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Antenna subtraction with initial state hadrons

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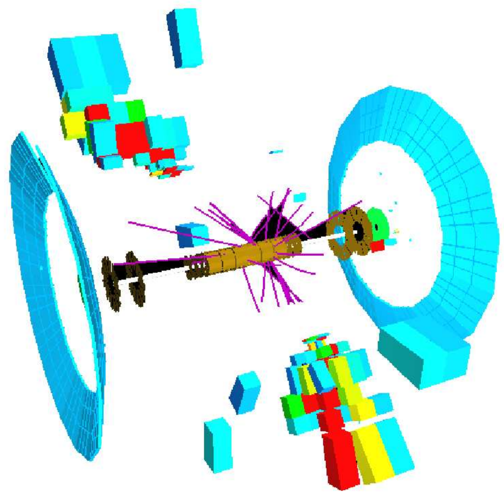
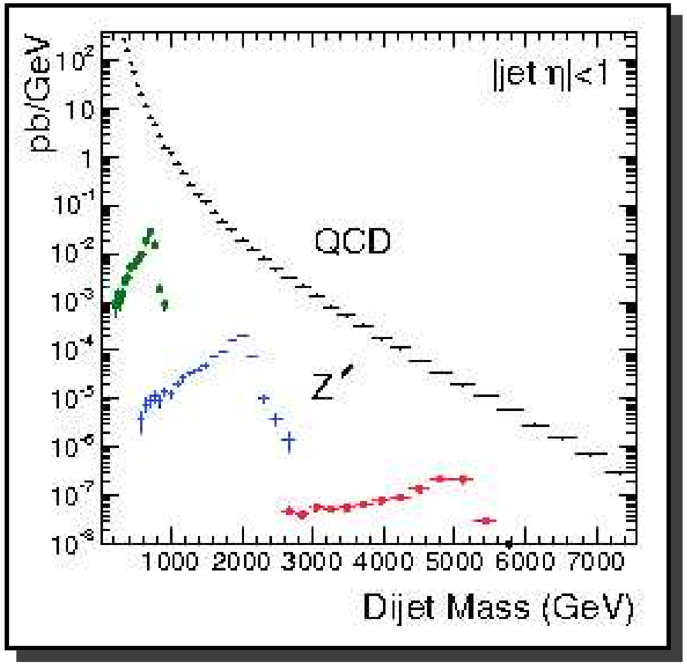
Why NNLO calculations are needed?

- great progress in NLO calculations:
  - new tecnologies to compute one loop integrals
  - well tested frameworks to treat infrared singularities
- few cross sections are known at LHC at NNLO
- high precision calculations needed for:
  - processes used to measure fundamental parameters
  - important backgrounds in the searches of new physics
- three basic processes at LHC:
  - dijet production
  - top pair production
  - vector boson pair production
- ‘standard candles’: background extrapolation

Jet production

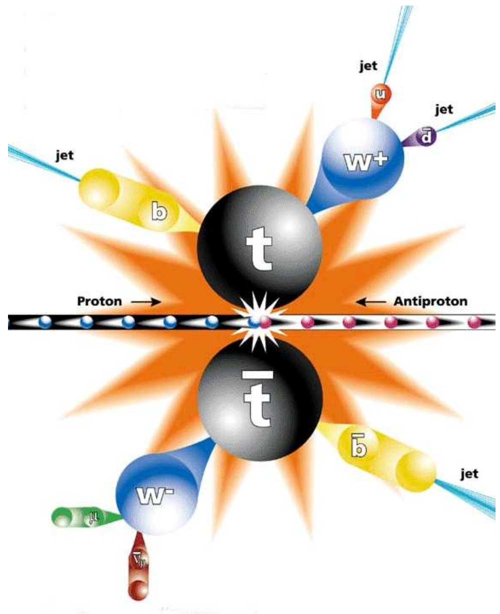


- dominat hard scattering process at LHC
- important input to constrain gluon PDFs and α_s
- rich in potential signals of new physics:
  - composite quarks
  - SUSY
  - extra gauge bosons, Z' and W'
  - Randall-Sundrum models (extra dimensions)



$t\bar{t}$ production

- third generation is the least known sector of the SM -except for the Higgs!-
- $t\bar{t}$ is key to measure top quark properties
- LHC will produce almost 1 $t\bar{t}$ per second at low luminosity!!
- $t\bar{t}$ is an important background for many searches of New Physics



$$\sigma(t\bar{t})_{\text{NLO}} \simeq 830 \text{ pb} \pm 15\% \text{ (scale+PDFs)}$$

