INFLATIONARY MODELS

and a Running Spectral Index

Laura Covi CERN (& DESY)

based on work in collaboration with

David Lyth, Alessandro Melchiorri and Carolina Odman

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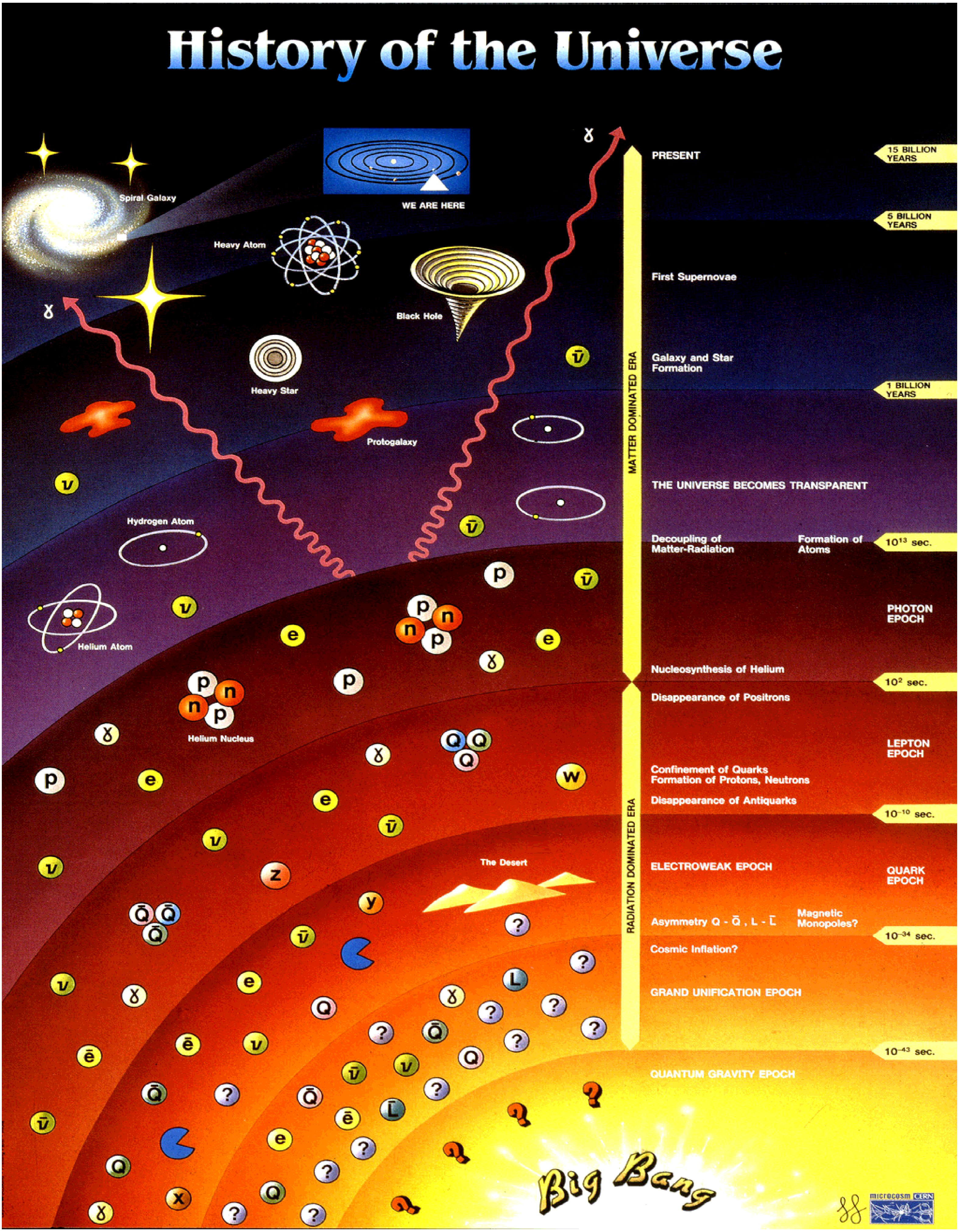
OUTLINE

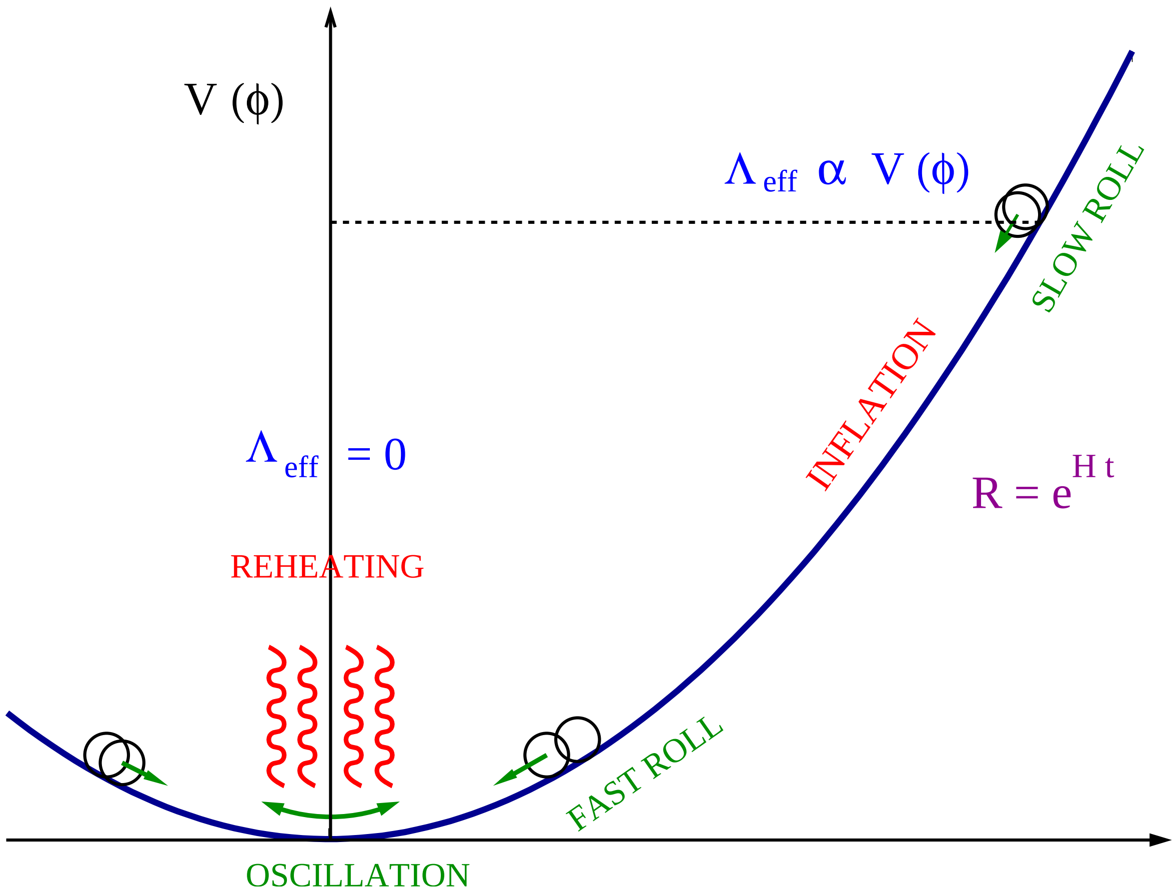
1. Introduction:
   - – single field inflationary models
   - – the primordial spectrum and its running
2. The running mass model(s):
   - – theoretical motivation
   - – characteristic predictions
3. Comparison with recent data:
   - – CMBR & LSS & Ly-α
   - – reionization constraints
4. Conclusions and Outlook


