CP-odd observables in MSSM at one loop level

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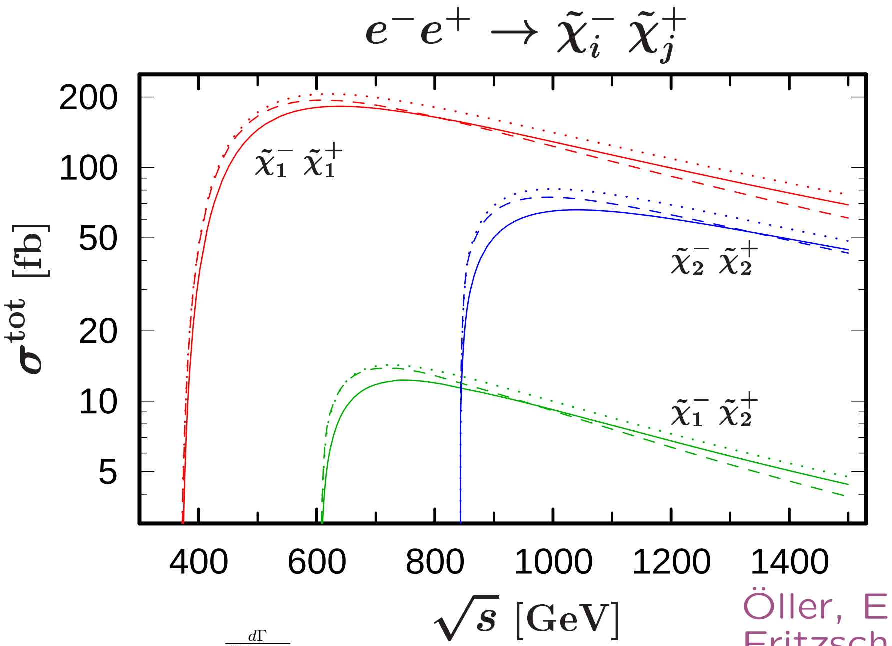
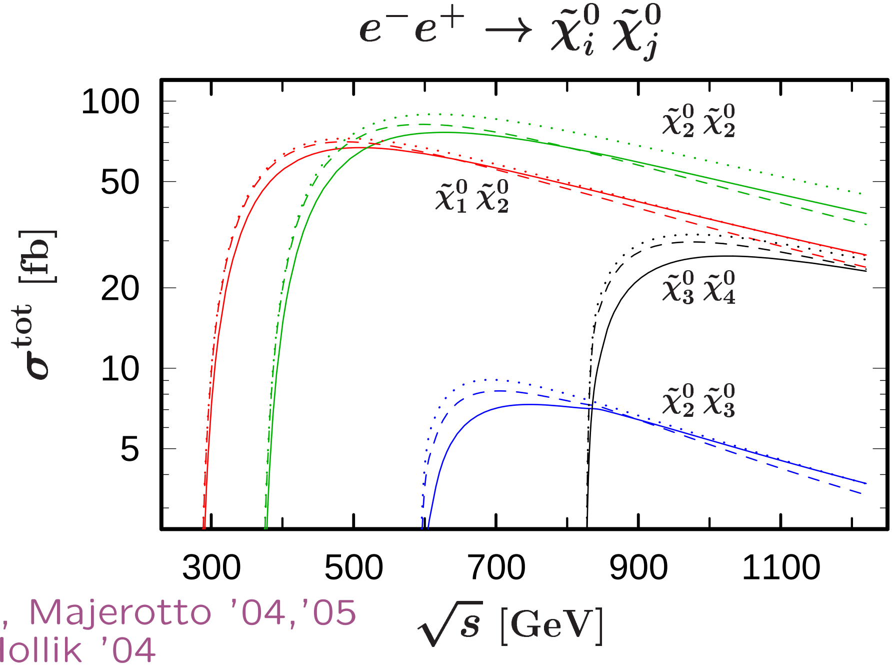
Motivation

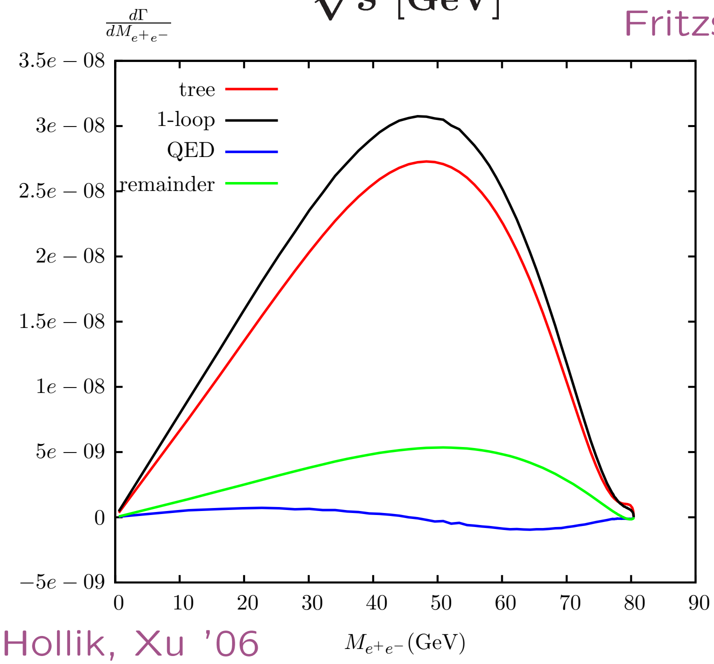
- radiative corrections in MSSM could be of order 20%
- so far only CP-conserving case at one loop thoroughly examined
- MSSM with CP violating phases:
  $M_1 = |M_1|e^{i\Phi_1}$, $\mu = |\mu|e^{i\Phi_\mu}$, $A_f = |A_f|e^{i\Phi_f}$
  $\rightarrow$ strong bounds on these phases from EDMs exist, however
  $\rightarrow$ large phases possible if accidental cancelations occur
  $\rightarrow$ or 1st and 2nd generation of squarks are heavy
  $\rightarrow$ Φ_1 poorly constrained
- calculation of radiative corrections to CP violating observables, e.g. asymmetries of triple products of momenta and/or spins
  $\rightarrow$ such observables provide unambiguous way of detecting CP violating phases
- here we analyze gaugino/higgsino sectors of complex MSSM at one loop level

