First order finite temperature restoration of the chiral symmetry of QCD

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- The chiral symmetry of the QCD, the familiar points of the phase diagram
- Description of the $SU_L(3) \times SU_R(3)$ symmetric linear sigma model ($L\sigma M$)
- Parameterization of the model at $T = 0$ for arbitrary m_π, m_K in accordance with the chiral perturbation theory $(ChPT)$
- Thermodynamics in the framework of optimized perturbation theory
- Results
- Conclusion, possible extensions

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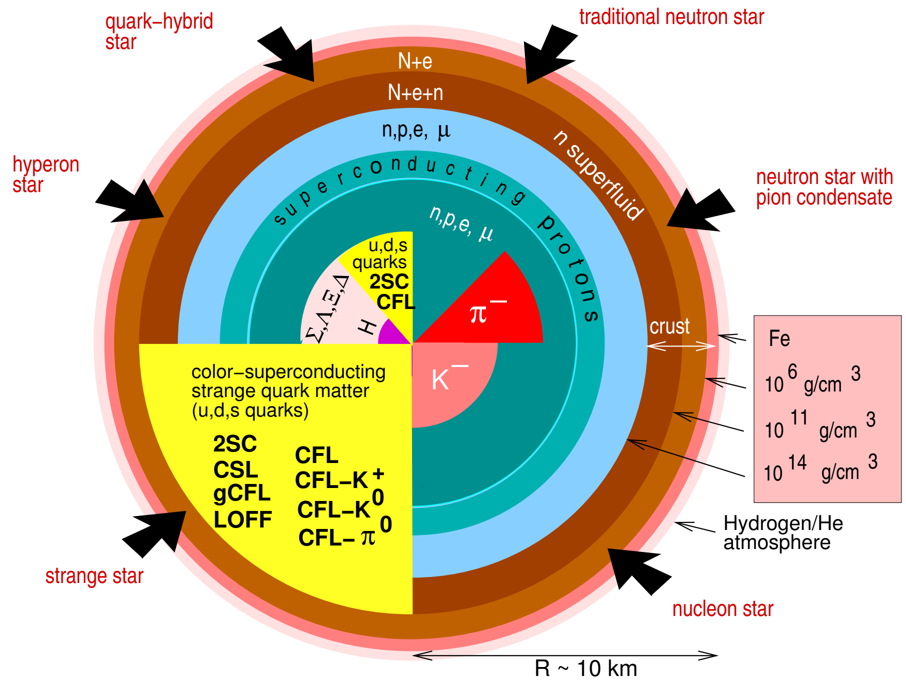
Motivation

Astrophysical interest in the accurate determination of QCD Equation of State.

- Possible cosmological effects of using realistic EoS of QCD
  (see, for example: Hindmarsh, Philipsen)

- Stucture of compact stars:



important task:
Exploration of the ground state of QCD under the variation of its parameters

This study: Determination of the shape of the phase boundary in the $m_\pi - m_K$ –plane, at zero chemical potential.

The phase diagram for $N_f = 3$, and $\mu = 0$

Order parameter for $m_i = 0$ ($N_f \geq 2$): $M^{ij} = \langle \bar{q}^i_L q^j_R \rangle$

$SU_L(N_f) \times SU_R(N_f) \times U_V(1)$ $\overset{T \leq T_c}{\Longrightarrow}$ breaks down $SU_V(N_f) \times U_V(1)$

Familiar points:

I. physical point: crossover

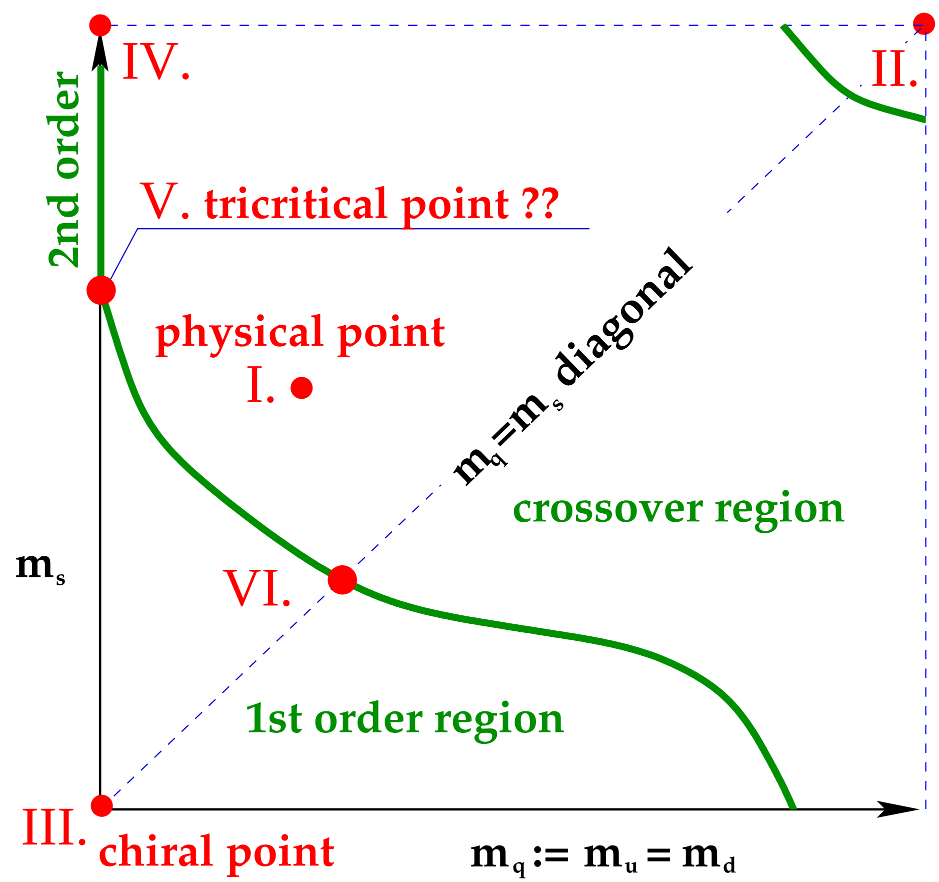
II. $m_s, m_q \to \infty$: 1st order transition

III. $m_{q,s} = 0$: 1st order transition

IV. $m_s \to \infty$, $m_q = 0$: 2nd order transition

V. tricritical point at finite m_s ?

VI. endpoint of 1st order region along the diagonal (on lattice: $m_{\pi\,diag} \leq 65$ MeV)



The $SU(3)_L \times SU(3)_R$ linear sigma model.

$\bar{q}^i_L q^j_R$ bound states $\rightarrow$ mesons transform as $(8,1) \oplus (1,8)$ multiplets

$\sigma_i(0^+)$: scalar nonet (1,8) | $\pi_i(0^-)$: pseudoscalar nonet (8,1)

$$M := \frac{1}{\sqrt{2}} \sum_{i=0}^{8} (\sigma_i + i\pi_i)\lambda_i$$

λ_i : $i = 1 \ldots 8$ *Gell-Mann* matrices, and $\lambda_0 := \sqrt{\frac{2}{3}}\mathbf{1}$

The most general $SU(3)_L \times SU(3)_R$ symmetric, renormalizable Lagrangian constructed with M is

$$\begin{aligned} L^{symm}(M) &= \frac{1}{2}\text{Tr}(\partial_\mu M^\dagger \partial^\mu M + \mu^2 M^\dagger M) - f_1 \left(\text{Tr}(M^\dagger M)\right)^2 - f_2 \text{Tr}(M^\dagger M)^2 - \\ &\quad -g\left(\det(M) + \det(M^\dagger)\right) \end{aligned}$$