History of the Universe
Spiral Galaxy
WE ARE HERE
Heavy Atom
Black Hole
Heavy Star
Protogalaxy
Hydrogen Atom
Helium Atom
Helium Nucleus
The Desert
Big Bang
PRESENT
First Supernovae
Galaxy and Star Formation
THE UNIVERSE BECOMES TRANSPARENT
Decoupling of Matter-Radiation
Formation of Atoms
Nucleosynthesis of Helium
Disappearance of Positrons
Confinement of Quarks Formation of Protons, Neutrons
Disappearance of Antiquarks
ELECTROWEAK EPOCH
Asymmetry Q - Q̄ , L - L̄
Magnetic Monopoles?
Cosmic Inflation?
GRAND UNIFICATION EPOCH
QUANTUM GRAVITY EPOCH
MATTER DOMINATED ERA
RADIATION DOMINATED ERA
15 BILLION YEARS
5 BILLION YEARS
1 BILLION YEARS
10^{13} sec.
PHOTON EPOCH
10^{2} sec.
LEPTON EPOCH
10^{-10} sec.
QUARK EPOCH
10^{-34} sec.
10^{-43} sec.
microcosm CERN

INFLATION: period of quasi-exponential expansion, explaining the FLATNESS, ISOTROPY, HOMOGENEITY of the Universe, the absence of UNWANTED RELICS and producing the initial SMALL PERTURBATIONS.

How to sustain inflation ???

$\Rightarrow$ USE THE POTENTIAL ENERGY OF A SCALAR FIELD ϕ AS AN EFFECTIVE COSMOLOGICAL CONSTANT



$\Rightarrow$ The scalar field has to slow roll in an ALMOST FLAT POTENTIAL such that

$$\ddot{\phi} \ll 3H\dot{\phi} \;\Rightarrow\; 3H\dot{\phi} = -V'$$

$\Rightarrow$ slow roll expansion

(Single field) inflationary model $\Leftrightarrow$ Flat Potential $V(\phi)$

- SLOW ROLL:
$$\begin{cases} \epsilon &= \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \\ |\eta| &= M_P^2 \frac{|V''|}{V} \ll 1 \end{cases}$$

- Enough expansion:
$$\mathcal{N} = \int_{t_i}^{t_f} dtH = \int_{\phi_f}^{\phi_i} d\phi \frac{V(\phi)}{M_P^2 V'(\phi)} > 50$$

- Sufficient reheating before Big Bang Nucleosynthesis: $T_{rh} > 1$ MeV

Huge number of models in the literature:

old inflation, new inflation, chaotic inflation, hybrid inflation, inverted hybrid inflation, smooth inflation, topological inflation, ...

Is that ALL ?

NOT REALLY: the Universe that we see is not perfectly uniform and homogeneous, it presents a rich structure of galaxies, clusters of galaxies, etc.. etc... BUT such inhomogeneities were small in the past and an initial fluctuation of the order of 10^{-5} is sufficient to produce the present Large Scale Structure. So the question is:

Was the universe during inflation perfectly homogeneous ???

NO !

Quantum Mechanics limits the homogeneity !

Quantum fluctuation of the inflaton

The inflaton rolls classically towards the minimum of the potential...

... but ϕ is a quantum field and in a de Sitter background its quantum fluctuations are given by

