Hadronic vacuum polarization contributions to $(g-2)_\mu$

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Vienna, December 2. 2006

Outline

$g_e = 2$ and related anomalies

$\vec{\mu}_e = g_e \frac{e}{2m_e} \vec{S}_e$ $\quad S_e = \frac{1}{2}$ $\quad g_e = 2$ $\quad$ Uhlenbeck-Goudsmit (26)

Dirac theory explained $g_e = 2$

Twenty years later deviations from $g_e = 2$ were detected

$$a_e \equiv \frac{g_e - 2}{2} = 0.00118 \pm 0.00003 \quad \text{Kusch and Foley (47)}$$

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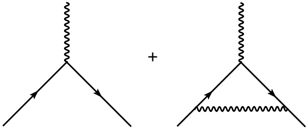

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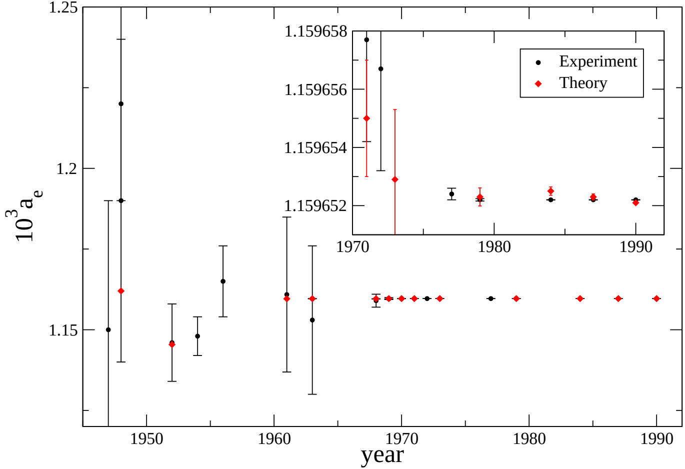
and provided one of the first strong confirmations of QED

Improvements in the last 60 years



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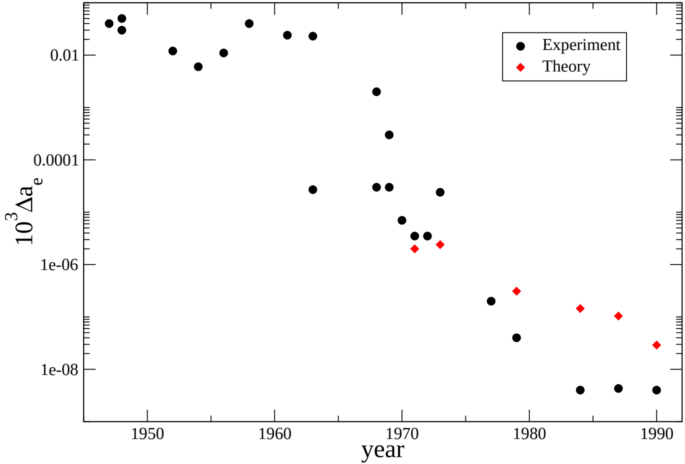
Experimental uncertainty on a logarithmic scale



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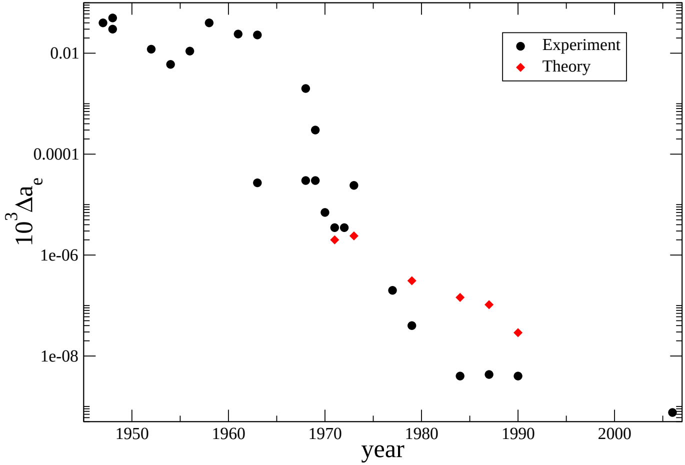

The electron $(g-2)$

The latest measurement of a_e gives [Gabrielse et al. (06)]

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$$a_e^{\mathrm{SM}} = \frac{\alpha}{2\pi} - 0.328478444\left(\frac{\alpha}{\pi}\right)^2 + 1.181234\left(\frac{\alpha}{\pi}\right)^3 - 1.7502\left(\frac{\alpha}{\pi}\right)^4 + \mathcal{O}(\alpha^5) + 1.7\times 10^{-12}$$

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α is not known to the needed accuracy

$\alpha^{-1} = 137.036\,000\,00(110)$ PRA73 (2006) 032504

$\alpha^{-1} = 137.035\,998\,78(91)$ PRL96 (2006) 033001

$\Rightarrow$ QED is tested only up to 6.7 ppb (4-loop level!)