- $\mu^2 > 0$ allows spontaneously broken ground states.
- Two independent quartic couplings (f_1, f_2).
- The determinant term breaks $U(3)$ symmetry $\rightarrow SU(3)$ ($U(1)_A$ anomaly)

Symmetry breaking

Explicit symmetry breaking terms:

$$L = L^{symm} + \epsilon_x \sigma_x + \epsilon_y \sigma_y + \epsilon_3 \sigma_3 \, , \quad \begin{pmatrix} \sigma_x \\ \sigma_y \end{pmatrix} := \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2} & 1 \\ 1 & -\sqrt{2} \end{pmatrix} \begin{pmatrix} \sigma_0 \\ \sigma_8 \end{pmatrix} ,$$

where σ_x and σ_y are the strange and non-strange combination of the fields.

The external fields are proportional to the quark masses:

$$\boxed{\epsilon_x \sim m_q, \, \epsilon_y \sim m_s, \, \epsilon_3 \sim m_u - m_d}$$

We restrict ourselves to the case $\epsilon_3 = 0$ (m_u, m_s are degenerate).

If $\epsilon_x \neq 0$ and/or $\mu^2 > 0$ then the order parameters are: $\boxed{x := \langle \sigma_x \rangle_0 \, , \; y =: \langle \sigma_y \rangle_0}$

PCAC relations can be obtained for the pion and kaon decay constants:

$$f_i m_{\pi_i}^2 := \langle 0 | \partial_\mu J_5^\mu | \pi_i \rangle \Longrightarrow \begin{cases} f_\pi m_\pi^2 = \epsilon_x & i = 1,2,3 \\ f_K m_K^2 = \frac{1}{2} \left(\epsilon_x + 2\sqrt{2} \epsilon_y \right) & i = 4,5,6,7 \, . \end{cases}$$

The Lagrangian of the shifted fields

Shifting the fields by vacuum expectation values: $\sigma_x \rightarrow \sigma_x - x\,; \quad \sigma_y \rightarrow \sigma_y - y$

- the vanishing of the 1^{st} order terms gives the two equations of states (EoS):

$$E_x := \frac{\partial L}{\partial \sigma_x}\Big|_0 = -\epsilon_x - \mu^2 + 2gxy + 4f_1 xy^2 + (4f_1 + 2f_2)x^3 = 0,$$
$$E_y := \frac{\partial L}{\partial \sigma_y}\Big|_0 = -\epsilon_y - \mu^2 + 2gx^2 + 4f_1 x^2 y + (4f_1 + 4f_2)y^3 = 0$$

- the coefficients of the 2^{nd} order terms are the mass squares of mesons :

$$M_{PS}(x,y) : m_\pi^2\,,\ m_K^2\,, \begin{pmatrix} m^2_{\eta 88} & m^2_{\eta 08} \\ m^2_{\eta 08} & m^2_{\eta 00} \end{pmatrix};\ M_S(x,y)\ :\ m^2_{a_0}\,,\ m^2_\kappa\,, \begin{pmatrix} m^2_{\sigma 88} & m^2_{\sigma 08} \\ m^2_{\sigma 08} & m^2_{\sigma 00} \end{pmatrix}$$

Note the mixing in the $(0$ - $8)$/$(x$ – $y)$ sectors.

After a straightforward calculation, two *Ward-identities* can be obtained:

$$\epsilon_x = {m_\pi}^2 x\,, \quad \epsilon_y = \frac{1}{\sqrt{2}}({m_K}^2 - {m_\pi}^2)x + {m_K}^2 y$$

which guarantee the Goldstone theorem,

and PCAC relations are simplified : $f_\pi = x\,,\ f_K = \frac{1}{\sqrt{2}}y + \frac{1}{2}x$

Determination of parameters for arbitrary (m_π, m_K)

The unknown parameters (μ_0, f_1, f_2, g, ϵ_x, ϵ_y) and the condensates (x, y) can be determined at tree level by using the mass spectra and the PCAC relations.

