2nd Vienna Central European Seminar on Particle Physics and Quantum Field Theory

Thoughts on Dark Energy
Acceleration without Dark Energy

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All work is the result of collaborations with
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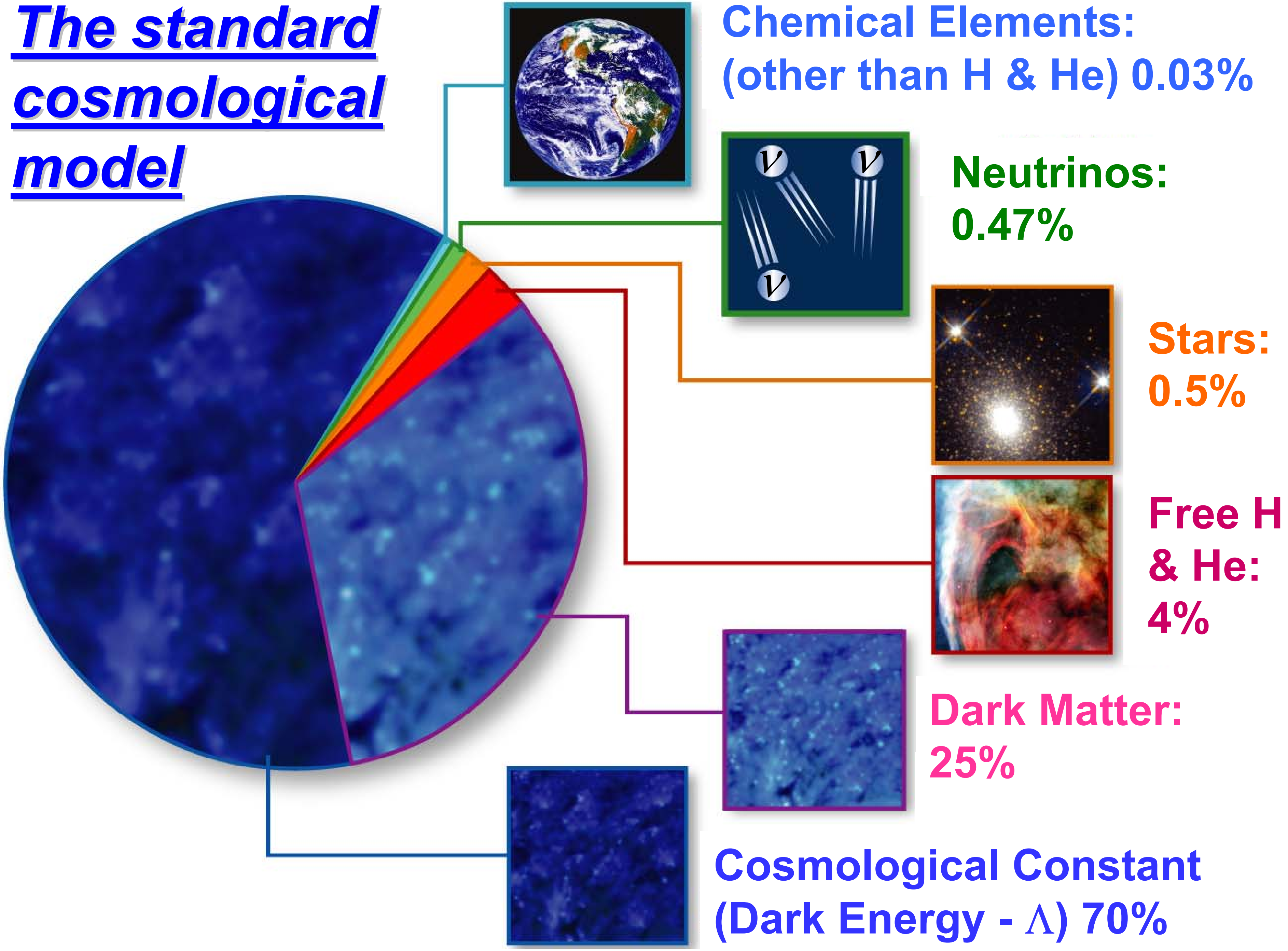
[and occasionally Alessio Notari (McGill)]

The standard cosmological model



Dark matter?

- neutrinos (hot dark matter)
- sterile neutrinos, gravitinos (warm dark matter)
- LSP (neutralino, axino, …) (cold dark matter)
- LKP (lightest Kaluza-Klein particle)
- axions, axion clusters
- solitons (Q-balls; B-balls; Odd-balls, Screw-balls….)
- supermassive wimpzillas

Mass range

$10^{-6}\,\mathrm{eV}\ \ (10^{-40}\,\mathrm{g})$ axions

$10^{-8}\,M_{\odot}\ \ (10^{25}\,\mathrm{g})$ axion clusters

Interaction strength range

Noninteracting: wimpzillas

Strongly interacting: B balls

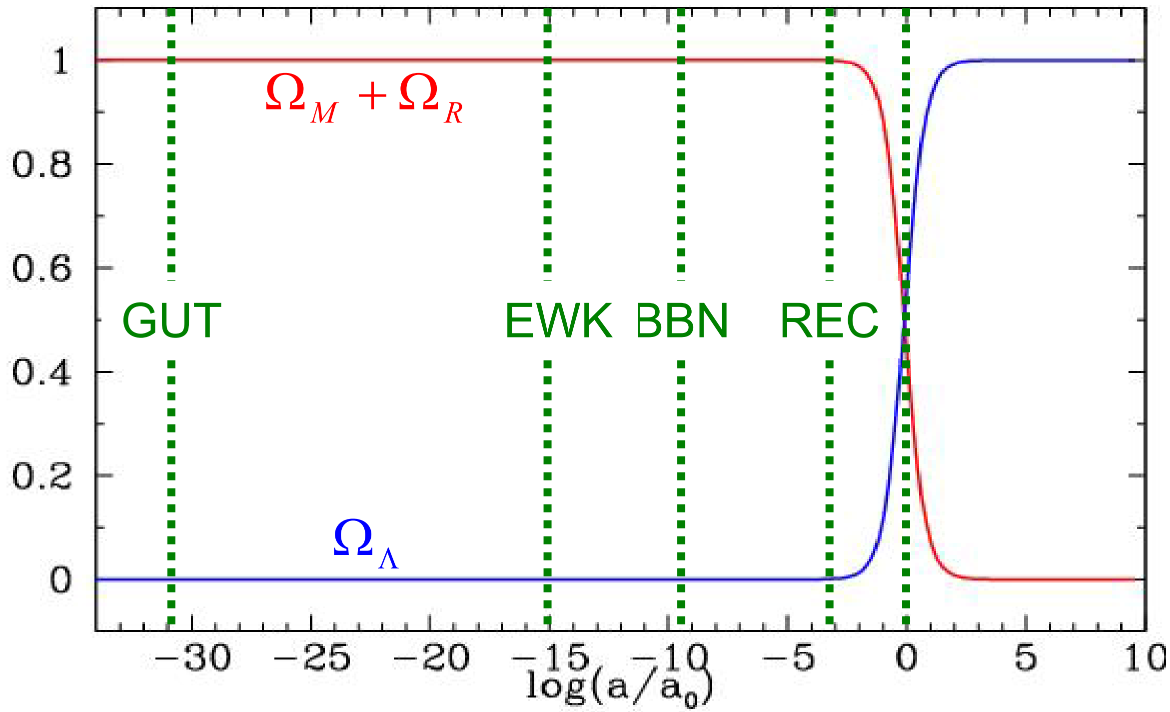
Cosmo-illogical constant?

Illogical magnitude (what's it related to?):

$$\rho_\Lambda \simeq 10^{-30}\,\mathrm{g\ cm^{-3}} \simeq \left(10^{-4}\ \mathrm{eV}\right)^4 \simeq \left(10^{-3}\ \mathrm{cm}\right)^{-4}$$

$$\Lambda = 8\pi G\rho_\Lambda \simeq \left(10^{29}\ \mathrm{cm}\right)^{-2} \simeq \left(10^{-33}\,\mathrm{eV}\right)^2$$

Illogical timing (why now?):



Friedmann-Lemâitre-Robertson-Walker (homogeneous/isotropic) model

RW metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \qquad H \equiv \frac{\dot{a}}{a}$$

expansion rate

deceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad q \equiv -\frac{\ddot{a}}{a}\frac{1}{H^2}$$