$\Rightarrow$ a_e provides the most precise determination of α to 0.7ppb

$$\alpha^{-1} = 137.035\,999\,710(96)$$

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Hadronic and weak contributions
are still below the experimental accuracy

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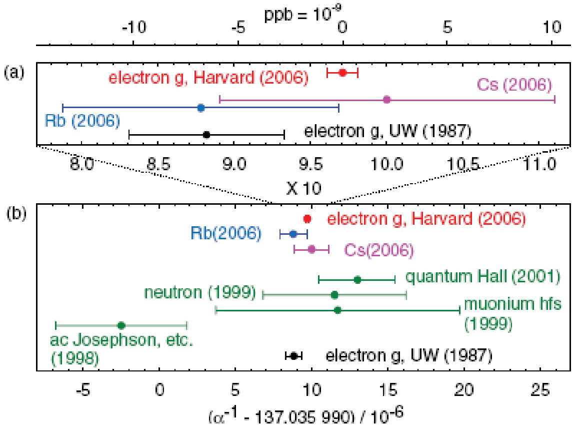

Figure from Gabrielse et al. PRL97 030802 (06)

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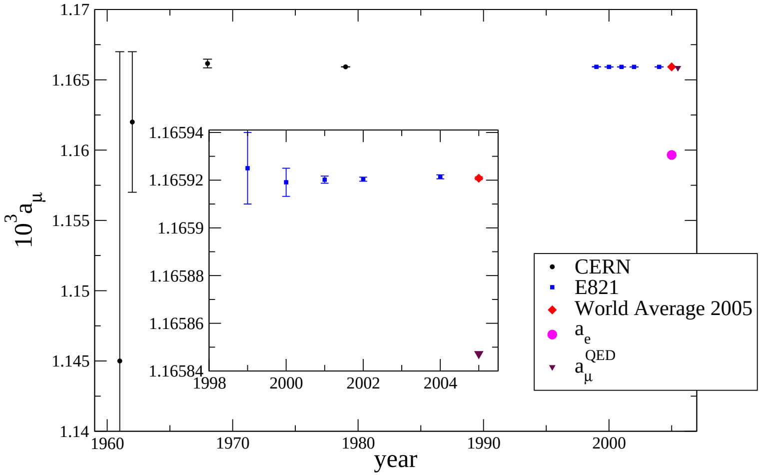
- Schwinger (1957) suggested to use a_μ to search for a field whose different coupling to μ and e could explain their mass difference
- In 1961 the first measurement of a_μ was carried out by Charpak, Farley, Garwin, Muller, Sens, Telegdi and Zichichi at CERN

  $$a_\mu = 0.001145 \pm 0.000022$$

  in good agreement with Schwinger's calculation:
  the leading correction is mass independent

History of a_μ measurements



a_μ, QED and the SM

Latest World Average

$$a_\mu^{\text{exp}} = (11\,659\,208 \pm 6) \times 10^{-10}$$

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$$a_\mu^{\rm had} \simeq 700 \times 10^{-10}$$

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- Weak contributions to a_μ

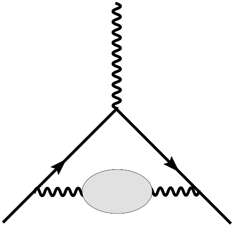
$$a_\mu^{\rm EW} = 15.4 \times 10^{-10} \simeq 2.5 \Delta a_\mu^{\rm exp}$$

Outline

How to calculate the hadronic contributions

Leading hadronic contribution:

- is of order α^2



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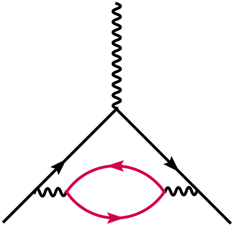

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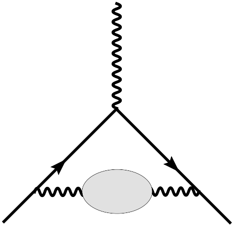

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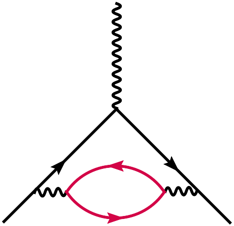

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Lepton $\ell \neq \mu$ vacuum polarization

$$a_\mu^{\ell vp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\ell^2}^{\infty} ds\, \frac{\hat{K}(s)}{s^2} R_\ell(s) \qquad R_\ell(s) = \frac{\sigma^{e^+e^-\to\ell^+\ell^-}(s)}{4\pi\alpha^2/3s}$$

$\hat{K}(0) = 0$ and grows monotonically to $\hat{K}(\infty) = 1$

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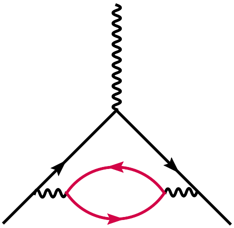
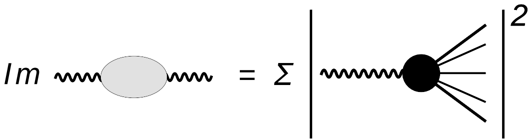


Lepton $\ell \neq \mu$ vacuum polarization

The formula follows from unitarity and analyticity...



