QCD developments for the LHC

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CERN

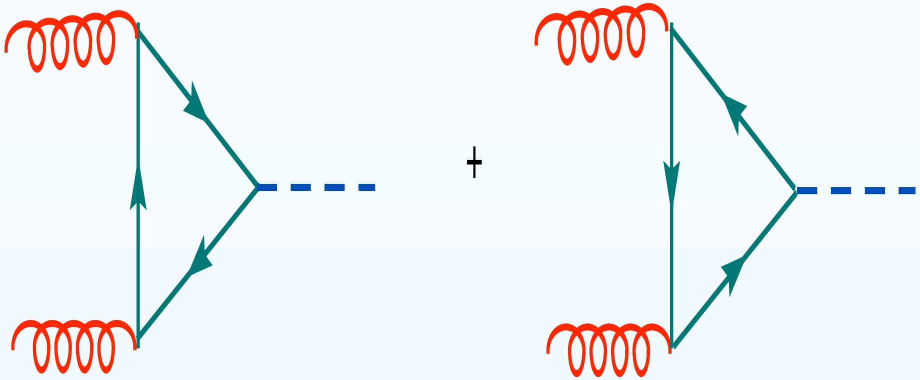
New phenomena at the LHC

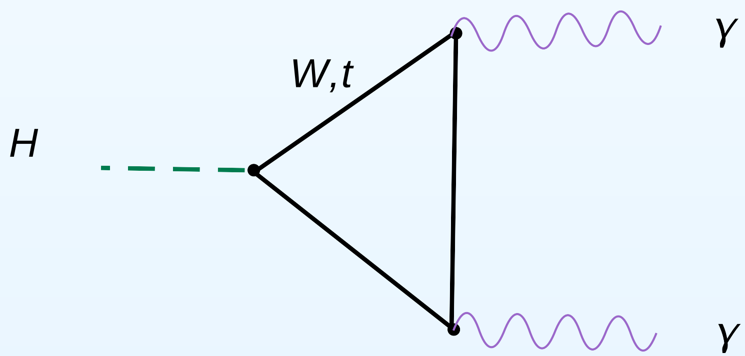
The LHC will give us a unique opportunity for new discoveries at TeV energies.

- Large energy and luminosity
  - Small statistical uncertainties.
  - Very good detectors; high rate calibration processes $\rightsquigarrow$ smaller systematic errors
- High rates could allow both discoveries, precision studies, and discoveries through precision.
- The LHC will also test how well we understand QCD effects.

An example of an “easy” experimental discovery

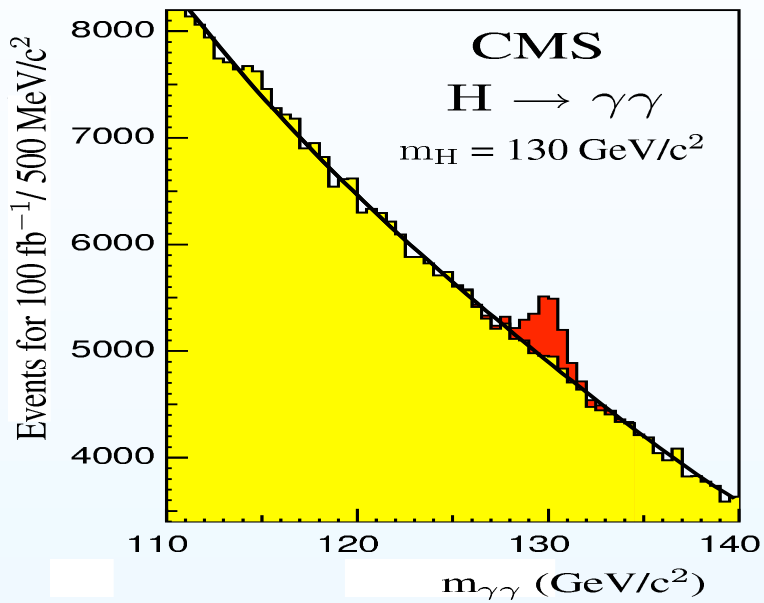
- The SM predicts a significant cross-section for a di-photon signal from a Higgs boson.





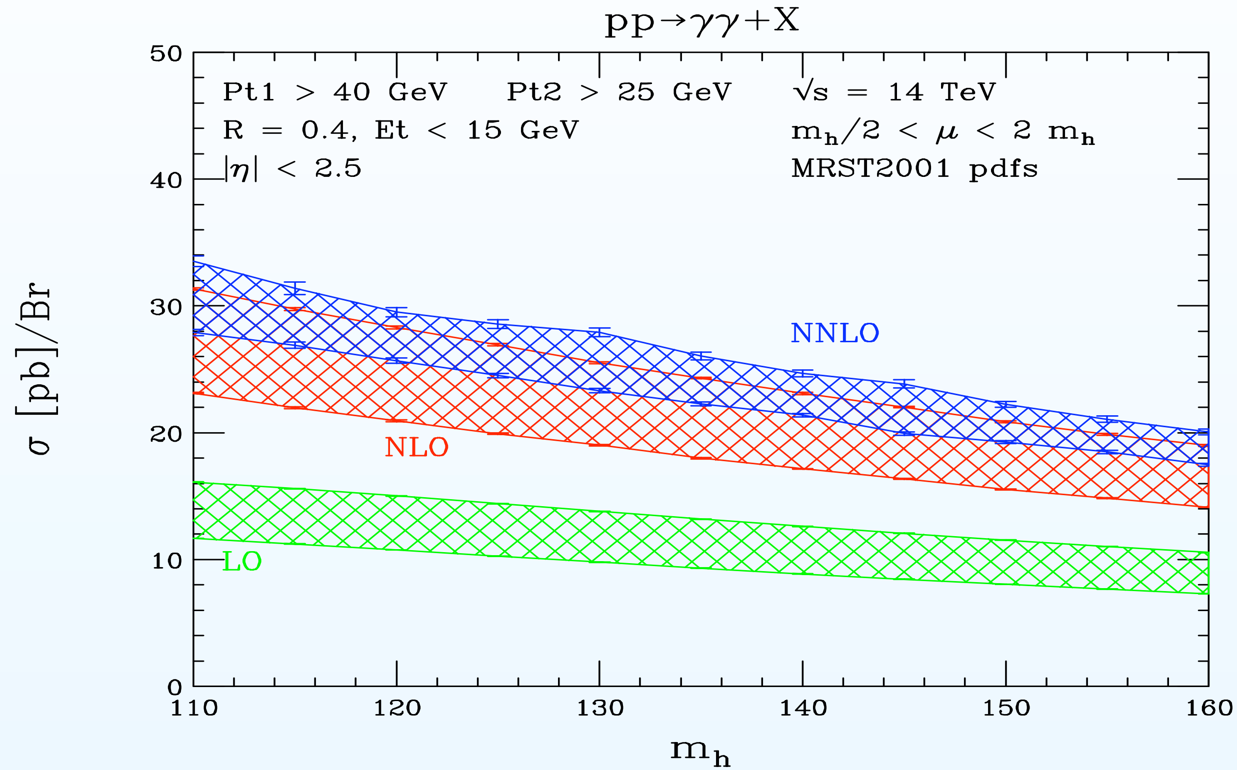
- Discovery of a resonance is a matter of purely (very hard) experimental work and collecting data.