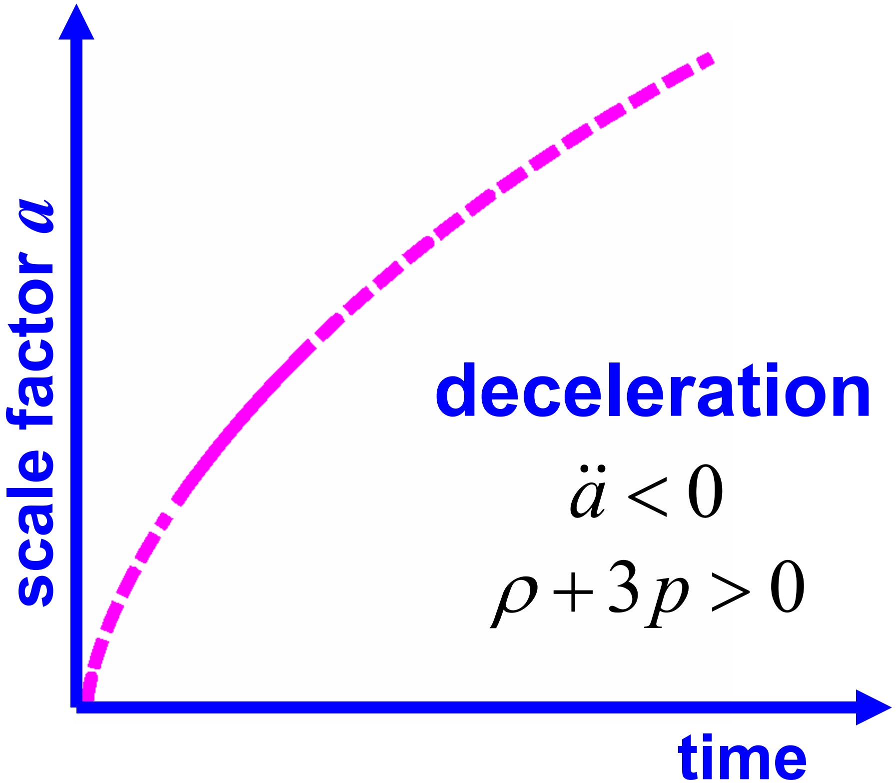
deceleration parameter

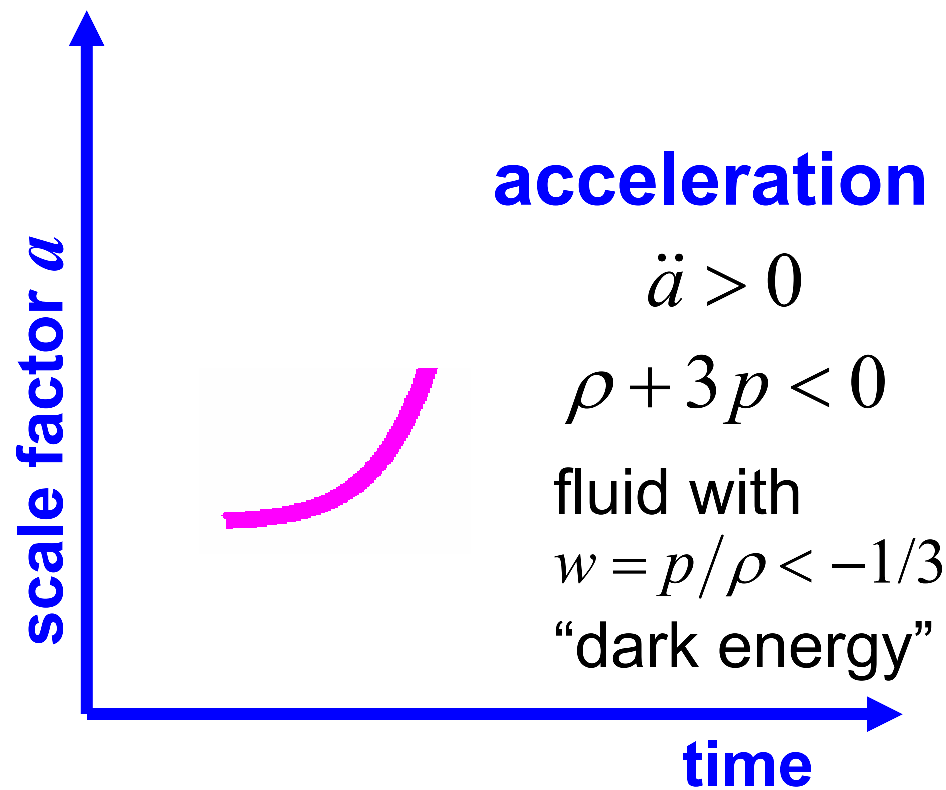
conservation of stress energy

$$T^{\mu\nu}{}_{;\nu} = 0 \qquad p = w\rho \qquad \rho \propto a^{-3(1+w)}$$

Matter:	$p_M = 0$	$w = 0$	$\rho_M \propto a^{-3}$
Radiation:	$p_R = \rho_R/3$	$w = 1/3$	$\rho_R \propto a^{-4}$
Cosmological constant:	$p_\Lambda = -1$	$w = -1$	$\rho_\Lambda \propto a^0$

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)$$





Do we "know" there is dark energy?

- Assume model cosmology:
  - Friedmann model: $H^2 + k/a^2 = 8\pi G\rho / 3$
  - Energy (and pressure) content: $\rho = \rho_M + \rho_R + \rho_\Lambda + \ldots$
  - Input or integrate over cosmological parameters: H_0, *etc*.
- Calculate observables $d_L(z)$, $d_A(z)$, $\ldots$
- Compare to observations
- Model cosmology fits with ρ_Λ, but not without ρ_Λ
- All evidence for dark energy is *indirect*: observed $H(z)$ is *not* described by $H(z)$ calculated from the Einstein-de Sitter model

Evolution of *H(z)* is a key quantity

Robertson–Walker metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

Many observables based on the coordinate distance *r(z)*

$$\int_0^{r(z)} \frac{dr'}{\sqrt{1-kr'^2}} = \int_0^t \frac{dt'}{a(t')} = \int_0^z \frac{dz'}{H(z')}$$

- Luminosity distance

  Flux = (Luminosity / $4\pi d_L^2$)

$$d_L(z) \propto r(z)(1+z)$$

- Angular diameter distance

  Angular diameter = (Physical size / d_A)

$$d_A(z) \propto \frac{r(z)}{(1+z)}$$

- Comoving number counts

  $N \propto V^{-1}(z)$

$$\frac{dV(z)}{dz\, d\Omega} \propto \frac{r^2(z)}{H(z)}$$

- Age of the universe

$$t(z) \propto \int_0^z \frac{dz'}{(1+z')H(z')}$$

Take sides!

- Can't hide from the data – ΛCDM too good to ignore
  - SNIa
  - Subtraction: $1.0 - 0.3 = 0.7$
  - Age
  - Large-scale structure
  - …

  } $H(z)$ not given by Einstein–de Sitter

$$3H^2 \neq 8\pi G\, \rho_{\mathrm{MATTER}}$$

- Dark energy (modify right-hand side of Einstein equations)
  - "Just" Λ, a cosmological constant?
  - If not constant, what drives dynamics (scalar field?)
- Gravity (modify left-hand side of Einstein equations)
  - Beyond Einstein (non-GR: branes, *etc.*)?
  - (Just) Einstein (GR: Back reaction of inhomogeneities)?

Modifying the left-hand side

- Braneworld modifies Friedmann equation — Binetruy, Deffayet, Langlois
- Friedmann equation modified today — Freese & Lewis

  $$H^2 = A\rho\left[1+\left(\rho/\rho_{\text{cutoff}}\right)^{n-1}\right]$$

- Gravitational force law modified at large distance — Deffayet, Dvali & Gabadadze

  Five-dimensional at cosmic distances

- Tired gravitons — Gregory, Rubakov & Sibiryakov; Dvali, Gabadadze & Porrati

  Gravitons metastable - leak into bulk

- Gravity repulsive at distance $R \approx \text{Gpc}$ — Csaki, Erlich, Hollowood & Terning
- *n=1* KK graviton mode very light, $m \approx (\text{Gpc})^{-1}$ — Kogan, Mouslopoulos, Papazoglou, Ross & Santiago
- Einstein & Hilbert got it wrong — Carroll, Duvvuri, Turner, Trodden

  $$S = (16\pi G)^{-1}\int d^4x\sqrt{-g}\left(R-\mu^4/R\right)$$

- Backreaction of inhomogeneities — Räsänen; Kolb, Matarrese, Notari & Riotto; Notari; Kolb, Matarrese & Riotto

Braneless cosmology

Old Friedmann law:

$$G_{00} = M_{Pl}^{-2}\, T_{00}$$

$$3H^2 = M_{Pl}^{-2}\rho$$

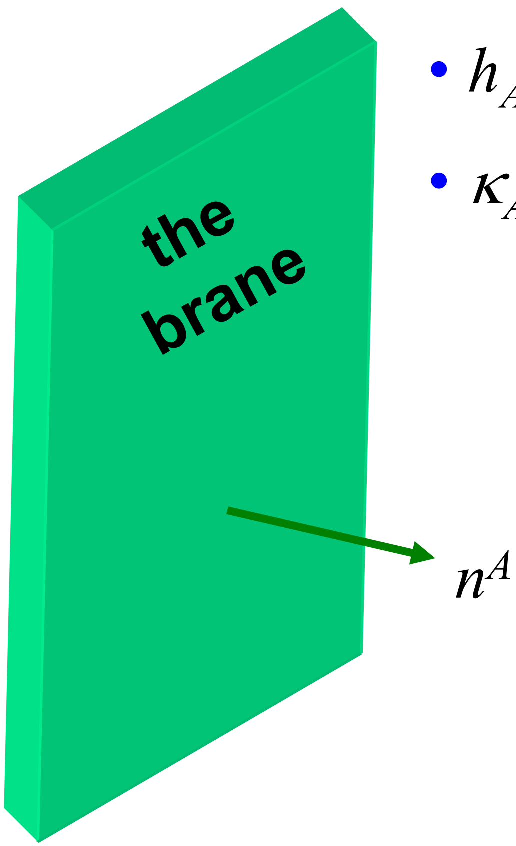
Friedmann (1921)

SNIa evidence for dark energy:

$$\int \frac{dz}{H(z)}$$

Brane cosmology

- Israel junction condition (Israel 1966)



- n_A unit vector normal to the brane
- $h_{AB} = g_{AB} - n_A n_B$ the induced metric
- $\kappa_{AB} = h_A{}^C \nabla_C n_B$ the extrinsic curvature

$$[\kappa_{\mu\nu}] = -M_*^{-3} \, T_{\mu\nu}{}^{\mathrm{BRANE}}$$

$[\ldots] =$ discontinuity across the brane

$$a'' = \langle \, a'' \, \rangle + [\, a' \,] \, \delta(y)$$

discontinuity in 2nd derivative of scale factor

Braneless cosmology

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$$\int \frac{dz}{H(z)}$$

Braneful cosmology

New Friedmann law:

Israel jump conditions

Binetruy, Deffayet, Langlois (2000)

$$3H^2 = \frac{\Lambda}{2} + \frac{M_*^{-6}}{12}\rho^2 + \frac{c}{a^4(t, y=0)}$$

Brane Cosmology

- New Friedmann law Binetruy, Deffayet, Langlois (2000)

$$3H^2 = \frac{\Lambda}{2} + \frac{M_*^{-6}}{12}\rho^2 + \frac{c}{a^4(t, y=0)}$$

- Possible solution Randall & Sundrum (2000)

Introduce a tension σ on the brane $\rho \to \rho + \sigma$

$$3H^2 = \left(\frac{\Lambda}{2} + \frac{M_*^{-6}}{12}\sigma^2\right) + \frac{M_*^{-6}}{6}\sigma\rho + \frac{M_*^{-6}}{12}\rho^2 + \frac{c}{a^4(t, y=0)}$$

cosmological constant (cancels?)

