Predicting neutrinoless double beta decay in the A_4 family symmetry model

Albert Villanova del Moral

Based on paper:

M. Hirsch, A. Villanova del Moral, J. W. F. Valle and E. Ma,

Phys. Rev. D **72**, 091301 (2005)

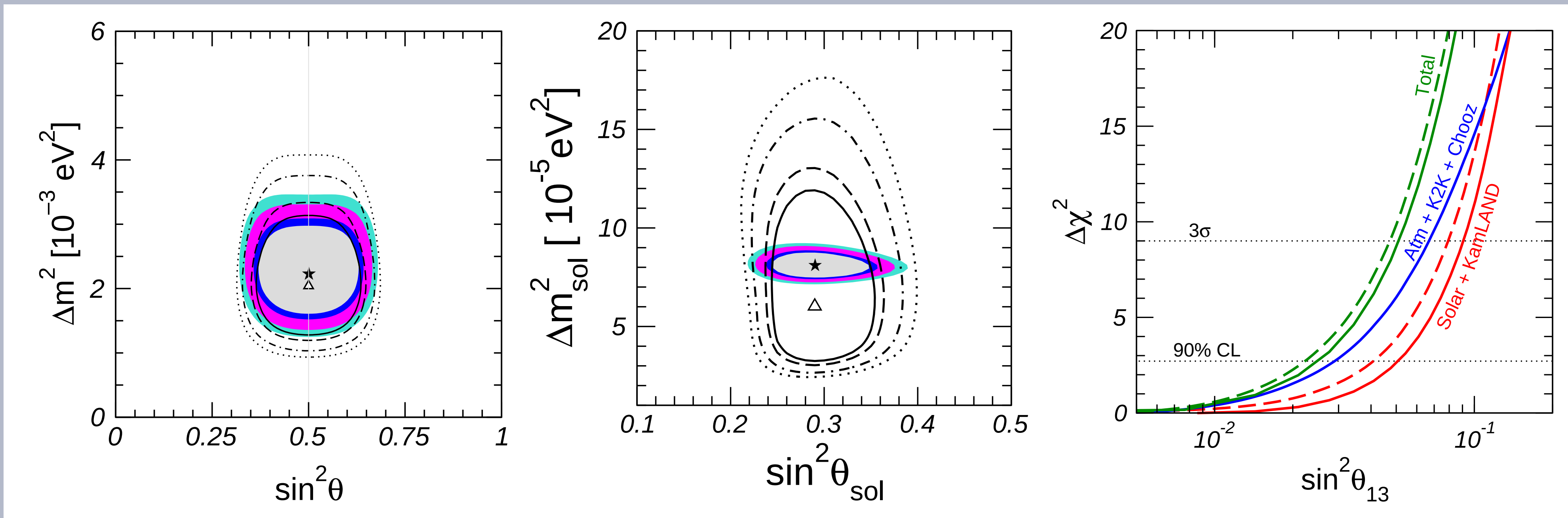
2nd Vienna Central European Seminar on Particle Physics and Quantum Field Theory

Wien, 25-27 November, 2005

Neutrino Physics Data

★ Neutrinos are massive

- Allowed parameter region from all neutrino experimental data: [⇛]



[M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **6**, 122 (2004)]

Type of Neutrino Masses

- Dirac neutrinos: $\nu \neq \overline{\nu}$

$$\mathcal{L}^{\mathrm{D}} = - \sum_{\ell\ell'} \overline{\nu}_{\ell'\mathrm{R}} \, M^{\mathrm{D}}_{\ell'\ell} \, \nu_{\ell\mathrm{L}} + \mathrm{h.c.}$$

Type of Neutrino Masses

- Dirac neutrinos: $\nu \neq \overline{\nu}$

$$\mathcal{L}^{\mathrm{D}} = -\sum_{\ell\ell'} \overline{\nu}_{\ell'\mathrm{R}}\, M^{\mathrm{D}}_{\ell'\ell}\, \nu_{\ell\mathrm{L}} + \mathrm{h.c.}$$