The di-photon signal



- It is not necessarily true that this peak is a SM Higgs boson.
- New physics beyond the SM can change significantly the height of the peak.
- So do higher order QCD corrections

Di-photon signal cross-section



CA, Melnikov, Petriello

- The cross-section at NNLO is 2 times the LO result.
- Scale uncertainty reduces from $\pm 15\%$ (NLO) to $\pm 7\%$ (NNLO).

A global approach to precision calculations

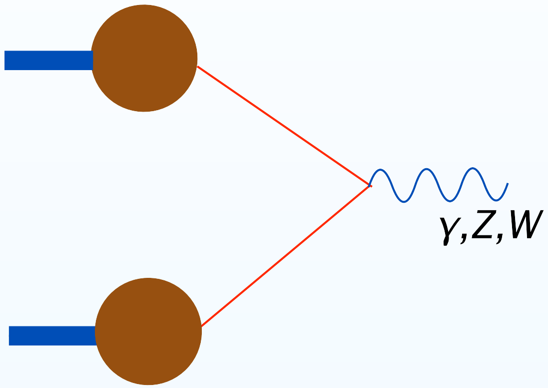
$$N = \mathcal{L} \times \left(\int f_i(x_1) f_j(x_2) \sigma(i + j \to H + X) \right) \times \frac{\Gamma(H \to \gamma\gamma)}{\Gamma_{total}}$$

- The measurement of the Higgs boson cross-section could become a tool for precision studies, if we know accurately:

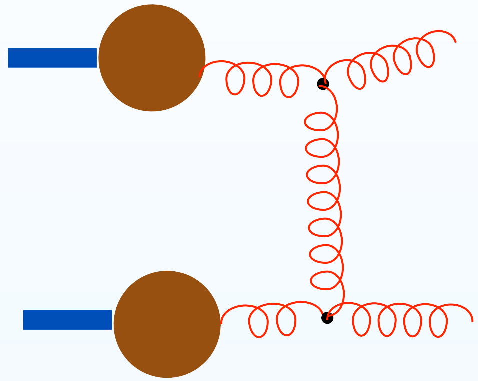
1. *Production cross-section and branching ratio*
2. *Strong coupling*
3. *Parton distribution functions*
4. *Luminosity (or partonic luminosities:* $\mathcal{L}_{ij}(x_1, x_2) = \mathcal{L} f_i(x_1) f_j(x_2)$*)*

ALL of the above require theory input!

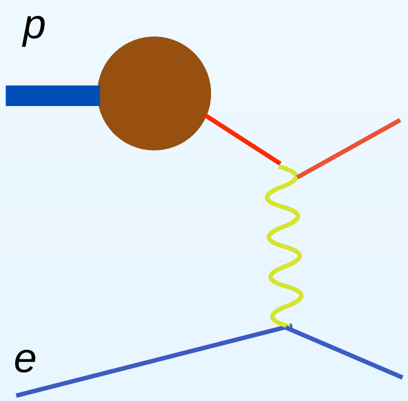
Standard candle processes



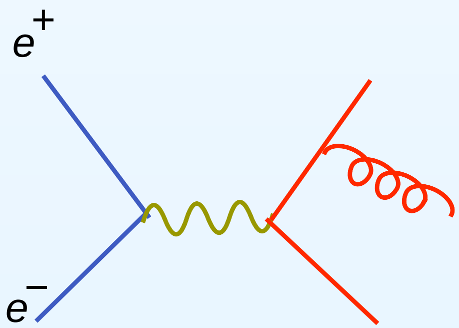
Luminosity measurement
Parton densities
weak mixing angle
W−mass



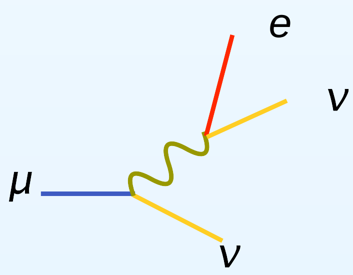
Gluon density



Parton densities



Strong coupling



Fermi constant

Luminosity monitoring

- Monitor luminosity with W production (Dittmar et al.). Two ways to improve on the standard NLO predictions.
  - Consistent merging of NLO+parton shower in MC@NLO (Frixione, Webber)
  - Fullly differential NNLO calculation with spin correlations complete (Melnikov, Petriello)
  - Cut 1: $p_T^e > 20$ GeV, $|\eta^e| < 2.5$, $\not{E}_T > 20$ GeV (LHC)
    Cut 2: $p_T^e > 40$ GeV, $|\eta^e| < 2.5$, $\not{E}_T > 20$ GeV (LHC)

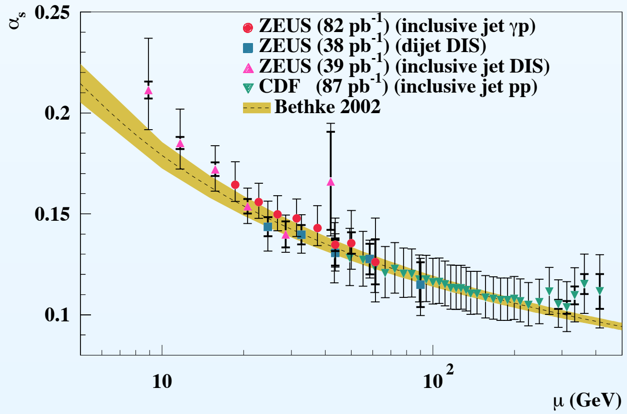
LHC	$\frac{\sigma_{MC@NLO}}{\sigma_{NLO}}$	$\frac{\sigma_{NNLO}}{\sigma_{NLO}}$
Cut 1	1.02	0.983
Cut 2	1.03	1.21

  - Large dependence of NNLO corrections on cuts.

$\Rightarrow$ extra hard emission at NNLO important! Not captured by the shower corrections in MC@NLO (off by 20%)

High multiplicity background processes

- Vital searches are more complicated. For example, SUSY models with R-parity conservation predict the production of a large number of jets and missing energy.
- Squark and gluino production is uncertain to 100% at leading order, and 30% at NLO. Beenakker, Höpker, Spira, Zerwas
- Standard Model multijet production processes are very sensitive to scale variations.



$$\sigma_{\text{N jets}} \sim \alpha_s^N(\mu) \times (\text{LO} + \ldots)$$

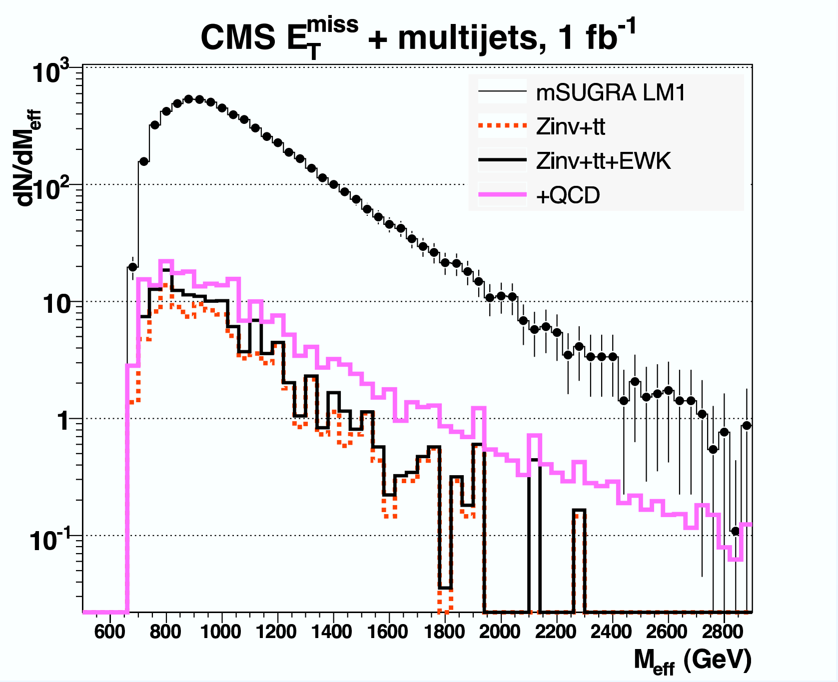
SUSY cross-sections are large

Susy Model	$m_{\tilde{q}}\ (GeV)$	$m_{\tilde{g}}\ (GeV)$	$\sigma\ (pb)$
LM1	558.61	611.32	54.86
LM3	625.65	602.15	45.47
LM5	809.66	858.37	7.75
LM7	3004.3	677.65	6.79
HM4	1815.8	1433.9	0.102

