |(\boldsymbol{c}\bar{\boldsymbol{d}})_{J=0}\rangle = |\boldsymbol{D}^+\rangle \\ |(\boldsymbol{c}\bar{\boldsymbol{s}})_{J=0}\rangle = |\boldsymbol{D}_{\boldsymbol{s}}\rangle \end{array}$$

$$\begin{split} |(\underline{b}\bar{u})_{J=1}\rangle &= |\underline{B}^{*-}\rangle \\ |(\underline{b}\bar{d})_{J=1}\rangle &= |\overline{B}^{*0}\rangle \\ |(\underline{b}\bar{s})_{J=1}\rangle &= |\overline{B}^{*}_{s}\rangle \end{split}$$

$$\begin{array}{l} |(\boldsymbol{c}\boldsymbol{\bar{u}})_{J=1}\rangle = |\boldsymbol{D}^{*0}\rangle \\ |(\boldsymbol{c}\boldsymbol{\bar{d}})_{J=1}\rangle = |\boldsymbol{D}^{*+}\rangle \\ |(\boldsymbol{c}\boldsymbol{\bar{s}})_{J=1}\rangle = |\boldsymbol{D}^{*}_{s}\rangle \end{array}$$

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Heavy Quark Limit Heavy Quark Symmetries Heavy Quark Effective Theory

### **Baryonic Ground States**

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### **Baryonic Ground States**

 $\left[(ud)_0 \mathbf{Q}\right]_{1/2} = |\Lambda_Q\rangle$ 

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### **Baryonic Ground States**

 $\left[(ud)_0 \mathbf{Q}\right]_{1/2} = |\Lambda_Q\rangle$  $\left[ (uu)_{1} \mathbf{Q} \right]_{1/2} \rangle, \left| \left[ (ud)_{1} \mathbf{Q} \right]_{1/2} \rangle, \left| \left[ (dd)_{1} \mathbf{Q} \right]_{1/2} \rangle = |\Sigma_{\mathbf{Q}} \rangle \right] \rangle$  $\left[(uu)_{1}\mathsf{Q}\right]_{3/2}\left\rangle,\left|\left[(ud)_{1}\mathsf{Q}\right]_{3/2}\right\rangle,\left|\left[(dd)_{1}\mathsf{Q}\right]_{3/2}\right\rangle=|\Sigma_{\mathsf{Q}}^{*}\rangle\right\rangle$ 

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## Baryonic Ground States

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Heavy Quark Limit Heavy Quark Symmetries Heavy Quark Effective Theory

## Wigner Eckart Theorem for HQS

HQS imply a "Wigner Eckart Theorem"

 $\left\langle H^{(*)}(v) \right| \mathcal{Q}_{v} \Gamma \mathcal{Q}_{v'} \left| H^{(*)}(v') \right\rangle = \mathcal{C}_{\Gamma}(v, v') \xi(v \cdot v')$ 

with  $H^{(*)}(v) = D^{(*)}(v)$  or  $B^{(*)}(v)$ 

- C<sub>r</sub>(v, v'): Computable Clebsh Gordan Coefficient
- $\xi(v \cdot v')$ : Reduced Matrix Element
- ξ(v · v'): universal non-perturbative Form Faktor: Isgur Wise Funktion
- Normalization of  $\xi$  at v = v':

#### $\xi(v \cdot v' = 1) = 1$

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Heavy Quark Limit Heavy Quark Symmetries Heavy Quark Effective Theory

# Heavy Quark Effective Theory

- The heavy mass limit can be formulated as an effective field theory
- Expansion in inverse powers of m<sub>Q</sub>
- Define the static field  $h_v$  for the velocity v

$$h_{v}(x) = e^{im_{Q}v \cdot x} \frac{1}{2}(1 + \psi)b(x)$$
  $p_{Q} = m_{Q}v + k$ 

HQET Lagrangian

$$\mathcal{L} = ar{h}_{v}(ar{i}v \cdot D)h_{v} + rac{1}{2m_{Q}}ar{h}_{v}(ar{D})^{2}h_{v} + \cdots$$

## Dim-4 Term: Feynman rules, loops, renormalization...

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# Heavy to Heavy: $B \rightarrow D \ell \bar{\nu}_{\ell}$ and $B \rightarrow D^* \ell \bar{\nu}_{\ell}$