  - Total L is conserved

Type of Neutrino Masses

- Dirac neutrinos: $\nu \neq \overline{\nu}$

$$\mathcal{L}^{\mathrm{D}} = - \sum_{\ell\ell'} \overline{\nu}_{\ell'\mathrm{R}} \, M^{\mathrm{D}}_{\ell'\ell} \, \nu_{\ell\mathrm{L}} + \mathrm{h.c.}$$

  - Total L is conserved

- Majorana neutrinos: $\nu = \overline{\nu}$

$$\mathcal{L}^{\mathrm{M}} = - \frac{1}{2} \sum_{\ell\ell'} \overline{\nu^{c}_{\ell'\mathrm{L}}} \, M^{\mathrm{M}}_{\ell'\ell} \, \nu_{\ell\mathrm{L}} + \mathrm{h.c.}$$

Type of Neutrino Masses

- Dirac neutrinos: $\nu \neq \overline{\nu}$

$$\mathcal{L}^{\mathrm{D}} = - \sum_{\ell\ell'} \overline{\nu_{\ell'\mathrm{R}}} \, M^{\mathrm{D}}_{\ell'\ell} \, \nu_{\ell\mathrm{L}} + \mathrm{h.c.}$$

  - Total L is conserved
- Majorana neutrinos: $\nu = \overline{\nu}$

$$\mathcal{L}^{\mathrm{M}} = - \frac{1}{2} \sum_{\ell\ell'} \overline{\nu^{c}_{\ell'\mathrm{L}}} \, M^{\mathrm{M}}_{\ell'\ell} \, \nu_{\ell\mathrm{L}} + \mathrm{h.c.}$$

  - Total L is violated
  - $0\nu\beta\beta$ is allowed

Double Beta Decay

$$(Z, A) \rightarrow (Z+2, A) + 2e^- + X$$

Double Beta Decay

$$(Z, A) \rightarrow (Z+2, A) + 2e^- + X$$

- If $X = 2\overline{\nu}_e, \quad \Rightarrow \quad 2\nu\beta\beta$
  - Allowed within the SM
  - Experimentally observed for 9 isotopes

