Dynamical breakdown of Abelian gauge chiral symmetry by strong Yukawa interactions

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based on P.B., Tomáš Brauner, Jiří Hošek: hep-ph/0605147 and CERN Yellow Book hep-ph/0608079

Motivations

The Standard Model:

- EW symmetry breaking by means of the Higgs mechanism
- Phenomenologically highly succesfull description

However:

- Difficult to interpret the wrong sign of the Higgs ‘mass’ squared ($M^2 < 0$)
- Fermion masses $\propto$ Yukawa couplings

Our proposal

The EW symmetry breaking can be achieved dynamically:

- Scalars and Yukawa couplings still present in theory
- However no condensing scalars ($M^2 > 0$)
- Fermion masses are nonlinear functions of Yukawa couplings

In this presentation we

- do not discuss the full $SU(2) \times U(1)$ theory of EW interactions
- present merely a simple $U(1)$ toy model

The model

The Lagrangian:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}_1 \mathrm{i}\not{D}\psi_1 + \bar{\psi}_2 \mathrm{i}\not{D}\psi_2 + (\mathrm{D}_\mu\phi)^\dagger(\mathrm{D}^\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &\quad -M^2\phi^\dagger\phi - \frac{1}{2}\lambda(\phi^\dagger\phi)^2 \\ &\quad +y_1(\bar{\psi}_{1L}\psi_{1R}\phi + \bar{\psi}_{1R}\psi_{1L}\phi^\dagger) + y_2(\bar{\psi}_{2R}\psi_{2L}\phi + \bar{\psi}_{2L}\psi_{2R}\phi^\dagger)\end{aligned}$$

$U(1)_A$ gauge invariance:

$$\begin{aligned}\psi_1 &\longrightarrow \mathrm{e}^{+\mathrm{i}\theta(x)\gamma_5}\psi_1 \\ \psi_2 &\longrightarrow \mathrm{e}^{-\mathrm{i}\theta(x)\gamma_5}\psi_2 \\ \phi &\longrightarrow \mathrm{e}^{-2\mathrm{i}\theta(x)}\phi\end{aligned}$$

The case of $M^2 < 0$

Standard Higgs mechanism takes place:

- Scalar develops VEV:

$$\langle\phi\rangle_0 = \sqrt{\frac{-M^2}{\lambda}}$$

- The spectrum:

fermions:
$$m_1 = y_1\langle\phi\rangle_0$$
$$m_2 = y_2\langle\phi\rangle_0$$

scalars: ($\phi = \frac{1}{\sqrt{2}}(\phi_1 + \mathrm{i}\phi_2)$)
$$M_1 = \sqrt{2\lambda}\langle\phi\rangle_0$$
$$(M_2 = 0)$$

gauge boson:
$$M_A = 2\sqrt{2}g\langle\phi\rangle_0$$

The case of $M^2 > 0$

Perturbatively, nothing interesting happens:

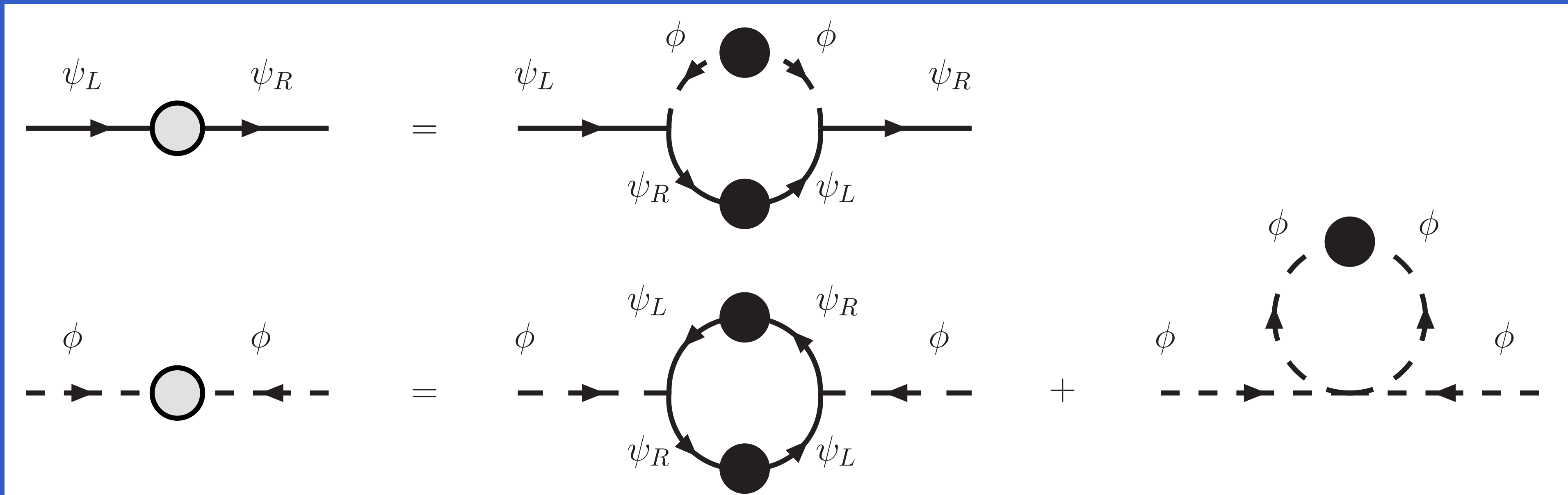
- $\langle\phi\rangle_0 = 0$
- Fermions remain massless
- No mass splitting in the scalar sector
- $\Rightarrow$ the $U(1)_A$ symmetry remains unbroken

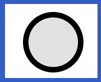
But what about some *nonperturbative* treatment?

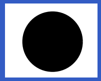
- Make use of the Schwinger-Dyson equations
- Consider only symmetry-breaking 2-point Green functions:

$$\langle\phi\phi\rangle, \langle\phi^\dagger\phi^\dagger\rangle, \langle\psi_{iL}\bar{\psi}_{iR}\rangle, \langle\psi_{iR}\bar{\psi}_{iL}\rangle$$

Schwinger-Dyson equations



 – 1PI propagator

 – full propagator

Schwinger-Dyson equations

$$\Sigma_{1,p} = \mathrm{i}y_1^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\Sigma_{1,k}}{k^2 - \Sigma_{1,k}^2} \frac{\Pi_{k-p}}{\left[(k-p)^2 - M^2\right]^2 - |\Pi_{k-p}|^2}$$

$$\Sigma_{2,p} = \mathrm{i}y_2^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\Sigma_{2,k}}{k^2 - \Sigma_{2,k}^2} \frac{\Pi_{k-p}^*}{\left[(k-p)^2 - M^2\right]^2 - |\Pi_{k-p}|^2}$$

$$\Pi_p = -\sum_{j=1,2} 2\mathrm{i}y_j^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\Sigma_{j,k}}{k^2 - \Sigma_{j,k}^2} \frac{\Sigma_{j,k-p}}{(k-p)^2 - \Sigma_{j,k-p}^2} + \mathrm{i}\lambda \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\Pi_k}{(k^2 - M^2)^2 - |\Pi_k|^2}$$

Notation: $\Sigma_{1,p} \equiv \Sigma_1(p^2)$, etc.

The spectrum

- Fermions: $m_1^2 = \Sigma_1^2(m_1^2)$
  $m_2^2 = \Sigma_2^2(m_2^2)$
- Scalars: $M_1^2 = M^2 + |\Pi(M_1^2)|$
  $M_2^2 = M^2 - |\Pi(M_2^2)|$
- Gauge boson:
  - Need to employ Ward identities
  - 'Would-be' Goldstone boson is composite of fermions and scalars
  - $M_A^2 = g^2 \times$ complicated integral of Σ_1, Σ_2, Π
- Unlike the Higgs case the masses are related through dependence on Σ_1, Σ_2, Π