Kinematic variable for a heavy quark: Four Velovity v
Differential Rates

$$\begin{split} \frac{d\Gamma}{d\omega} (B \to D^* \ell \bar{\nu}_\ell) &= \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega) (\mathcal{F}(\omega))^2 \\ \frac{d\Gamma}{d\omega} (B \to D \ell \bar{\nu}_\ell) &= \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2 \end{split}$$

- with  $\omega = vv'$  and
- $P(\omega)$ : Calculable Phase space factor
- $\mathcal{F}$  and  $\mathcal{G}$ : Form Factors

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# Heavy Quark Symmetries

- Normalization of the Form Factors is known at vv' = 1 from Heavy Quark Symmetries:
- Corrections can be calculated / estimated in HQET

$$\mathcal{F}(\omega) = \eta_{\text{QED}} \eta_A \left[ 1 + \delta_{1/\mu^2} + \cdots \right] + (\omega - 1)\rho^2 + \mathcal{O}((\omega - 1)^2)$$
  
$$\mathcal{G}(1) = \eta_{\text{QED}} \eta_V \left[ 1 + \mathcal{O} \left( \frac{m_B - m_D}{m_B + m_D} \right) \right]$$

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• Parameter of HQS breaking:  $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$ •  $\eta_A = 0.960 \pm 0.007, \eta_V = 1.022 \pm 0.004, \delta_{1/\mu^2} = -(8 \pm 4)\%, \eta_{\text{QED}} = 1.007$ 

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## Form Factors from the Lattice

- Unquenched Calculations become available!
- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

$$\mathcal{F}(1) = 0.91^{+0.03}_{-0.04}$$

 $G(1) = 1.074 \pm 0.018 \pm 0.016$ 

A. Kronfeld et al.

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## $B ightarrow D^* \ell ar{ u}_\ell$



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## $B \rightarrow D \ell \bar{ u}_{\ell}$



Thomas Mannel, University of Siegen Theoretical Tools for Heavy Quark Physics

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# $V_{\textit{cb,excl}} = (39.4 \pm 0.87^{+1.56}_{-1.24}) imes 10^{-3}$

Bob Kowalewski @ ICHEP06

Possible Improvements:

More precise Lattice calculations

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#### • Possible Improvements:

More precise Lattice calculations

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Operator Product Expansion Twist Expansion

# Inclusive Decays: Using OPE

### Operator Product Expansion = Heavy Quark Expansion

(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$\begin{split} &\Gamma \propto \sum_{X} (2\pi)^{4} \delta^{4} (P_{B} - P_{X}) |\langle X | \mathcal{H}_{eff} | B(v) \rangle|^{2} \\ &= \int d^{4} x \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}_{eff}^{\dagger}(0) | B(v) \rangle \\ &= 2 \, \operatorname{Im} \int d^{4} x \, \langle B(v) | T \{ \mathcal{H}_{eff}(x) \mathcal{H}_{eff}^{\dagger}(0) \} | B(v) \rangle \\ &= 2 \, \operatorname{Im} \int d^{4} x \, e^{-im_{b} v \cdot x} \langle B(v) | T \{ \widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}_{eff}^{\dagger}(0) \} | B(v) \rangle \end{split}$$

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Operator Product Expansion Twist Expansion

• Perform an OPE: *m<sub>b</sub>* is much larger than any scale appearing in the matrix element

$$\int d^{4}x e^{im_{b}vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}_{eff}^{\dagger}(0)\}$$
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2m_{Q}}\right)^{n} C_{n+3}(\mu) \mathcal{O}_{n+3}$$

 $\rightarrow$  The rate for  $B \rightarrow X_c \ell \bar{\nu}_\ell$  can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

• The  $\Gamma_i$  are power series in  $\alpha_s(m_Q)$ :  $\rightarrow$  Perturbaton theory!