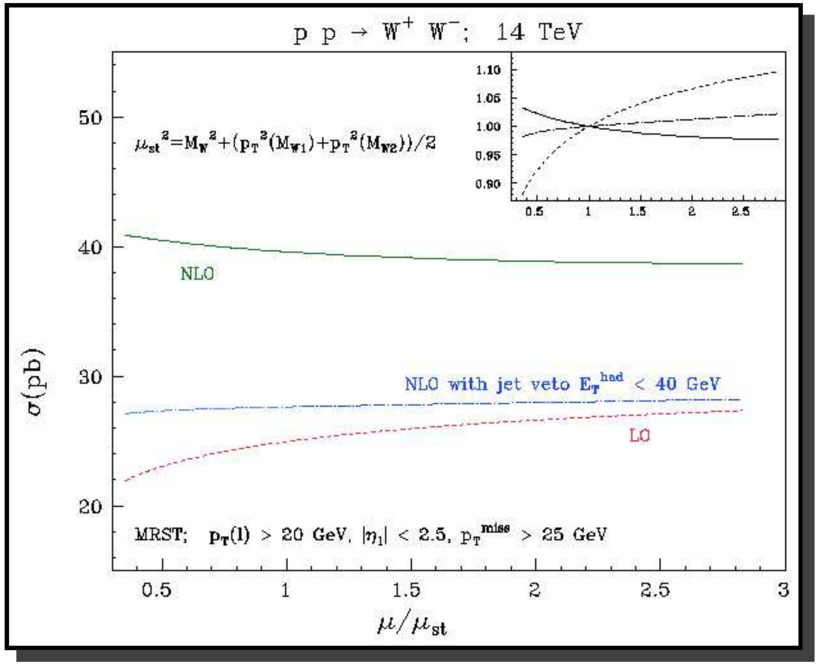
[Bonciani *et al.*;Cacciari]

	$\Delta\hat{\sigma}_{t\bar{t}(\mu)}/\hat{\sigma}_{t\bar{t}(\mu)}$		
	$1\,\text{fb}^{-1}$	$5\,\text{fb}^{-1}$	$10\,\text{fb}^{-1}$
Simulation samples (ϵ_{sim})	0.6%		
Simulation samples (F_{sim})	0.2%		
Pile-Up (30% On-Off)	3.2%		
Underlying Event	0.8%		
Jet Energy Scale (light quarks) (2%)	1.6%		
Jet Energy Scale (heavy quarks) (2%)	1.6%		
Radiation (Λ_{QCD}, Q_0^2)	2.6%		
Fragmentation (Lund b, σ_q)	1.0%		
b-tagging (5%)	7.0%		
Parton Density Functions	3.4%		
Background level	0.9%		
Integrated luminosity	10%	5%	3%
Statistical Uncertainty	1.2%	0.6%	0.4%
Total Systematic Uncertainty	13.6%	10.5%	9.7%
Total Uncertainty	13.7%	10.5%	9.7%

[CMS Physics TDR]

Vector boson pair production

- unique opportunity to probe the non-Abelian gauge symmetry of the Standard Model
- test the presence of anomalous couplings $\longrightarrow$ New Physics
- important backgrounds for Higgs and SUSY searches
- mild NLO corrections with jet veto
- important contributions from high p_T region



[Dixon, Kunszt, Signer]

QCD corrections: the infrared problem

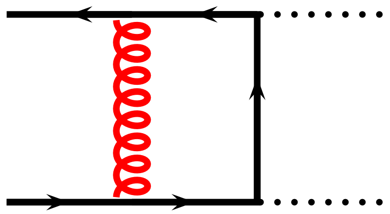
amplitudes are singular due to soft and collinear radiation

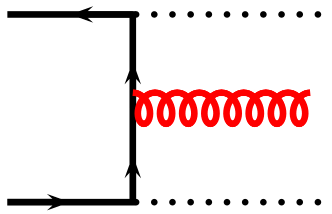
Virtual corrections

explicit singularites (loop integration)

Real corrections

“potential” singularites (phase space)





Singularites are guaranteed to cancel between real and virtual contributions

... but only after phase space integration...

and phase space integration is either not possible -e.g. jets- or not appropriate -e.g. differential cross sections-

how to extract the singularities from the real contributions?