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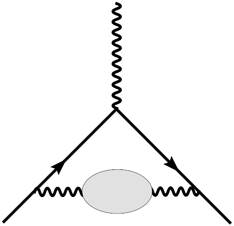

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Hadronic vacuum polarization

... and is therefore valid for hadrons too

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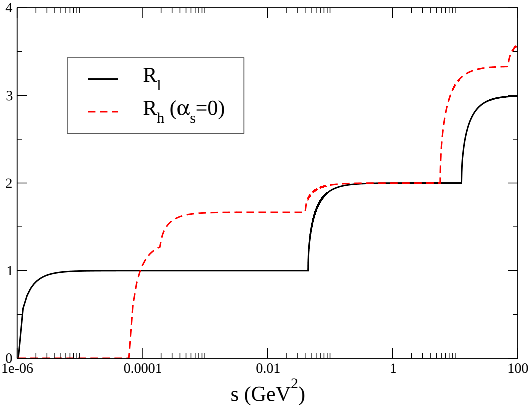
Bouchiat and Michel (61), Durand (62)

The ratio *R*

for free leptons and quarks



The ratio R

for real-world hadrons

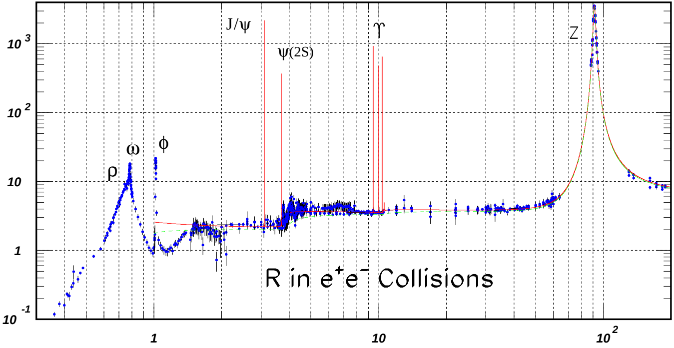
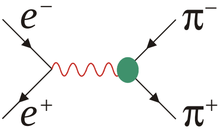
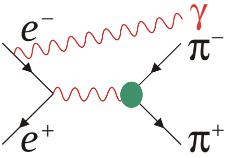


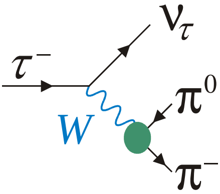
Figure from the COMPETE Coll.

Sources of information on $\sigma(e^+e^- \to \text{hadrons})$

Direct measurement	radiative return	τ decays + isospin corr.





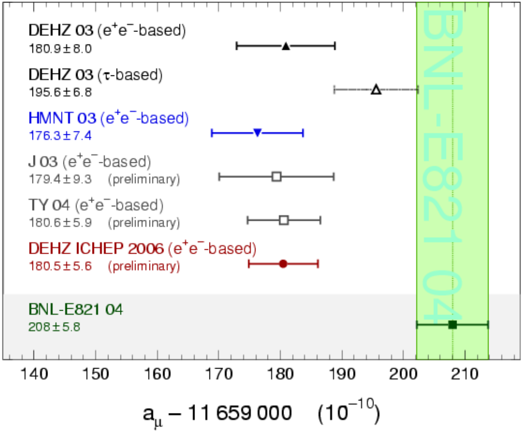


CMD-2, SND

KLOE (BABAR, BELLE)

ALEPH, CLEO, OPAL

Status of a_μ



J = Jegerlehner, DEHZ = Davier, Eidelman, Höcker, Zhang

Figure from M. Davier (06)

HMNT = Hagiwara, Martin, Nomura, Teubner, TY = de Tróconiz, Ynduráin

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- the current experimental error is about 0.9% of $a_\mu^{\rm hvp}$
- the challenge is the evaluation of $a_\mu^{\rm hvp}$ to 1% or better
- the evaluation of the hadronic contribution at order α^3 is also nontrivial (e.g. hadronic light-by-light) but its size is of the order of the current experimental error

Summary of e^+e^- data

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– three different sets of e^+e^- data (CMD-2(old), SND, CMD-2(new)) are in perfect mutual agreement;
– one set of data (KLOE) disagrees somewhat with the others in shape – the integral, however, is the same