$$\frac{M_*^{-6}}{18}\sigma = \frac{8\pi G}{3}$$

Friedmann equation

unconventional corrections

Acceleration from inhomogeneities

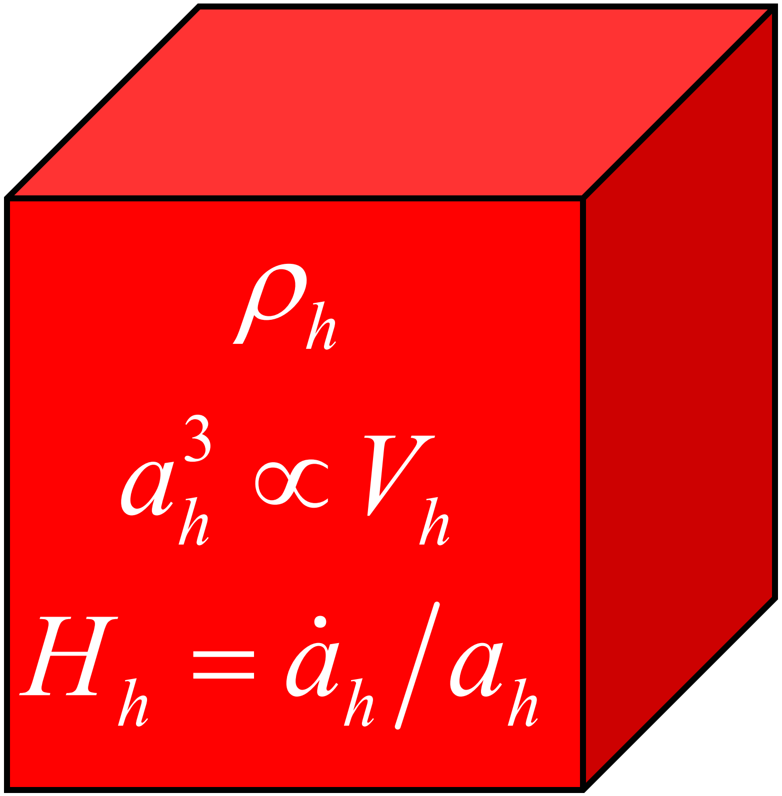
- Most conservative approach — nothing new
  - no new fields (like $10^{-33}\,\mathrm{eV}$ mass scalars)
  - no extra long-range forces
  - no modification of general relativity
  - no modification of Newtonian gravity at large distances
  - no Lorentz violation
  - no extra dimensions, bulks, branes, etc.
  - no faith-based (anthropic) reasoning

- Magnitude?: calculable from observables related to $\delta\rho/\rho$

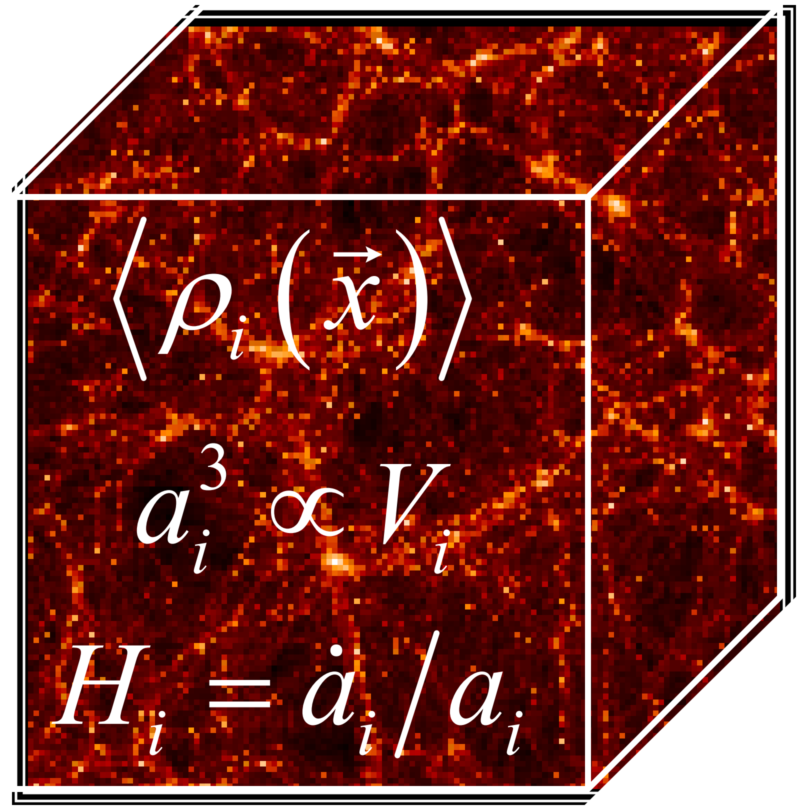
- Why now?: acceleration triggered by era of non-linear structure