Different approaches

- phase space slicing [Giele, Glover; Giele, Glover, Kosower]
  - split the phase space volume into singular and non-singular regions
  - in singular regions, matrix elements are approximated by their soft/collinear limits
  - these pieces are integrated analytically
  - and they cancel the explicit singularities of virtual components
  - in the non-singular regions it is safe to integrate numerically
- sector decomposition [Anastasiou, Melnikov, Petriello]
  - use sector decomposition of phase space integrals to isolate singularities
  - explicit poles in ϵ are extracted before integration
  - the finite coefficients are integrated numerically
  - cancellations of poles take place after numerical integration
  - delivers results at NNLO!!
- methods based on subtraction

Subtraction methods

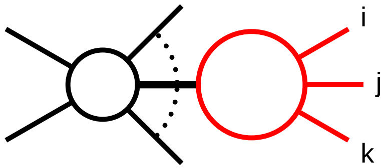
- Ellis-Kunszt-Soper method
- dipole subtraction [Catani, Seymour]
- iterative construction of counterterms [Grazzini, Frixione; Somogyi, Trocsanyi, del Duca]
- antenna subtraction [Kosower]

All based on the factorization properties of amplitudes and matrix elements in soft and collinear limits

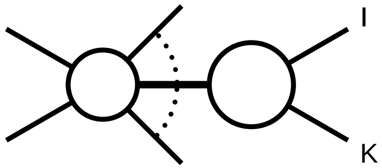
$$|M(\cdots, a, b, c, \cdots)|^2 \xrightarrow{a||b||c} P_{abc\to X}\, |M(\cdots, X, \cdots)|^2 + \text{ang.}$$

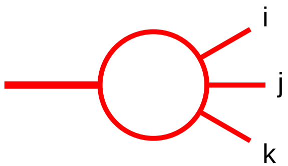
$$|M(\cdots, a, b, c, d, \cdots)|^2 \xrightarrow{c,d\to 0} S_{abcd}\, |M(\cdots, a, d, \cdots)|^2$$

Antenna subtraction [Gehrmann-De Ridder, Gehrmann, Glover]



$$\longrightarrow \quad U_{ijk} \times$$





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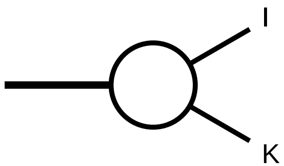
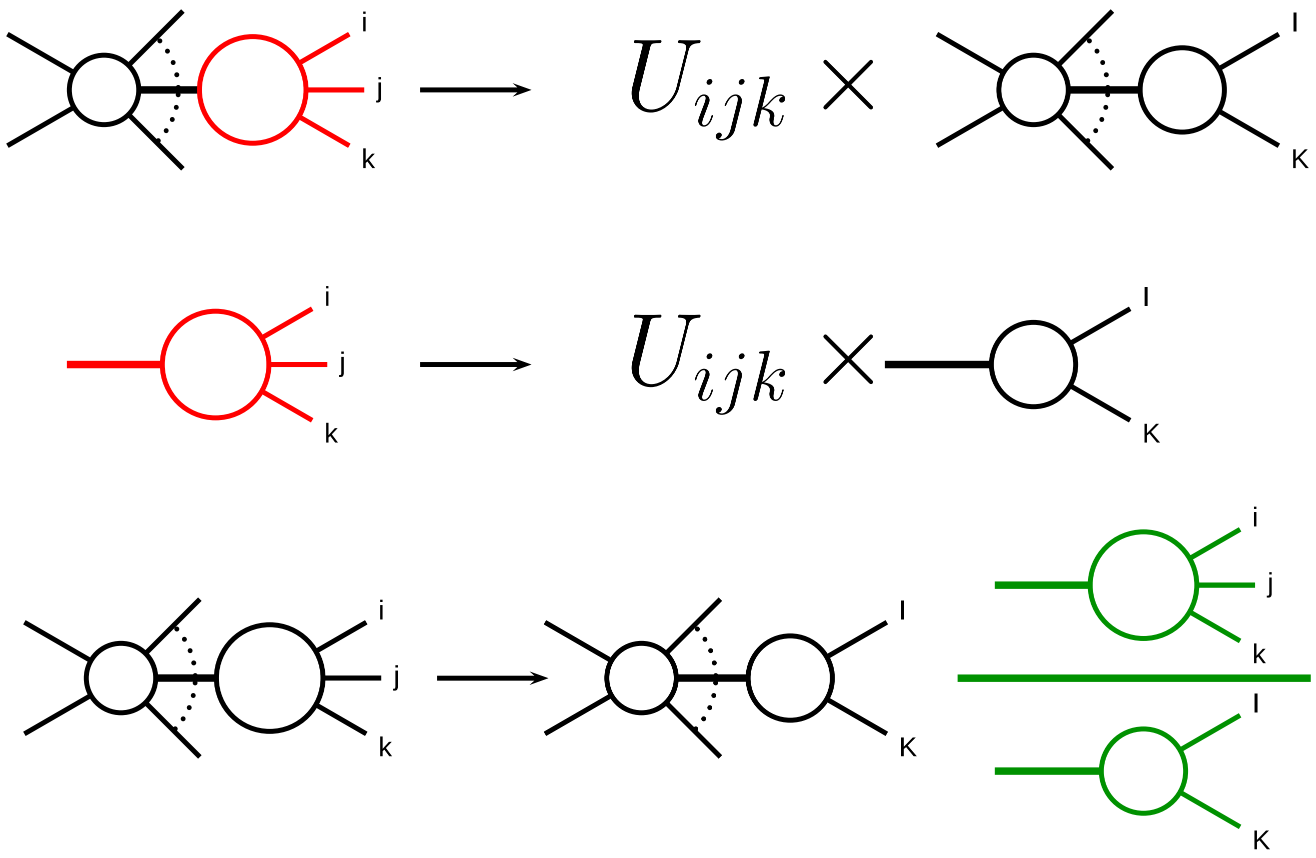