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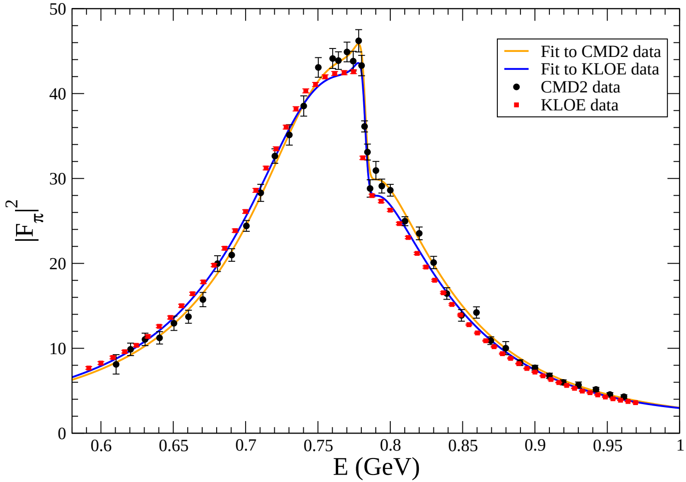


Outline

Introduction
- $(g-2)_e$ tests QED
- $(g-2)_\mu$ tests the standard model

Hadronic contribution to a_μ
- Introduction
- Present status of e^+e^- data

Can theory help?

New Physics in $(g-2)_\mu$?

Summary

Hadronic vacuum polarization contribution

Breakdown of a_μ^{hvp} in contributions of different energy regions

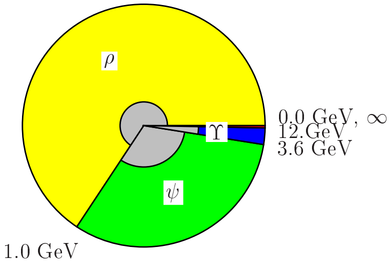
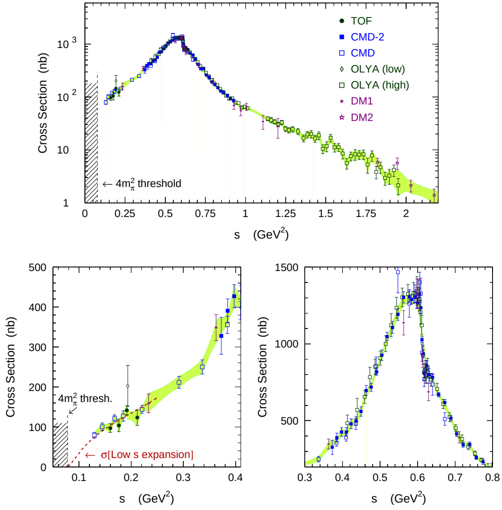


Figure from F. Jegerlehner

$$a_\mu^{\text{hvp}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\ell^2}^{\infty} ds\, \frac{\hat{K}(s)}{s^2} R_h(s)$$

The region below 1 GeV is the most important

Hadronic vacuum polarization contribution



The calculation of a_μ^{hvp} is done using

- trapezoidal rule
- Taylor exp. at low energy

Figure from Davier et al. (03)

Can theory help?

- The contributions to a_μ^{hvp} up to 1 GeV are dominated by the two-pion contribution

$$\sigma(e^+e^- \to \text{hadrons})|_{s \leq 1\text{GeV}} \sim |F_\pi(s)|^2$$

- Analyticity and unitarity relate very strongly the pion form factor and the P-wave $\pi\pi$ phase shift
- The $\pi\pi$ interaction at low energy is known theoretically to high precision

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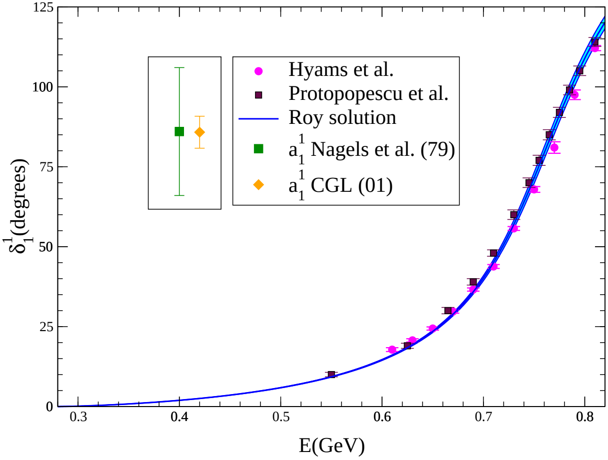
$\Rightarrow$ Use the knowledge on $\delta_{\pi\pi}$ to constrain $F_\pi(s)$