CMS TDR, using PROSPINO

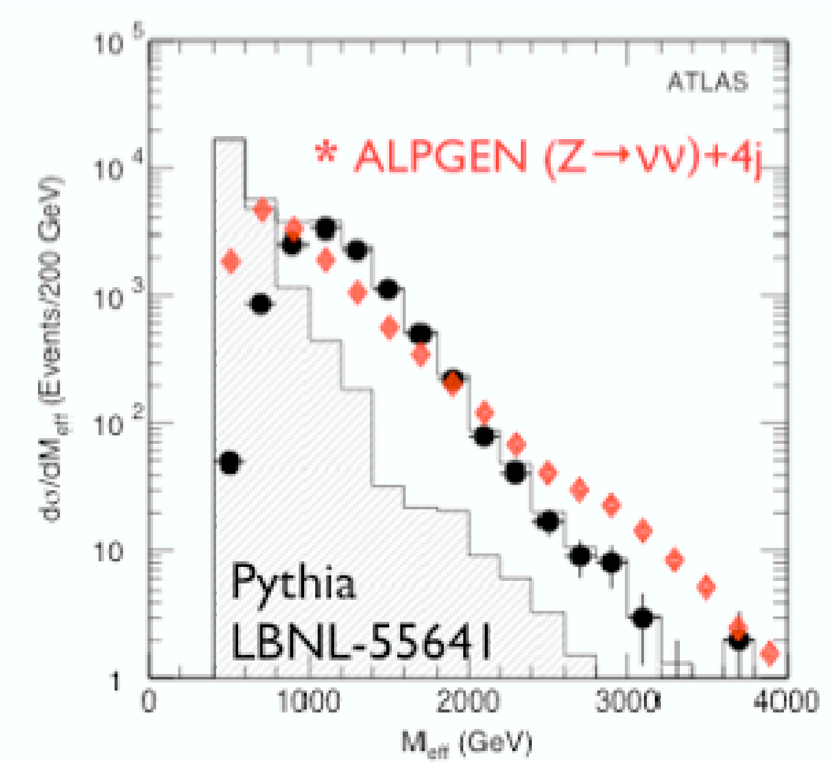
- Cross-sections can vary a lot in its versions.
- But “SUSY signals should be spectacular!”

SUSY signals could be spectacular



- CMS TDR: analysis with full detector simulation
- SM backgrounds with PYTHIA (parton-shower)

Or not?



Mangano

- Shower fails to simulate hard jets!
- We need exact LO matrix-elements for $2 \to 3, 4, 5$ processes.

Do we need NLO?

Leading order scale variation for $pp \to \nu\bar{\nu} + N$jets Select high $p_t > 80\,\mathrm{GeV}$, central $|\eta| < 2.5$ jets. Let us assume that a reasonable scale is:

$$\mu^2 = M_Z^2 + \sum_{jet} p_{t,jet}^2$$

and allow to vary: $\mu_R = \mu_F = \mu/2 - 2\mu$

N	$\sigma(2\mu)[pb]$	$\sigma(\mu/2)[pb]$	variation
1	182	216	17%
2	47.1	75.4	46%
3	6.47	13.52	70%
4	0.90	2.48	93%

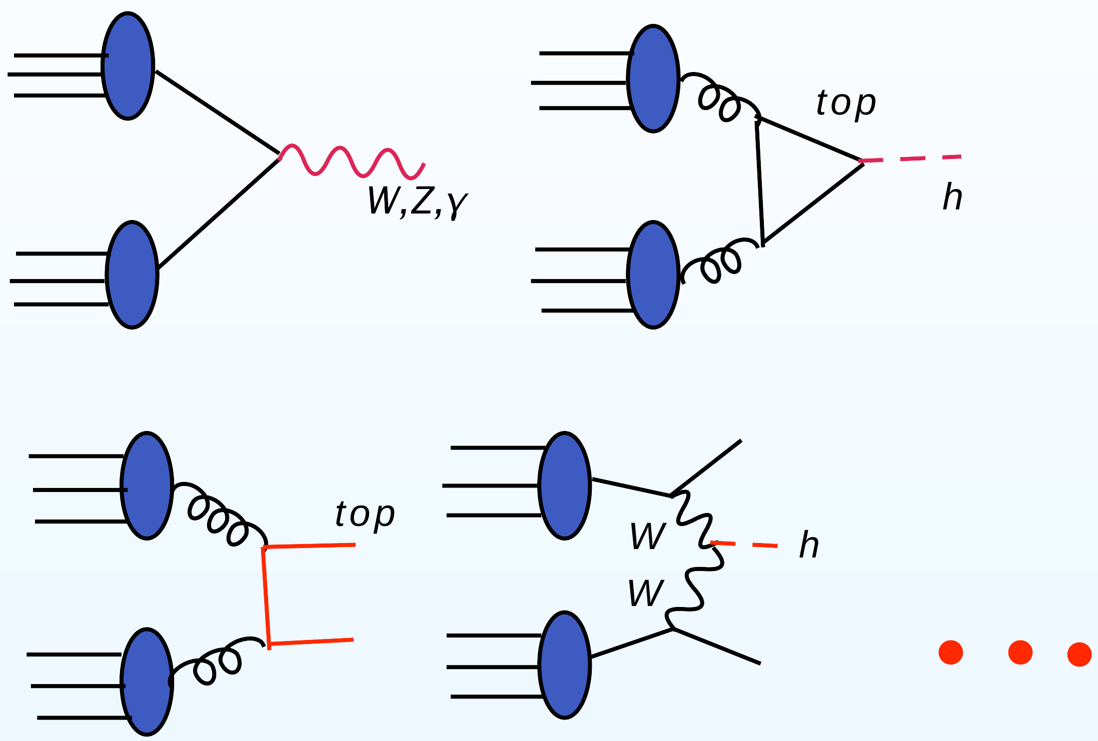
ALPGEN

For a 5σ discovery with LO magnitudes: $\rightsquigarrow$ Signal > 2.5 Background

Conclusions I

- At the LHC we could get clear signals over very well measured backgrounds *(new reasonances)* or negligible backgrounds
  - $\rightsquigarrow$ Precise calculation of the signal
  - $\rightsquigarrow$ Flexibility to include interactions of new models in our calculations.
- We also anticipate not so spectacular signals with difficult to measure backgrounds
  - $\rightsquigarrow$ Precise calculation of the signal and background (to consolidate or compete with the precision of the experimental measurement of the background)
- Standard caddle (LHC, LEP, Tevatron, Hera, . . .) processes for luminosity monitoring, α_s, parton densities . . .
  - $\rightsquigarrow$ Very precise calculation of cross-sections
- We need to look at the same process in more than one ways (e.g. parton shower vs fixed-order, . . .)