Double Beta Decay

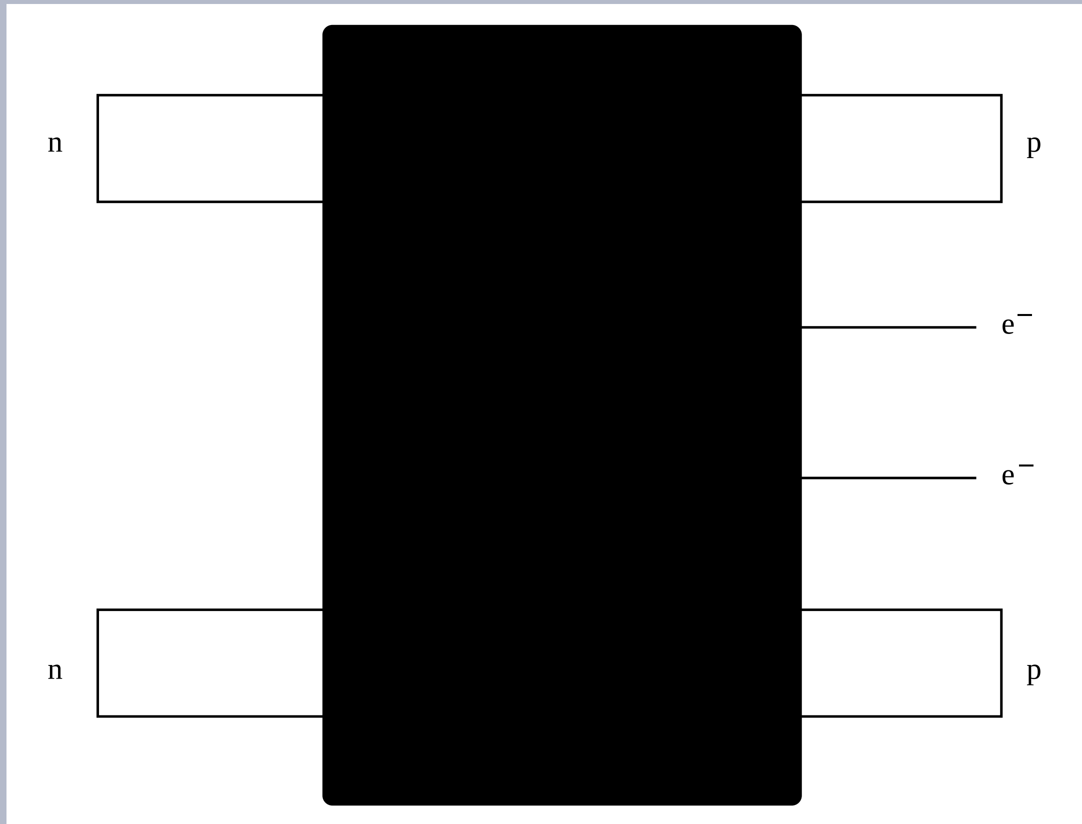
$$(Z, A) \to (Z+2, A) + 2e^- + X$$

- If $X = 2\overline{\nu}_e, \quad \Rightarrow \quad 2\nu\beta\beta$
  - Allowed within the SM
  - Experimentally observed for 9 isotopes
- If $X = 0\overline{\nu}_e, \quad \Rightarrow \quad 0\nu\beta\beta$
  - L is violated in 2 units
  - Experimentally not observed
  - Current experimental limits:

$$m_{ee} \leq 0.3 - 1\,\mathrm{eV}$$

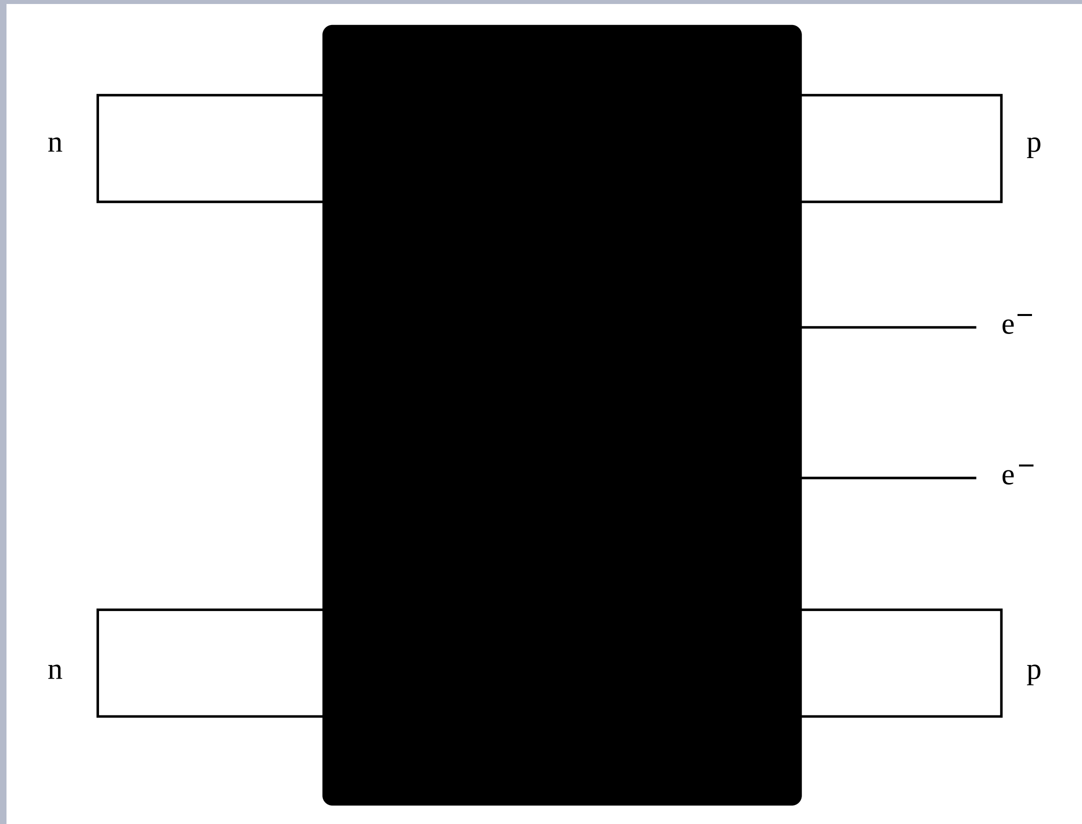
[L. Baudis *et al.*, Phys. Rev. Lett. **83**, 41 (1999). C. E. Aalseth *et al.* [IGEX Collaboration], Phys. Rev. **D65**, 092007 (2002).]