$$\delta\phi \simeq \frac{H}{2\pi}$$

GAUSSIAN

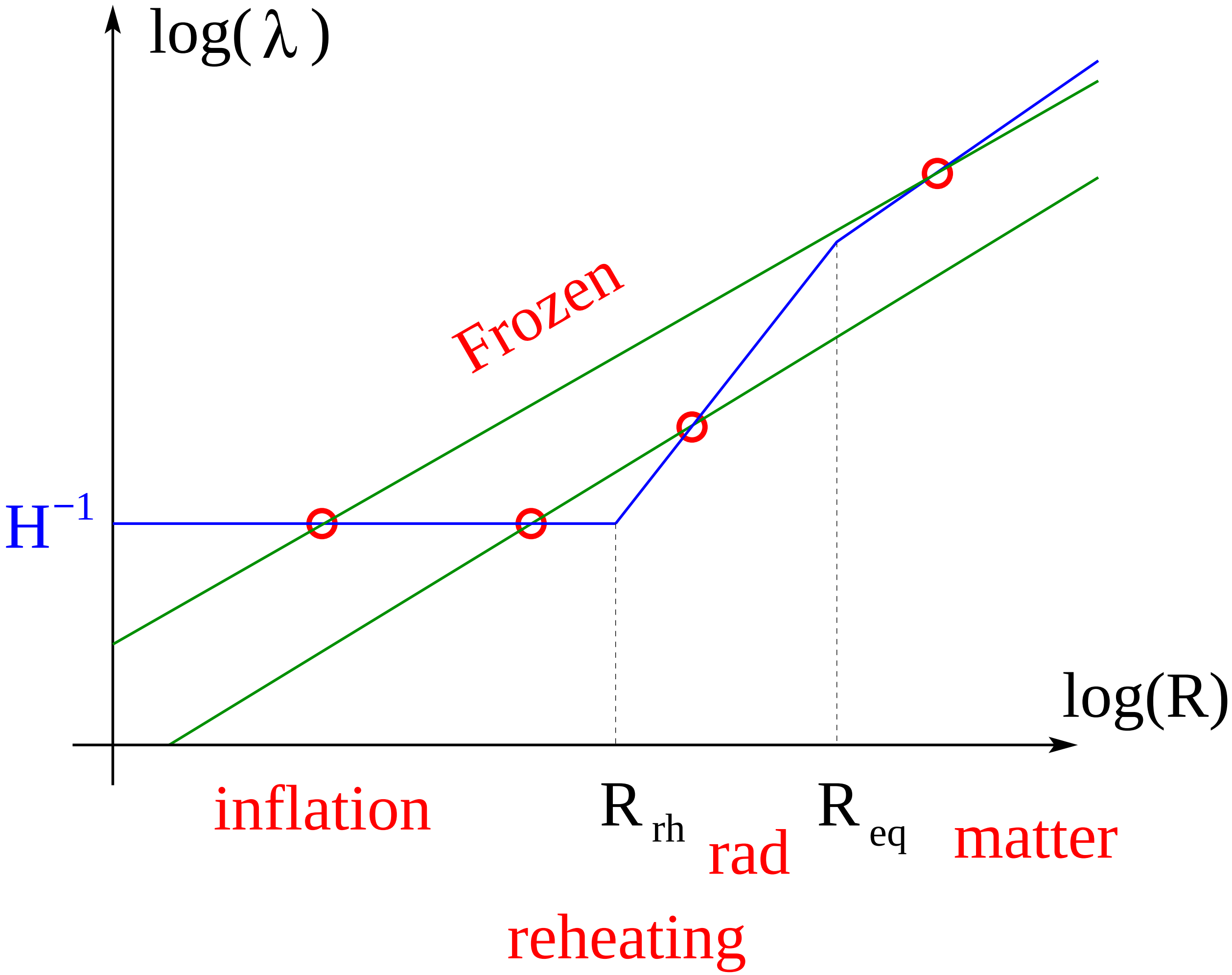
So the dynamics of the inflaton is slightly different in different parts of the universe and this

generates density fluctuations: $$\frac{\delta\rho}{\rho} \simeq H\delta t \simeq H\frac{\delta\phi}{\dot{\phi}} \simeq \frac{H^2}{2\pi\dot{\phi}}$$

$$\Downarrow$$

The amplitude of the density fluctuations at a scale λ is given by

$\frac{H^2}{2\pi\dot{\phi}}$ computed at the time when $\lambda = H^{-1}$ during inflation.



If H and $\dot{\phi}$ were exactly constant during inflation, all the wavelengths would have exactly the same amplitude

$\Rightarrow$ SCALE INVARIANT SPECTRUM

(nearly so in slow roll !!!)

In this limit, the power spectrum is determined by the inflaton potential:

$$\frac{H^2}{\dot{\phi}} \propto \frac{V^{3/2}}{V' M_P^3}$$

Initial condition for structure formation !

Testing inflation: Single field inflation $\Longleftrightarrow$ Flat Potential $V(\phi)$

The scalar power spectrum is given by

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{12\pi^2 M_P^6} \frac{V^3}{V'^2}\bigg|_{k=aH} \qquad \propto k^{n-1}$$

and its spectral index is:

$$n(k)-1 = \frac{d\log(\mathcal{P}_{\mathcal{R}})}{d\log(k)}\bigg|_{k=aH} = 2\eta - 6\epsilon + \ldots$$

The tensor power spectrum is given by

$$\mathcal{P}_{grav}(k) = \frac{1}{6\pi^2} \frac{V}{M_P^4}\bigg|_{k=aH}$$

and its spectral index is

$$n_{grav}(k) = \frac{d\log(\mathcal{P}_{grav})}{d\log(k)}\bigg|_{k=aH} = -2\epsilon + \ldots$$

where the slow roll parameters are

$$\epsilon = \frac{M_P^2}{16\pi}\frac{(V')^2}{V^2} \qquad \eta = \frac{M_P^2}{8\pi}\frac{V''}{V} \qquad \left(\text{and} \qquad \xi = \frac{M_P^4}{64\pi^2}\frac{V'V'''}{V^2}\right)$$

At first order in the slow roll expansion that is all, BUT n' arises at 2nd order in SLOW ROLL:

$$n'(k) = \frac{2}{3}\left((n-1)^2 - 4\eta^2\right) + 2\xi$$

so naively for a safe perturbative expansion, we expect $n' \propto (n-1)^2 < |n-1|$!!!

Breakdown of slow-roll expansion for large $|n'|$???

NO, $|n'|$ can still be larger than expected if ξ dominates.

Surprisingly WMAP seemed to require a large running...; possible for large ξ, but are there “natural” models giving it ? $\Rightarrow$ RUNNING MASS model !

Before discussing a particular model, a couple of general remarks:

- is $|n'| < |n-1|$ a requirement of slow roll ???

  Not really, slow roll can be a perfectly good approximation, even for "large" n'.

  E.g. a pathological potential

$$V(\phi) \propto \phi^m \quad \rightarrow \quad n - 1 = 2\eta - 6\epsilon = \left(2m(m-1) - 3m^2\right) \frac{M_P^2}{\phi^2}$$

  $\Rightarrow$ for $m = -2$ the first order vanishes exactly, but slow roll still holds for $\phi \gg M_P$!

  In this case both $n-1$ and n' are 2^{nd} order and are expected to be similar.

- at which scale k should the inequality hold ???

  If the potential changes curvature, $V''' \neq 0$, the spectral index can naturally cross 1 and there we must have $|n'| > |n-1| \simeq 0$!!! The only way to keep a scale dependent spectral index very close to 1, as required by the data, is to have $n-1$ change sign !

  Only the simple polynomial potentials give a fixed sign for $n-1$...

Running mass model: theoretical motivation

SUSY broken in inflation $\Longrightarrow$ SUGRA !