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Operator Product Expansion Twist Expansion

- Γ<sub>0</sub> is the decay of a free quark ("Parton Model")
- Γ<sub>1</sub> vanishes due to Heavy Quark Symmetries
   Γ<sub>2</sub> is expressed in terms of two parameters

$$2M_{H}\mu_{\pi}^{2} = -\langle H(v)|\bar{Q}_{v}(iD)^{2}Q_{v}|H(v)\rangle$$
  
$$2M_{H}\mu_{G}^{2} = \langle H(v)|\bar{Q}_{v}\sigma_{\mu\nu}(iD^{\mu})(iD^{\nu})Q_{v}|H(v)\rangle$$

μ<sub>π</sub>: Kinetic energy and μ<sub>G</sub>: Chromomagnetic moment
Γ<sub>3</sub> two more parameters

 $2M_{H}\rho_{D}^{3} = -\langle H(v)|\bar{Q}_{v}(iD_{\mu})(ivD)(iD^{\mu})Q_{v}|H(v)\rangle$  $2M_{H}\rho_{LS}^{3} = \langle H(v)|\bar{Q}_{v}\sigma_{\mu\nu}(iD^{\mu})(ivD)(iD^{\nu})Q_{v}|H(v)\rangle$ 

 $\rho_D$ : Darwin Term and  $\rho_{LS}$ : Chromomagnetic moment

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Operator Product Expansion Twist Expansion

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Operator Product Expansion Twist Expansion

New:  $1/m_b^4$  Contribution  $\Gamma_4$  (Dassinger, Turczyk, M.)

#### • Five new parameters:

 $\begin{array}{ll} \langle \vec{E}^2 \rangle : & \mbox{Chromoelectric Field squared} \\ \langle \vec{B}^2 \rangle : & \mbox{Chromomagnetic Field squared} \\ \langle (\vec{p}^2)^2 \rangle : & \mbox{Fourth power of the residual } b \mbox{ quark momentum} \\ \langle (\vec{p}^2)(\vec{\sigma} \cdot \vec{B}) \rangle : & \mbox{Mixed Chromomag. Mom. and res. Momentum} \\ \langle (\vec{p} \cdot \vec{B})(\vec{\sigma} \cdot \vec{p}) \rangle : & \mbox{Mixed Chromomag. field and res. helicity} \end{array}$ 

Some of these can be estimated in naive factorization

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Operator Product Expansion Twist Expansion

### Heavy to Heavy: $B \rightarrow X_c \ell \bar{\nu}_\ell$

Determine the HQE parameters from

- Charged lepton energy spectrum
- Hadronic invariant mass spectrum
- From the theoretical side: Calculation of moments of the spectra

$$\langle M_X^n \rangle = \frac{1}{\Gamma} \int dM_X M_X^n \int_{E_{\text{cut}}} dE_\ell \frac{d^2 \Gamma}{dM_x dE_\ell} \langle E_\ell^n \rangle = \frac{1}{\Gamma} \int dM_X \int_{E_{\text{cut}}} dE_\ell \frac{E_\ell^n}{dM_x dE_\ell}$$

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Operator Product Expansion Twist Expansion

#### Hadronic Invariant Mass Moments (Buchmüller, Flächer)



Thomas Mannel, University of Siegen

Theoretical Tools for Heavy Quark Physics

**Operator Product Expansion** 

#### Lepton Energy Moments I (Buchmüller, Flächer)



Thomas Mannel, University of Siegen

Theoretical Tools for Heavy Quark Physics

Operator Product Expansion Twist Expansion

### Lepton Energy Moments II (Buchmüller, Flächer)



Operator Product Expansion Twist Expansion

$$V_{\textit{cb,incl}} = (41.96 \pm 0.23_{\textit{exp}} \pm 0.35_{\textit{HQE}} \pm 0.59_{\Gamma_{sl}}) imes 10^{-3}$$

O. Buchmüller, HQL2006

Thomas Mannel, University of Siegen Theoretical Tools for Heavy Quark Physics

Operator Product Expansion Twist Expansion

#### **Twist Expansion**

Calculation of spectra within the OPE



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Operator Product Expansion Twist Expansion

• In the massless case this becomes for  $B \to X_u \ell \bar{\nu}_\ell$ 

$$\frac{d\Gamma}{dy} \stackrel{y \to 1}{=} \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \\ \left[\Theta(1-y) + \frac{\mu_\pi^2 - \mu_G^2}{6m_b^2} \delta(1-y) + \frac{\mu_\pi^2}{6m_b^2} \delta'(1-y) + \cdots \right]$$

• Likewise for  $B \to X_s \gamma$  ( $x = \frac{2E_\gamma}{m_b}$ )