Neutrinoless Double Beta Decay



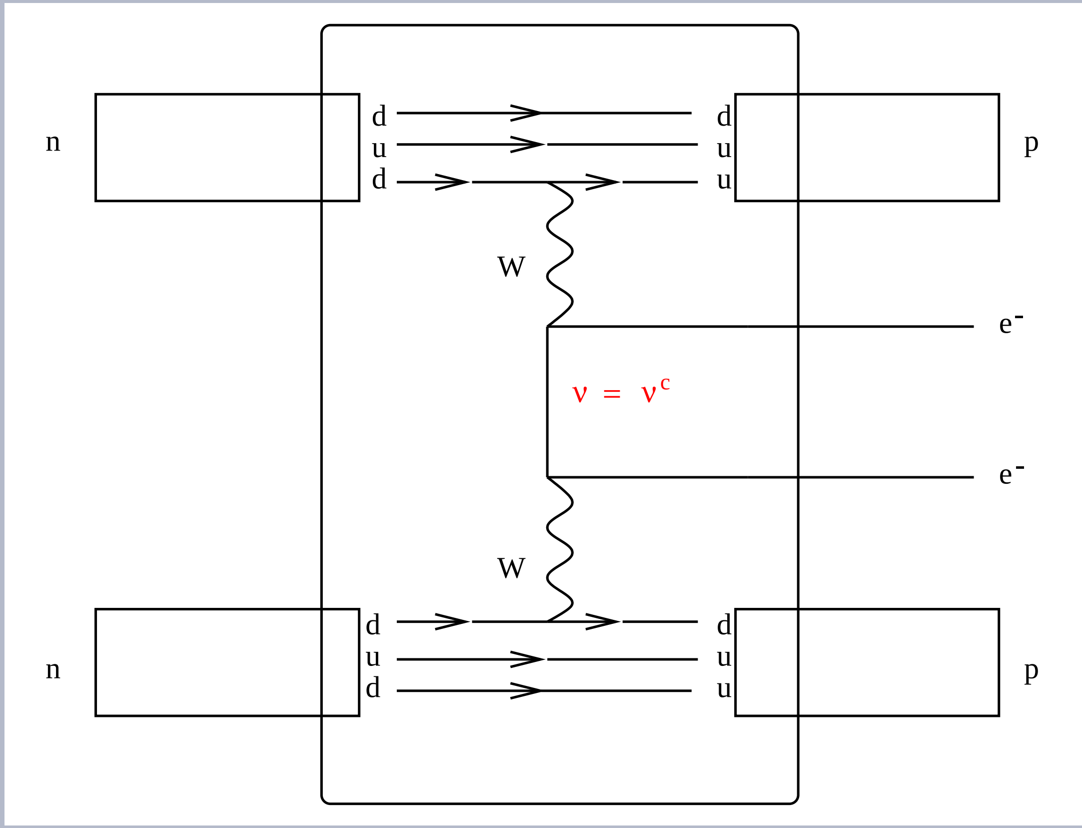
- How can it happen?