Acceleration from inhomogeneities

Homogeneous model

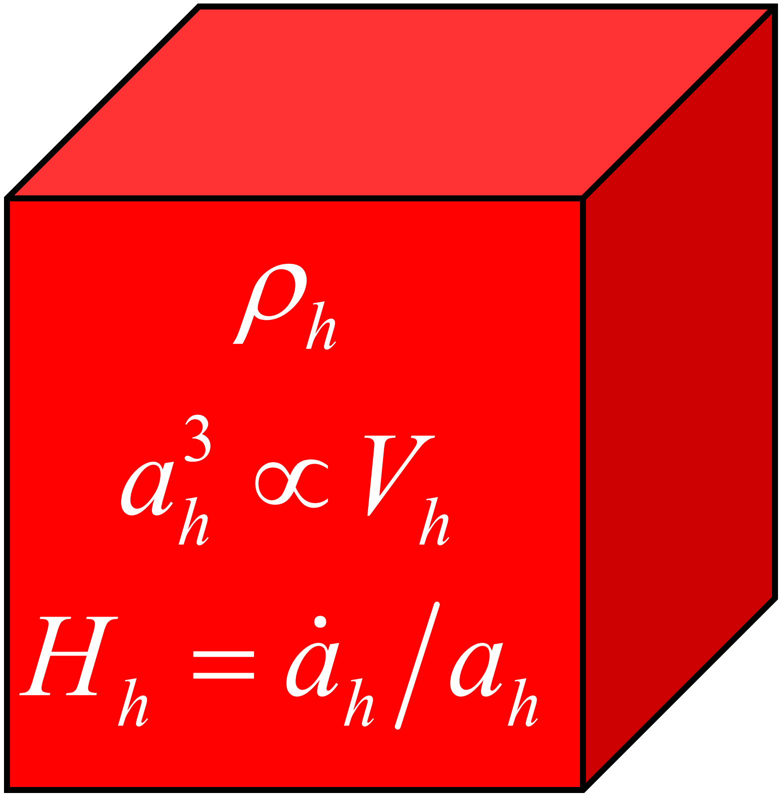


Inhomogeneous model

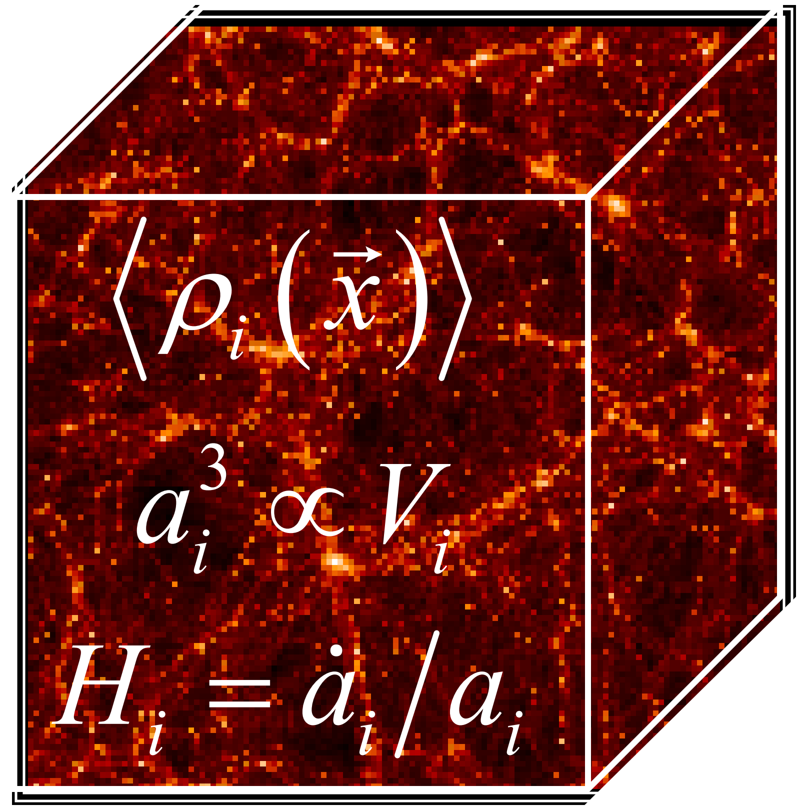


Acceleration from inhomogeneities

Homogeneous model



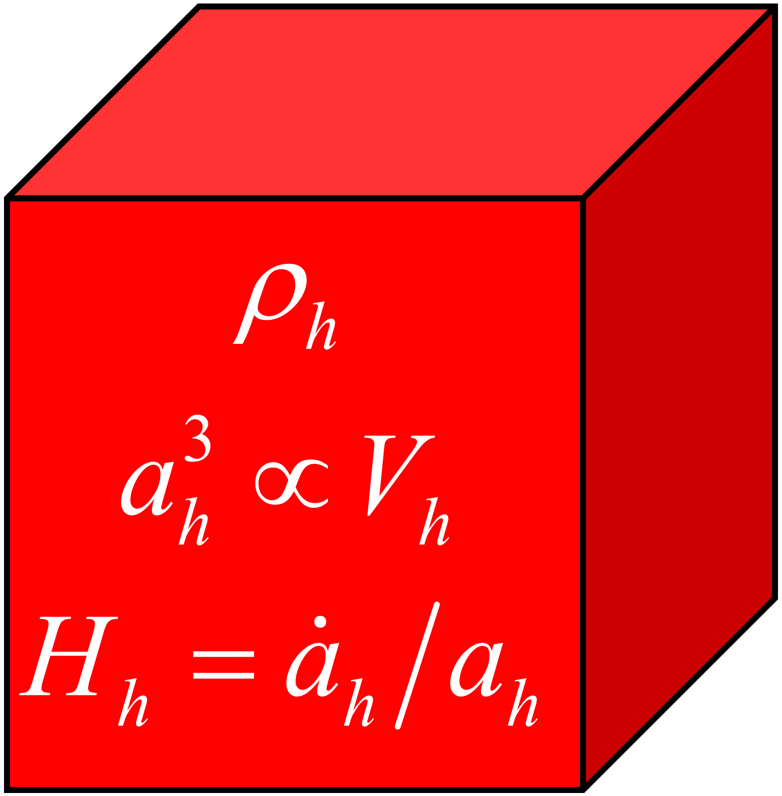
Inhomogeneous model



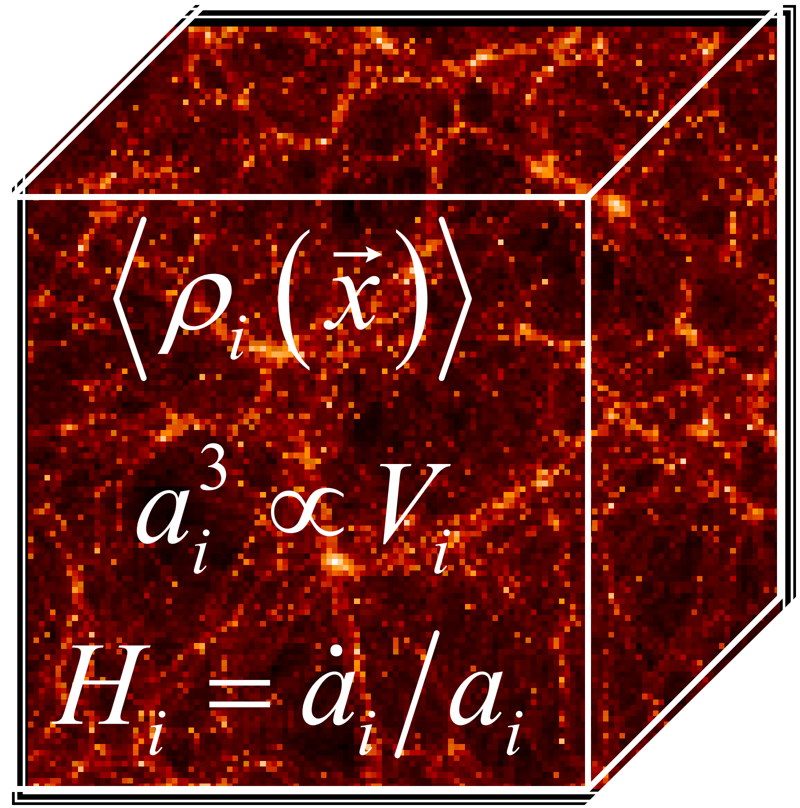
$$\rho_h = \langle \rho_i(\vec{x}) \rangle \Rightarrow H_h = H_i \ ?$$

Acceleration from inhomogeneities

Homogeneous model



Inhomogeneous model



$$\rho_h = \langle \rho_i(\vec{x}) \rangle \Rightarrow H_h = H_i \ ?$$

We think not!

Acceleration from inhomogeneities

- View scale factor as zero-momentum mode of gravitational field
- In homogeneous/isotropic model it is the only degree of freedom
- Inhomogeneities: non-zero modes of gravitational field
- Non-zero modes interact with and modify zero-momentum mode

Acceleration from inhomogeneities

- View scale factor as zero-momentum mode of gravitational field
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Cosmology $\leftrightarrow$ scalar field theory analogue

	cosmology	scalar-field theory
zero-mode	a	$\langle\phi\rangle$ (vev of a scalar field)
non-zero modes	inhomogeneities	thermal/finite-density bkgd.
physical effect	modify $a(t)$ e.g., acceleration	modify $\langle\phi(t)\rangle$ e.g., phase transitions

Acceleration from inhomogeneities

- We operate under assumption that observables (d_A, d_L, z, etc.) are modified if effective scale factor is modified.

- We can only show this for unrealistic models.

- We must assume that there will be no (or little) anisotropy (shear).

Different approaches

Standard approach	Our approach
• Model an inhomogeneous Universe as a homogeneous Universe model with $\rho = \langle\rho\rangle$	• Expansion rate of an inhomogeneous Universe $\neq$ expansion rate of homogeneous Universe with $\rho = \langle\rho\rangle$
• Zero mode [$a(t)$] is zero mode of homogeneous model with $\rho = \langle\rho\rangle$	• Inhomogeneities modify zero-mode [effective scale factor is $a_D \equiv V_D^{1/3}$]
• Inhomogeneities only have a local effect on observables	• Effective scale factor has a (global) effect on observables
• Cannot account for observed acceleration	• Potentially can account for acceleration without dark energy or modified GR

Acceleration from inhomogeneities

- We *do not* use super-Hubble modes for acceleration.
- We do not depend on large gravitational potentials such as black holes and neutron stars.
- We assert that the back reaction should be calculated in a frame comoving with the matter—other frames can give spurious results.
- We demonstrate large corrections in the gradient expansion, but the gradient expansion technique can not be used for the final answer—so we have indications (not proof) of a large effect.
- The basic idea is that small-scale inhomogeneities "renormalize" the large-scale properties.

Inhomogeneities–cosmology

- Our Universe is inhomogeneous
- Can define an average density $\langle \rho \rangle$
- The expansion rate of an *inhomogeneous* universe of average density $\langle \rho \rangle$ is *NOT!* the same as the expansion rate of a *homogeneous* universe of average density $\langle \rho \rangle$!
- Difference is a new term that enters an effective Friedmann equation — the new term need not satisfy energy conditions!
- We deduce dark energy because we are comparing to the wrong model universe (*i.e.,* a homogeneous/isotropic model)

Inhomogeneities–example

Kolb, Matarrese, Notari & Riotto

- Perturbed Friedmann–Lemâitre–Robertson–Walker model:

$$G_{\mu\nu}\left(\vec{x},t\right) = G_{\mu\nu}^{\mathrm{FLRW}}\left(t\right) + \delta G_{\mu\nu}\left(\vec{x},t\right)$$

$$G_{00}^{\mathrm{FLRW}}\left(t\right) + \delta G_{00}\left(\vec{x},t\right) = 8\pi G T_{00}\left(\vec{x},t\right)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\langle\rho\rangle - \frac{3}{8\pi G}\langle\delta G_{00}\rangle\right]$$

- $(\dot{a}/a)^2$ *is not* $8\pi G\,\langle\rho\rangle/3$
- ($\dot{a}/a$ *is not even the expansion rate*)
- Could $\langle\delta G_{00}\rangle$ play the role of dark energy?

Inhomogeneities–cosmology

- For a general fluid, four velocity $u^\mu = (1,\vec{0})$
  (local observer comoving with energy flow)

- For irrotational dust, work in synchronous and comoving gauge
  $$ds^2 = -dt^2 + h_{ij}(\vec{x},t)dx^i dx^j$$

- Velocity gradient tensor
  $$\Theta^i{}_j = u^i{}_{;j} = \tfrac{1}{2}h^{ik}\dot{h}_{kj} = \Theta\delta^i{}_j + \sigma^i{}_j \qquad (\sigma^i{}_j \text{ is traceless})$$

- Θ is the volume-expansion factor and $\sigma^i{}_j$ is the shear
  (shear will have to be small)

- For flat FLRW, $h_{ij}(t) = a^2(t)\delta_{ij}$
  $\Theta = 3H$ and $\sigma^i{}_j = 0$

Inhomogeneities and acceleration

- Local deceleration parameter positive: Hirata & Seljak; Flanagan; Giovannini; Alnes, Amarzguioui & Gron

$$q = -\frac{\left(3\dot{\Theta} + \Theta^2\right)}{\Theta^2} = 6\left(\sigma^2 + 2\pi G\rho\right) \geq 0$$

- However must course-grain over some finite domain:

$$\langle F \rangle_D = \frac{\int_D \sqrt{h}\, F\, d^3x}{\int_D \sqrt{h}\ d^3x}$$

- Evolution and smoothing do not commute: Buchert & Ellis; Kolb, Matarrese & Riotto

$$\langle F \rangle_D^{\bullet} = \langle F^{\bullet} \rangle_D + \langle F\Theta \rangle_D - \langle \Theta \rangle_D \langle F \rangle_D$$

$$\langle \Theta \rangle_D^{\bullet} = \langle \Theta^{\bullet} \rangle_D + \langle \Theta^2 \rangle_D - \langle \Theta \rangle_D^2 \geq \langle \Theta^{\bullet} \rangle_D$$

- $\langle \Theta \rangle_D^{\bullet} \neq \langle \Theta^{\bullet} \rangle_D$ although $\langle \Theta^{\bullet} \rangle_D$ can't accelerate, $\langle \Theta \rangle_D^{\bullet}$ can!