Processes at the LHC



- A vast experimental program: pointless to give a list of "interesting" processes.
- We hope to discover many new BSM processes. But even within the SM, there is a lot to do!

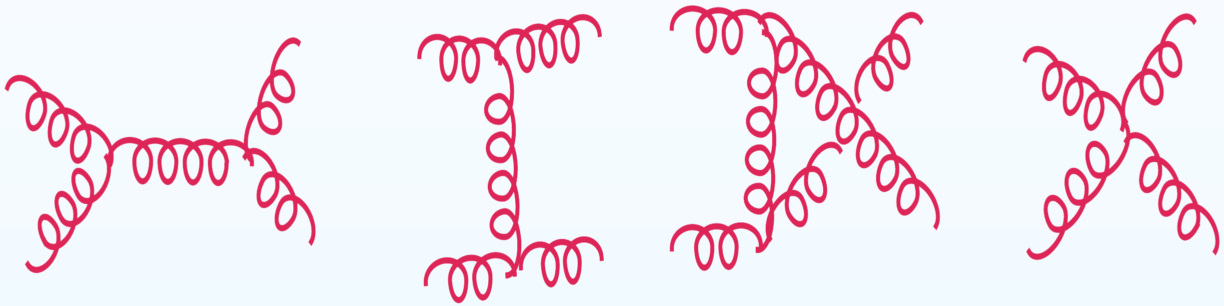
What is available?

Many new techniques!

- Perturbative QCD is a very active field in recent years.
- We have made progress in every aspect of it:
  - Leading order, Next to LO, NNLO
  - Resummation, merging fixed order calculations and parton showers.
  - All orders!
- Progress has been made with the generation of very good new ideas. Not just by turning the crank!
- Refreshing influx of ideas and people from other fields (string theory)
- A very competitive research area, with many challenges to be taken up.

Leading order perturbation theory

- It provides a rough estimate for cross-sections.
- Usually, it involves the calculation of tree diagrams:



  - Derive Feynman rules from Lagrangian.
  - Write down diagrams.
  - Perform Dirac and colour algebra.
  - Numerically integrate over the phase-space.

- A conceptually solved problem (like most in pQCD)! But in practice we need to be more clever.

Algebraic explosion

- For example, in $gg \to N$ gluons we need to compute:

N	diagrams
2	4
4	220
6	$34,300$
8	$10,525,900$

- Feynman rules in gauge theory

$$\mathcal{V}_{ggg} = f^{abc}\left[g_{\mu_1\mu_2}(p_1-p_2)^{\mu_3} + g_{\mu_2\mu_3}(p_2-p_3)^{\mu_1} + g_{\mu_3\mu_1}(p_3-p_1)^{\mu_2}\right]$$

- Algebra of γ matrices, colour algrebra, etc.

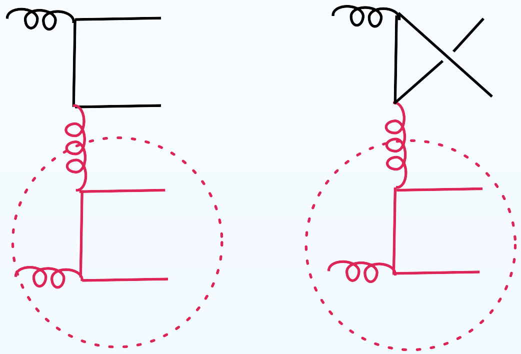
$$\begin{aligned} Tr(\gamma^{\mu_1}\gamma^{\mu_2}) &= \text{1 term} \\ Tr(\gamma^{\mu_1}\cdots\gamma^{\mu_8}) &= \text{105 terms} \\ Tr(\gamma^{\mu_1}\cdots\gamma^{\mu_{14}}) &= \text{26,931 terms} \end{aligned}$$

Tree-level Monte-Carlo generators

- HELAC/PHEGAS: EWK+QCD $10-12$ final state particles
  Kanaki, Papadopoulos
- ALPGEN: e.g. $Zt\bar{t}+(\leq 4)$jets, $(W,Z)+(\leq 6)$jets, inclusive ≤ 6 jets, $\ldots$, Mangano, Moretti, Piccinini, Pittau, Polosa
- COMPHEP, $2\rightarrow N(\leq 4)$ Puckhov et al.
- GRACE/GR@PPA, e.g. $W+4$ jets Ishikawa et al./Sato et al.
- PHASE, 6 final state fermions Accomando, Ballestero, Maina
- AMEGIC++, up to 6 external legs Krauss et al.
- MADGRAPH/MADEVENT, up to 1000 diagrams Long,Maltoni, Stelzer

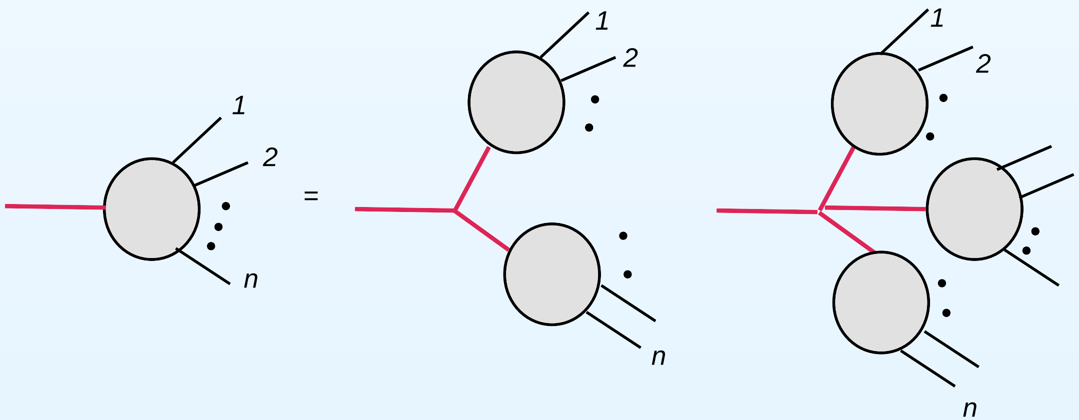
Recursion at tree-level

- Feynman diagrams contain sub-parts which we compute over and over.



- It is possible to organize the evaluation of tree amplitudes recursively

e.g. Berends, Giele



Britto Cachazo Feng Witten recursion finding trick

- Amplitudes are functions of external momenta

$$A(p_1, p_2, \ldots, p_n)$$

- For massless particles $p^\mu \to p_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}}$; this can be written as the product of two spinors:

$$p = \lambda^a \tilde{\lambda}^{\dot{a}}$$

- Then they considered a more general object, extending two of the momenta to be complex but preserving momentum conservation:

$$p_1 = p_1 + z\lambda_1^a \tilde{\lambda}_4^{\dot{a}} \qquad p_4 = p_4 - z\tilde{\lambda}_4^{\dot{a}} \lambda_1^a$$