Neutrinoless Double Beta Decay



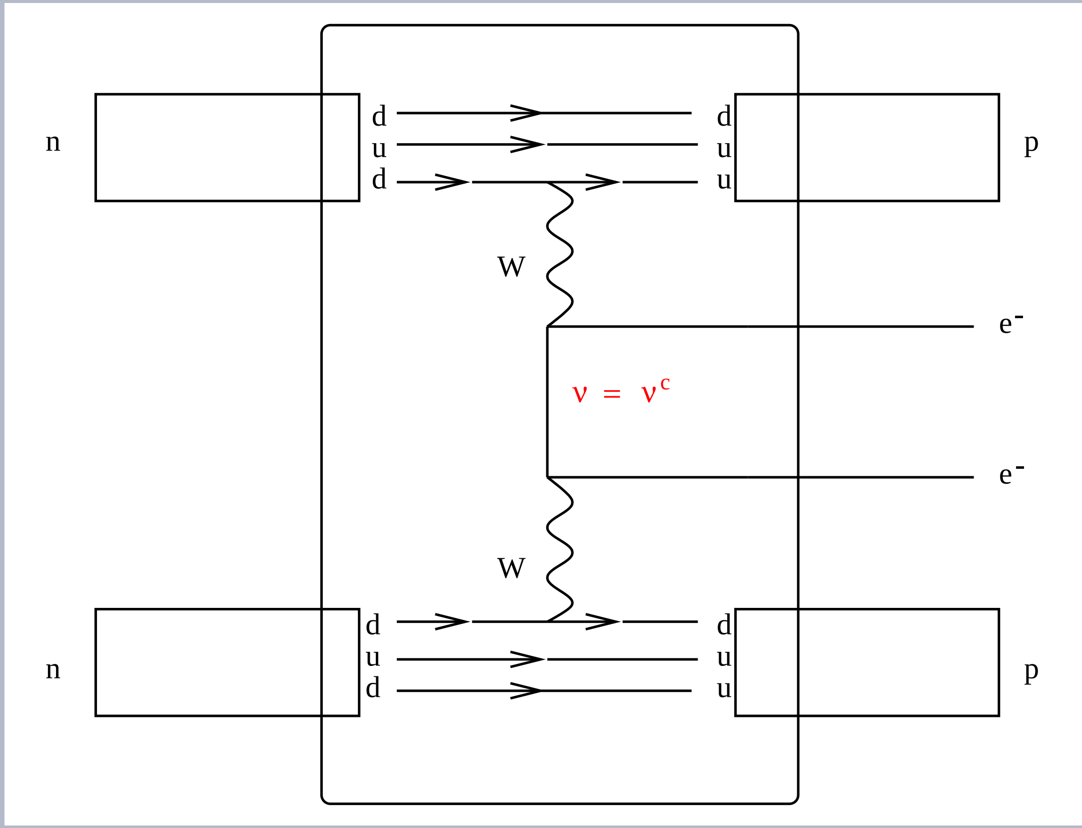
- How can it happen?
  - Mass mechanism:
    exchange of massive Majorana neutrinos

Neutrinoless Double Beta Decay



- How can it happen?
  - Mass mechanism:
    exchange of massive Majorana neutrinos

Neutrinoless Double Beta Decay



- How can it happen?
  - Mass mechanism:
    exchange of massive Majorana neutrinos
  - However, it is not the only mechanism! [⇛]