Introduction

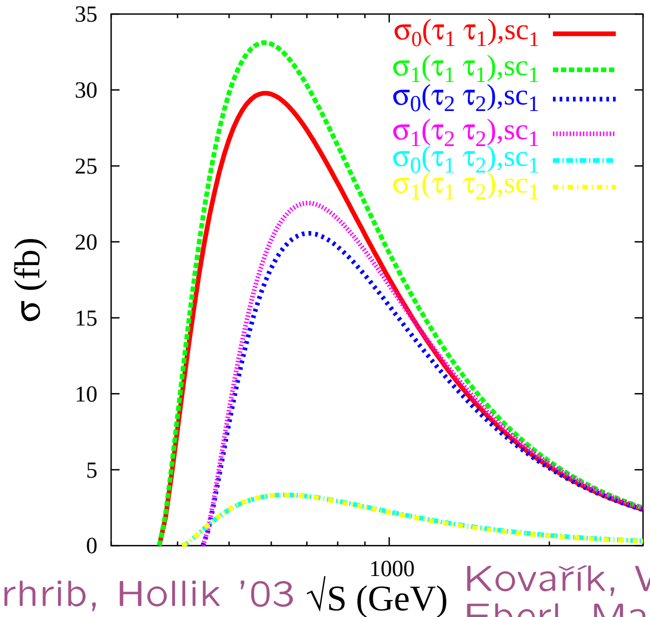




Öller, Eberl, Majerotto '04,'05
Fritzsche, Hollik '04



Drees, Hollik, Xu '06



Arhrib, Hollik '03

Kovařík, Weber,
Eberl, Majerotto '04, '05

Neutralino sector at tree level

- neutralino mass matrix

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix}$$

- mass matrix diagonalization

$$\mathrm{diag}(m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_3}, m_{\tilde{\chi}^0_4}) = N^* M_{\tilde{\chi}^0} N^{-1}$$

- mass eigenstates Weyl spinors χ^0_i and Majorana spinors $\tilde{\chi}^0_i$ $(i = 1, 2, 3, 4)$

$$\begin{pmatrix} \chi^0_1 \\ \chi^0_2 \\ \chi^0_3 \\ \chi^0_4 \end{pmatrix} = N \begin{pmatrix} \tilde{B} \\ \tilde{W}^0 \\ \tilde{H}^0_d \\ \tilde{H}^0_u \end{pmatrix} \qquad \tilde{\chi}^0_i = \begin{pmatrix} \chi^0_i \\ \bar{\chi}^0_i \end{pmatrix}$$

Sfermion sector at tree level

- mass matrix

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m_f^2 + m_{LL}^2 & m_{LR}^* m_f \\ m_{LR} m_f & m_f^2 + m_{RR}^2 \end{pmatrix}$$

- with

$$\begin{aligned} m_{LL}^2 &= \tilde{M}_L^2 + m_Z^2 \cos 2\beta \, (I_3^f - Q_f s_W^2) \\ m_{RR}^2 &= \tilde{M}_R^2 + m_Z^2 \cos 2\beta \, Q_f s_W^2 \\ m_{LR} &= A_f - \mu^* (\tan\beta)^{-2I_3^f} \end{aligned}$$

- after diagonalization with unitary $U_{\tilde{f}}$ matrix

$$U_{\tilde{f}} \mathcal{M}^2_{\tilde{f}} U^\dagger_{\tilde{f}} = \mathrm{diag}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2})$$

- we obtain mass eigenstates $\tilde{f}_1$ and $\tilde{f}_2$

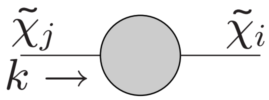
$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = U_{\tilde{f}} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{f}} & \sin\theta_{\tilde{f}} e^{-i\phi} \\ -\sin\theta_{\tilde{f}} e^{i\phi} & \cos\theta_{\tilde{f}} \end{pmatrix} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}$$

Renormalization scheme

- regularization by dimensional reduction
- we choose to work in an on-shell scheme
  $\longrightarrow$ masses physical
  $\longrightarrow$ no mixing between particles on-shell
- renormalization is performed after rotation of fields to mass eigenstates basis
- no direct renormalization of MSSM parameters, but renormalization of fields and mixing matrices

Renormalization of chargino and neutralino sectors

- 1PI renormalized Green's function



$$= \hat{\Gamma}^{\tilde{\chi}}_{ij} = i(\not{k} - m_{\tilde{\chi}_i})\delta_{ij} + i\hat{\Sigma}^{\tilde{\chi}}_{ij}(k^2)$$

- where self-energy can be decomposed into

$$\Sigma^{\tilde{\chi}}_{ij}(k^2) = \not{k} P_L \Sigma^{L}_{ij}(k^2) + \not{k} P_R \Sigma^{R}_{ij}(k^2) + P_L \Sigma^{SL}_{ij}(k^2) + P_R \Sigma^{SR}_{ij}(k^2)$$

- renormalization conditions
  poles at $k^2 = m^2_{\tilde{f}}$, residues equal 1 and no mixing on-shell

$$\widetilde{\mathrm{Re}}\, \hat{\Gamma}^{\tilde{\chi}}_{ij}(k) u_j(k)\Big|_{k^2 = m^2_{\chi_j}} = 0 \qquad \widetilde{\mathrm{Re}}\, \bar{u}_i(p) \hat{\Gamma}^{\tilde{\chi}}_{ij}(k)\Big|_{k^2 = m^2_{\chi_i}} = 0$$

$$\lim_{k^2 \to m^2_{\chi_j}} \frac{\not{k} + m_{\chi_j}}{k^2 - m^2_{\chi_j}} \widetilde{\mathrm{Re}}\, \hat{\Gamma}^{\tilde{\chi}}_{ii}(k) u_i(k) = i u_i(k)$$

$$\lim_{k^2 \to m^2_{\chi_j}} \bar{u}_i(k) \widetilde{\mathrm{Re}}\, \hat{\Gamma}^{\tilde{\chi}}_{ii}(k) \frac{\not{k} + m_{\chi_j}}{k^2 - m^2_{\chi_j}} = i \bar{u}_i(k)$$