A model is defined by superpotential $W(\Phi)$ & Kähler potential $K(\Phi, \bar{\Phi})$

$$\mathcal{L} = K_{n^* m} \partial_\mu \bar{\Phi}^n \partial^\mu \Phi^m - V(\Phi, \bar{\Phi})$$

$$V(\Phi, \bar{\Phi}) = e^{K(\Phi, \bar{\Phi})} \left(\mathcal{F}_m K^{mn^*} \mathcal{F}_n^* - 3|W|^2 \right) + V_D$$

where $\mathcal{F}_m = \frac{\partial W}{\partial \Phi_m} + \frac{\partial K}{\partial \Phi_m} W(\phi)$, $\quad K_{n^* m} = \frac{\partial^2 K}{\partial \Phi_m \partial \bar{\Phi}_{n^*}}$ $\quad K^{mn^*} = \left(K^{-1}\right)^{mn^*}$

Take a canonical Kähler $K = \Phi_n \bar{\Phi}_n$ and we have

$$V = e^{|\Phi_n|^2} \left(\left| \frac{\partial W}{\partial \Phi_n} + \bar{\Phi}_n W \right|^2 - 3|W|^2 \right) + V_D$$

so that from the exponential one obtains

$$V'' = V + ... \quad \rightarrow \eta \simeq 1$$

η problem

NO SLOW ROLL POSSIBLE IN SUGRA ?!

There are a couple of ways out... ... one of them: the running mass !

[Stewart '96, '97]

The running mass model:

$\phi \longrightarrow$ flat direction of the SUSY potential $\quad V'_{SUSY}(\phi) = 0$

SUSY breaking generates a soft mass for ϕ: $\quad V(\phi) = V_0 + \frac{1}{2} m^2 \phi^2 + \ldots$ for $\phi < M_P$.

At tree level, for a generic scalar field one has naturally $|m^2| \simeq V_0 / M_P^2$ $\quad \eta$ problem !

$\rightarrow V(\phi)$ is NOT flat at high scale

But if the inflaton field interacts not so weakly, the one loop corrections to the potential give

$$m^2 \rightarrow m^2(Q = \phi) \quad \text{running mass}$$

The running of the mass can flatten the potential somewhere in the region $\phi \ll M_P$.

$\Downarrow$

Slow roll inflation

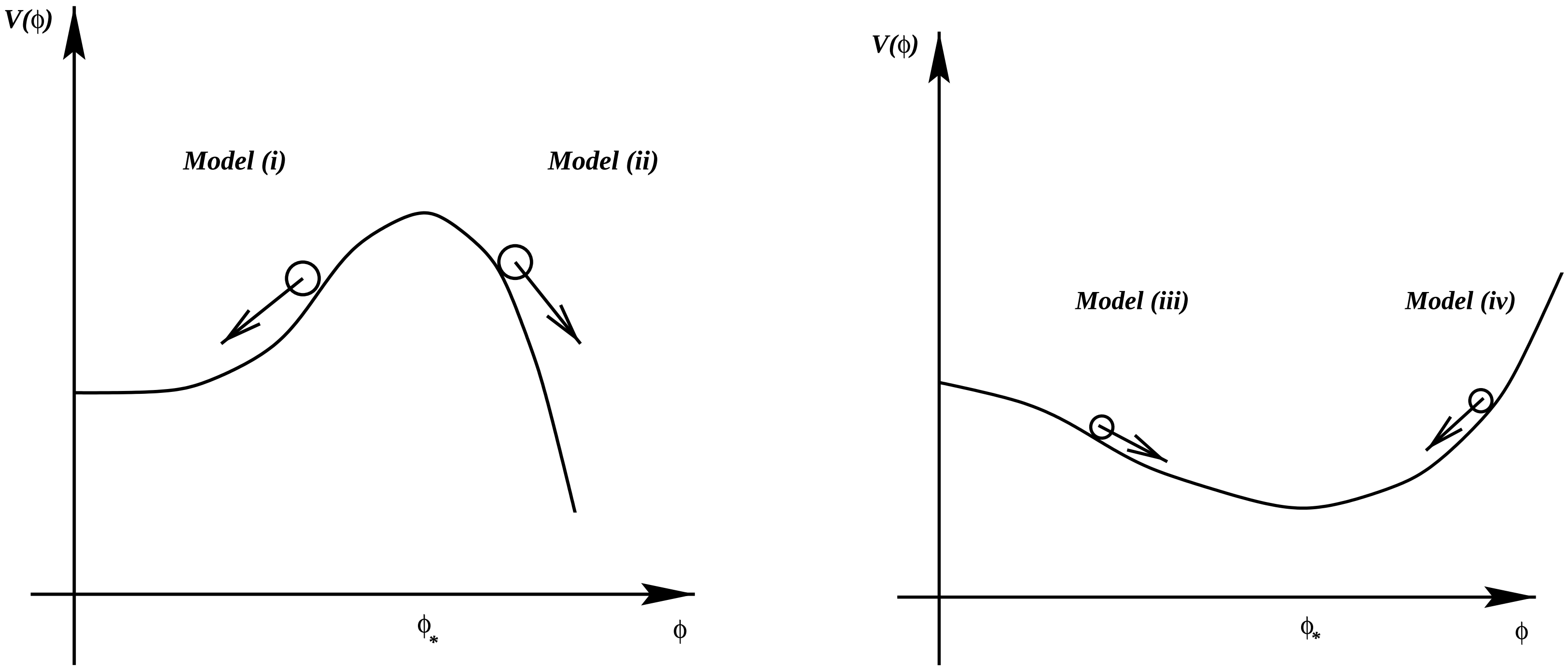
In general any type of coupling can be responsible for the inflaton's mass running:

$$\frac{dm^2}{d\log(Q)} = -\frac{2C}{\pi}\alpha\tilde{m}^2 + \frac{D}{16\pi^2}|\lambda|^2 m_s^2$$

gauge Yukawa

The inflaton has to couple sufficiently strongly, but still in the perturbative regime...

Different models exist depending on the sign of the running and the initial conditions:



What are the observable consequences of non-weakly coupled inflaton ???