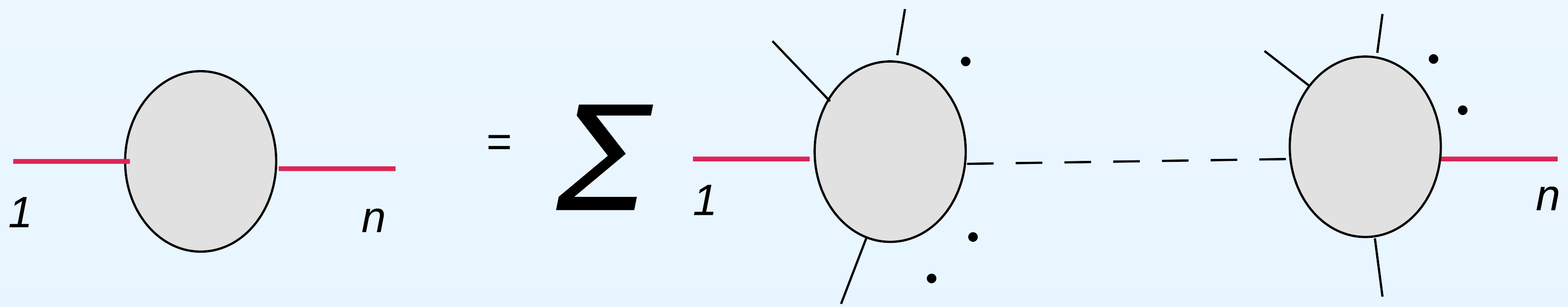
Analytic extension of tree amplitudes

- The generalized tree-amplitudes, if $(A(z \to \infty) = 0)$, have simple poles; all possible poles that can be found in propagators of Feynman diagrams.

$$A(z) = \sum_{p_{i\ldots j}} \frac{c_{ij}(z)}{\tilde{p}^2_{i\ldots j} - z}$$

- The physical amplitude is:

$$A(0) = \sum_{p_{i\ldots j}} \frac{c_{ij}(0)}{\tilde{p}^2_{i\ldots j}}$$



Quantitative predictions at NLO

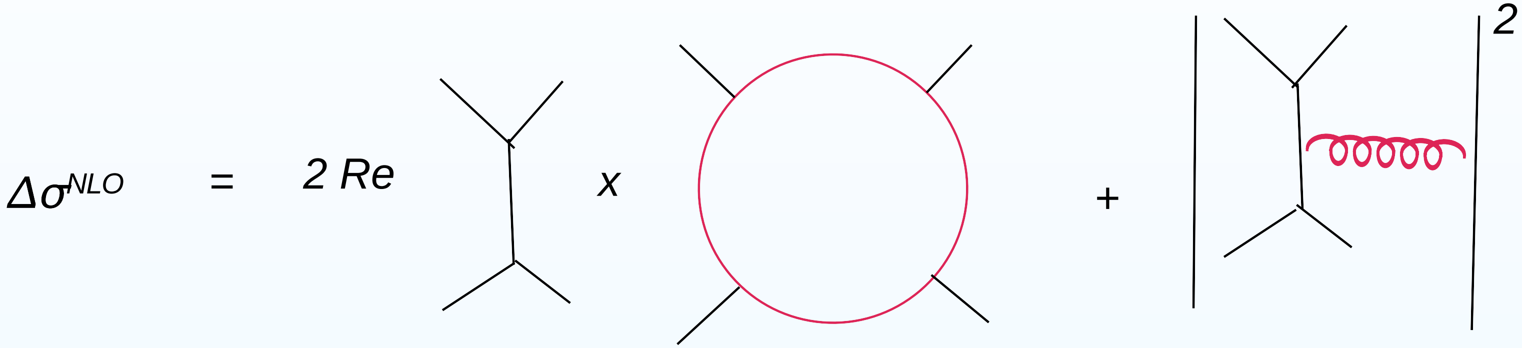
- Reduced senitivity in factorization and renormalization scales

$$\frac{\partial \alpha_s}{\partial \log(\mu)} = -\beta_0 \alpha_s^2 + \mathcal{O}(\alpha_s^3)$$

$$\frac{\partial f(x,\mu)}{\partial \log(\mu)} = \alpha_s \int_z^1 \frac{dy}{y} P_{ab}(y) f(x/y,\mu) + \mathcal{O}(\alpha_s^2)$$

- New channels: For example, in Higgs production we included the processes $gg \longrightarrow hg$, $qg \longrightarrow hq$ and $q\bar{q} \longrightarrow hg$.
- More realistic cover of the phase-space. At leading order, the Higgs boson has no transverse momentum. At NLO, $p_t \geq 0$.
- We have seen many examples where NLO corrections cannot be neglected ($gg \longrightarrow h$, Drell-Yan production, squark and gluino production,W-pair production, $\ldots$)

NLO computations



- Exploit universality of infrared singularities. We always cancel the same divergences.

$$\Delta\sigma_{NLO} = \int dPS_m (2 Tree_m Loop_m) Obs_m$$
$$+ \int dPS_{m+1} |Tree_{m+1}|^2 Obs_{m+1}$$

Cancelation of infrared divergences at NLO

Ellis, Ross, Terrano, Giele, Glover; Giele Glover, Kosower; Kunst, Soper; Frixione, Kunszt, Signer; Catani, Seymour; . . .

- The single infrared limit (one soft or two collinear partons) of tree amplitudes is universal “antennae” functions:

$$|Tree_{m+1}|^2 \to \text{infrared limit} \to |Tree_m|^2 \times Antenna$$

- I can rearrange:

$$\Delta\sigma_{NLO} = \int dPS_{m+1}\left[|Tree_{m+1}|^2 \mathbf{Obs_{m+1}} - |Tree_m|^2 \times Antenna\mathbf{Obs_m}\right]$$

$$+\int dPS_m (2Tree_m Loop_m) Obs_m + \int dPS_m |Tree_m|^2 \times Obs_m \int PS_{1\to 2} Antenna$$

Loop integral relations

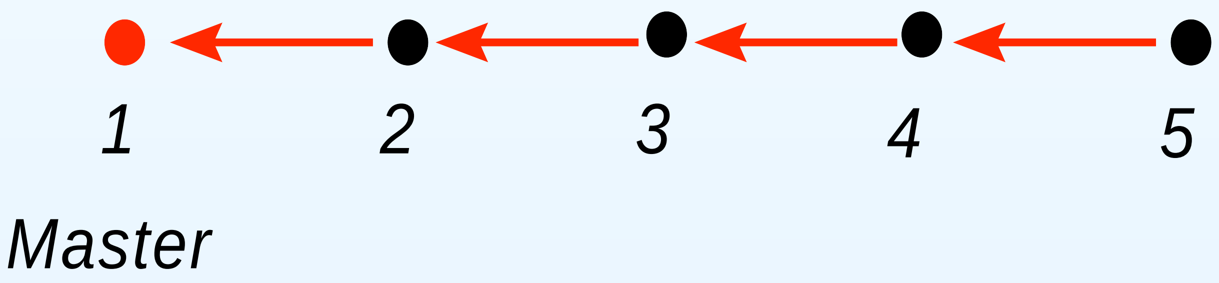
- Loop integrals are not independent:

$$\int d^dk \frac{\partial}{\partial k_\mu} \frac{k_\mu}{k^2 - M^2} = 0$$

Chetyrkin, Tkachov

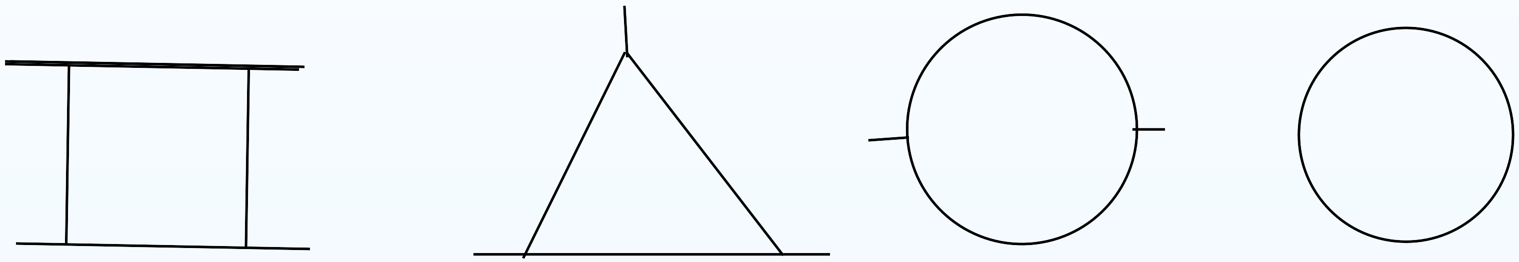
$$M^2 \int d^dk \frac{1}{(k^2 - M^2)^{2}} + \left(\frac{d}{2} - 1\right) \int d^dk \frac{1}{(k^2 - M^2)^{1}} = 0$$

- We need to compute less!