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts}V_{tb^*}|^2 |C_7|^2 \\ \left(\delta(1-x) - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \delta'(1-x) + \frac{\mu_\pi^2}{6m_b^2} \delta''(1-x) + \cdots\right)$$

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Operator Product Expansion Twist Expansion

# Shape- or Light-Cone Distribution Functions

 Resummation into a shape function or light cone distribution function (Bigi, Shifman, Uraltsev, Neubert, M., ...)

$$2M_{B}f(\omega) = \langle B(v)|\bar{b}_{v}\delta(\omega + i(n \cdot D))|B(v)\rangle$$

such that

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \int d\omega \,\Theta(m_b(1-y)-\omega) f(\omega)$$

and

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Operator Product Expansion Twist Expansion

#### • General Structure:

$$\frac{d\Gamma}{dx} = \Gamma_0 \left[ \sum_i a_i \left( \frac{1}{m_b} \right)^i \delta^{(i)}(1-x) + \mathcal{O}((1/m_b)^{i+1} \delta^{(i)}(1-x)) \right]$$

• Coefficients *a<sub>i</sub>* are the moments of the spectrum:

• Moment Expansion of *f* in terms of HQE parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_{\pi}^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{18m_b^3}\delta'''(\omega) + \cdots$$

• Twist Expansion in complete analogy to deep inelastic scattering

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Operator Product Expansion Twist Expansion

• General Structure:

$$\frac{d\Gamma}{dx} = \Gamma_0 \left[ \sum_i a_i \left( \frac{1}{m_b} \right)^i \delta^{(i)}(1-x) + \mathcal{O}((1/m_b)^{i+1} \delta^{(i)}(1-x)) \right]$$

- Coefficients *a<sub>i</sub>* are the moments of the spectrum:
- Moment Expansion of *f* in terms of HQE parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_{\pi}^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{18m_b^3}\delta'''(\omega) + \cdots$$

• Twist Expansion in complete analogy to deep inelastic scattering

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Soft Collinear Effective Theory Towards Understanding Nonleptonic Decays

Recent Developments: Soft Collinear Effective Theory

- Problem: How to calculate corrections to the shape functions?
- More than two scales involved!
- Inclusive Rates in the Endpoint become (Korchemski, Sterman)

 $d\Gamma = H * J * S$ 

with \* = Convolution

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- *H*: Hard Coefficient Function, Scales  $\mathcal{O}(m_b)$
- J: Jet Function, Scales  $\mathcal{O}(\sqrt{m_b \Lambda_{\text{QCD}}})$
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Soft Collinear Effective Theory Towards Understanding Nonleptonic Decays

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### **Basics of Soft Collinear Effective Theory**

• Heavy-to-light decays:

Kinematic Situations with energetic light quarks hadronizing into jets or energetic light mesons  $p_{fin}$ : Momentum of a light final state meson

$$p_{ ext{fin}}^2 \sim \mathcal{O}(\Lambda_{ ext{QCD}} m_b) \quad v \cdot p_{ ext{fin}} \sim \mathcal{O}(m_b)$$

• Use light-cone vectors  $n^2 = \bar{n}^2 = 0$ ,  $n \cdot \bar{n} = 2$ :

$$p_{\mathrm{fin}} = rac{1}{2}(n \cdot p_{\mathrm{fin}})ar{n}$$
 and  $v = rac{1}{2}(n + ar{n})$ 

$$p_{\text{light}} = \frac{1}{2}[(n \cdot p_{\text{light}})\bar{n} + (\bar{n} \cdot p_{\text{light}})n] + p_{\text{light}}^{\perp}$$

Soft Collinear Effective Theory Towards Understanding Nonleptonic Decays

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Soft Collinear Effective Theory Towards Understanding Nonleptonic Decays

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- Define the parameter  $\lambda = \sqrt{\Lambda_{\rm QCD}/m_b}$
- The light quark invariant mass (or virtuality) is assumed to be

$$p_{ ext{light}}^2 = (n \cdot p_{ ext{light}})(ar{n} \cdot p_{ ext{light}}) + (p_{ ext{light}}^{\perp})^2 \sim \lambda^2 m_b^2$$

• The components of the quark momentum have to scale as

$$(n \cdot p_{ ext{light}}) \sim m_b \quad (ar{n} \cdot p_{ ext{light}}) \sim \lambda^2 m_b \qquad p_{ ext{light}}^\perp \sim \lambda m_b$$

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### A brief look at SCET (Bauer, Stewart, Pirjol, Beneke, Feldmann ...)