$n(k) - 1 \ll 1$ on cosmological scales $\Rightarrow$ linear expansion around pivot ϕ_0 $(\leftrightarrow k_0)$

So expand the running mass around ϕ_0 as $m^2(\phi) \simeq m^2(\phi_0) + c * \log\left(\frac{\phi}{\phi_0}\right)$ Then we can write the potential as a function of two parameters s and c as

$$\frac{V}{3H_I^2 M_P^2} \simeq 1 + \frac{1}{2}\left(s + \frac{1}{2}c - c\log\left(\frac{\phi}{\phi_0}\right)\right)\frac{\phi^2}{M_P^2}$$

Note that s, c are related to physical parameters of the lagrangian of specific models rescaled by the inflationary Hubble scale H_I^2:

$$c \equiv -\frac{\beta_m(\phi_0)}{3H_I^2} \qquad\qquad s + \frac{1}{2}c \equiv \frac{m^2(\phi_0)}{3H_I^2}$$

NOTE: this is equivalent to stopping the perturbative expansion to one loop and neglecting the change of β_m... , but a higher order can be used to run from M_P down to the scale ϕ_0.

Connect to simple supersymmetric examples:

$\longrightarrow$ gauge coupling α dominance for ϕ in the adjoint representation of $SU(N)$

$$c = \frac{2N\alpha(M_P)}{\pi} \frac{\tilde{m}^2(M_P)}{3H_I^2} \frac{\alpha^3(\phi_0)}{\alpha^3(M_P)} \qquad \tilde{m} \quad \text{gaugino mass}$$

$$s = -\frac{c}{2} + \frac{m^2(M_P) - 2\tilde{m}^2(M_P)}{H_I^2} + \frac{2\tilde{m}^2(M_P)}{3H_I^2} \frac{\alpha^2(\phi_0)}{\alpha^2(M_P)} \qquad m \quad \text{inflaton mass}$$

$\longrightarrow$ Yukawa coupling λ dominance

$$c = -\frac{\lambda^2(M_P)}{12\pi^2} \left[\frac{1}{1 - \frac{3}{8\pi^2}\lambda^2(M_P)\log\left(\frac{\phi_0}{M_P}\right)} \right]^2 \quad \text{for} \quad m_{scalars} \simeq H_I$$

$$s = -\frac{c}{2} + \frac{2}{3}\left[\frac{1}{1 - \frac{3}{8\pi^2}\lambda^2(M_P)\log\left(\frac{\phi_0}{M_P}\right)} - \frac{1}{2} \right]$$

Result for the spectrum: a ”strongly” scale-dependent $n(k)$ and $\mathcal{P}_{\mathcal{R}}(k)$!

In fact the slow-roll parameters for this potential become:

$$\epsilon \simeq \frac{s^2\phi^2}{M_P^2} e^{2c\Delta N} \qquad \eta \simeq s e^{c\Delta N} - c \qquad \xi \simeq -cs e^{c\Delta N}$$

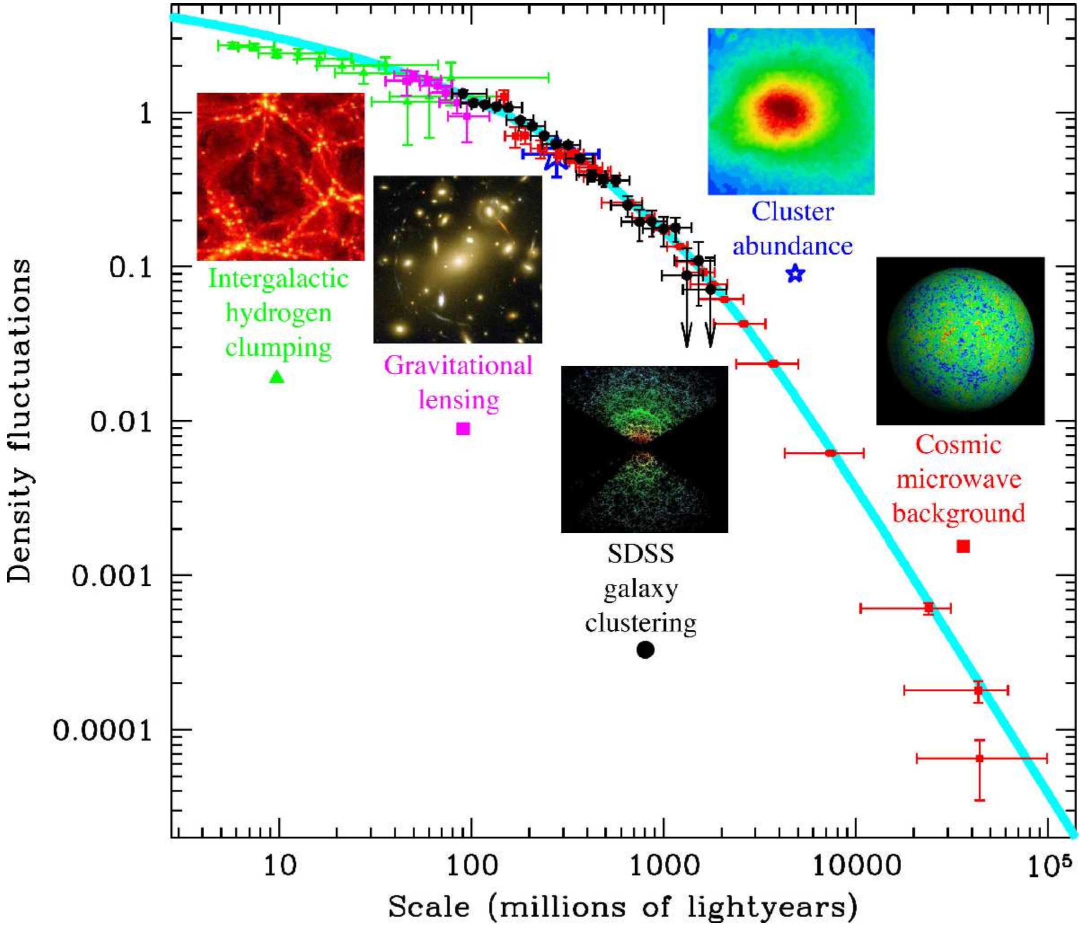
where $\Delta N = \log\left(\frac{k}{k_0}\right) = \frac{1}{c}\log\left[1 - \frac{c}{s}\log\left[\frac{\phi}{\phi_0}\right]\right].$

c suppressed by a coupling, s also to have slow roll... Then we have for the spectral index

$$\frac{n(k)-1}{2} = s\left(\frac{k}{k_0}\right)^c - c \quad \text{and} \quad n'(k) = 2sc\left(\frac{k}{k_0}\right)^c \rightarrow \xi!$$

“Strong (exponential !)” scale dependence !!