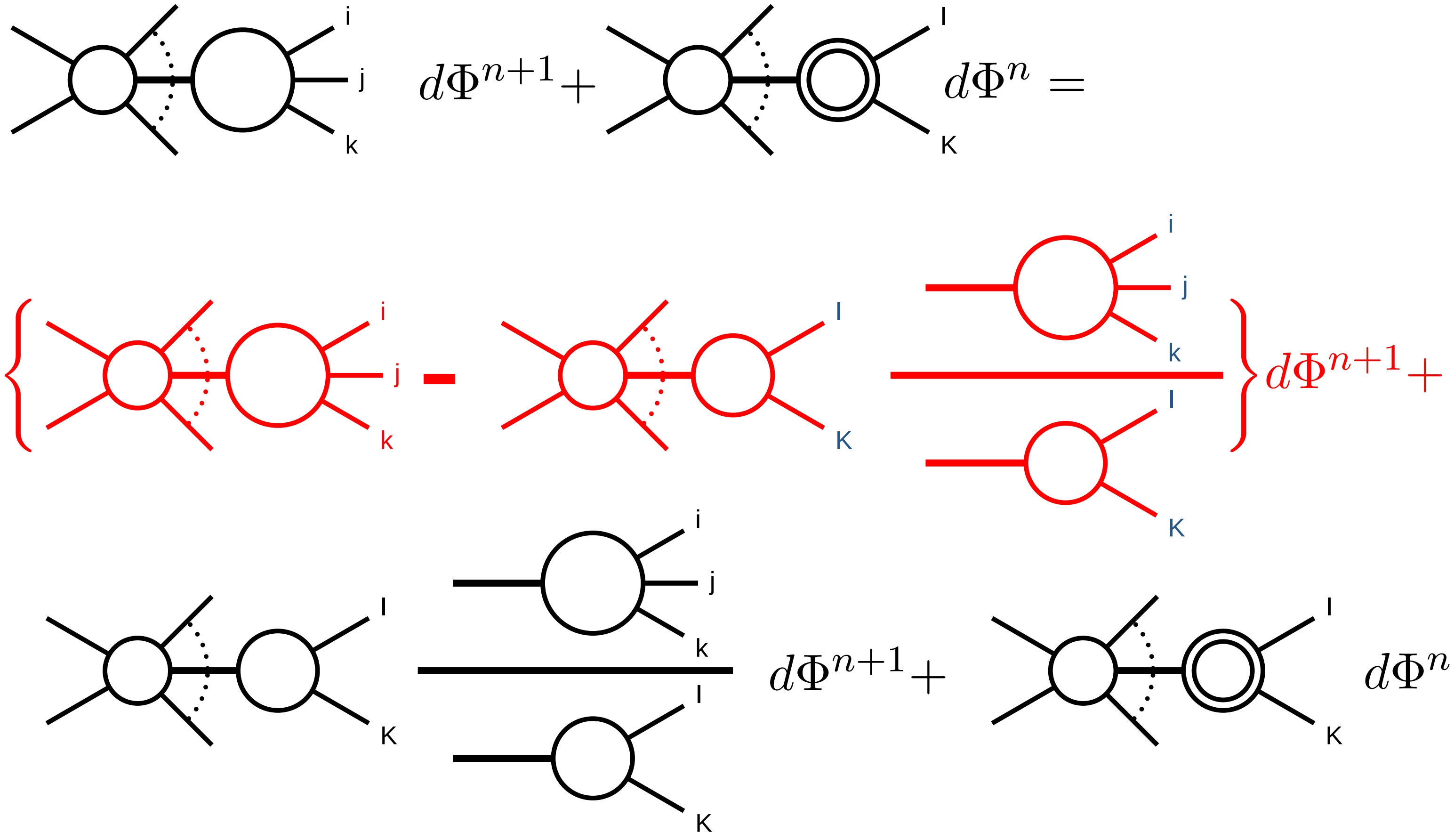
Antenna subtraction [Gehrmann-De Ridder, Gehrmann, Glover]