Inhomogeneities and smoothing

- Define an course-grained scale factor:

Kolb, Matarrese & Riotto astro-ph/0506534; Buchert & Ellis

$$a_D \equiv \left(V_D / V_{D0}\right)^{1/3} \qquad V_D = \int_D d^3x \sqrt{h}$$

- Course-grained Hubble rate:

$$H_D = \frac{\dot{a}_D}{a_D} = \tfrac{1}{3}\left\langle \Theta \right\rangle_D$$

- Effective evolution equations:

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G}{3}\left(\rho_{\rm eff} + 3 p_{\rm eff}\right) \qquad \rho_{\rm eff} = \left\langle \rho \right\rangle_D - \frac{Q_D}{16\pi G} - \frac{\left\langle R \right\rangle_D}{16\pi G}$$

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3}\rho_{\rm eff} \qquad p_{\rm eff} = -\frac{Q_D}{16\pi G} + \frac{\left\langle R \right\rangle_D}{48\pi G}$$

not described by a simple $p = w\,\rho$

- Kinematical back reaction: $Q_D = \tfrac{2}{3}\left(\left\langle \Theta^2 \right\rangle_D - \left\langle \Theta \right\rangle_D^2\right) - 2\left\langle \sigma^2 \right\rangle_D$

Inhomogeneities and smoothing

- Kinematical back reaction: $Q_D = \frac{2}{3}\left(\left\langle \Theta^2 \right\rangle_D - \left\langle \Theta \right\rangle_D^2\right) - 2\left\langle \sigma^2 \right\rangle_D$
- For acceleration: $\rho_{\text{eff}} + 3p_{\text{eff}} = \left\langle \rho \right\rangle_D - \frac{Q_D}{4\pi G} < 0$
- Integrability condition (GR): $\left(a_D^6 Q_D\right)^{\bullet} + a_D^4 \left(a_D^2 \left\langle R \right\rangle_D\right)^{\bullet} = 0$
- Acceleration is a pure GR effect:
  - curvature vanishes in Newtonian limit
  - Q_D will be exactly a pure boundary term, and small

Inhomogeneities and integrability

- Integrability condition: $\left(a_D^6 Q_D\right)^{\bullet} + a_D^4 \left(a_D^2 \langle R \rangle_D\right)^{\bullet} = 0$
- General solution: $\langle R \rangle_D = -Q_D + \frac{6k_D}{a_D^2} - \frac{4}{a_D^2}\int_0^{a_D} da\, a\, Q_D(a)$
- H_D and q_D: $H_D^2 = \frac{8\pi G}{3}\langle \rho \rangle_D - \frac{k_D}{a_D^2} + \frac{2}{3a_D^2}\int_0^{a_D} da\, a\, Q_D(a)$

$$\rho_{\text{eff}} + 3p_{\text{eff}} = \langle \rho \rangle_D - \frac{Q_D}{4\pi G}$$

- Particular solution: If $Q_D = 0$ or $Q_D \propto a_D{}^{-6}$
  - integrability condition: $\langle R \rangle_D = 6k_D / a_D{}^2$
  - curvature dominated: can have $q \to 0$, but no acceleration
- Particular solution: $3Q_D = -\langle R \rangle_D =$ const.
  - *i.e.*, $\Lambda_{\text{eff}} = Q_D$, so Q_D acts as a cosmological constant)

Inhomogeneities

- Now specialize: $h_{ij}(\vec{x},t) = a^2(t)\, e^{-2\Psi(\vec{x},t)}\left[\delta_{ij} + \chi_{ij}(\vec{x},t)\right]$

  $a \sim t^{2/3}$ is the usual FRW scale factor

  Ψ is a scalar perturbation: $\Psi = \Psi_\ell + \Psi_s$ ℓ = long, s = short (wrt: D)

  χ_{ij} is a traceless tensor with scalar, vector, & tensor d.o.f.

- Absorb Ψ_s into $\tilde{h}_{ij}(\vec{x},t)$: $h_{ij}(\vec{x},t) = a^2(t)\, e^{-2\Psi_\ell(t)}\tilde{h}_{ij}(\vec{x},t)$

- In terms of metric functions: $\langle R\rangle_D = a^{-2}e^{2\Psi_\ell}\left\langle \tilde{R} + 4\tilde{\nabla}^2\Psi_\ell - 2\tilde{\nabla}^i\Psi_\ell\tilde{\nabla}_i\Psi_\ell\right\rangle$

$$Q_D = \frac{2}{3}\left\langle\tilde{\Theta}^2\right\rangle_D - 2\left\langle\tilde{\sigma}^2\right\rangle_D$$

- *Only* super-Hubble modes: Q_D vanishes

  integrability condition $\rightarrow \langle R\rangle_D \propto a_D{}^{-2}$

  can have $q \rightarrow 0$, but no acceleration

Gradient expansion

Lifsitz, Khalatnikov, Tomita, Salopek, Stewart, Comer, Deruelle, Langlois, Parry, Nambu, Taruya, Bruni, Sopuerta, Croudace, …

- Local curvature expanded in powers of gradients of perturbations
- Lowest-order solution is "seed" long-wavelength approximation
- Successively add higher-order gradient terms
- Up to two gradients: $\nabla^2\phi = 4\pi G\,\delta\rho$

$$\Psi = \frac{5}{3}\phi + \frac{1}{18}\frac{a}{a_0}\left(\frac{2}{H_0}\right)^2 e^{10\phi/3}\left[\nabla^2\phi - \frac{5}{6}(\nabla\phi)^2\right]$$

$$\chi^i{}_j = -\frac{1}{3}\frac{a}{a_0}\left(\frac{2}{H_0}\right)^2 e^{10\phi/3}\left[D^i{}_j\phi + \frac{5}{3}\left(\phi^{,i}{}_{,j} - \frac{1}{3}(\nabla\phi)^2\delta^i{}_j\right)\right]$$

Sub-Hubble instabilities

Kolb, Notari, Matarrese, & Riotto

- Result in 2nd-order perturbation (in ϕ) theory:

$$\frac{\langle \theta = \Theta - 3H \rangle}{3H} = -\frac{20\tau^2}{9}\langle \nabla^2\phi \rangle - \frac{100\tau^2}{9}\langle \phi \rangle \langle \nabla^2\phi \rangle - \frac{23\tau^4}{54}\langle \nabla^2\phi \rangle \langle \nabla^2\phi \rangle$$

$$+ \frac{130\tau^2}{27}\langle \phi^{,i}\phi_{,i} \rangle + \frac{20\tau^2}{3}\langle \phi\nabla^2\phi \rangle + \frac{4\tau^4}{27}\left(\langle \nabla^2\phi\nabla^2\phi \rangle - \langle \phi^{,ij}\phi_{,ij} \rangle \right)$$

$$\rightarrow \frac{130\tau^2}{27}\langle \phi^{,i}\phi_{,i} \rangle + \frac{4\tau^4}{27}\left(\langle \nabla^2\phi\nabla^2\phi \rangle - \langle \phi^{,ij}\phi_{,ij} \rangle \right)$$

- Each derivative accompanied by conformal time $\tau = 2/aH$
- Each factor of τ accompanied by c.
- Highest derivative is highest power of $\tau \propto c$: "Newtonian"
- Lower derivative terms $\propto c^{-n}$: "Post-Newtonian"
- ϕ and its derivatives can be expressed in terms of $\delta\rho / \rho$

$\Delta^2(k,a)$: power spectrum of $\delta\rho/\rho$

- Amplitude $A = 1.9 \times 10^{-5}$ and transfer function $T^2(k)$:

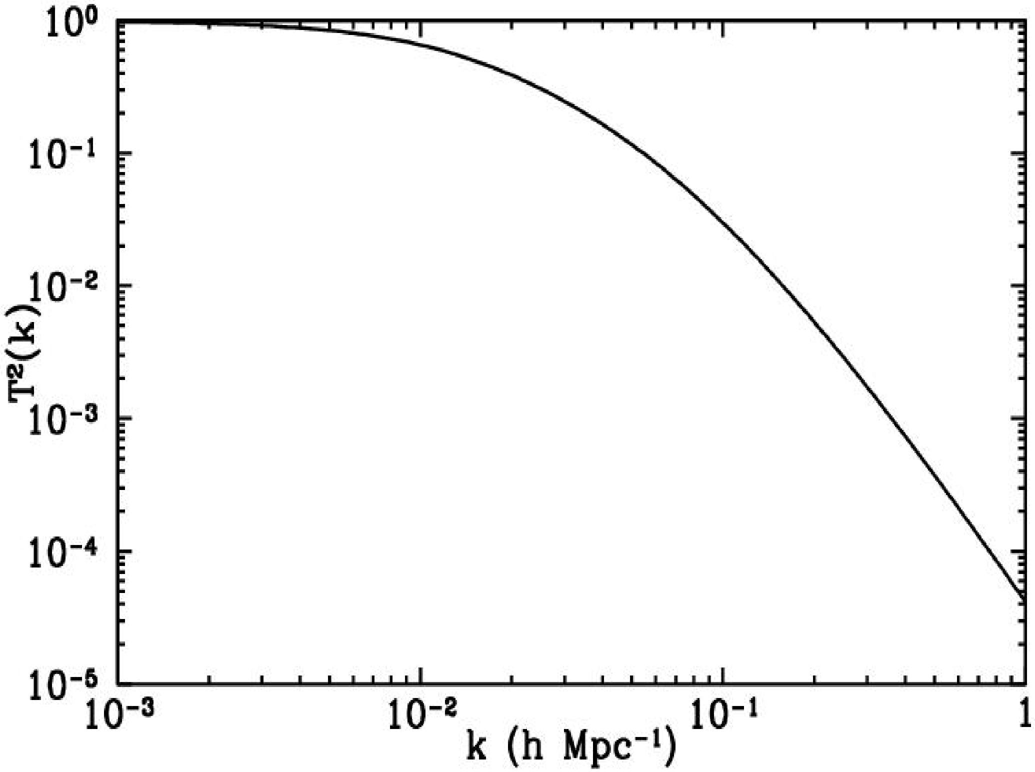
$$\Delta^2(k,a) = A^2 \left(\frac{k}{aH}\right)^4 T^2(k)$$

Harrison–Zel'dovich spectrum

- Use CDM transfer function:

$$T^2(k) \to \begin{cases} 1 & k \to 0 \\ k^{-4} \ln^2(k) & k \to \infty \end{cases}$$

$$\Delta^2(k) \to \begin{cases} k^4 & k \to 0 \\ \ln^2(k) & k \to \infty \end{cases}$$



Some examples

- $$\tau^2 \left\langle \nabla\phi \cdot \nabla\phi \right\rangle = -\frac{4}{a^2H^2} \int_{V(R)} \frac{d^3x}{V(R)} \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} k_1 \cdot k_2 \, \overline{\phi_{\vec{k}_1} \phi_{\vec{k}_2}} \; e^{i\left(\vec{k}_1+\vec{k}_2\right)\cdot\vec{x}}$$

$$\simeq A^2 \frac{1}{a^2H^2} \int_0^\infty dk\, k\, T^2(k) \sim 10^{-5} \frac{a}{a_0}$$

- $$\tau^4 \left\langle \nabla^2\phi \nabla^2\phi \right\rangle = -\frac{1}{a^4H^4} \int_{V(R)} \frac{d^3x}{V(R)} \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} k_1^2 k_2^2 \, \overline{\phi_{\vec{k}_1} \phi_{\vec{k}_2}} \; e^{i\left(\vec{k}_1+\vec{k}_2\right)\cdot\vec{x}}$$

$$\simeq A^2 \frac{1}{a^4H^4} \int_0^\infty dk\, k^3\, T^2(k) \sim 10^0 \left(\frac{a}{a_0}\right)^2$$