Black-box Theorem

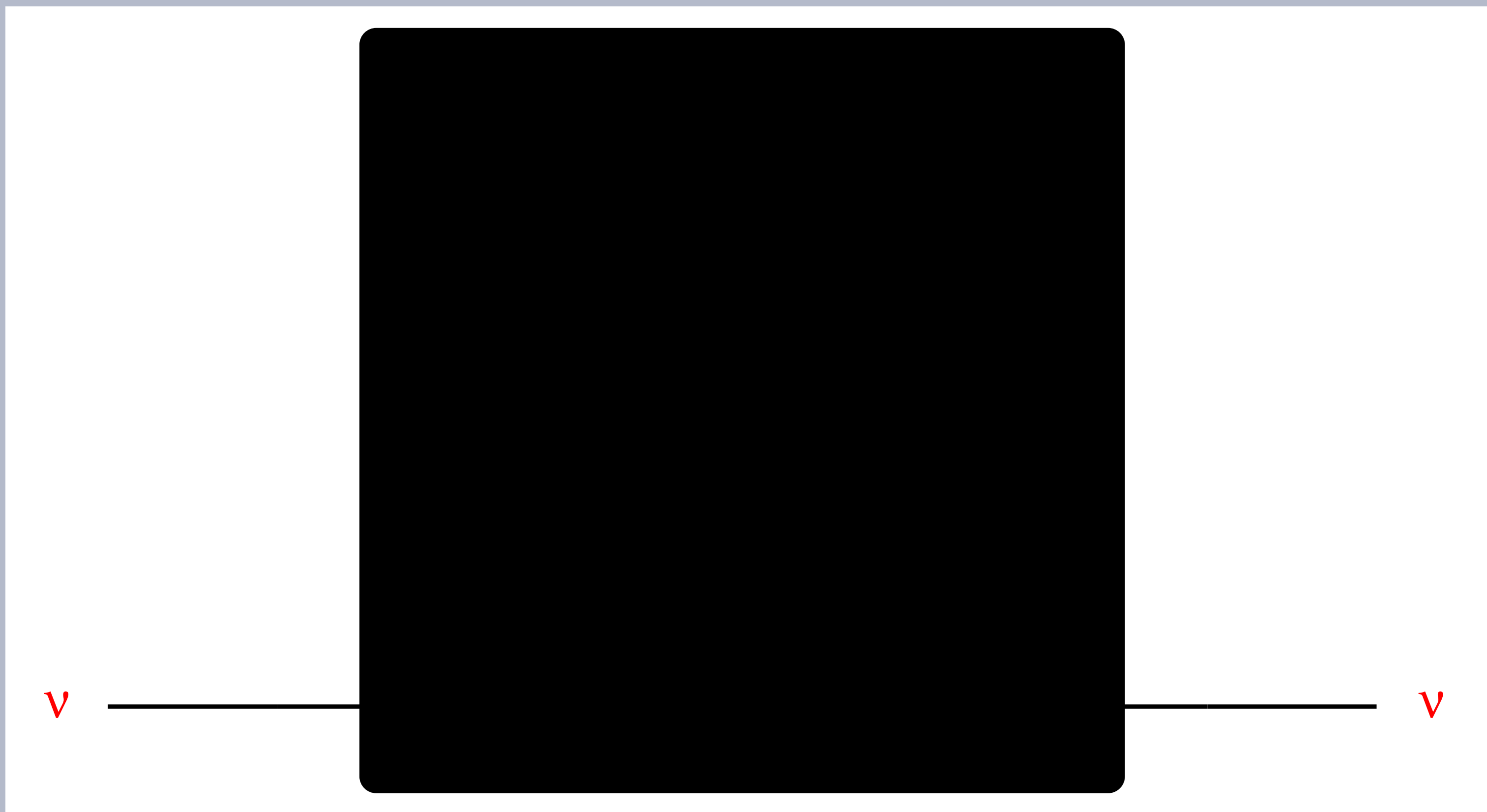
"In any gauge theory, whatever the mechanism for inducing $0\nu\beta\beta$ is, it is bound to also yield a Majorana neutrino mass at some level."

[J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 2951 (1982)]

Black-box Theorem

"In any gauge theory, whatever the mechanism for inducing $0\nu\beta\beta$ is, it is bound to also yield a Majorana neutrino mass at some level."

