HOLOGRAPHIC DARK ENERGY AND IR SCALE OF THE UNIVERSE

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Elements of holography

IR cutoff (dimension of a box): $L = 1/\mu$
UV cutoff (minimal length): $l = 1/\Lambda$
entropy ($\sim$ number of the degrees of freedom $\propto$ volume)

$$S \sim L^3 \Lambda^3$$

Black holes have a quantity analogous to entropy, but proportional to the surface (not the volume)

$$S_{BH} \sim L^2 M_{Pl}^2$$

Two possibilities:

1. (more conservative) - the analogy is merely an analogy, there is no deeper physics in it.

2. (more interesting) - S_{BH} is really *the* entropy of the black hole.

I will discuss some physical consequences that emerge from the assumption that the 2. possibility is the right one.

Why black hole entropy is proportional to the surface (holography)?
A possible answer - Bekenstein bound:

$$S \leq S_{BH} \quad \Rightarrow \quad L^3\Lambda^3 \lesssim L^2 M_{Pl}^2$$

- a relation between UV and IR cutoff.
The Bekenstein bound motivated introduction of various similar bounds.
In particular, Cohen, Kaplan and Nelson (1999) introduce the following bound:
In order to prevent the formation of black holes in vacuum, the vacuum energy in a box should be smaller than the mass needed for a formation of a black hole.
Mass of a black hole with radius L: $\sim LM_{Pl}^2$
Vacuum energy-density (the cosmological constant): $\rho_\Lambda = \Lambda^4$
Vacuum energy inside L: $V\rho_\Lambda = L^3\Lambda^4$
$\Rightarrow$ (Cohen, Kaplan and Nelson)

$$L^3\Lambda^4 \lesssim LM_{Pl}^2$$

This bound is even stronger than the Bekenstein bound.
It can also be written as

$$\rho_\Lambda \lesssim \mu^2 G_N^{-1}$$

Thus a holography relation relevant to cosmology is:

$$\boxed{\rho_\Lambda = \kappa\mu^2 G_N^{-1}}$$

where $\kappa \sim 1$ is a constant.
Can μ^{-1} be identified with some cosmological distance?
Natural candidates are age of the Universe, Hubble distance, past horizon (i.e. particle horizon), future horizon (i.e. event horizon). All these scales are (today) of the same order of magnitude.
Moreover, (today) all these scales satisfy the holography relation with $\kappa \sim 1$.
Thus, holography might explain the value of the cosmological constant!!!
Our *aim* is to explore this possibility in more details, by exploring the corresponding cosmological implications.

All these scales *depend on time* $\Rightarrow \mu = \mu(t)$.
Consequently, ρ_Λ, or G_N, or both, also must depend on time. The cosmological constant or/and the Newton constant - is not really a constant.
Instead, they are *running coupling constants* $\rho_\Lambda(\mu(t))$, $G_N(\mu(t))$ (the phenomenon known from renormalization group analysis in quantum field theory).
To understand the dependence on time, we need dynamical equations of motion - the Einstein equations.

Einstein equations

Einstein equation:

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

From Bianchy identity

$$\nabla^\mu G_{\mu\nu} \equiv 0$$

$\Rightarrow$ conservation of $G_N T_{\mu\nu}$

$$\nabla^\mu (G_N T_{\mu\nu}) = 0$$

In a Robertson-Walker Universe

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

the conservation equation leads to

$$\dot{G}_N(\rho_\Lambda + \rho_m) + G_N\dot{\rho}_\Lambda + G_N(\dot{\rho}_m + 3H\rho_m) = 0$$

where $H \equiv \dot{a}/a =$ inverse Hubble distance.
Set the present value $a_0 = 1$.
Use

$$\frac{d}{dt} = \dot{\mu}\frac{d}{d\mu}$$

Assume matter scales canonically

$$\rho_m = \rho_{m0} a^{-3}$$

$\Rightarrow$

$$G'_N(\rho_\Lambda + \rho_m) + G_N \rho'_\Lambda = 0$$

where $' \equiv d/d\mu$.
Combining with the holography relation $\Rightarrow$
scale-setting relation

$$\mu = -\frac{G'_N \rho_m}{2\kappa}$$

The time evolution is governed by the Friedmann equation

$$H^2 = \frac{8\pi G}{3}(\rho_\Lambda + \rho_m) - \frac{k}{a^2}$$

The scale-setting relation, together with other equations above, still does not fix the scale μ uniquelly.
The model still alows and requires an additional assumption on μ.

SMALL-μ EXPANSION AND COSMOLOGICAL IMPLICATIONS

Renormalization group applied to the vacuum energy – contributions from zero-point energy and condensates (Shapiro et al, 2003; Babić et al 2002) $\Rightarrow$

$$\rho_\Lambda(\mu) = c_0 + c_2\mu^2 + \mathcal{O}(\mu^4)$$

Neglecting $\mathcal{O}(\mu^4)$, the model can be solved completely.
This finishes the list of physical assumptions. The rest is mathematics.

The results (for $k = 0$):

$$c_0 = -\frac{\rho_{\Lambda 0}}{r_0}$$

$$c_2 = \frac{\kappa}{G_0}\frac{1 + r_0}{r_0}$$

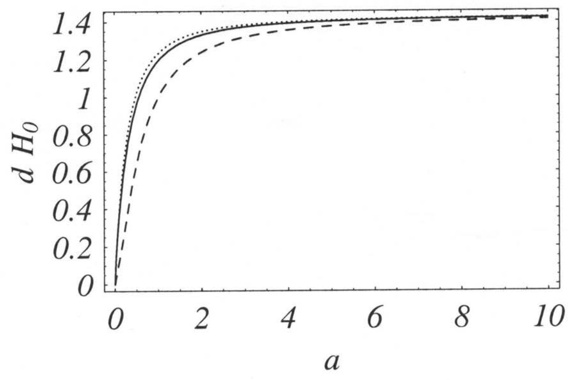
where $r_0 \equiv \rho_{m0}/\rho_{\Lambda 0}$.
The ratio of densities:

$$\frac{\rho_m}{\rho_\Lambda} = r_0 a^{-3/2}$$

The relation between the IR scale and the cosmological Hubble scale:

$$\mu = \sqrt{\frac{3}{8\pi\kappa(r_0+1)}}\sqrt{H_0 H}$$

We take $r_0 = 3/7$ and $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$.



solid - $\mu^{-1}\sqrt{3/8\pi\kappa}$
dotted - future event horizon $d = a\int_t^\infty dt/a$
dashed - Hubble distance

Time evolution of the Universe:

$$a(t) = r_0^{2/3}\left(e^{\frac{3\beta}{2}t} - 1\right)^{2/3}$$

where $\beta \equiv H_0/(1+r_0)$.
Asimptotically de Sitter: $a(t) \simeq r_0^{2/3} e^{\beta t}$
Age of the Universe: $t_0 = 16$ billion years.
Transition from deceleration to acceleration:
$\ddot{a} = 0$ at $z = a^{-1} - 1 = 1.793$ (experiment: $z < 0.72$)
Scaling of the gravitational "constant":

$$G_N = G_{N0}\frac{r_0 + a^{3/2}}{r_0 + 1}$$

Time dependence of G_N

$$\frac{\dot{G}_N}{G_N} = H_0 \frac{3}{2(1+r_0)}$$

- one order of magnitude larger than the experimental bound.

Conclusion

The presented simple cosmological model based on holographic dark energy has qualitative behaviour in agreement with observations. Perhaps better quantitative agreement might be obtained with matter that does not scale canonically.