antenna factor contain ALL the singular limits associated to ijk

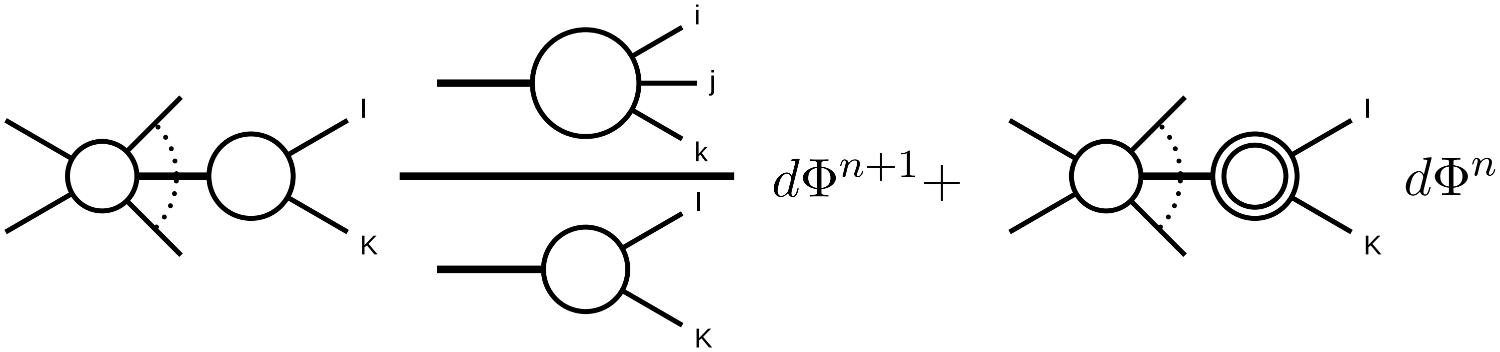
need a mapping $\{i, j, k\} \rightarrow \{I, K\}$ that interpolates between these limits

Antenna subtraction: the counterterms

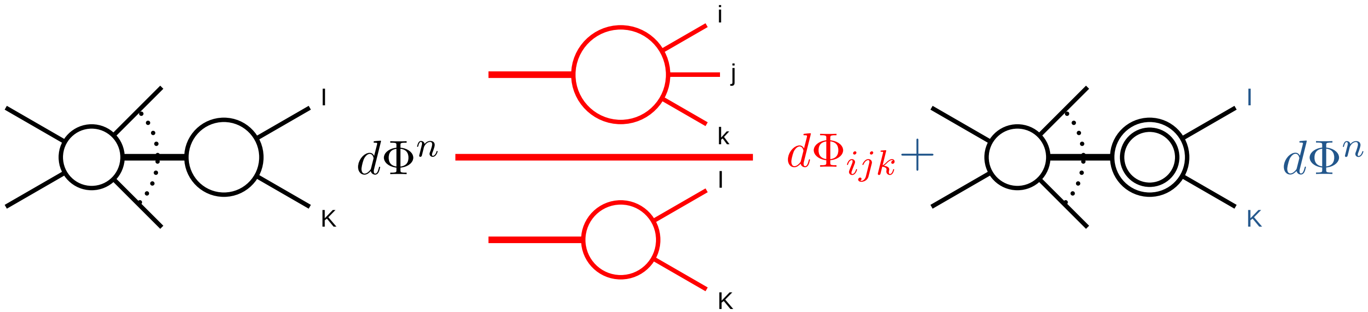


first line on the rhs is finite in all $n+1$ parton configurations:
it can be integrated numerically