input:		output:		predictions:
f_π, f_K	$\Longrightarrow$	x, y		
m_π, m_K, M_η^2	$\Longrightarrow$	g, f_2, M^2	$\Longrightarrow$ (from x, y, g, f_2, M^2)	m_η, $m_{\eta'}$, θ_η, m_{a_0}, m_κ
A1 & M^2, A2 & M^2	$\Longrightarrow$	μ^2, f_1	$\Longrightarrow$	m_σ, m_{f_0}, θ_σ
$E_x = 0$, $E_y = 0$	$\Longrightarrow$	ϵ_x, ϵ_y		

where $M_\eta^2 = m_{\eta_{00}}^2 + m_{\eta_{88}}^2$

- At tree level, μ_0, f_1 appear only in the combination:

$$M^2 = -\mu^2 + 4 f_1 (x^2 + y^2)$$

– except for the admixed scalar masses – m_π, m_K dependence of m_σ, m_{f_0} not known $\Longrightarrow$ assumptions is needed (A1, A2)

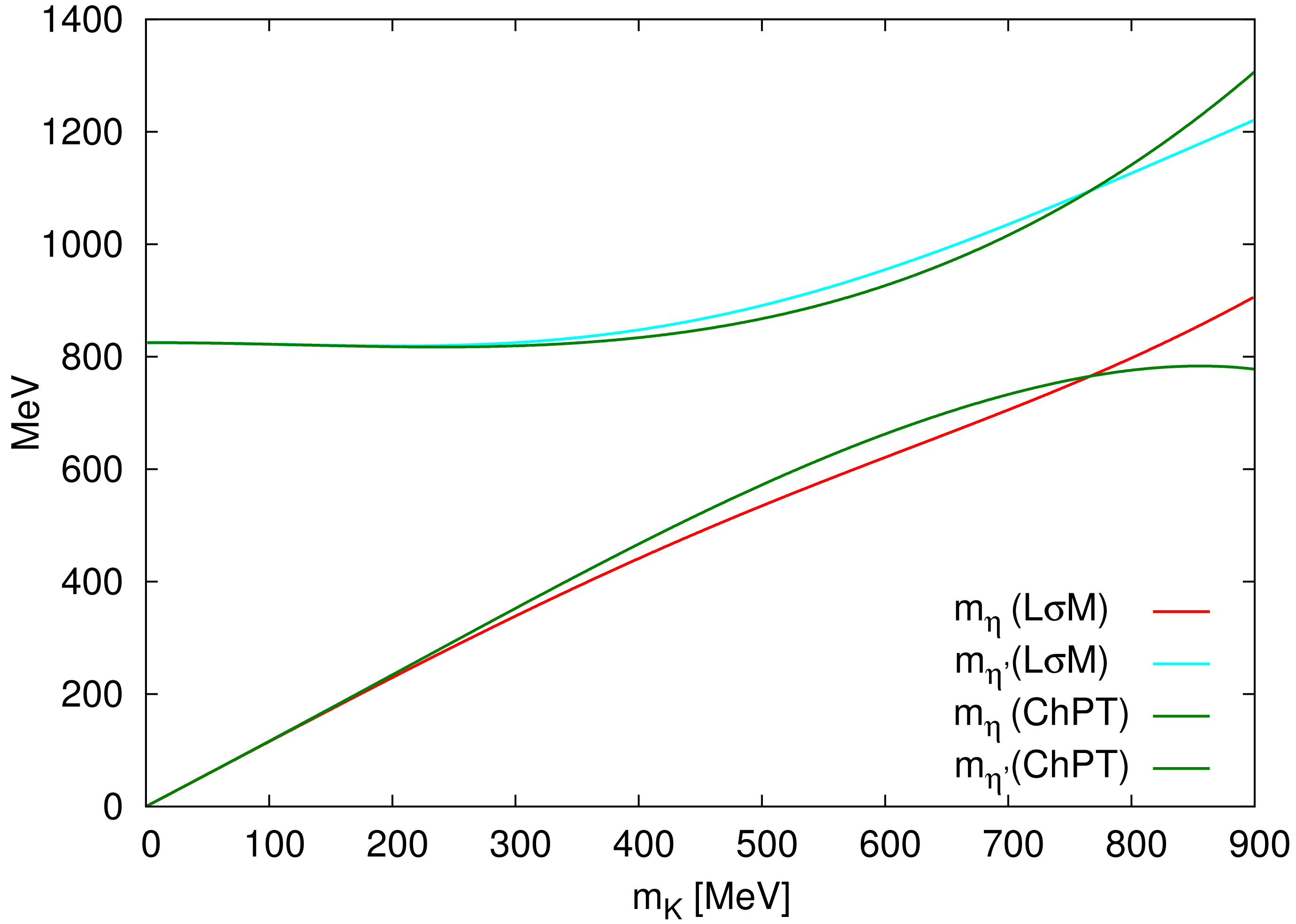
- ϵ_x, ϵ_y, x, y, f_1, f_2, g, μ_0, are known as the function of m_π, m_K, f_π, f_K, M_η, But going away from the physical point, we keep in mind their pion, kaon mass dependence:

$$f_\pi(m_\pi, m_K), \;\; f_K(m_\pi, m_K), \;\; M_\eta(m_\pi, m_K)$$

provided by the U(3) $ChPT$.

Predictions after fixing the parameters

Comparison of the predicted m_K dependence of the η, $\eta^{'}$ mass by $L\sigma M$ with the results of U(3) ChPT.



Remarkable agreement up to $m_K \approx 800$ MeV even for $m_\pi = 0$.

Thermodynamics at one-loop level

• tree level mass squares: $m_i^2 \to -\mu^2$ as T is increasing $\Longrightarrow$ Perturbation theory fails at finite T $\Longrightarrow$ resummation is needed

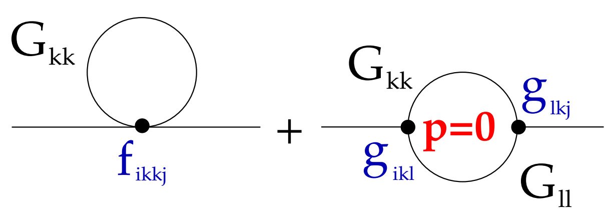
• Optimized perturbation theory (OPT): the mass term of the Lagrangian is reshuffled by a temperature dependent effective mass $M(T)$:

$$L_{mass} = -\frac{1}{2}M^2(T)\mathrm{Tr}M^\dagger M + \frac{1}{2}\overbrace{(\mu_0^2 + M^2(T))\mathrm{Tr}M^\dagger M}^{\Delta m^2 \mathrm{Tr} M^\dagger M\text{: one-loop counterterm}} \ .$$

• $-\mu^2 \to M^2(T)$ replacement in the propagators $\to$ preserving the relations among tree-level masses $\to$ Goldstone theorem, renormalization are assured

One-loop masses: $$\boxed{M_{ij}^2(T) = iG(\mathbf{p},T)_{ij}^{-1}\Big|_{\mathbf{p}=0} = m_{ij}^2 - \Delta m^2 + \Sigma_{ij}(\mathbf{0},T)}$$

Self-energy: $\Sigma_{ij}(\mathbf{0},T) = \sum_{k\,l}$



$$M_i^2(T) = m_i^2(T,x,y) - \Delta m^2(T) + \sum_i c_j^i I_{TP}(m_j(T,x,y),T)$$

The principle of minimal sensitivity (PMS) $\rightarrow$ determination of M(T):