Renormalization of chargino and neutralino sectors

- in Lagrangian we introduce wave function and mass counter terms

$$\tilde{\chi}_i \to (\delta_{ij} + \frac{1}{2}\delta\tilde{Z}^L_{ij}P_L + \frac{1}{2}\delta\tilde{Z}^R_{ij}P_R)\tilde{\chi}_j, \qquad m_{\tilde{\chi}_i} \to m_{\tilde{\chi}_i} + \delta m_{\tilde{\chi}_i}$$

- using renormalization conditions they can be related to the corresponding self-energies

$$\delta\tilde{Z}^L_{ii} = \widetilde{\mathrm{Re}}\Big(-\Sigma^L_{ii}(m^2_{\tilde{\chi}_i}) - m^2_{\tilde{\chi}_i}\Big(\Sigma^{L'}_{ii}(m^2_{\tilde{\chi}_i}) + \Sigma^{R'}_{ii}(m^2_{\tilde{\chi}_i})\Big) + \frac{1}{2m_{\tilde{\chi}_i}}\Big(\Sigma^{SL}_{ii}(m^2_{\tilde{\chi}_i}) - \Sigma^{SR}_{ii}(m^2_{\tilde{\chi}_i})\Big) - m_{\tilde{\chi}_i}\Big(\Sigma^{SL'}_{ii}(m^2_{\tilde{\chi}_i}) + \Sigma^{SR'}_{ii}(m^2_{\tilde{\chi}_i})\Big)\Big)$$

$$\delta\tilde{Z}^L_{ij} = \frac{2}{m^2_{\tilde{\chi}_i} - m^2_{\tilde{\chi}_j}}\widetilde{\mathrm{Re}}\Big(m^2_{\tilde{\chi}_j}\Sigma^L_{ij}(m^2_{\tilde{\chi}_j}) + m_{\tilde{\chi}_i}m_{\tilde{\chi}_j}\Sigma^R_{ij}(m^2_{\tilde{\chi}_j}) + m_{\tilde{\chi}_i}\Sigma^{SL}_{ij}(m^2_{\tilde{\chi}_j}) + m_{\tilde{\chi}_j}\Sigma^{SR}_{ij}(m^2_{\tilde{\chi}_j})\Big)$$

$$\delta m_{\tilde{\chi}_i} = \frac{1}{2}\widetilde{\mathrm{Re}}\Big(m_{\tilde{\chi}_i}\Big(\Sigma^L_{ii}(m^2_{\tilde{\chi}_i}) + \Sigma^R_{ii}(m^2_{\tilde{\chi}_i})\Big) + \Sigma^{SL}_{ii}(m^2_{\tilde{\chi}_i}) + \Sigma^{SR}_{ii}(m^2_{\tilde{\chi}_i})\Big)$$

with: $\delta\tilde{Z}^R_{ii} = \delta\tilde{Z}^L_{ii}(L \leftrightarrow R)$, $\delta\tilde{Z}^R_{ij} = \delta\tilde{Z}^L_{ij}(L \leftrightarrow R)$

Renormalization of chargino and neutralino sectors

- we define renormalized mixing matrices for neutralinos and charginos

$$N \to N + \delta N, \qquad U \to U + \delta U, \qquad V \to V + \delta V$$

- the counterterms can be expressed in terms of wave function renormalization constants as

$$\delta N_{ij} = \frac{1}{4} \sum_{k=1}^{4} \left(\delta \tilde{Z}_{ik}^{0,L} - \delta \tilde{Z}_{ki}^{0,R} \right) N_{kj}$$

$$\delta U_{ij} = \frac{1}{4} \sum_{k=1}^{2} \left(\delta \tilde{Z}_{ik}^{\pm,R} - (\delta \tilde{Z}_{ki}^{\pm,R})^* \right) U_{kj}$$

$$\delta V_{ij} = \frac{1}{4} \sum_{k=1}^{2} \left(\delta \tilde{Z}_{ik}^{\pm,L} - (\delta \tilde{Z}_{ki}^{\pm,L})^* \right) V_{kj}$$

- due to the structure of counterterms, renormalized matrices N, U, V are unitary

Renormalizing sfermion sector

- kinetic and mass Lagrangian for sfermions

$$\mathcal{L} = \partial_\mu \begin{pmatrix} \tilde{f}_1^* & \tilde{f}_2^* \end{pmatrix} \partial^\mu \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} - \begin{pmatrix} \tilde{f}_1^* & \tilde{f}_2^* \end{pmatrix} \begin{pmatrix} m_{\tilde{f}_1}^2 & 0 \\ 0 & m_{\tilde{f}_2}^2 \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}$$

- substitute renormalized fields and masses

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{11}^{\tilde{f}} & \frac{1}{2}\delta Z_{12}^{\tilde{f}} \\ \frac{1}{2}\delta Z_{21}^{\tilde{f}} & 1 + \frac{1}{2}\delta Z_{22}^{\tilde{f}} \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}$$

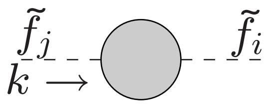
$$m_{\tilde{f}_i}^2 \to m_{\tilde{f}_i}^2 + \delta m_{\tilde{f}_i}^2$$

- and renormalized $U_{\tilde{f}}$ matrix in interaction vertices

$$U_{ij}^{\tilde{f}} \to U_{ij}^{\tilde{f}} + \delta U_{ij}^{\tilde{f}}$$

Renormalization scheme – sfermion sector

- 1PI renormalized Green's function



$$= \hat{\Gamma}^{\tilde{f}}_{ij} = i(k^2 - m^2_{\tilde{f}_i})\delta_{ij} + i\hat{\Sigma}^{\tilde{f}}_{ij}(k^2)$$

- renormalization conditions
  poles at $k^2 = m^2_{\tilde{f}}$, residues equal 1 and no mixing on-shell:

$$\widetilde{\text{Re}}\,\hat{\Gamma}_{ij}(k)|_{k^2=m^2_{\tilde{f}_j}} = 0 \qquad i,j = 1,2$$

$$\lim_{k^2 \to m^2_{\tilde{f}_i}} \frac{1}{k^2 - m^2_{\tilde{f}_i}} \widetilde{\text{Re}}\,\hat{\Gamma}_{ii}(k) = i$$