– Mean of linear terms vanish: $\langle \nabla^2\phi \rangle = \langle \phi \rangle = 0$

– Individual Newtonian terms large, *i.e.*, $\langle \nabla^2\phi \nabla^2\phi \rangle = \mathcal{O}(1)$ Räsänen

– <u>*But*</u> total Newtonian term vanishes $\langle \nabla^2\phi \nabla^2\phi \rangle = \langle \phi^{,ij} \phi_{,ij} \rangle$

– Post-Newtonian: $\langle \nabla\phi \cdot \nabla\phi \rangle = \mathcal{O}(10^{-5})$ ***huge!*** (large k^2/a^2H^2)

Sub-Hubble instabilities

- First term in gradient expansion (2 spatial derivatives):

  $\langle R \rangle_D \propto a_D^{-2} \qquad Q_D = 0 \rightarrow$ no acceleration

- In general, gradient expansion gives   Notari; Kolb, Matarrese, & Riotto

$$\langle R \rangle_D = \sum_{n=1}^{\infty} r_n a^{n-3} \qquad \left(r_n = \sum_{m=n}^{2n} \left(2n \text{ derivatives}\right) \phi^m \right)$$

$$Q_D = \sum_{n=2}^{\infty} q_n a^{n-3} \qquad \left(q_n = \sum_{m=n}^{2n} \left(2n \text{ derivatives}\right) \phi^m \right)$$

- Newtonian terms, $(\nabla^2 \phi)^n \sim (k/aH)^{2n} \phi^n$, _individually_ are large, but only appear as surface terms, hence small in total
- Post-Newtonian terms, $(\nabla \phi)^{2n} \sim (k/aH)^{2n} \phi^{2n}$, individually are small, but do not appear as surface terms
- Dominant term is combination: $(\nabla^2 \phi)^{n-1} (\nabla \phi)^2 \sim (k/aH)^{2n} \phi^{n+1}$

$(\nabla^2 \phi)^{n-1} (\nabla \phi)^2 \sim (k/aH)^{2n} \phi^{n+1}$

- $\phi \to A = 2 \times 10^{-5}$
- $(aH)^{2n} = a_0^{2n} H_0^{2n} (a_0/a)^n$
- $H_0^{-1} = 3000 h^{-1}$ Mpc
- $(k/aH)^{2n} \phi^{n+1} \sim (3\times10^3)^{2n} (k/h\ \mathrm{Mpc}^{-1})^{2n} (2\times10^{-5})^{n+1}$
  - $n = 1$: $4\times10^{-3}\ (k/h\ \mathrm{Mpc}^{-1})^2 (a/a_0)$ $\times\, a^{-3}$: curvature
  - $n = 2$: $6\times10^{-1}\ (k/h\ \mathrm{Mpc}^{-1})^4 (a/a_0)^2$ $\times\, a^{-3}$: ?
  - $n = 3$: $9\times10^{1}\ (k/h\ \mathrm{Mpc}^{-1})^6 (a/a_0)^3$ $\times\, a^{-3}$: Λ
- Of course have to include transfer function, integrate over k, *etc*.

Sub-Hubble instabilities

- Gradient expansion: $\langle R \rangle_D = \sum_{n=1}^{\infty} r_n a^{n-3}$ $\qquad Q_D = \sum_{n=2}^{\infty} q_n a^{n-3}$

- Lowest-order term to make big contribution is $n = 3$ (6 derivatives)

- Disconnected fourth-order moment of ϕ: $\left\langle \frac{\left(\nabla^2 \phi\right)^2}{H_0^4} \right\rangle \left\langle \frac{\left(\nabla \phi\right)^2}{H_0^2} \right\rangle$

- Notice $n = 3$ contributes to Q_D and $\langle R \rangle_D$ terms $\propto a^0$, *i.e.*,
  expansion as if driven by a cosmological constant !!!

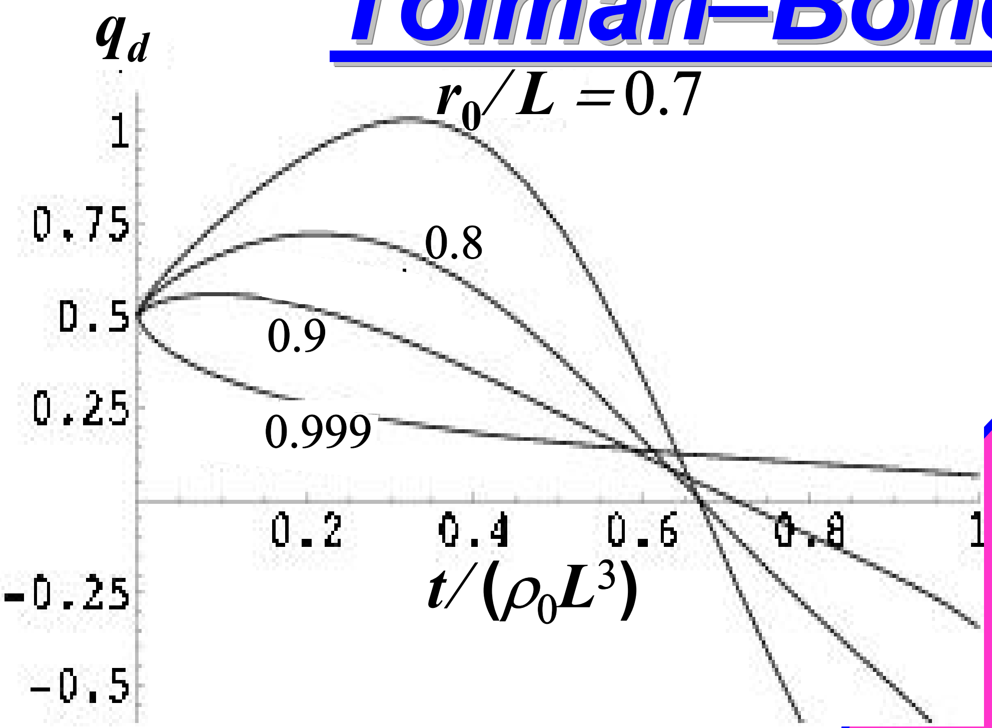
- But why stop at $n = 3$?????

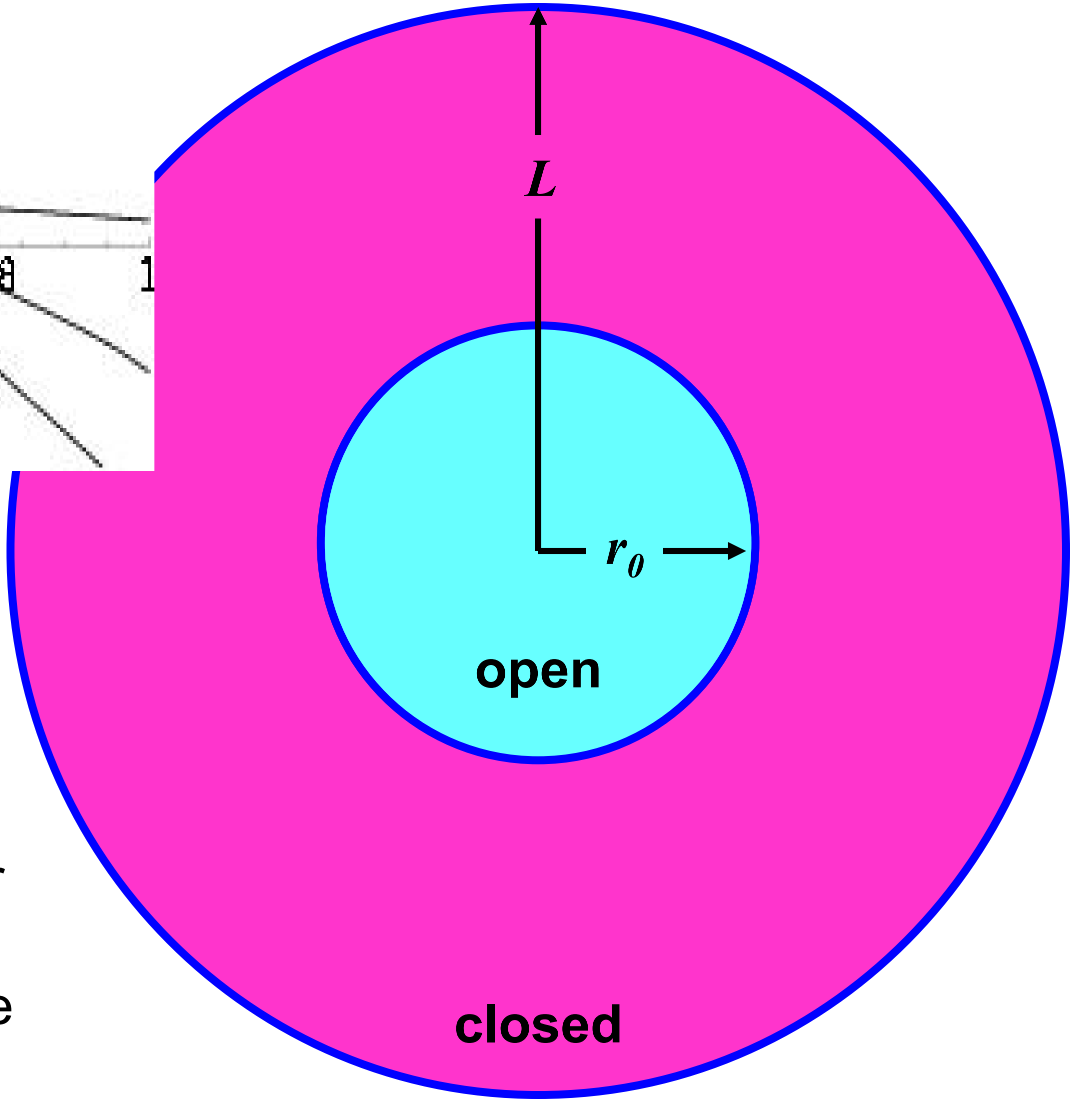
Inhomogeneities

- Does this have anything to do with our universe?
- Have to go to non-perturbative limit!
- How to relate observables ($d_L(z)$, $d_A(z)$, $H(z)$, …) to Q_D & $\langle R \rangle_D$?
- Can one have large effect and isotropic expansion/acceleration? (*i.e.,* will the shear be small?)
- What about gravitational instability?
- Toy model proof of principle: Tolman-Bondi dust model
  Nambu & Tanimoto (gr-qc/0507057)