Look for such strong scale dependence in the data, trying to extend the lever arm as far as possible:



[Figure by M. Tegmark]

- CMB: first year WMAP data

  astro-ph/0302207

- LSS: Sloan Digital Sky Survey results for the galaxy power spectrum

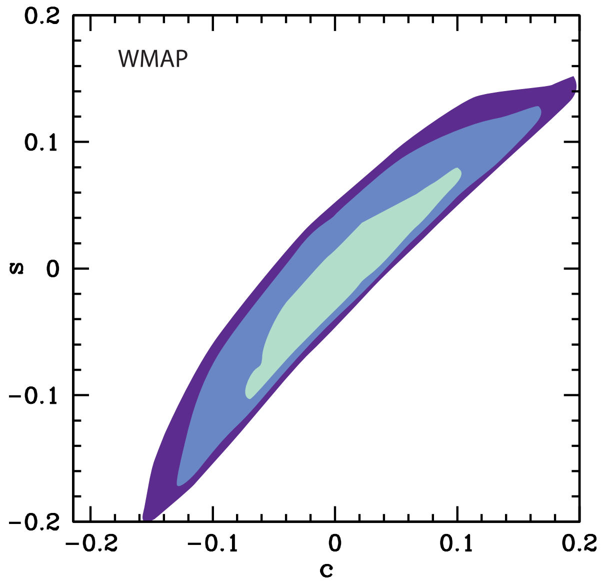
  astro-ph/0310723

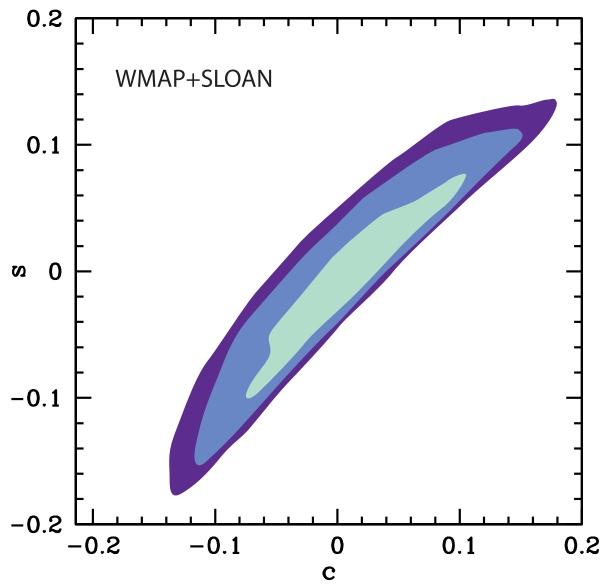
- LSS: Sloan Digital Sky Survey results on Lyman-α

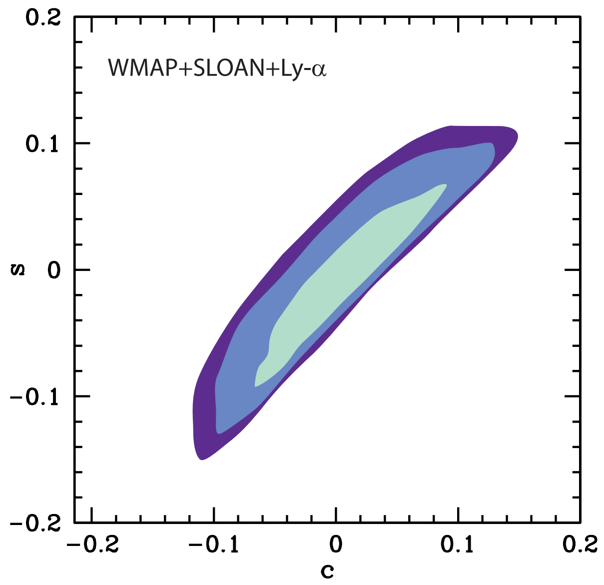
  astro-ph/0405013 & 0407372

What are the constraint from the new data for s, c in such models ?

[LC, Lyth, Melchiorri & Odman astro-ph/0408129]