- for self-energies $\hat{\Sigma}^{\tilde{f}}_{ij}(k^2)$ we obtain

$$\widetilde{\text{Re}}\,\hat{\Sigma}^{\tilde{f}}_{ij}(m^2_{\tilde{f}_j}) = 0 \qquad \widetilde{\text{Re}}\,\frac{\partial\hat{\Sigma}^{\tilde{f}}_{ii}(k^2)}{\partial k^2}\Big|_{k^2=m^2_{\tilde{f}_i}} = 0$$

Renormalization scheme – sfermion sector

- from renormalization conditions we can read renormalization constants in sfermion sector

$$\begin{aligned} \delta Z^{\tilde{f}}_{ii} &= -\frac{\partial \Sigma^{\tilde{f}}_{ii}(k^2)}{\partial k^2}\Big|_{k^2=m^2_{\tilde{f}_i}} \\ \delta Z^{\tilde{f}}_{ij} &= \frac{2}{m^2_{\tilde{f}_i}-m^2_{\tilde{f}_j}}\Sigma^{\tilde{f}}_{ij}(m^2_{\tilde{f}_j}) \quad \text{for} \quad i \neq j \\ \delta m^2_{\tilde{f}_i} &= \Sigma^{\tilde{f}}_{ij}(m^2_{\tilde{f}_i}) \end{aligned}$$

- and renormalization constant for mixing matrix

$$\delta U^{\tilde{\ell}}_{ij} = \frac{1}{4}\sum_{k=1,2}\underbrace{(\delta Z^{\tilde{\ell}}_{ik} - \delta Z^{\tilde{\ell}*}_{ki})}_{u} U^{\tilde{\ell}}_{kj}$$

- due to antihermiticity of u renormalized mixing matrix $U_{\tilde{f}}$ is unitary

Renormalization of other parameters

- Standard Model fields and masses

Denner '93

$$m_W^2 \to m_W^2 + \delta m_W^2 \qquad m_Z^2 \to m_Z^2 + \delta m_Z^2$$

$$f_L \to (1 + \frac{1}{2}\delta Z_L) f_L \qquad f_R \to (1 + \frac{1}{2}\delta Z_R) f_R$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{ZZ} & \frac{1}{2}\delta Z_{ZA} \\ \frac{1}{2}\delta Z_{AZ} & 1 + \frac{1}{2}\delta Z_{AA} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}$$

- electric charge defined in the Thomson limit $\alpha = e^2/4\pi \simeq 1/137.036$

$$\frac{\delta e}{e} = -\frac{1}{2}\delta Z_{AA} - \frac{s_W}{2c_W}\delta Z_{ZA}$$

- renormalization of $\tan\beta = v_2/v_1$

$$\frac{\delta \tan\beta}{\tan\beta} = \frac{1}{m_Z \sin 2\beta} \, \mathrm{Im}\left[\widetilde{\mathrm{Re}}\, \Sigma_{A^0 Z}(m_{A^0}^2)\right]$$

$\to$ no mixing of pseudoscalar Higgs A_0 and Z vector boson for on-shell momenta