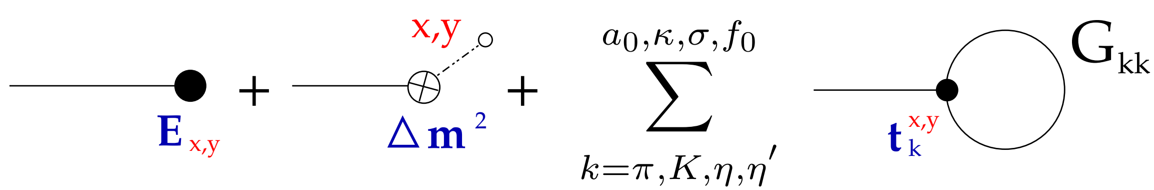
$$M_\pi^2(T) \overset{!}{=} m_\pi^2(T) \Longrightarrow \boxed{M(T)^2 = -\mu_0^2 + \sum_i c_i^\pi I_{TP}(m_i(T, x, y), T)}$$

this can be rewritten into a Gap equation for the pion mass:

$$\boxed{{m_\pi}^2 = -\mu_0^2 + (4f_1 + 2f_2)x^2 + 4f_1 y^2 + 2gy + \sum_i c_i^\pi(x,y) I_{TP}(m_i(m_\pi, x, y), T)}$$

The one point functions of σ_x and σ_y gives the equations of states:

$\sigma_{x,y}$:



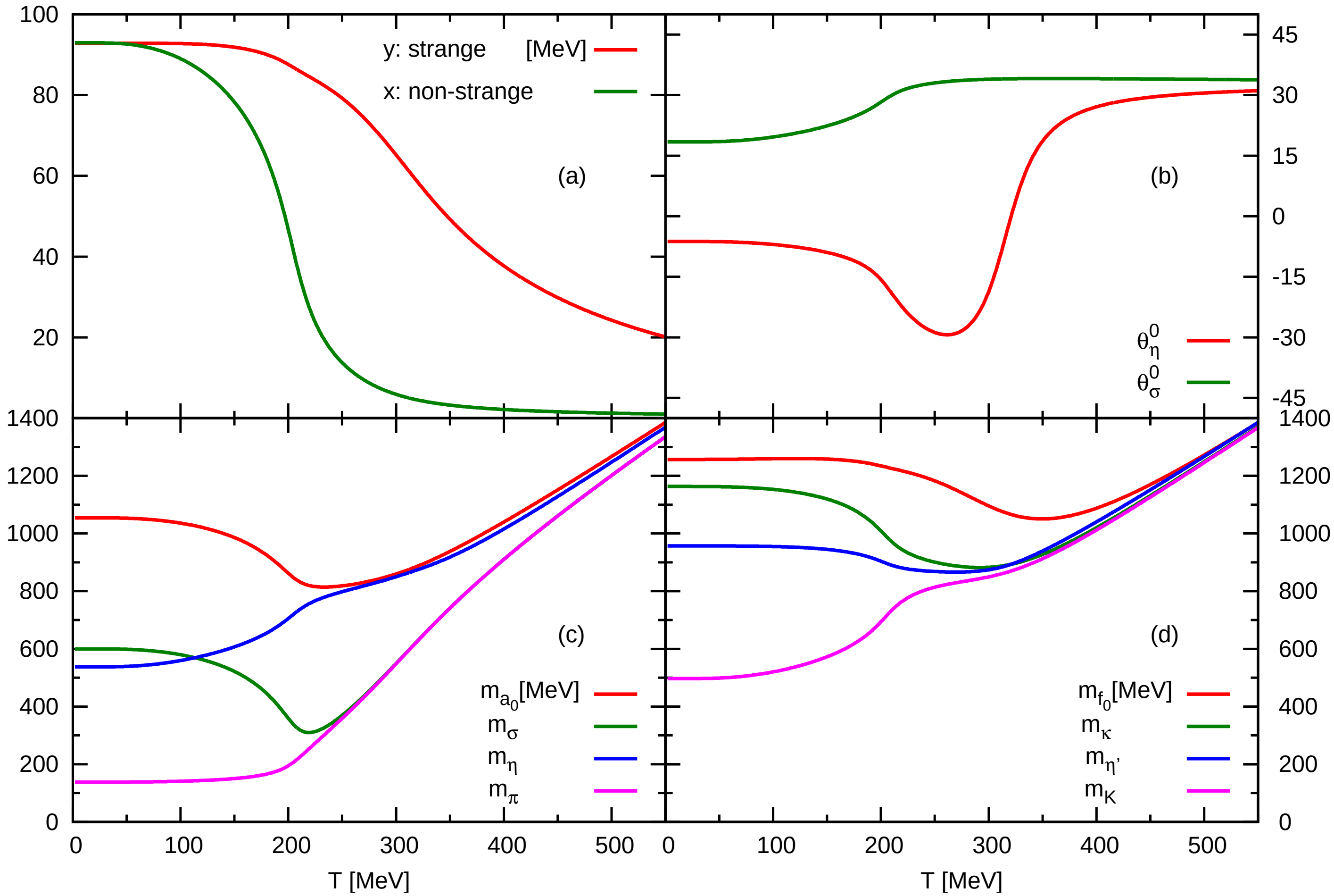
$= 0,$

$$\boxed{\begin{aligned} \epsilon_x &= -\mu_0^2 + 2gxy + 4f_1 xy^2 + (4f_1 + 2f_2)x^3 + \sum_i t_i^x(x,y) I_{TP}(m_i(m_\pi, x, y), T) \\ \epsilon_y &= -\mu_0^2 + 2gx^2 + 4f_1 x^2 y + (4f_1 + 4f_2)y^3 + \sum_i t_i^y(x,y) I_{TP}(m_i(m_\pi, x, y), T) \end{aligned}}$$

Tadpole: $I_{TP}(m,T) = \underbrace{\frac{1}{16\pi^2} m^2 \ln\frac{m^2}{l^2}}_{I_{TP}^l(m)} + \underbrace{\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\left(\frac{1}{\omega^2}\right)\frac{1}{e^{\beta\omega} - 1}}_{I_{TP}^T(m,T)}, \; I_{TP}^T(m, 0) = 0$

We omit the zero temperature contribution to avoid the one-loop redefinition of parameters $\longrightarrow$ quasiparticle approximation $I_{TP} = I_{TP}^T$.

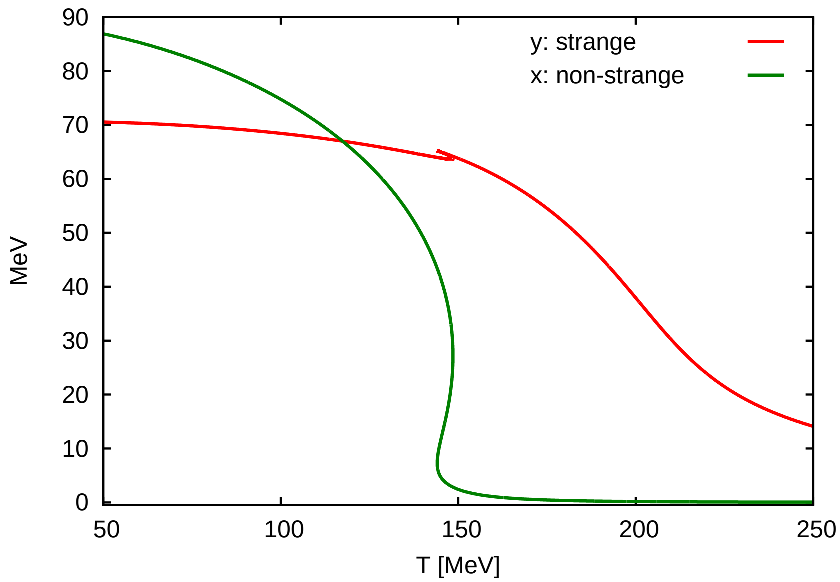
RESULTS I. Solution at the physical point: crossover