- QCD quark field Q is split into a collinear component
- The Lagrangian  $\mathcal{L}_{OCD} = \bar{q}(i\mathcal{D})q$  is rewritten in terms

$$\mathcal{L} = \frac{1}{2}\bar{\xi}\not\!/_+(in_-D)\xi - \bar{\xi}i\not\!/_\perp\frac{1}{in_+D + i\epsilon}\frac{\not\!/_+}{2}i\not\!/_\perp\xi$$

• Expansion according to the above power couning:

$$in_+D = in_+\partial + gn_+A_c + gn_+A_{us} = in_+D_c + gn_+A_{us}$$

Leading L becomes non-local: Wilson lines

Soft Collinear Effective Theory

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Soft Collinear Effective Theory Towards Understanding Nonleptonic Decays

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- Allows us to calculate radiative corrections systematically
- Extremely important for the determination of V<sub>ub</sub>
- Shape functions are modelled or taken from  $B \rightarrow X_s \gamma$

### $V_{ub,incl} = (4.48 \pm 0.20_{exp} \pm 0.27_{m_b,theo}) imes 10^{-3}$

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### Results for V<sub>ub</sub>



Thomas Mannel, University of Siegen

Theoretical Tools for Heavy Quark Physics

# **Towards Understanding Non-leptonic Decays**

• Non-leptonic decays require the calculation of hadronic matrix elements of four-quark operators, e.g. for a decay like  $B \rightarrow \pi\pi$ 

$$\mathcal{M} = \langle \pmb{B} | (ar{\pmb{b}} \gamma_{\mu} (\pmb{1} - \gamma_5) \pmb{q}) (ar{\pmb{q}}' \gamma_{\mu} (\pmb{1} - \gamma_5) \pmb{q}'') | \pi \pi 
angle$$

 In the large *m<sub>b</sub>* limit a factorization theorem has been proven (QCD-Factorization, similar to SCET)

(Beneke, Buchalla, Neubert, Sachrajda)



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- Leading term is (contains) naive factorization
- Non-perturbative qunatities are the soft form factor and the light cone distributions of the light hadrons and of the *B* meson
- The strong phases of the matrix elements are either perturbative (O(α<sub>s</sub>(m<sub>b</sub>))) or power suppressed (O(Λ<sub>QCD</sub>/m<sub>b</sub>))
- → The strong phases are predicted to be small(ish)
- $\bullet \rightarrow$  Important for the calculation of CP Asymmetries
- Does it work?

Soft Collinear Effective Theory Towards Understanding Nonleptonic Decays

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(Beneke, Buchalla, Neubert, Sachrajda, 2001)

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#### Update on the BR's by Neubert (CKM 2005, San Diego)



Thomas Mannel, University of Siegen

Theoretical Tools for Heavy Quark Physics

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#### Update on the CP Asymmetries by Neubert (CKM 2005)



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Theoretical Tools for Heavy Quark Physics

Soft Collinear Effective Theory Towards Understanding Nonleptonic Decays

- Effective field theory methods made precise calculations in heavy quark physics possible
- Starting about 1989 HQET and HQE put heavy quark physics on a model independent basis
- → Model dependence often appears only at subleading orders
- SCET is an ansatz to undestand also exclusive non-leptonic decays systematically
- I did not talk about Lattice QCD calculations: Enormous progress due to better algorithms and to stronger computers
- Heavy Flavour Physics has become (in some corners) a precision field.

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# Summary II: The problems

- The key question is:
  - Is a small "deviation" due to physics beyond the standard model or due to our lack of understanding of QCD dynamics?
- Subleading terms are under reasonable control only for inclusive semileptonic decays
- Exclusive non-leptonic decays still have uncertainties of typically  $\mathcal{O}(10\%)$
- Up to now no (large) effects have been observed that contradict the CKM description of flavour mixing and CP violation
- Still the progress has been dramatic over the last 15 years ...

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### 1988: Pre-historic Unitarity Triangle


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## 2006: Todays Unitarity Triangle



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#### There are interesting tensions ...



Thomas Mannel, University of Siegen

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#### There are interesting tensions ...



# ... we shall see .

Thomas Mannel, University of Siegen

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