Numerical computation

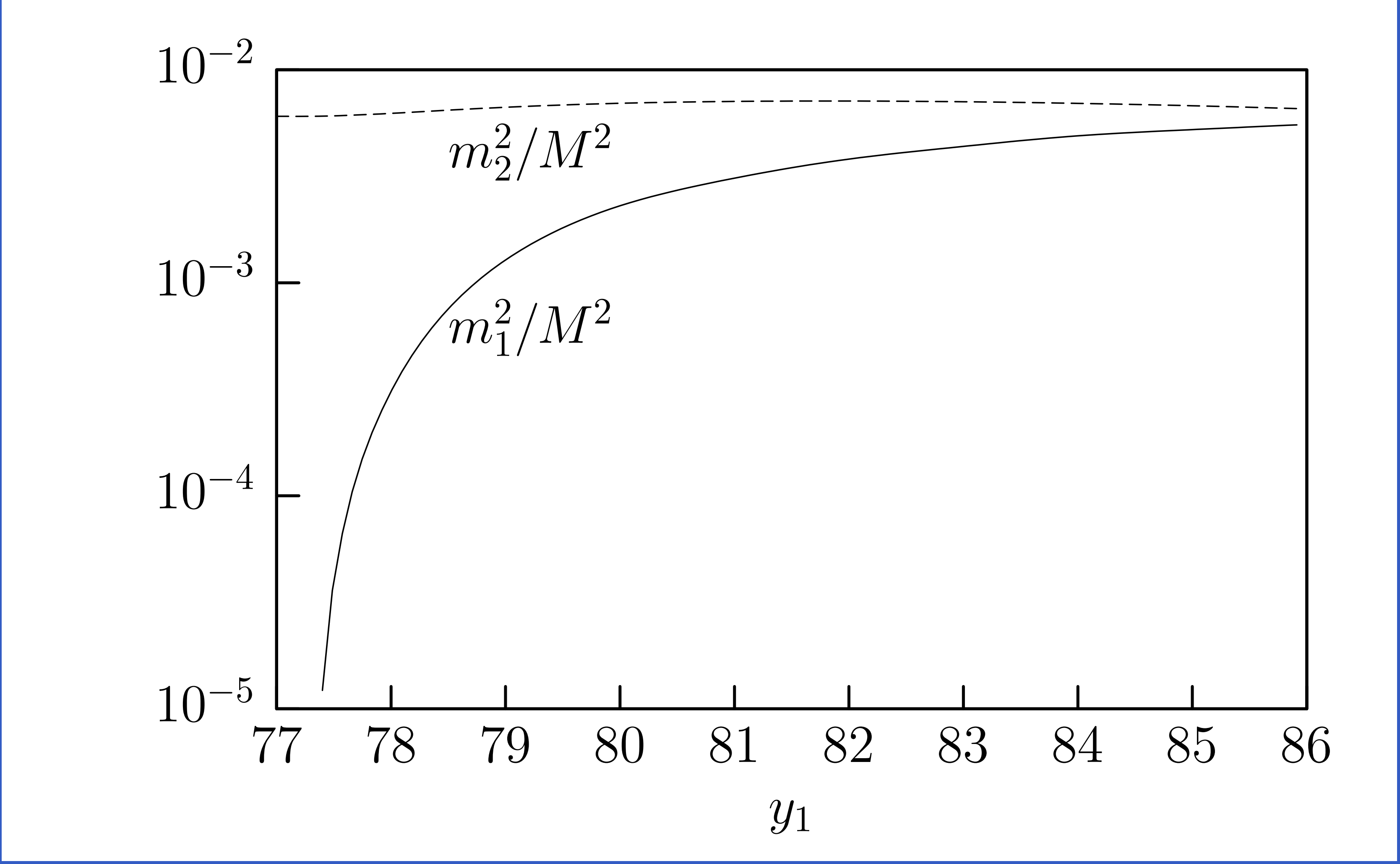
SD equations still too difficult, need some simplifications:

- Switch to Euclidean metric
  $\Rightarrow$ some poles removed (but not all)
- Set $\lambda = 0$

Then:

- For some values (y_1, y_2) the solution to the SD equations was found
- Self-energies Σ_1, Σ_2, Π are rapidly decreasing functions
- The $y_{1,2}$-dependence of the spectrum was probed
- The case of fermions is most interesting . . .

Fermionic spectrum



Calculated for fixed $y_2 = 88$

Summary and outlook

We have found that

- the Model is capable of providing SSB
- the ratio of dynamically generated masses can be large for not vastly different Yukawa couplings

But still much to do:

- Get some analytical insight into the above results
- Try to find solution to the SD equations in the Minkowski space
- Work out the realistic $SU(2) \times U(1)$ model (sketch already present in `hep-ph/0407339`)