One-loop master integrals

All one-loop integrals are reduced to a few known master integrals:



These are known analytically, or known how to compute, for all cases needed.

- Generic reductions now work for multi-loop calculations too

Laporta, Gehrmann, Remiddi; CA, Lazopoulos, . . .

Available next to leading order calculations

- Numerous calculations have been made at NLO.
- Anything you can think of for $2 \to 2$ processes: $pp \to WW$, $pp \to \gamma\gamma$, $pp \to t\bar{t}, \ldots$
- Many but not all, $2 \to 3$ processes: $pp \to \leq 3$jets, $pp \to W, Z+ \leq 2$jets, $pp \to qqh$, $pp \to tth, \ldots$
- No $2 \to 4$ process for the LHC. Only example of close enough complexity $e^+e^- \to 4$ fermions , Denner, Dittmaer, et al.

  High multiplicity Standard Model processes (more than two particles in the final state) are baqckrounds to new physics $2 \to 2$ production processes. E.g. in supersymmetry with R-parity conservation sparticles are always pair produced.

- What is the problem? Gigabyte sized expressions!

New attempts to solve the problem

- Modified reductions to compactify expressions, avoid fake singularities, . . ., Denner, Dittmaer
- Numerical reduction to master integrals, Giele, Glover; Ellis, Giele, Zanderighi
- Numerical evaluation in the complex plane, CA, Daleo
- Subtraction method for loop amplitude, Nagy, Soper
- Improved unitarity method I Bern, Berger, Dixon, Forde, Kosower Xiao, Yang, Zhu; Binoth, Gulliet, Heinrich
- Improved unitarity method II Britto, Cachazo, Feng, CA, Mastrolia,Kunszt

Recent breakthrough

del Aguila, Pittau; Ossola, Papadopoulos, Pittau

- Discovered a miraculous functional form for generic loop integrands!

$$Amplitude = \int d^d k \left(A_1 \frac{1}{Den_1 Den_2 \ldots Den_n} + B_1 \frac{Spurious_1(k)}{Den_1 Den_2 \ldots Den_n} + \sim \text{60 more terms} \right)$$

- first term (A_1) inegrates to a single master integral
- Spurious term (B_1) integrates to zero
- Determine A_1, B_1 by evaluating the INTEGRAND at a sufficient number of values for the loop momentum.

Ossola, Papadopoulos, Pittau method

- Choose momenta corresponding to the unitarity cuts of the loop amplitude
- Isolates one master integral at the time
- Setup a numerical evaluation approach

Message

- All master integral coefficients are simply sums of products of tree amplitudes. Simple algebraic substitution!
- Makes great connection with the developments at tree-level!
- Watch this space! Great news and timing for the LHC.

Is NNLO needed?

- We have seen that, in Higgs production, the NLO corrections are very large ($\sim 80\%$). NNLO is needed to justify the perturbative calculation.
- NNLO calculations for observables which can be measured very well and be used for high precision studies:
  - *cross-sections for resonances (Higgs boson, W,Z, new gauge bosons, . . .)*
  - *High rate processes, e.g. inclusive jet cross-section, top-quark cross-section, etc*

What is available

- NNLO results:
  - *Drell-Yan total cross-section* — *Matsuura, Hamberg, van Neerven (1991)*; *Harlander, Kilgore (2002)*
  - *Higgs boson (h,A) total cross-section* — *Harlander, Kilgore (2002)*; *CA, Melnikov (2002)*; *Ravindran, Smith, van Neerven (2003)*
  - *Drell-Yan rapidity distribution* — *CA, Dixon, Melnikov, Petriello (2003)*
  - *Splitting functions* — *Moch, Vogt, Vermaseren (2004)*
  - *Higgs boson fully differential cross-section* — *CA, Melnikov, Petriello (2004)*
  - *W-boson fully differential cross-section* — *Melnikov, Petriello*
  - *Two-loop amplitudes (but not yet the cross-sections) for* $pp \to 1jet + X$, $pp \to \gamma\gamma$, $pp \to \gamma jet$, $pp \to W, Z + 1jet$, $pp \to h + 1jet$ — *CA, Glover, Oleari, Tejeda-Yeomans; Bern, Dixon, De Freitas, Ghinculov; Garland, Glover, Gehrmann, Koukoutsakis, Remiddi*

Towards a subtraction method at NNLO

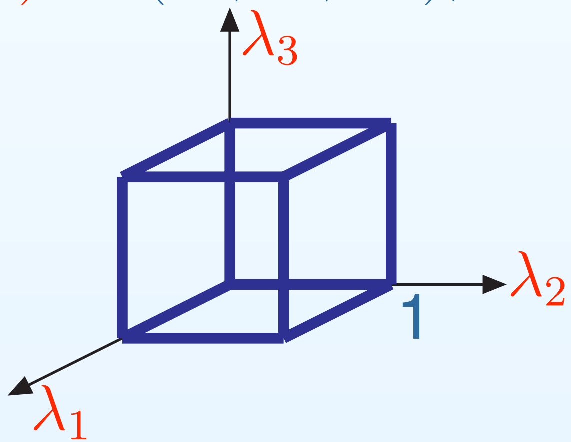
- New subtraction methods are now under completion.
  (Weinzierl; Kosower; Gehrmann-de Ridder, Gehrmann, Glover, Heinrich; Kilgore; Frixione, Grazzini; Somogyi, Trocsanyi, del Duca)
- Significant progress in understanding the infrared structure of perturbation theory at the second and higher orders.
- Subtraction algorithms satisfy all criteria to be sussesfull.
- Implementation phase is on. Still a significant amount of work is required

Singularities in a form amenable to algorithms

CA, Melnikov, Petriello

- Singularities have a very complicated form in momentum space (beyond NLO)
- Map phase-space volume to the unit hypercube

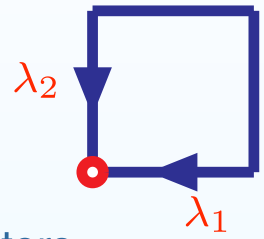
$$(E, p_x, p_y, p_z) \to (\lambda_1, \lambda_2, \ldots), \quad 0 \le \lambda_i \le 1$$



  - Simple geometry ⇝ (automatization)
  - Easy to spot singular regions ⇝ the edges!