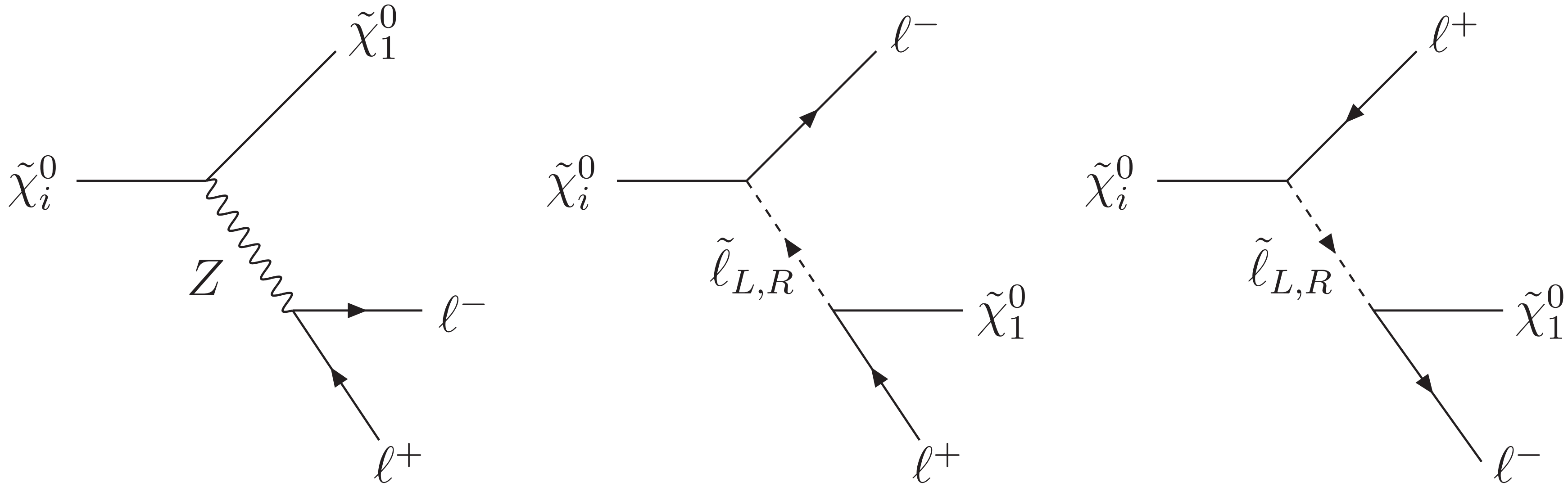
Example of renormalized coupling

$$
\begin{aligned}
C(Z,\tilde{\chi}^0_i,\tilde{\chi}^0_j) &= \frac{ie}{2c_W s_W}\left(P_L\left(-N^*_{j,3}N_{i,3}+N^*_{j,4}N_{i,4}\right)+P_R\left(N^*_{i,3}N_{j,3}-N^*_{i,4}N_{j,4}\right)\right)\\
\delta C(Z,\tilde{\chi}^0_i,\tilde{\chi}^0_j) &= P_L\Bigg(C_L(Z,\tilde{\chi}^0_i,\tilde{\chi}^0_j)\left(\left(\frac{\delta Z_{ZZ}}{2}+\delta e+\delta s_W\frac{s_W^2-c_W^2}{s_W c_W^2}\right)\right.\\
&+ \left.\frac{ie}{2c_W s_W}\left(-\delta N^*_{j,3}N_{i,3}+\delta N^*_{j,4}N_{i,4}-N^*_{j,3}\delta N_{i,3}+N^*_{j,4}\delta N_{i,4}\right)\right)\\
&+ \frac{1}{2}\sum_{n1=1}^{4}\left(\delta\tilde{Z}^{0L}_{n1,i}\right)^* C_L(Z,\tilde{\chi}^0_{n1},\tilde{\chi}^0_j)+\frac{1}{2}\sum_{n2=1}^{4}\delta\tilde{Z}^{0L}_{n2,j}C_L(Z,\tilde{\chi}^0_i,\tilde{\chi}^0_{n2})\Bigg)\\
&+ P_R\Bigg(C_R(Z,\tilde{\chi}^0_i,\tilde{\chi}^0_j)\left(\left(\frac{\delta Z_{ZZ}}{2}+\delta e+\delta s_W\frac{s_W^2-c_W^2}{s_W c_W^2}\right)\right.\\
&+ \left.\frac{ie}{2c_W s_W}\left(\delta N^*_{i,3}N_{j,3}-\delta N^*_{i,4}N_{j,4}+N^*_{i,3}\delta N_{j,3}-N^*_{i,4}\delta N_{j,4}\right)\right)\\
&+ \frac{1}{2}\sum_{n1=1}^{4}\left(\delta\tilde{Z}^{0R}_{n1,i}\right)^* C_R(Z,\tilde{\chi}^0_{n1},\tilde{\chi}^0_j)+\frac{1}{2}\sum_{n2=1}^{4}\delta\tilde{Z}^{0R}_{n2,j}C_R(Z,\tilde{\chi}^0_i,\tilde{\chi}^0_{n2})\Bigg)
\end{aligned}
$$

- renormalized couplings included in *FeynArts*

Application: decay of neutralino with CP phase

- 3-body decay of polarized $\tilde{\chi}_2^0(\hat{n}) \to \tilde{\chi}_1^0(p) + \ell^+(q_+) + \ell^-(q_-)$



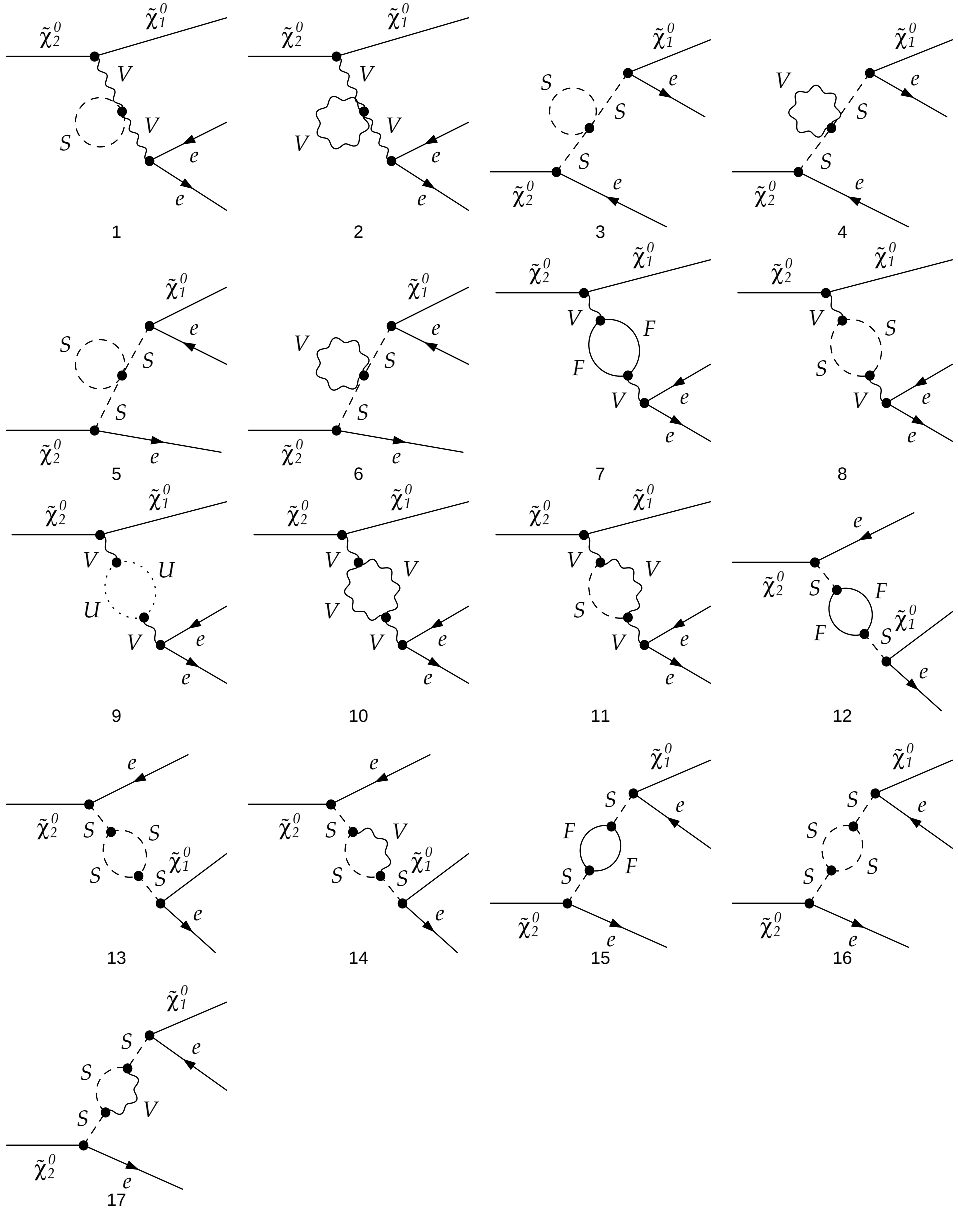
- differential decay rate at tree level   Choi, Chung, Kalinowski, Kim, KR '05