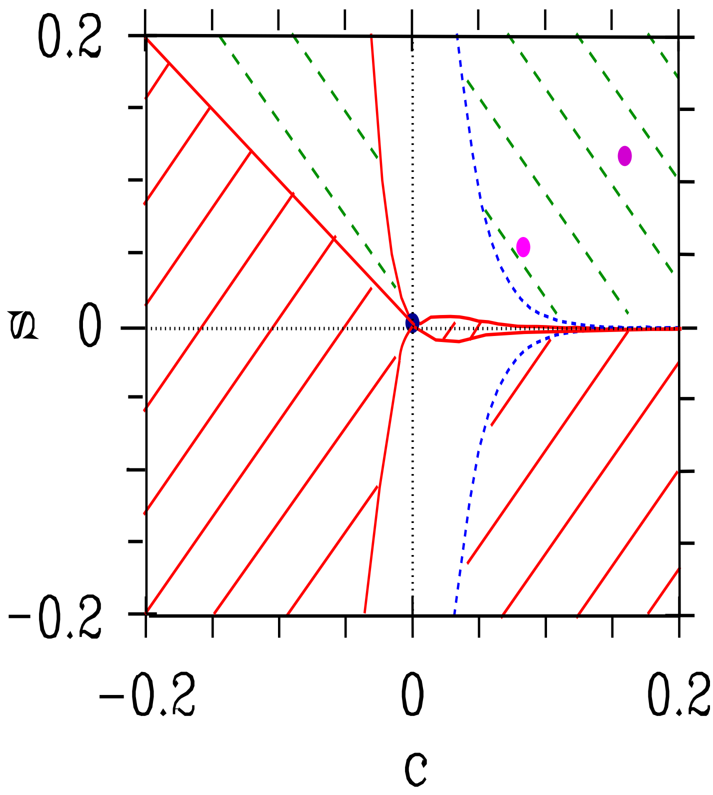






WMAP strongly constrains along the direction $s = c$, i.e. $n(k_0) - 1 = 0$

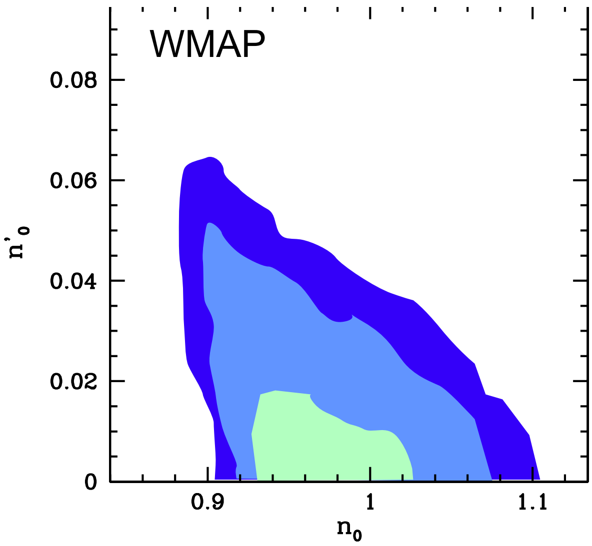
Theoretically expected region

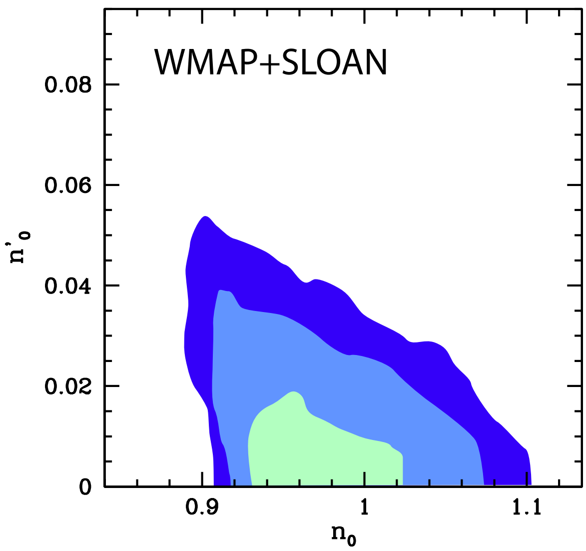


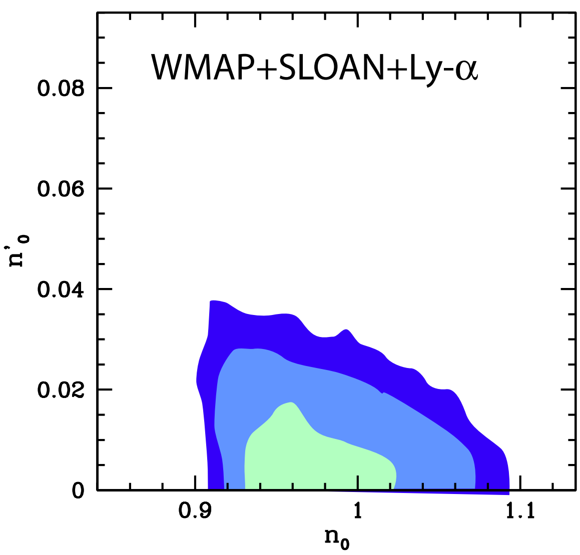
Ly–α data tighten the bound on scale dependence and require

$$|c| \leq 0.12$$

Look at the constraints in the n_0' vs n_0 plane instead







NOTE: negative n_0' is allowed by the model only for

$$n_0' \leq -\frac{(n_0-1)^2}{4}$$

due to the dependence on s, c.

The rest of the parameter space is unphysical !

Fitting for arbitrary n_0, n_0' is not equivalent as fitting for the running mass model !

Again the most stringent bound on n_0' comes from Ly–α data giving

$$n_0' \leq 0.2$$

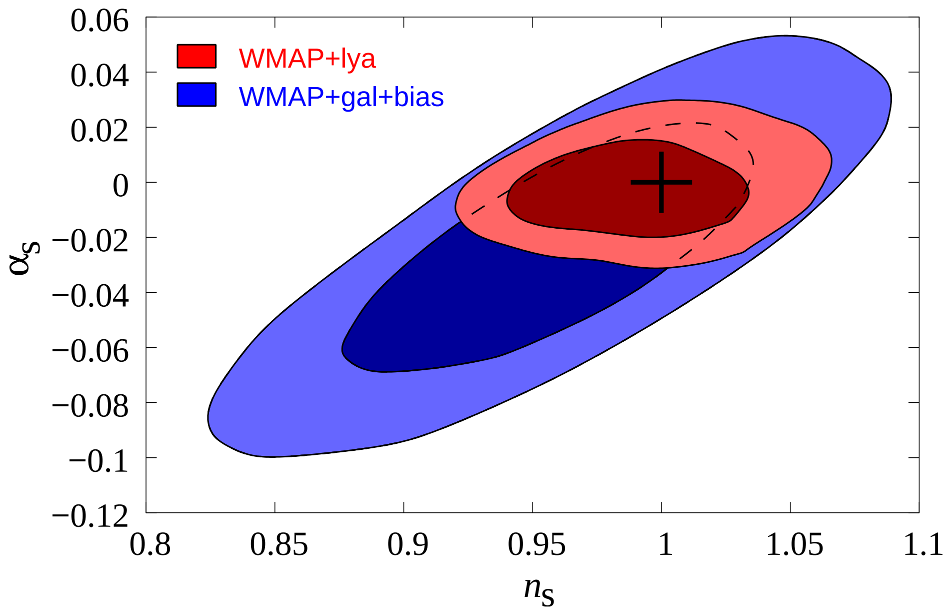
Compare the result with the fit for a general Taylor expansion: $n(k) = n_s + \alpha_s \log\left(\frac{k}{k_0}\right)$.

Using the same data Seljak et al. (astro-ph/0407372) find, contrary to WMAP,

$$n_s = n_0 = 0.977^{+0.025}_{-0.021}$$

$$\alpha_s = n_0' = -0.003 \pm 0.010$$

NO RUNNING !