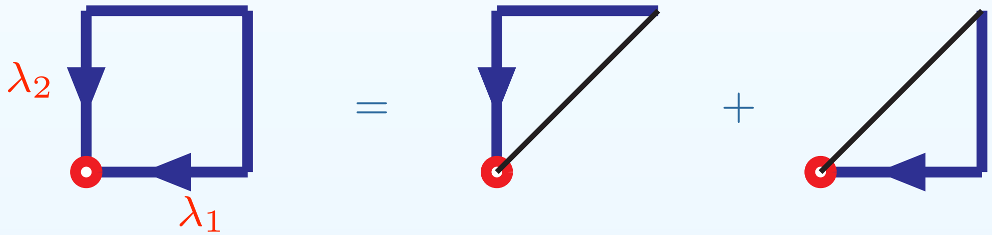
Overlapping singularities

- Singularity when two (or more) variables reach the same corner

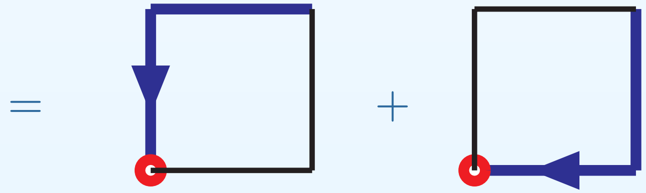


$$: \quad \frac{\lambda_1^\epsilon \lambda_2^\epsilon}{(\lambda_1+\lambda_2)^2} f(\lambda_1, \lambda_2; Obs(\lambda_1, \lambda_2))$$

- Split into sectors — Binoth, Heinrich; Denner, Roth; Hepp

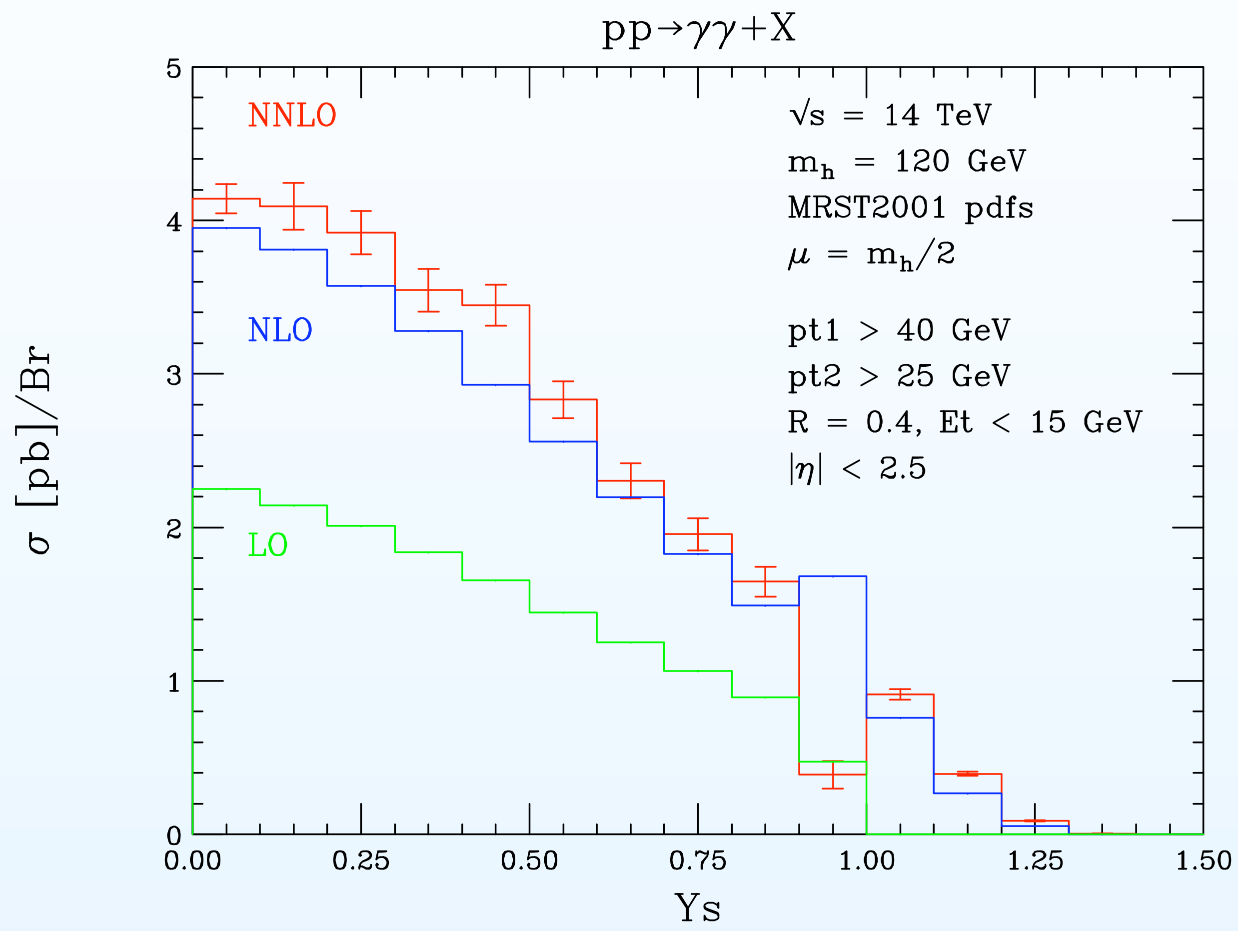


- map each sector to $[0, 1]$



- Repeat until singularities are fully factorized in all phase-space variables.

Fully differential Higgs production



Unexploited properties of gauge theories

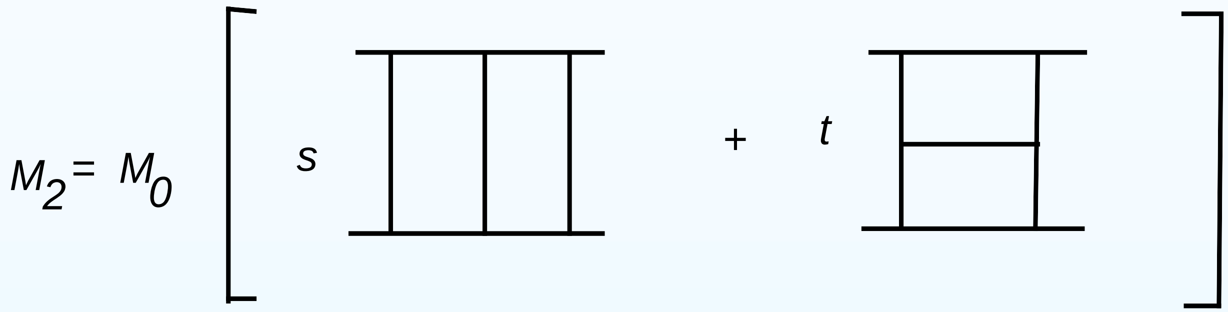
- We have made enormous progress in perturbative computations.
- We know, however, that our methods are primitive!
- The results seem to be disproportionally simpler than our efforts to compute them.
- For example, we know that multi-loop amplitudes factorize simply in their infrared limit. *Catani; Sterman, Tejeda-Yeomans*

$$\mathcal{M}_{\text{all orders}} = \left(\prod_{all \;\; legs} J^{leg}(\alpha_s, \epsilon) \right) \mathbf{Soft}(\alpha_s, \epsilon) Hard(\alpha_s)$$

- Still, we do not know how to compute $Hard$ on its own.

2-loop 4-point amplitudes in $\mathcal{N} = 4$ super Yang-Mills

- These amplitudes can be expressed in terms of one only integral in the planar limit. This was known already in 97.
  Bern, Rosowsky, Yan



- The same integral enters the expression for QCD amplitudes, together with another $\sim 10,000$.
- Many people tried and failed to compute it.
- In a breakthrough, Smirnov solved the problem in 1999.