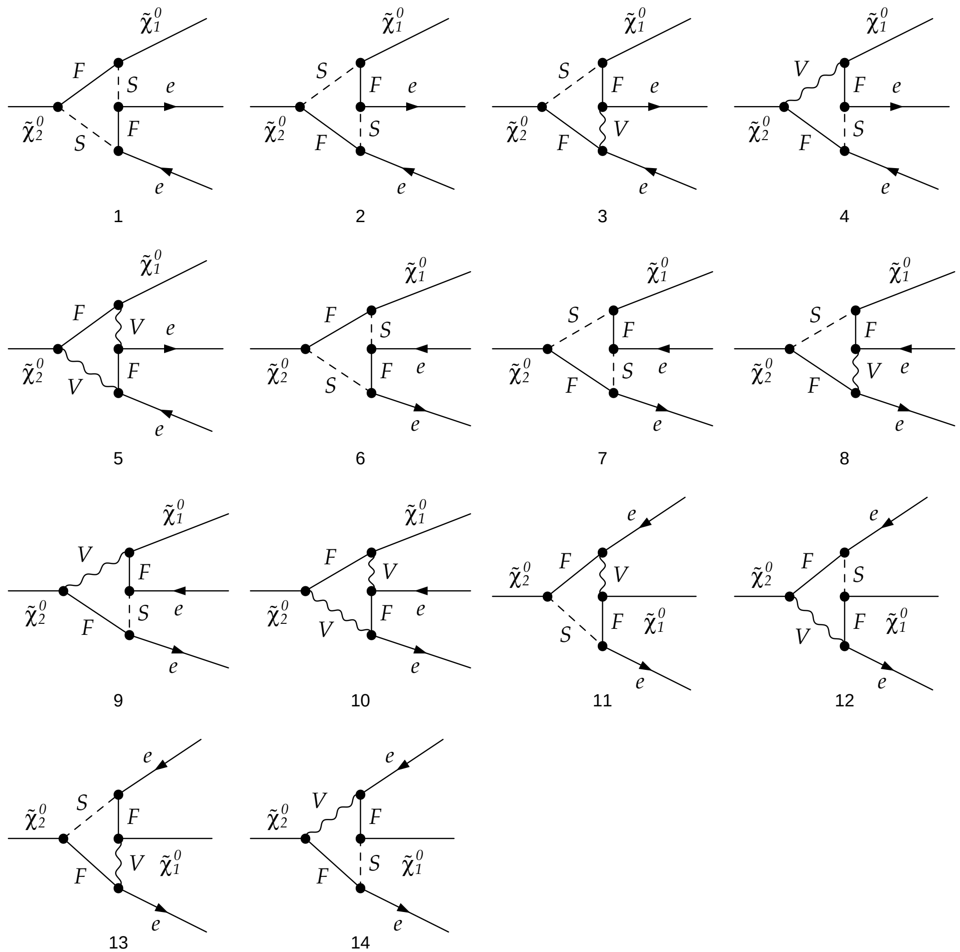
$$\frac{d^4\Gamma}{dx_- dx_+ d\cos\theta\, d\varphi} = \frac{\alpha^2 m_2}{16\pi^2}\,[F_0(x_-, x_+) + (\hat{q}_- \cdot \hat{n})\, F_1(x_-, x_+) + (\hat{q}_+ \cdot \hat{n})\, F_2(x_-, x_+) + \hat{n} \cdot (\hat{q}_- \times \hat{q}_+)\; F_3(x_-, x_+)\,]$$

- with kinematic functions $F_i(x_-, x_+)$ and $x_\pm = 2E_{\ell^\pm}/m_2$

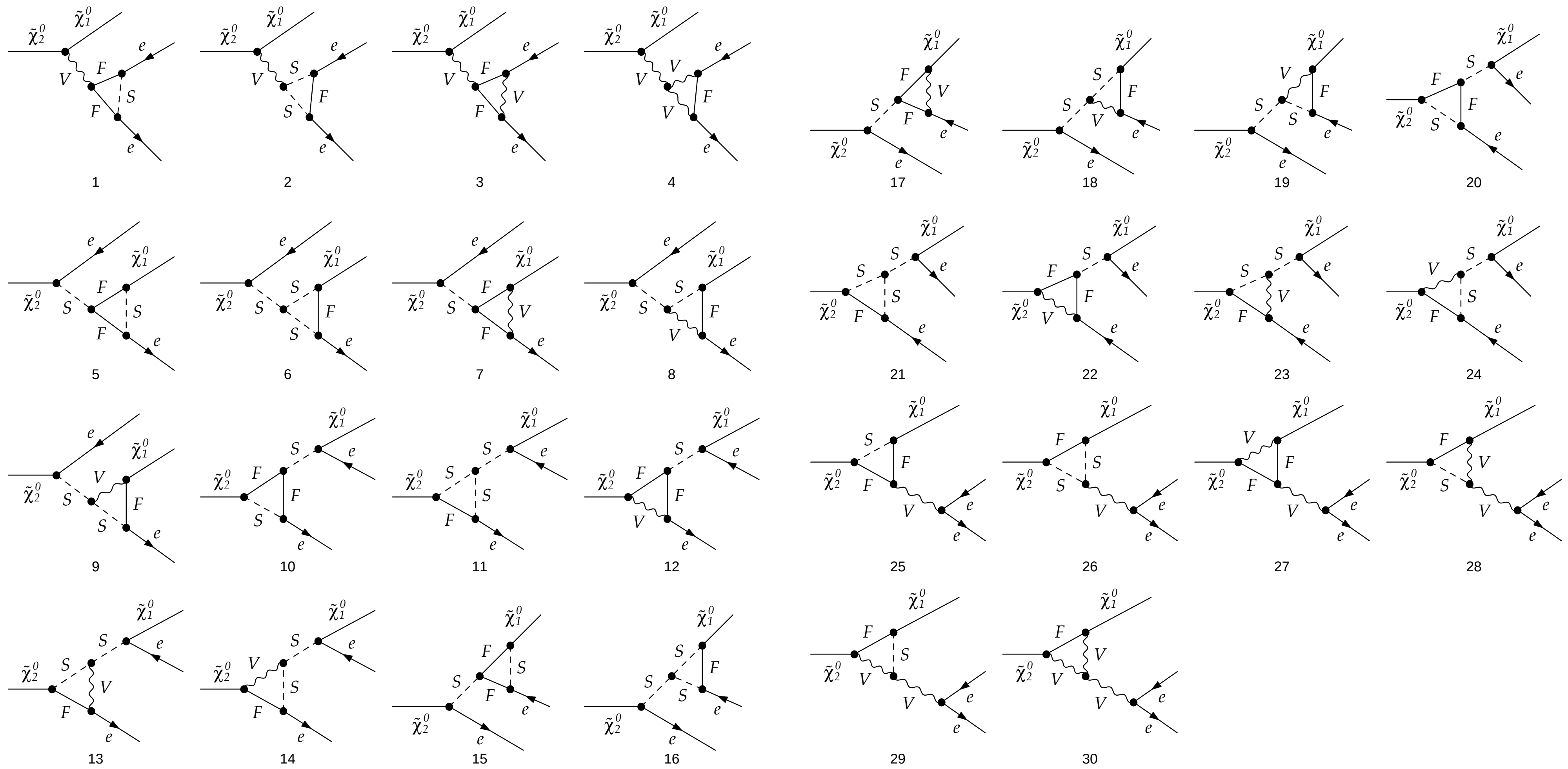
- Majorana nature of neutralinos leads to $F_3(x_-, x_+) \xrightarrow{CP} -F_3(x_+, x_-)$