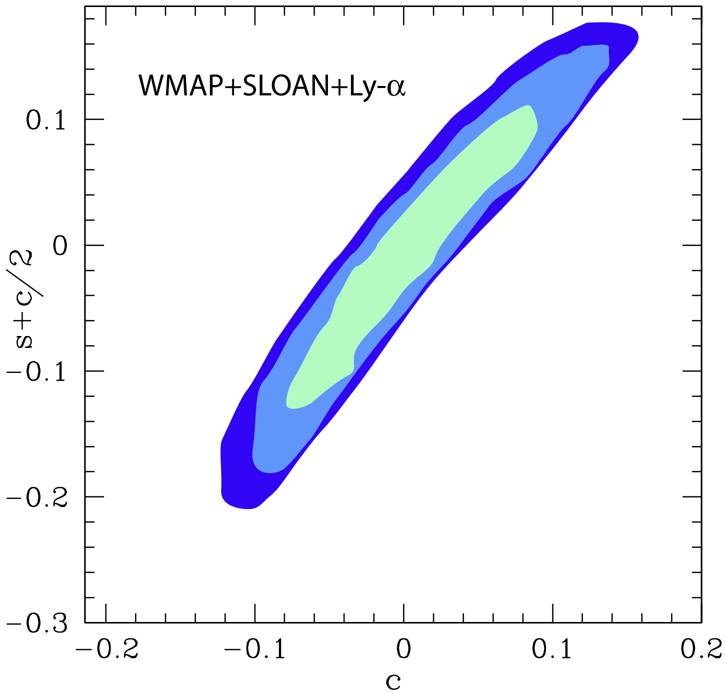


That is fully compatible with our result; the data cannot yet distinguish between the different parameterizations ! In fact $\alpha_s \leq 0.2$ is similar to our result...

NOTE: the blue data "like" a spectral index crossing unity at a scale $\log\left(\frac{k}{k_0}\right) \simeq -\frac{n-1}{\alpha_s}$.

What are the bounds on the “physical parameters” ?

$\frac{m^2(\phi_0)}{3H_I^2}$



$c \propto \frac{m'^2(\phi_0)}{3H_I^2}$

Strong running, i.e. large inflationary scale, is disfavoured... From the WMAP normalization

$$H_I = 2\pi \mathcal{P}_{\mathcal{R}}^{1/2} |\phi_0||s| \sim 3 \times 10^{-4} |\phi_0||s|$$

and assuming linear running from M_P

$$m^2(\phi_0) \simeq 0 \to \frac{\phi_0}{M_P} \sim \exp\left(-\frac{1}{|c|}\frac{|m^2(M_P)|}{3H_I^2}\right)$$

i.e. $|c| \leq 0.1$ and $|m^2| = 3H_I^2$

$$\hookrightarrow \phi_0 \leq 10^{-5} M_P.$$

Small $|c|$ implies $\phi_0 \ll M_P$ and therefore also $H_I \ll \phi_0 \ll M_P$ ... H_I highly sensitive to c !

Another hint for a running index: REIONIZATION..... ???

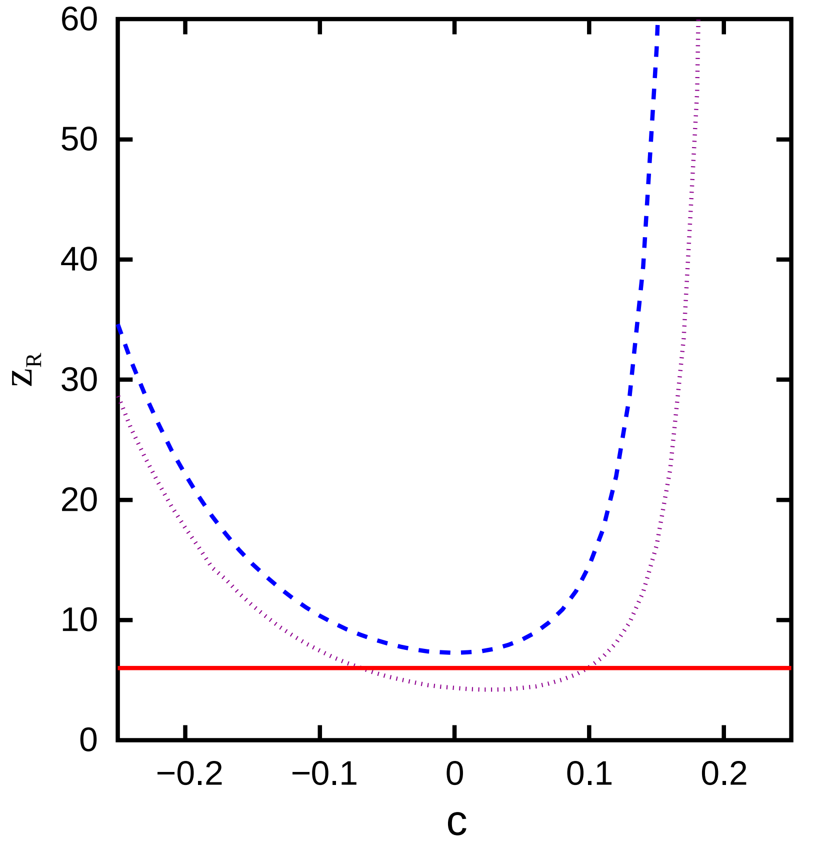
Estimate the reionization epoch z_R using the Press-Schechter formula as the epoch of collapse of a fraction f of matter into objects of mass $10^6 M_\odot$:

$$1 + z_R \simeq \frac{\sqrt{2}\sigma(10^6 M_\odot)}{1.7 g(\Omega_M)} \,\mathrm{erfc}^{-1}(f)$$

where σ is the present linear rms density contrast computed from the primordial spectrum and the CDM transfer function and $g(\Omega_M)$ accounts for the suppression of the growth when $\Omega_M < 1$.

There is a strong correlation between c and z_R:
z_R grows cery quickly for large c

s has been fixed to c and $c - 0.05$.
The red line corresponds to $z_R = 6$.



Conclusions and Outlook

- The simple (single field) inflationary paradigm is very successful in describing present observations. Unfortunately it is still not clear which model of the many proposed is favoured..., therefore let us try to look for specific observational signatures, e.g. investigate the scale-dependence of the spectral index !

- The running mass model is very well-motivated within particle physics and has a very characteristic expression for the spectral index.

- Present data allow still a relatively strong scale dependence and cannot yet exclude this type of models. MORE DATA are expected soon (WMAP...?!?) and then we will know more !