Finding simplicity: $\mathcal{N}=4$ supersymmetric Yang-Mills

- The full 2-loop 4-point MHV amplitudes obey the same factorization as the infrared limit *CA, Bern, Dixon, Kosower (2003)*

$$M_4^{(2)}(\epsilon) = \frac{1}{2}\left(M_4^{(1)}(\epsilon)\right)^2 + f^{(2)}(\epsilon) M_4^{(1)}(2\epsilon) - \frac{5}{4}\zeta_4.$$

Unlikely to be an accident

- All two-loop amplitudes obey the same relation *collinear factorization*

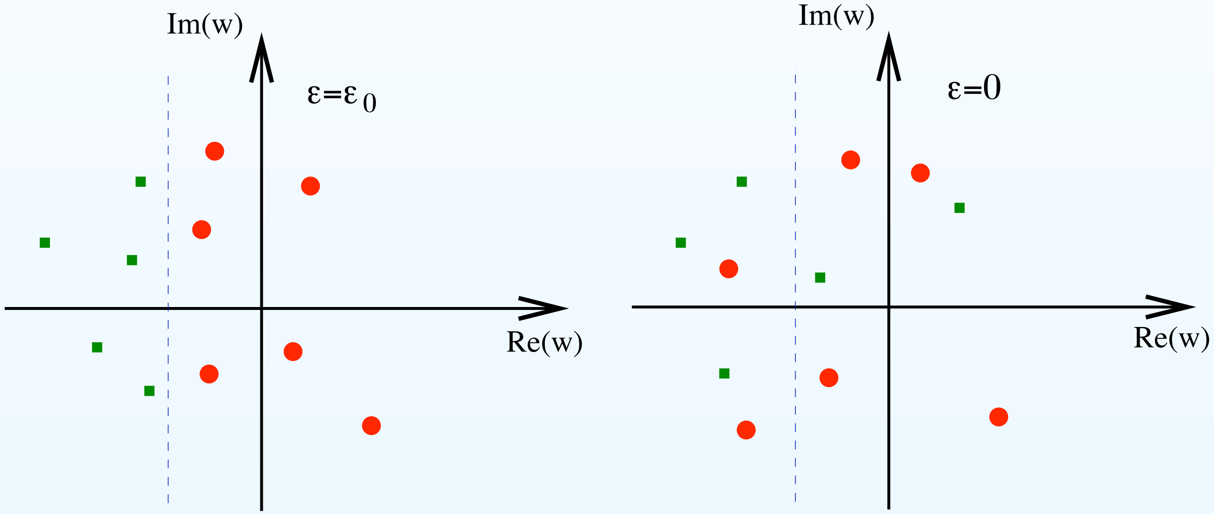
$$M_n^{(2)}(\epsilon) = \frac{1}{2}\left(M_n^{(1)}(\epsilon)\right)^2 + f^{(2)}(\epsilon) M_n^{(1)}(2\epsilon) - \frac{5}{4}\zeta_4.$$

- Are multi-loop amplitudes polynomials of the one-loop amplitude?

Complex representations of arbitrary loop integrals

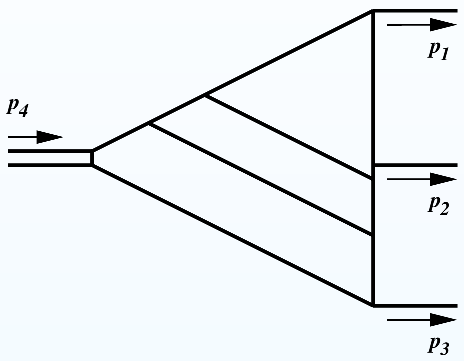
Smirnov; Tausk; CA, Daleo; Czakon

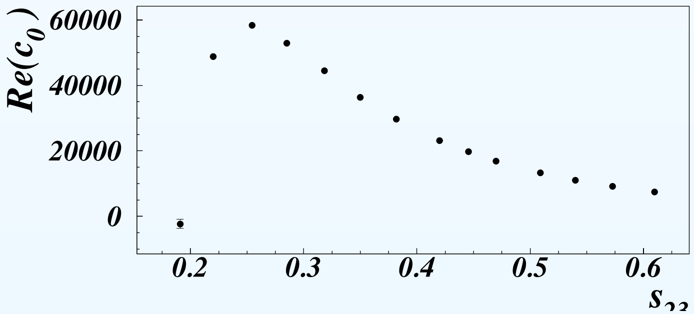
- All loop integrals can be written as complex contour (Mellin-Barnes) integrals

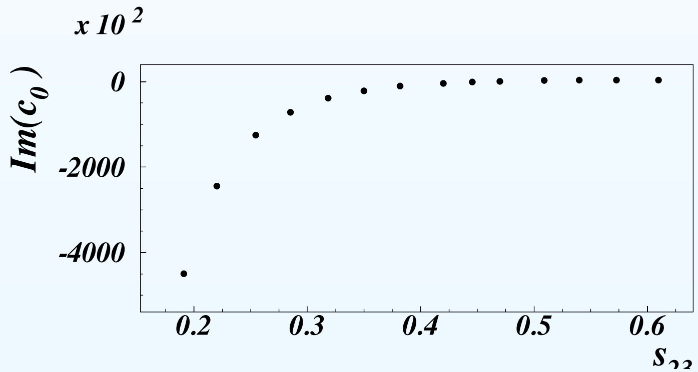


- The infrared divergences are localized on poles that can be extracted automatically with the Cauchy theorem (Smirnov; Tausk)
- Numerical integration on the complex contour (CA, Daleo; Czakon)

An “impossible” loop integral to compute:







In half an hour!

One can outdate this title very easily . . .

3-loop amplitudes in the planar limit

In a tour de force calculation, Bern, Dixon and Smirnov proved analytically:

$$M_4^{(3)}(\epsilon) = -\frac{1}{3}\left(M_4^{(1)}(\epsilon)\right)^3 + M_4^{(1)}(\epsilon)M_4^{(2)}(\epsilon) + f^{(3)}(\epsilon)M_4^{(1)}(3\epsilon) + C.$$

and proposed the ansatz:

$$M_n(\epsilon) = \exp\left(\sum_{l=0}^{\infty} a^l \left[f^{(l)}(\epsilon)M_n^{(1)}(l\epsilon) + h^{(l)}\right]\right)$$

Cachazo, Spradlin, Volovic proved with numerical Mellin-Barnes integrations that the parity even part of two-loop 5-point MHV amplitudes satisfy the conjecture.
Bern, Czakon, Kosower, proved with Mellin-Barnes numerical integrations that the full two-loop 5-point MHV amplitudes satisfy the conjecture.

Is the perturbative expansion solvable?

AdS/CFT correspondence

- strongly coupled $N = 4$ SYM is dual to weakly coupled gravity.
- Anomalous dimensions $f(l)$ can be conjectured from string theory and integrability (Eden, Staudacher; Kotikov, Lipatov, Velizhanin).
- NEW: Bern, Czakon, Dixon, Kosower and Smirnov computed numerically the infrared poles of the 4-loop 4-point amplitudes, probing directly these conjectures.

Conclusions

- At the LHC, the path to new physics goes through QCD production processes.
- We know a lot about QCD and the SM; to the level that it was considered very boring and unatractive.
- It is phenomenologically absolutely essential. But it is also very fun. A very energetic field, which will interface experiment and ideas for new physics at the TeV energy regime.
- Room for new ideas!