Tolman–Bondi–Lemaître

Nambu & Tanimoto (gr-qc/0507057)
[also Moffet]

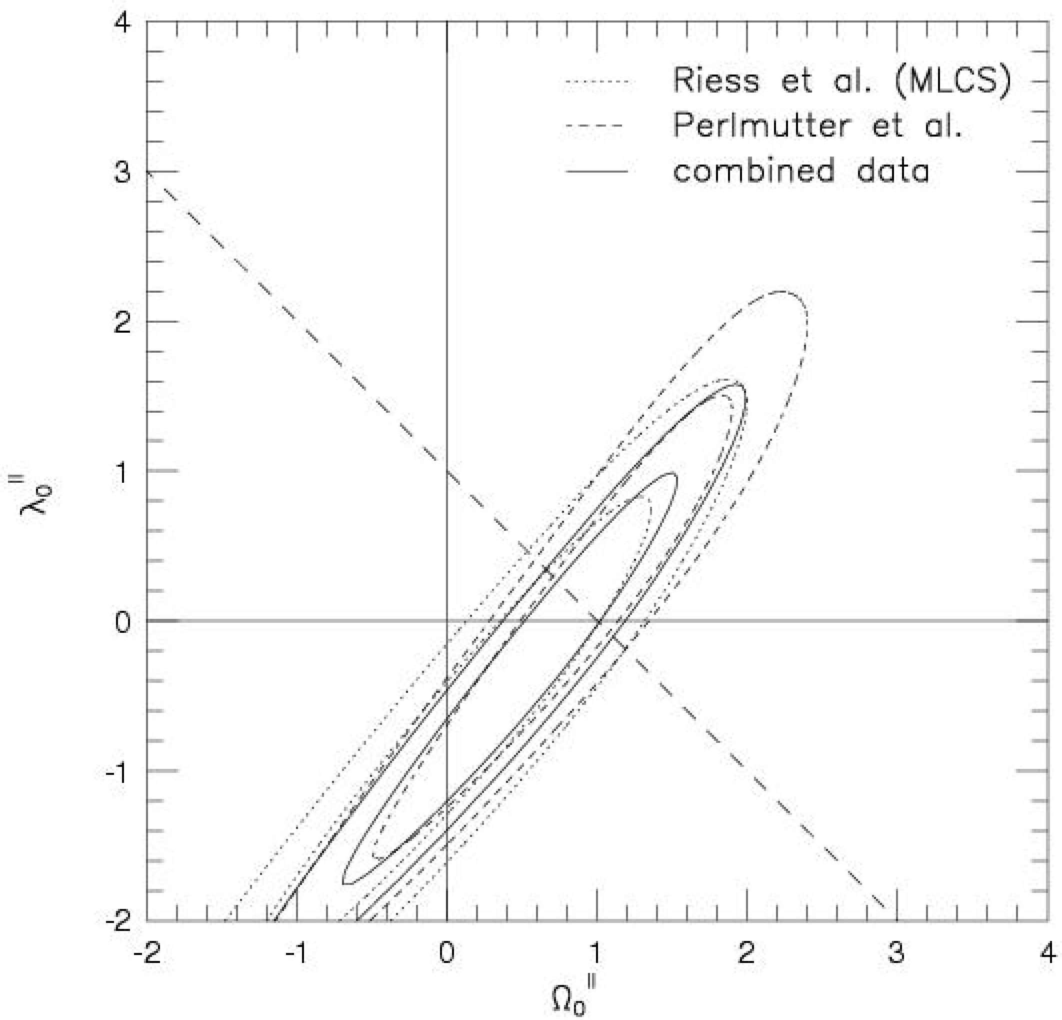




Acceleration in our local Hubble patch if the mean rarefaction factor (w.r.t. the underlying FRW model) grows fast enough to overshoot the FRW background evolution.

Observational consequences

- Spherical model
- Overall Einstein–de Sitter
- Inner underdense 200 Mpc region
- Compensating high-density shell
- Calculate $d_L(z)$
- Compare to SNIa data
- Fit with $\Lambda = 0$!



Tomita, 2001

Comments

- *"Do you believe?"* is not the relevant question
- Acceleration of the Universe is important; this must be explored
- How it could go badly wrong:
  - Backreaction should not be calculated in frame comoving with matter flow
  - Series re-sums to something harmless
  - No reason to stop at first large term
  - Synchronous gauge is tricky
    - ☹ Residual gauge artifacts
    - ☹ Synchronous gauge develops coordinate singularities at late time (shell crossings)
    - ☺ Problem could be done in Poisson gauge

Conclusions

- Must properly smooth inhomogeneous Universe
- In principle, acceleration possible even if "locally" $\rho + 3p > 0$
- Super-Hubble modes, of and by themselves, cannot accelerate
- Sub-Hubble modes have large terms in gradient expansion
  - Newtonian terms can be large but combine as surface terms
  - Post-Newtonian terms are not surface terms, but small
  - Mixed Newtonian $\times$ Post-Newtonian terms can be large
  - Effect from "mildly" non-linear scales
- The first large term yields effective cosmological constant
- No reason to stop at first large term
- Can have $w < -1$?
- Advantages to scenario:
  - No new physics
  - "Why now" due to onset of non-linear era

2nd Vienna Central European Seminar
on Particle Physics and Quantum Field Theory

Thoughts on Dark Energy: *Acceleration without Dark Energy*

Rocky Kolb
Fermilab &
University of Chicago

All work is the result of collaborations with
Sabino Matarrese and Antonio Riotto (Padova)
[and occasionally Alessio Notari (McGill)]

Shell Crossing

- Gradient terms:
  - Shell-crossing instabilities imply *divergent* gradient terms.
  - Our effect comes from *infinite* number of *finite* gradient terms
- Newtonian terms:
  - Shell crossing instabilities lead to infinite Newtonian terms
  - Our effect has small Newtonian terms
- Caustics:
  - Caustics carry small amount of mass
  - They can be smoothed

Poisson gauge

- The weak-field form of metric:

  $$ds^2 = a^2(t)\,[-(1-2\psi_P)\,dt^2 + (1-2\psi_P)\,\delta_{ij}\,dx^i dx^j\,]$$

  $\psi_P = \Phi_N/c^2$ is the Newtonian gravitational potential, related to $\delta\rho$ by the Poisson equation: $\nabla^2\Phi_N = 4\pi G\,a^2\,\delta\rho$

- Kinematical back reaction will contain a term $\langle N^2\Theta^2\rangle_D$

  N is the lapse function relating Poisson-gauge coordinate time $t_P = \int d\tau\,a(\tau)$ as a function of the proper time t of comoving observers; N contains $(\nabla\Phi_N)^2$

- Q_D will contains terms like $\langle\,(\nabla^2\Phi_v)^2\,(\nabla\Phi_N)^2\,\rangle$
  - Velocity potential Φ_v related to gravitational potential
  - Non-linear (non-Gaussian) nature $\rightarrow$ average has disconnected terms as before

How Do We Sort It Out?

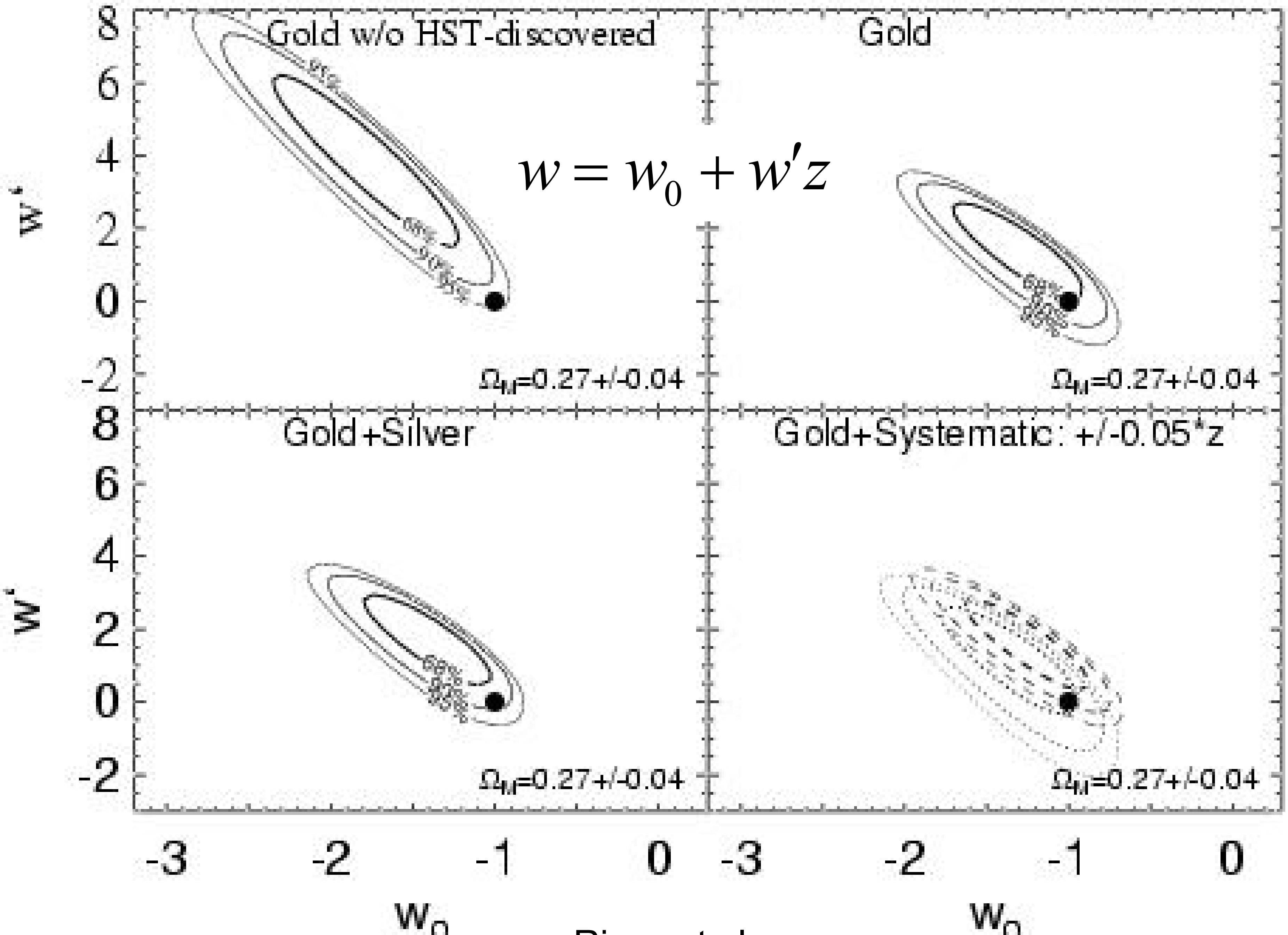
- Something is established-ΛCDM too good to ignore
  - SNIa
  - Subtraction
  - Age
  - Large-scale structure
  - .....
- Left-hand side or right-hand side?