$\pi\pi$ scattering

Using analyticity, unitarity ($\equiv$ Roy eqs.) and chiral symmetry



GC, Gasser and Leutwyler, NPB 01

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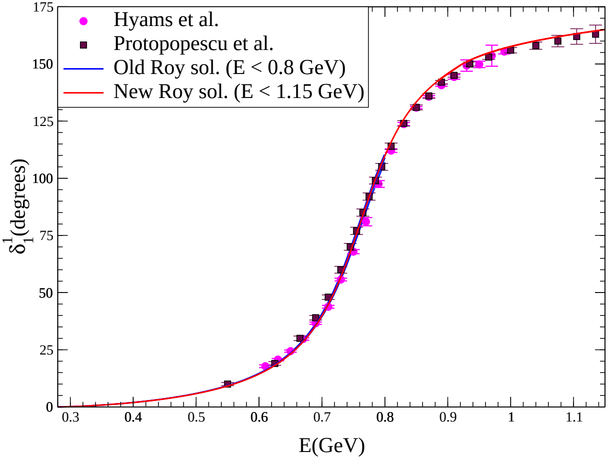

The uncertainties above 0.8 GeV are being evaluated (work in progress GC and Leutwyler)

How theory can help

- Analyticity $\Rightarrow$ $\delta(s) \Leftrightarrow F_\pi(s)$ (Omnés)

$$F_\pi(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \, \frac{\delta(s')}{s'\,(s'-s)}\right] \,, \qquad F_\pi(0) = 1$$

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- Unitarity and experiments (Eidelman-Lukaszuk)
  $\Rightarrow$ $s_{\rm in} = (M_\pi + M_\omega)^2$

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- Unitarity and experiments (Eidelman-Lukaszuk)
  $\Rightarrow$ $s_{\rm in} = (M_\pi + M_\omega)^2$

Representation of $F_\pi(s)$ which automatically satisfies unitarity, analyticity and chiral symmetry

[Heyn and Lang 81, de Trocóniz and Ynduráin 02]

[Caprini, GC, Leutwyler and Smith work in progr.]

An improved representation of the form factor

- Omnés representation (57)

$$F_\pi(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s)}\right] \equiv \Omega(s)$$

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- Split elastic from inelastic contributions

$$\delta = \delta_{\pi\pi} + \delta_{\rm in} \quad \Rightarrow \quad F_\pi(s) = \Omega_{\pi\pi}(s)\Omega_{\rm in}(s)$$

Eidelman-Lukaszuk: unitarity bound on $\delta_{\rm in}$

$$\sin^2\delta_{\rm in} \le \frac{1}{2}\left(1-\sqrt{1-r^2}\right) \qquad r = \frac{\sigma^{I=1}_{e^+e^-\to\neq 2\pi}}{\sigma_{e^+e^-\to 2\pi}}$$

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$$\Rightarrow \quad {\rm Im}\Omega_{\rm in}(s) \simeq 0 \qquad s \le (M_\pi + M_\omega)^2$$

- $\rho-\omega$–mixing must also be explicitly taken into account

$$F_\pi(s) = \Omega_{\pi\pi}(s)\Omega_{\rm in}(s)G_\omega(s)$$

Free parameters

$$\Omega_{\pi\pi}(s) \Rightarrow \begin{cases} \phi_0 = \delta_{\pi\pi}(E_0) & E_0 = 0.8\text{GeV} \\ \phi_1 = \delta_{\pi\pi}(E_1) & E_1 = 1.15\text{GeV} \end{cases}$$

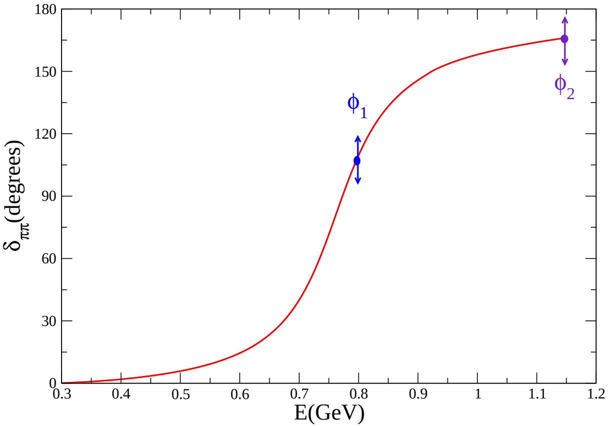
$$G_\omega(s) \Rightarrow \begin{cases} \epsilon & \omega - \rho \text{ mixing} \\ M_\omega \end{cases}$$

$$\Omega_{\text{in}}(s) \Rightarrow \begin{cases} c_1 \\ \vdots \\ c_P \end{cases} \qquad \text{Im}\Omega_{\text{in}}(s) = 0 \quad s \le s_{\text{in}}$$