Self-energy and box corrections





Vertex corrections



Decay width

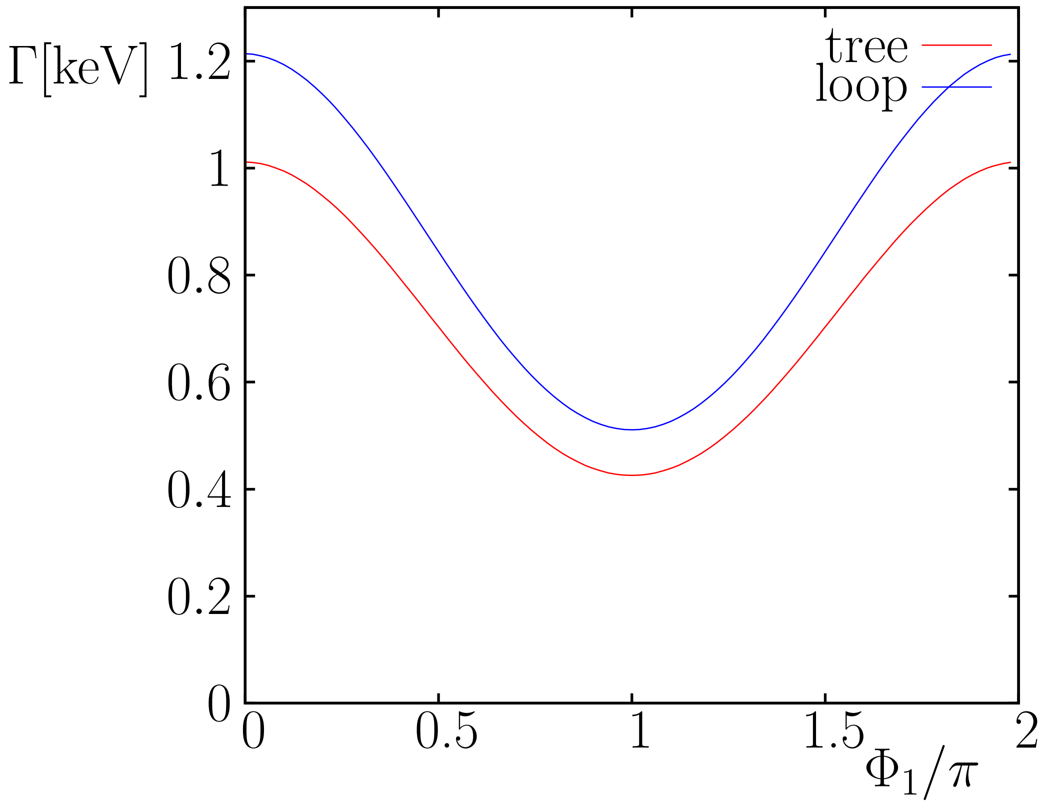
- $m_{\tilde{\chi}^0_1} = 78.1$ GeV, $m_{\tilde{\chi}^0_2} = 148.5$ GeV, $m_{\tilde{e}_L} = 208$ GeV, $m_{\tilde{e}_R} = 173$ GeV $m_{\tilde{\nu}_e} = 192$ GeV

- only genuine three-body decay allowed: $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+ \ell^-$, $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 q\bar{q}$, $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \nu\bar{\nu}$