Left-hand side:

- Growth of structure
- New gravity?
  - solar-system effects
  - short-range effects
  - branes (accelerator effects)
- Inhomogreneities?

Right-hand side:

- $w = -1$
  "just" Λ?
- $w \neq -1$
  what is dynamics?
- Scalars
  long-range forces?


Gold w/o HST-discovered
Gold
Gold+Silver
Gold+Systematic: +/-0.05*z
Ω_M=0.27+/-0.04
Ω_M=0.27+/-0.04
Ω_M=0.27+/-0.04
Ω_M=0.27+/-0.04
w'
w'
w_o
w_o
$w = w_0 + w'z$

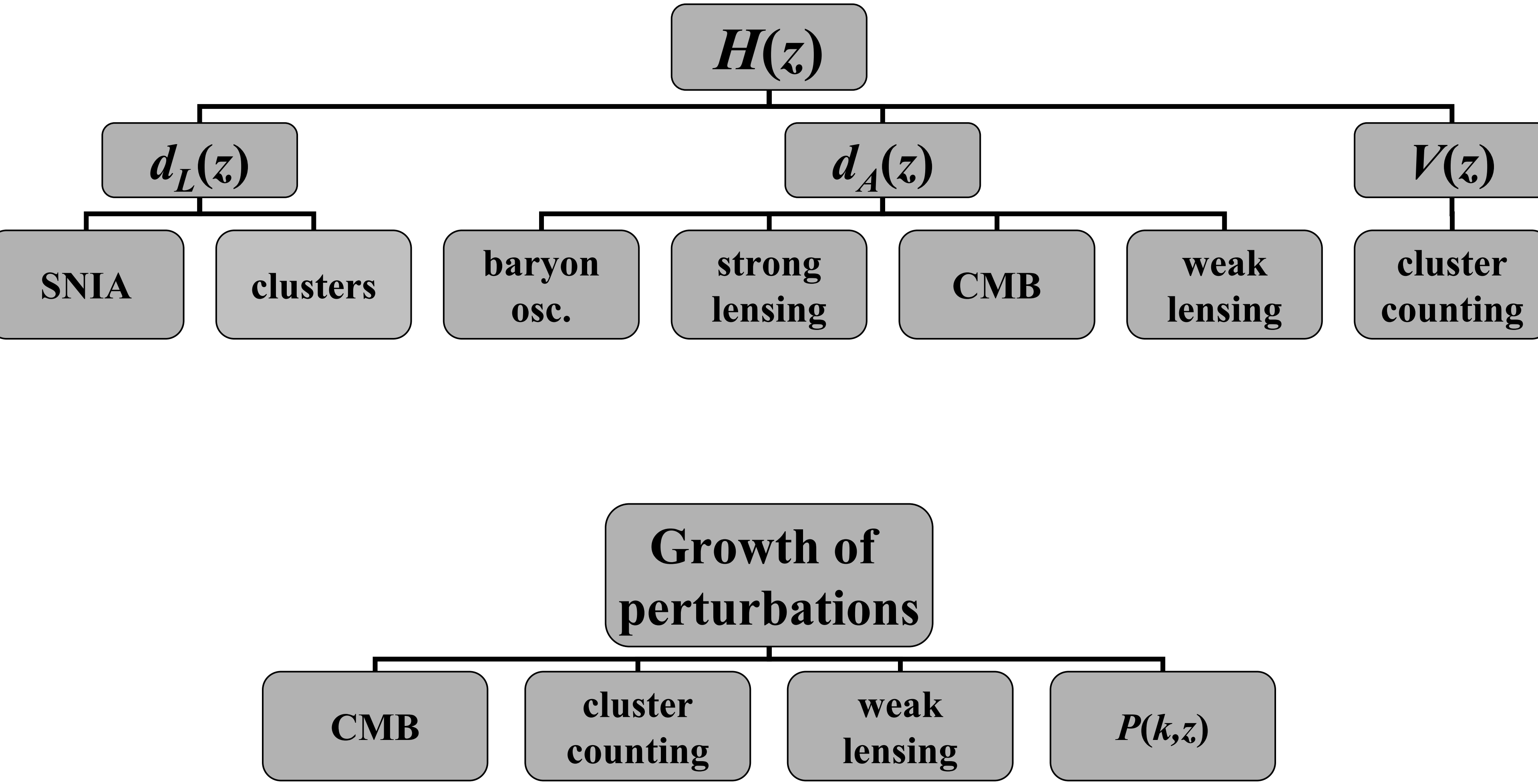
Riess et al.

Caution in Interpretation

Always read the fine print:

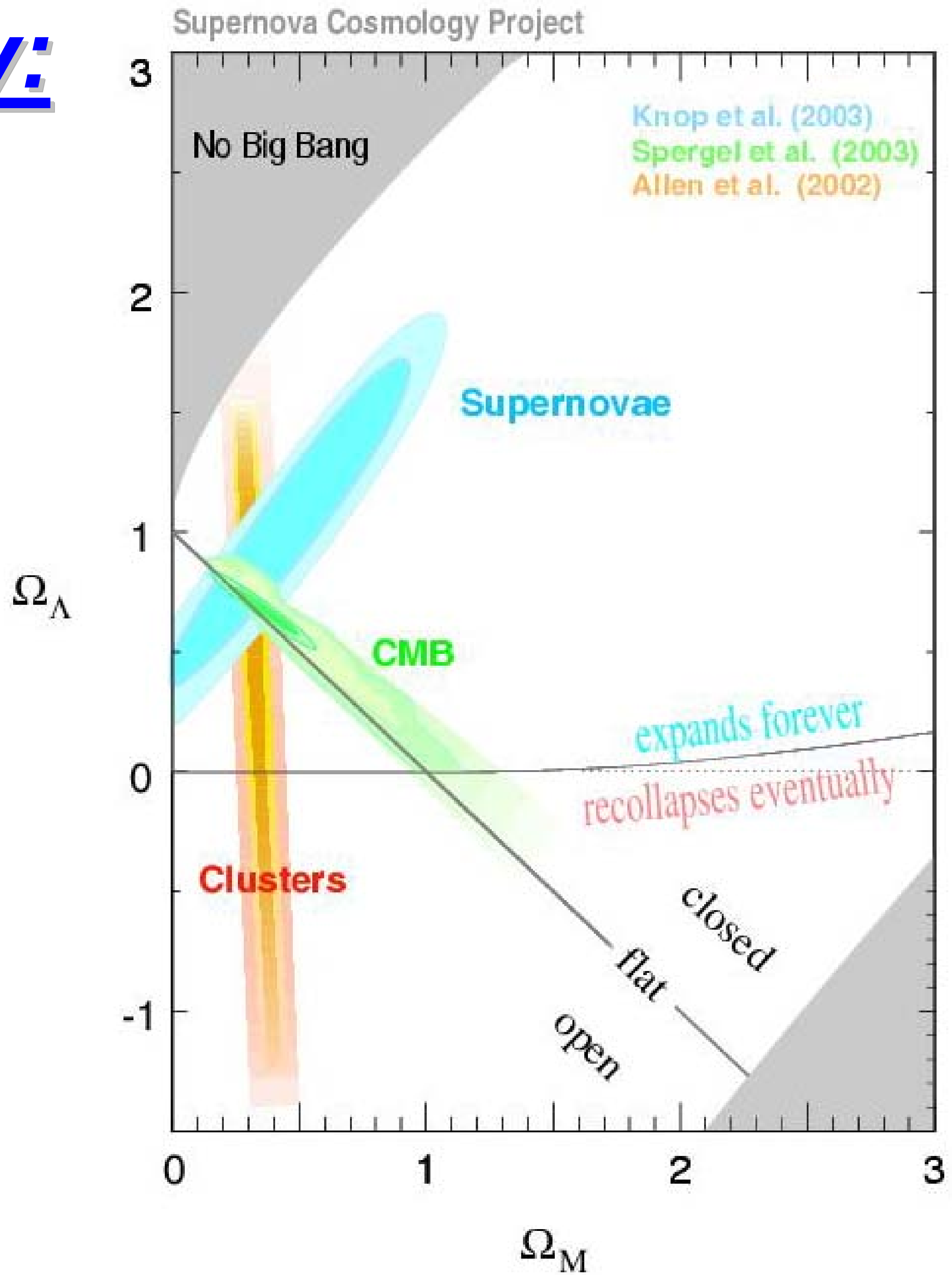
- Astrophysical systematic errors
- What are the model assumptions?
  - w = constant? w', w_a
  - assume Ω_Λ?
- What are the priors?
  - Ω_M, Ω_B, H_0, …

How Do We Sort It Out?



Complementarity: Reason #1

- Don't focus on any one particular error contour
- Focus on fact that error contours for different methods are not parallel



Complementarity: Reason #2

- If right-hand side, measure w associated with $H(z)$.

- If left-hand side, measure w associated with $H(z)$, ***AND*** w associated with growth of structure.

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_0\delta = \begin{cases} 0\,? \\ \text{source term?} \end{cases}$$

- w deduced from methods sensitive only to $H(z)$ ***NEED NOT*** agree with w deduced from methods sensitive to growth.