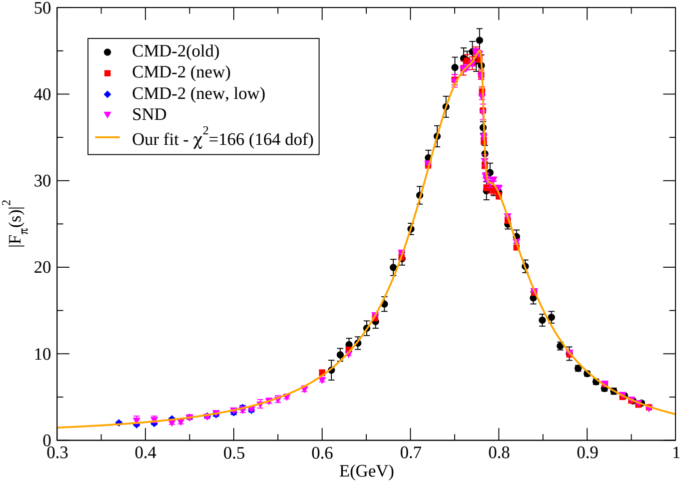
Output:

$$a_{\rho,2M_K} = 10^{10}\left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4M_\pi^2}^{s_\rho,4M_K^2} ds\,\frac{\hat{K}(s)R(s)}{s^2} \qquad s_\rho = (0.81\text{GeV})^2$$

Free parameters



Outcome of the fit



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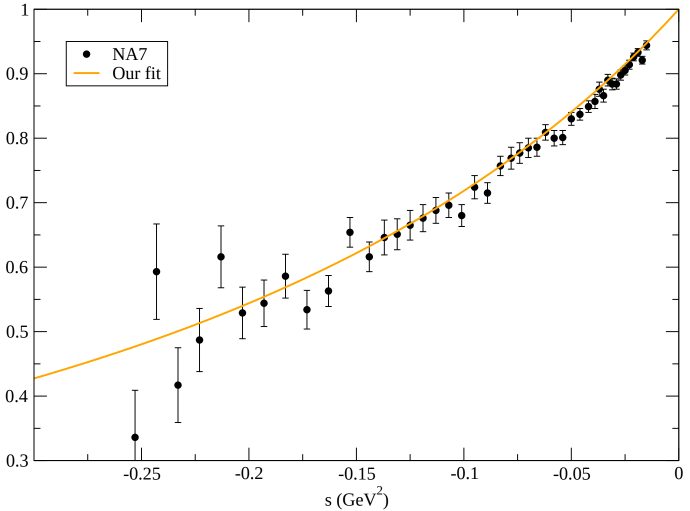

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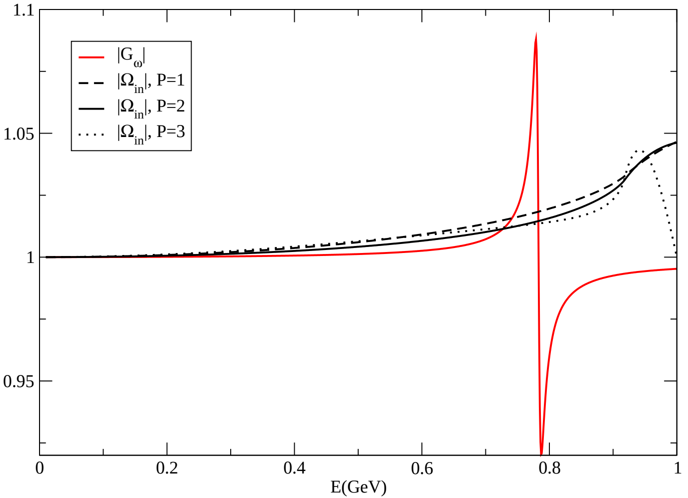

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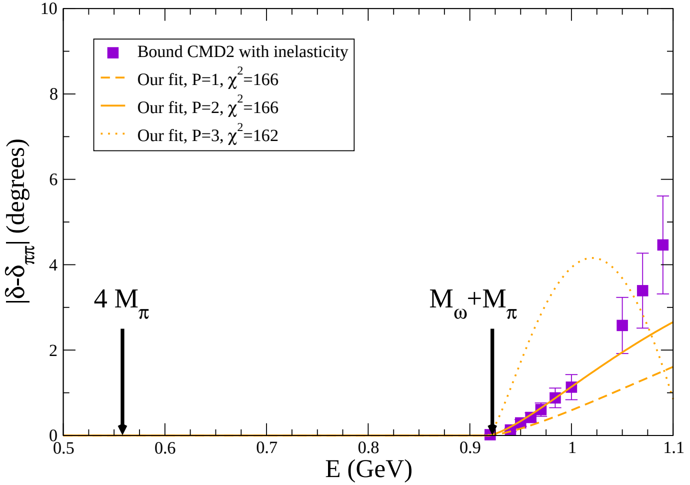

P = number of parameters in Ω_{in}

Discussion

- Reduced statistical error in the evaluation of the integral

P	χ^2/d.o.f.	a_ρ	a_{2M_K}
0	80.5/83	420.9 ± 2.3	490.6 ± 2.2
1	76.2/82	423.2 ± 2.6	493.6 ± 2.6
2^*	75.0/81	422.0 ± 2.8	492.2 ± 3.0
3^*	73.6/80	422.3 ± 2.8	492.2 ± 3.0