Antenna subtraction: phase space factorization

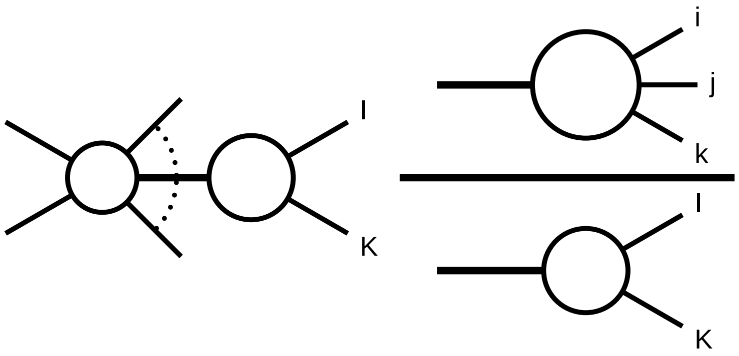


with adequate mappings, the $n+1$ particles phase space factorizes:

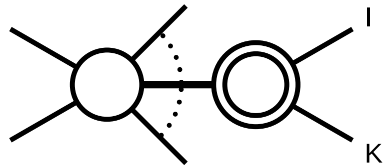


- observable do not depend on individual momenta i, j and k
- antenna factors can be integrated analytically

Antenna subtraction: phase space factorization



$d\Phi^{n+1}+$



$d\Phi^{n}$

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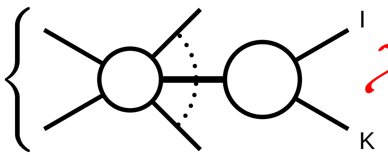
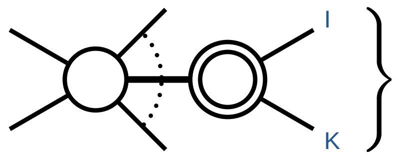


$\mathcal{X}_{ijk}+$



$d\Phi^{n} =$ finite

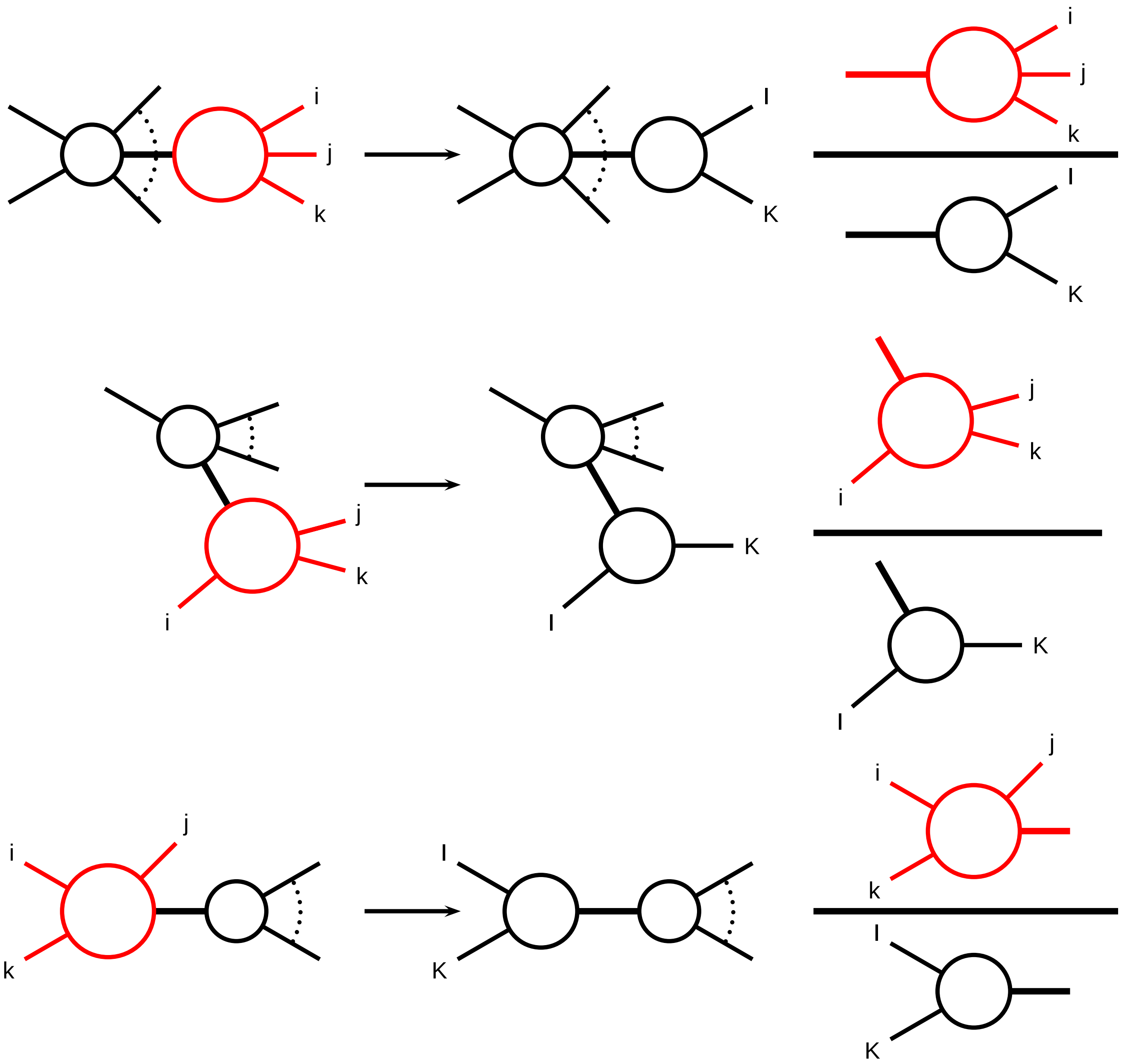
- observable do not depend on individual momenta i, j and k
- antenna factors can be integrated analytically
- explicit poles cancel between one loop and integrated antenna contributions
- integration over phase space can be done numerically