- decay width ~ 1 keV
  $\to$ strongly depends on Φ_1

- correction significant $\sim 20\%$

- strong dependence on Φ_1 also for BR's

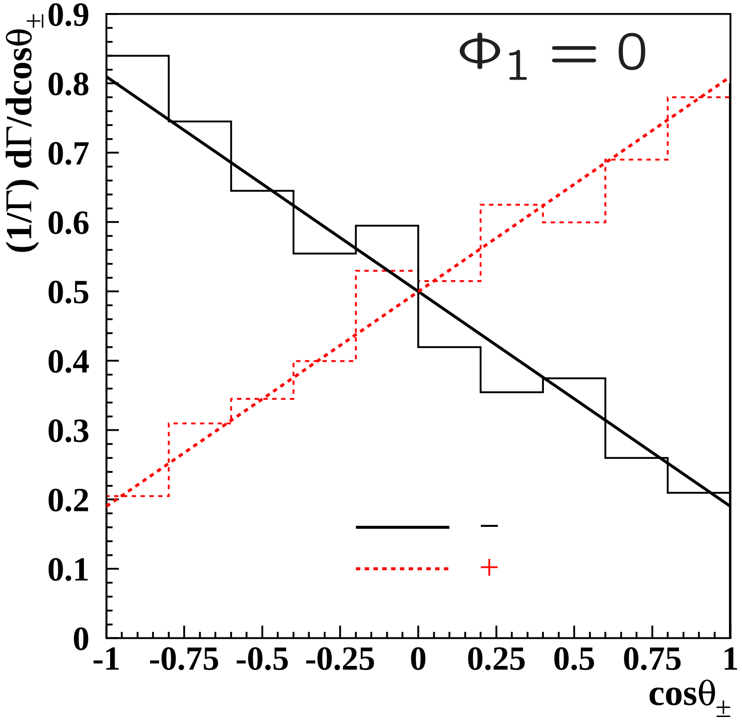


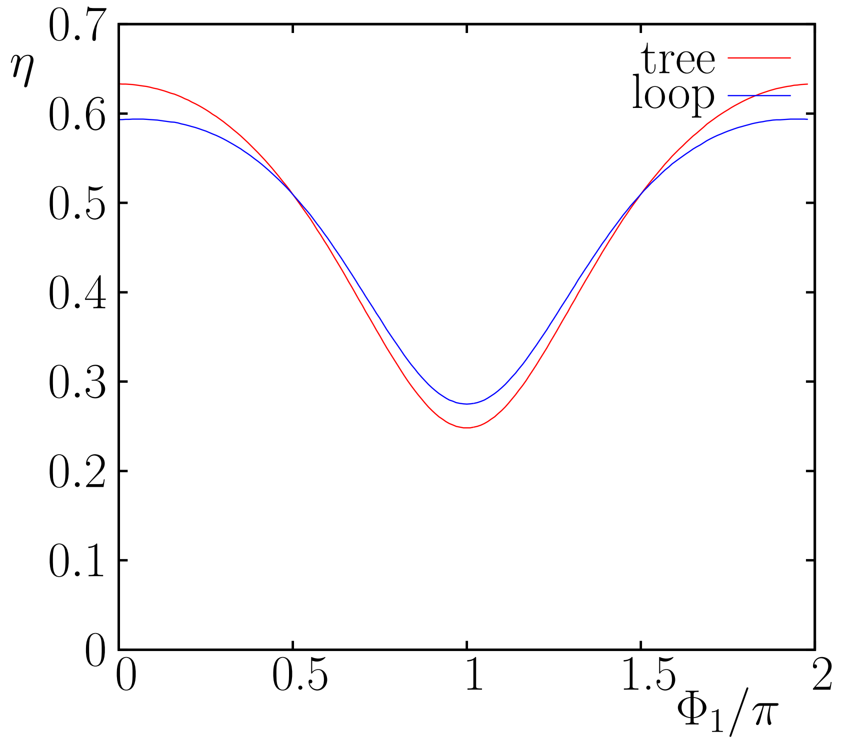
Lepton angular distribution

- lepton angle distribution with respect to the neutralino polarization vector

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\pm} = \frac{1}{2}(1 \pm \eta_\pm \cos\theta_\pm)$$

  with $\cos\theta_\pm = \hat{q}_\pm \cdot \hat{n}$
- from CP$\tilde{\mathrm{T}}$ invariance and Majorana nature of neutralinos: $\eta_- = \eta_+$
- $\eta_\pm$ strongly depends on CPV phase Φ_1





CP-odd triple spin/momentum product

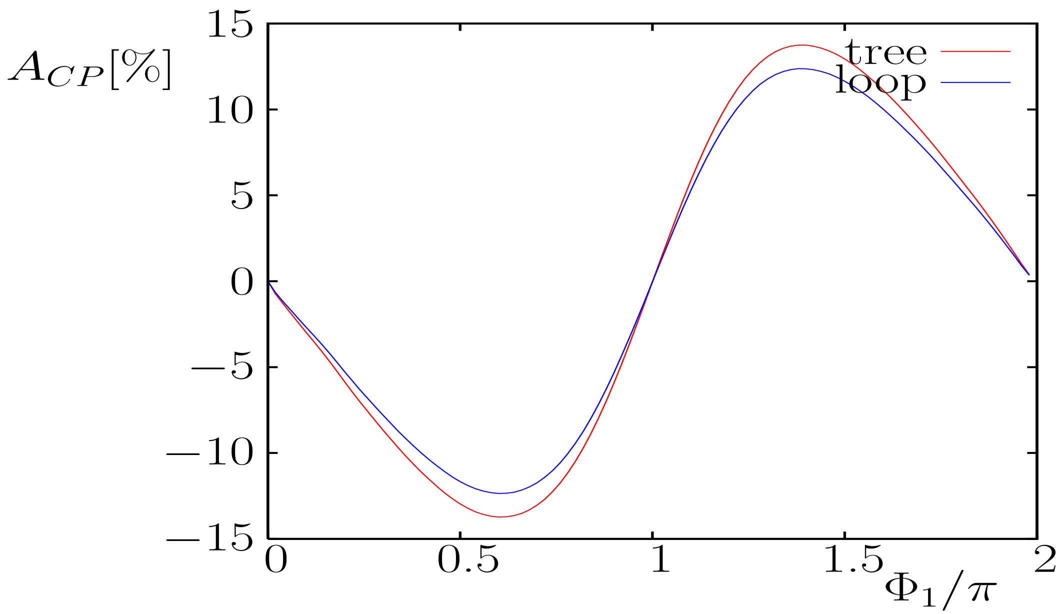
- a CP-odd and CPT-even distribution

$$F_{\mathrm{CP}}(x_-, x_+) = \frac{1}{2}\Big[F_3(x_-, x_+) + F_3(x_+, x_-) \Big]$$

- a CP-odd quantity related to the above CP-odd distribution

$$O_{\mathrm{CP}} = \hat{n} \cdot (\hat{q}_+ \times \hat{q}_-)$$

$$A_{\mathrm{CP}} \equiv \frac{N(O_{\mathrm{CP}} > 0) - N(O_{\mathrm{CP}} < 0)}{N(O_{\mathrm{CP}} > 0) + N(O_{\mathrm{CP}} < 0)} = \frac{\int_{\mathcal{D}} \frac{1}{2} \sin\chi \, F_{\mathrm{CP}}(x_-, x_+)\, dx_- dx_+}{\int_{\mathcal{D}} F_0(x_-, x_+)\, dx_- dx_+}$$



Conclusions and outlook

- renormalization scheme for complex MSSM tested
- corrections $\mathcal{O}(\alpha)$ to some production processes and decays calculated
- first results of loop corrections for CP-odd observables
- for determination of CP-violating phases corrections to full process: production + decay required