* The $P = 2, 3$ fit violate the Eidelman-Lukaszuk bound

Cf. Jegerlehner (03) (using the trapezoidal rule):

$$a_\rho = 429.02 \pm 4.95 \;\; \text{(stat.)}$$

Difference in central value mostly due to FS radiation, not included in our analysis

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The extrapolation down to threshold is almost for free (the total uncertainty barely increases):

$$a_\mu^{\mathrm{hvp}}{}_{(0.6\mathrm{GeV} \le \sqrt{s} \le 2M_K)} = (385.3 \pm 2.3) \cdot 10^{-10}$$

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$$\Delta a_\rho = 4.95 \rightarrow 2.6$$

- Being able to fit a set of data with this parametrization is quite nontrivial and provides a check on the data

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- The analysis is work in progress with I. Caprini, H. Leutwyler and C. Smith

Fit to all data sets

Stat. and syst. error added in squares

P	χ^2/d.o.f.	a_ρ	a_{2M_K}
0	246/165	416.4 ± 1.1	486.5 ± 1.0
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2	166/163	421.3 ± 1.3	492.1 ± 1.5
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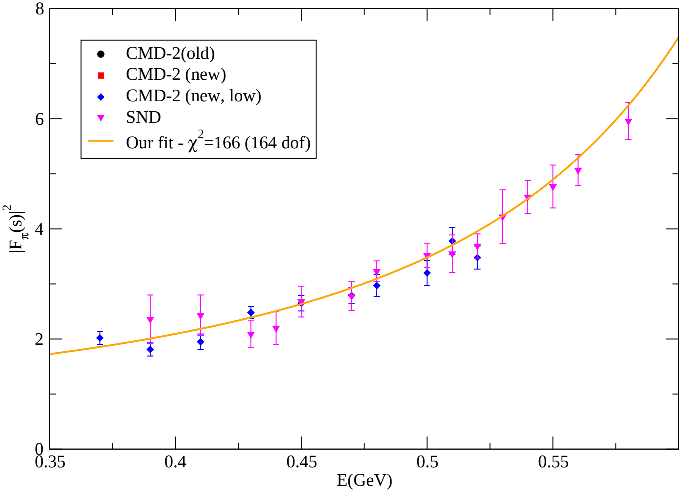
Breakdown of the χ^2 in the different data sets (P=2 fit):

	NA7	CMD-2 [old]	CMD-2 [new]	CMD-2 [new,low]	SND	total
N. data	45	43	29	10	45	172
χ^2	42.5	36.1	33.2	15.4	38.5	166

Fit to all data sets



Adding KLOE data to the fit

KLOE uncertainties are dominated by the systematic error

No correl. matrix, stat. and syst. errors added in squares

P	$\chi^2/$d.o.f.	a_ρ	a_{2M_K}
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	NA7	CMD-2 [old]	CMD-2 [new]	CMD-2 [new,lo]	SND	KLOE	total
N. dt.	45	43	29	10	45	60	232
χ^2	41.6	42.4	26.2	17.4	53.5	56.2	237

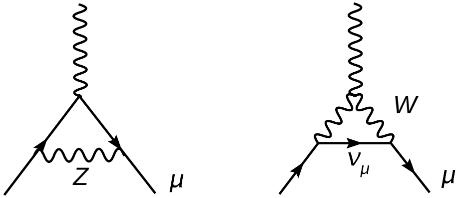
Open questions

- different e^+e^- data sets can be fitted simultaneously; the tension between KLOE data and the others should be understood
- isospin corrections between τ and e^+e^- data not yet understood
- treatment of photons – at the moment radiative corrections applied to the data are based on scalar electrodynamics. At this level of precision this is not satisfactory
- light-by-light is still controversial

Outline

SUSY contributions to a_μ

Electroweak contributions in the standard model



$$a_\mu^{\mathrm{EW,1loop}} = 19.5 \cdot 10^{-10}$$

Jackiw-Weinberg, Altarelli-Cabibbo-Maiani, Bars-Yoshimura, Fujikawa-Lee-Sanda (72)

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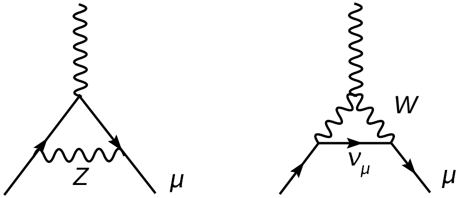

$$a_\mu^{\mathrm{EW,1loop}} = 19.5 \cdot 10^{-10}$$

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Two-loop corrections are found to be rather large (and negative):

$$a_\mu^{\mathrm{EW}} = (15.4 \pm 0.2) \cdot 10^{-10}$$

Czarnecki-Krause-Marciano (96), Czarnecki-Marciano-Vainshtein (03)