The antenna subtraction toolkit

- antenna functions:
  - obtained from physical matrix elements
  - contain all the singular configurations
  - use color ordered matrix elements to simplify the limits
- phase space mapping:
  - $n+1 \to n$ at NLO, $n+2 \to n$ at NNLO
  - must interpolate between all the relevant singular configurations
  - must allow the factorization of the phase space measure
- integrated antenna functions:
  - obtained by integrating the antennae over the factorized phase space
  - explicit poles in ϵ cancel the loop integration ones

Antenna subtraction with colored initial states

[A.D., T. Gehrmann, D. Maître, in preparation]



final-final

initial-final

initial-initial

NLO phase space mappings and antennae

- phase space mappings:
  - initial-final: $\{p, k_j, k_k\} \to \{\tilde{p} = x\, p, K_K\}$

$$
\begin{aligned}
x &= \frac{s_{ij} + s_{ik} + s_{jk}}{s_{ij} + s_{ik}}\,, \\
K_K &= k_j + k_k - (1 - x)p\,,
\end{aligned}
$$

  - initial-initial: a bit more involved, all momenta must be boosted
- simple generalizations to NNLO
- full set of NLO antennae: $0 \to 3$, $1 \to 2$, $2 \to 1$
- simple phase space integrations at NLO

NNLO Antennae with one parton in the initial state [A.D., in preparation]