a) order parameters, b) mixing angles c) and d) tree-level masses

RESULTS II. 1^{st} order transition and the phase boundary

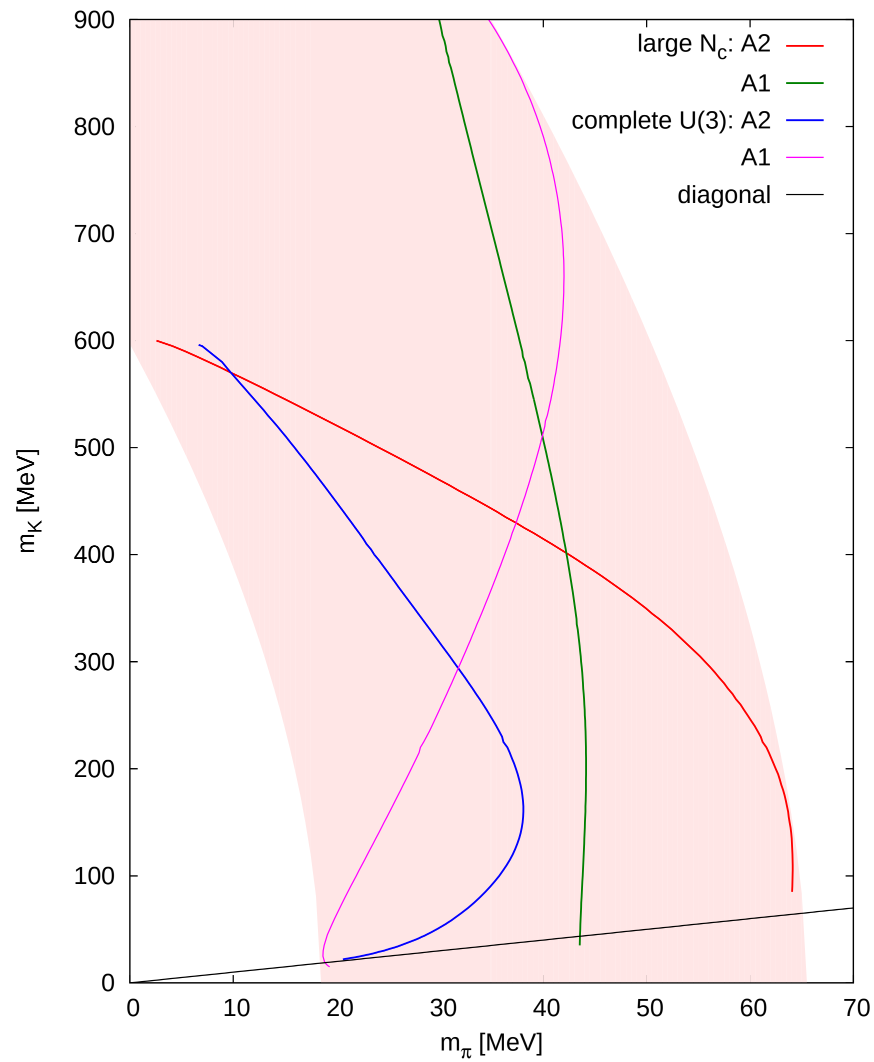
Decreasing pion mass:
$\longrightarrow$ 1^{st} order transition



Typical 1^{st} order transition for $m_\pi = 30$ MeV, $m_K = 400$ MeV

- x drives the transitions
- as the kaon mass decreases, the crossover in y is getting more sharp
- critical temperature: $T_c \approx 150$ MeV

Phase boundaries for different parameteriztion



Conclusions and further extension

- Sensivity of the 1st order $-$ crossover boundary to the scalar spectra
- Physical point is rather distant from the the 1st order region
- $m_{crit} \approx 42 \pm 23$MeV for $m_\pi = m_K$ (in agreement with lattice results)

- Analougus investigation of the 3-flavour chiral quark model is in progress $\Longrightarrow$ (T, μ) phase diagram of the physical space