SUSY contributions to a_μ

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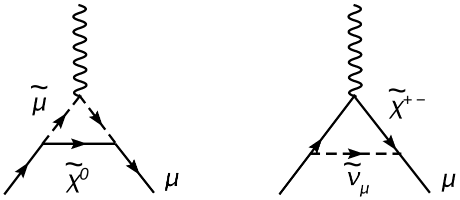

$$a_\mu^{\text{SUSY,1loop}} = \text{sign}(\mu) \tan\beta \left(\frac{100\text{GeV}}{M_{\text{SUSY}}}\right)^2 \times 13 \cdot 10^{-10}$$

Moroi (96) Czarnecki-Marciano (01)

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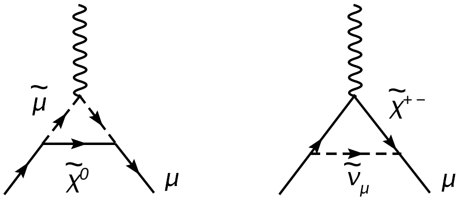

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Moroi (96) Czarnecki-Marciano (01)

In the simplest approximation (all masses equal) the most important two-loop contribution is

Heinemeyer, Stöckinger and Weiglein (04)

$$a_\mu^{\mathrm{SUSY},\chi\mathrm{2loops}} = \mathrm{sign}(\mu) \left(\frac{\tan\beta}{50}\right) \left(\frac{100\mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 \times 11 \cdot 10^{-10}$$

"Harbinger of new physics"?

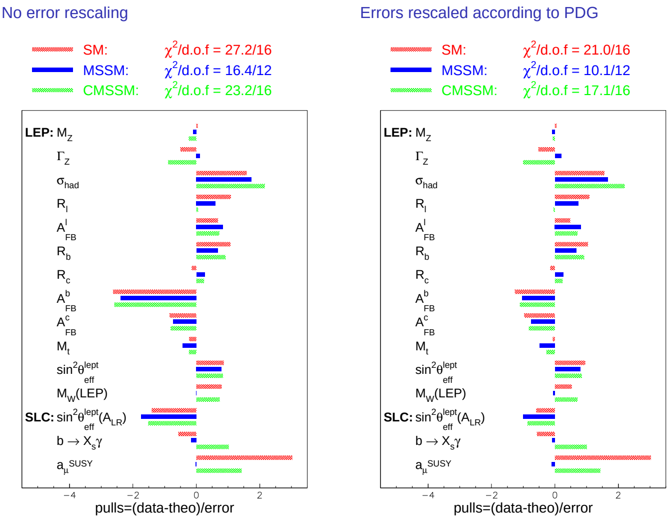

Figure from de Boer and Sander PLB (04)

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  this is also favoured by other data ($b \to s\gamma$)
- a_μ plays an important role among other precision observables as a test of the SM or estensions thereof
- if the discrepancy will disappear in the future a_μ will still provide strong constraints on the MSSM parameter space

"Harbinger of new physics"?

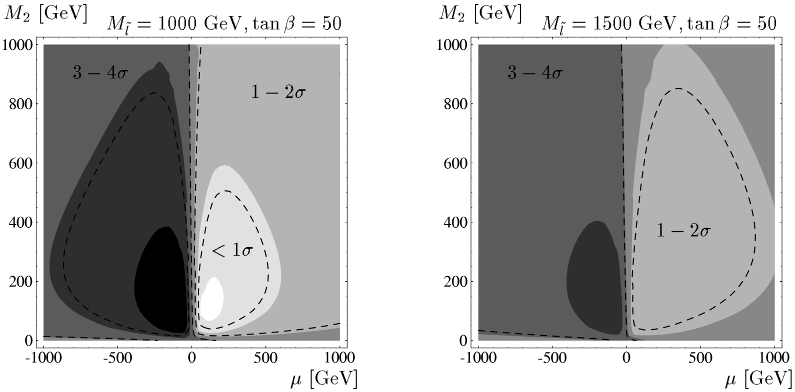

Figure from Heinemeyer, Stöckinger and Weiglein (04)

Outline

Summary

- a possible discrepancy between the standard model prediction and the measured value of $(g-2)_\mu$ might be one of the few signs of new physics we have;
- the delicate part in the standard model calculation are the hadronic contributions, and particularly the leading one: the hadronic vacuum polarization
- theory [$\equiv$ analyticity, unitarity and χ-symmetry] can help in the evaluation of the integral $a_\mu^{\rm hvp}$ by providing a controlled framework in which to analyze the data below 1 GeV and make the extrapolation down to threshold
- the machinery is working and ready to be used with all data sets

work in progress, Caprini, GC, Leutwyler and Smith