- tree antennae $1 \to 3$
- 1 loop antennae $1 \to 2$

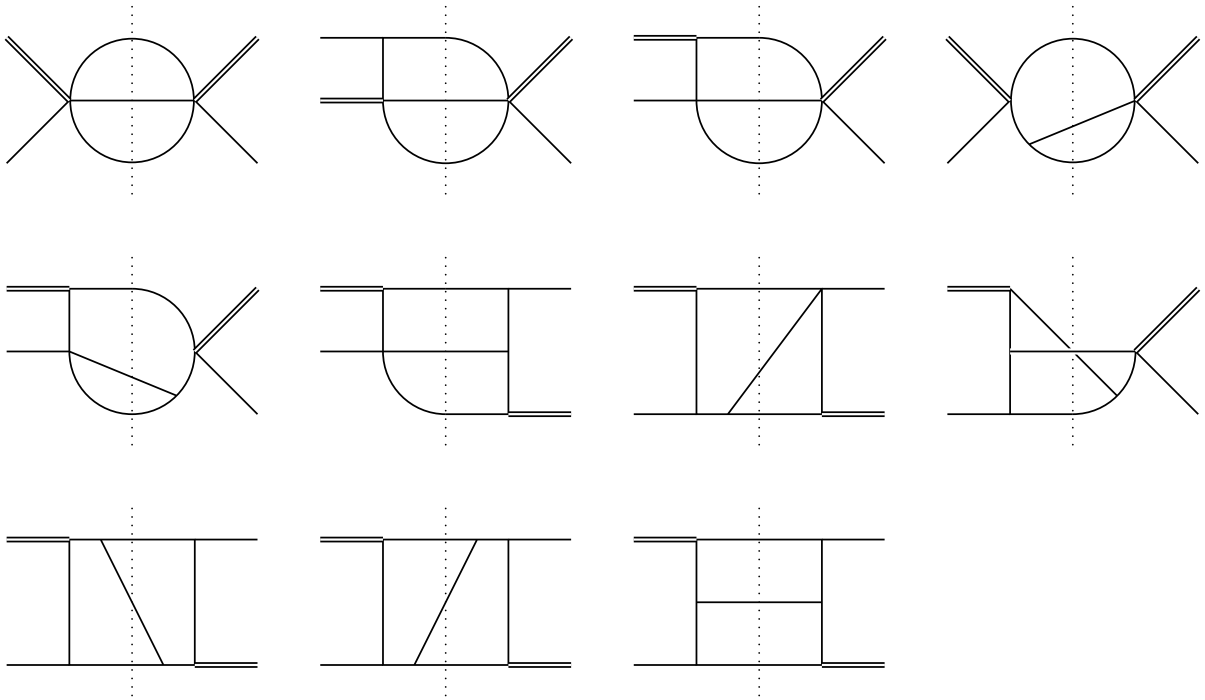
i.e. quark initiated antennae

$A_4^0(q, ggq)$	$\gamma q \to ggq$
$B_4^0(q, qq'\bar{q}')$	$\gamma q \to qq'\bar{q}'$
$D_4^0(q, ggg)$	$\chi\tilde{g} \to ggg$
$D_4^0(q, ggg)$	$\chi\tilde{g} \to ggg$
$E_4^0(q, q'\bar{q}'g)$	$\chi\tilde{g} \to q'\bar{q}'g$
$E_4^0(q, qq'g)$	$\chi q \to q\tilde{g}g$
$F_4^0(q, ggq)$	$hq \to ggq$
$G_4^0(q, qq'\bar{q}')$	$hq \to qq'\bar{q}'$

$A_3^1(q, gq)$	$\gamma q \to gq$
$D_3^1(q, gg)$	$\chi\tilde{g} \to gg$
$D_3^1(q, ggg)$	$\chi\tilde{g} \to ggg$
$E_3^1(q, q'\bar{q}')$	$\chi\tilde{g} \to q'\bar{q}'$
$F_3^1(q, gq)$	$hq \to gq$

Integration over antenna phase space

- antennae derived from matrix elements only involve propagator like denominators
- use of integration by parts identities and Laporta algorithm to reduce all phase space integrals to a small set of masters:



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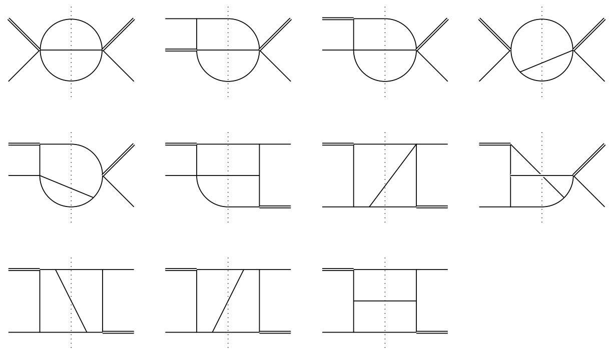

- master integrals computed with differential equations method
- agreement with existing results [Zijlstra, van Neerven]
- checks with direct numerical integration using sector decomposition to deal with singularites
- checks with explicit integration using Feynman parameters

Summary

- NNLO fully differential calculations are very important in the LHC era!
- antenna subtraction provides a general method to deal with singular configurations of real radiation contributions
- in hadron collisions three singular configurations arise:
  – final-final: already known from e^+e^-
  – initial-final: this presentation
  – initial-initial: still missing
- phase space mappings interpolating all the singular limits for the three configurations are known
- full set of antennae for subtraction of initial-final singular configurations is now known