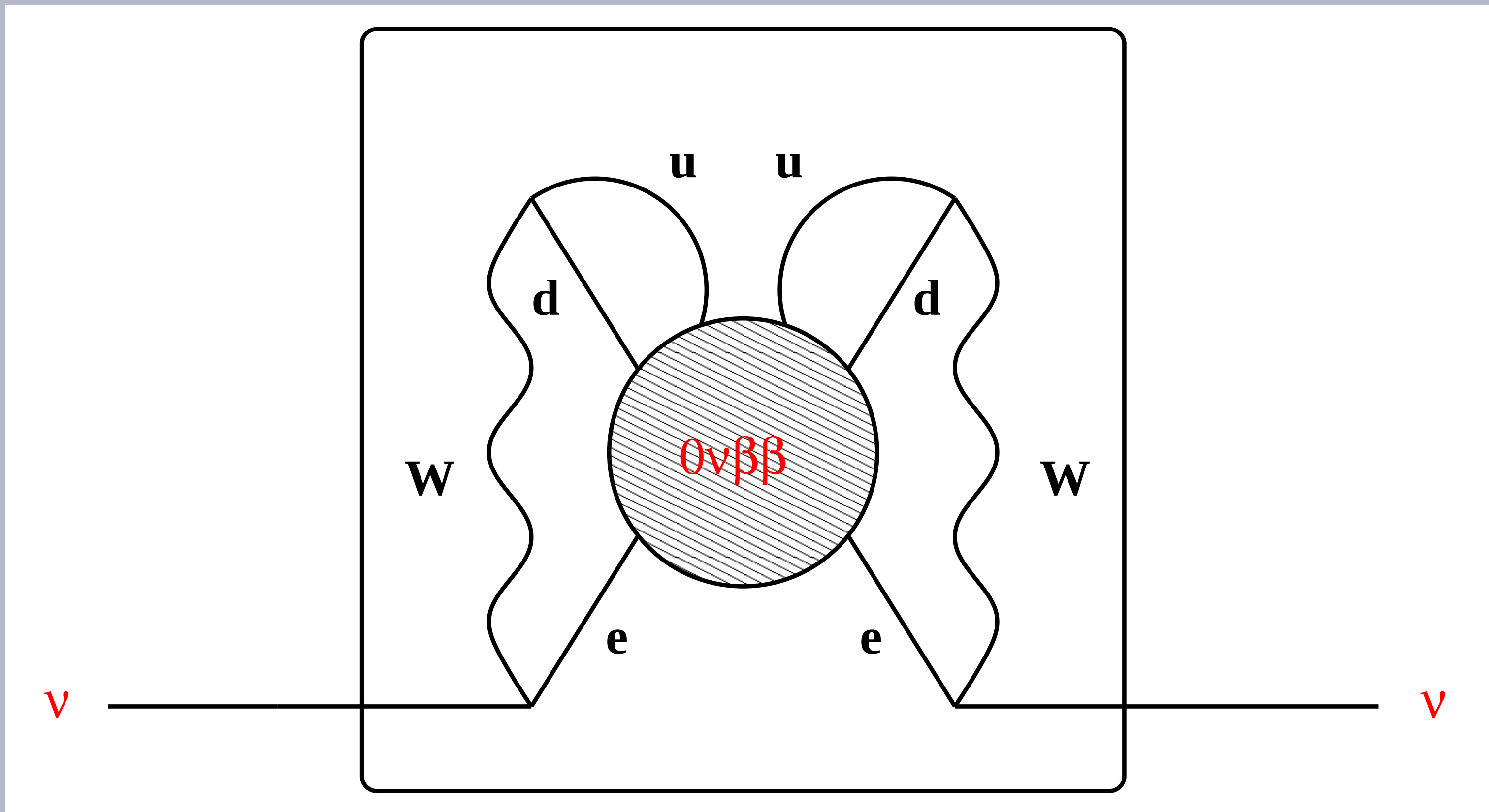
[J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 2951 (1982)]



Black-box Theorem

"In any gauge theory, whatever the mechanism for inducing $0\nu\beta\beta$ is, it is bound to also yield a Majorana neutrino mass at some level."

[J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 2951 (1982)]



Standard Model

- SM neutrinos are massless

Standard Model

- SM neutrinos are massless since:
  - Right-handed neutrinos do not exist
  - Lepton number is “accidentally” conserved
  - Higgs triplets do not exist

Standard Model

- SM neutrinos are massless since:
  - Right-handed neutrinos do not exist
  - Lepton number is “accidentally” conserved
  - Higgs triplets do not exist

$\Rightarrow$ SM must be extended in some sector:
  - Particles
  - Symmetries
  - or both

Standard Model

- SM neutrinos are massless since:
  - Right-handed neutrinos do not exist
  - Lepton number is “accidentally” conserved
  - Higgs triplets do not exist

$\Rightarrow$ SM must be extended in some sector:

- Particles
- Symmetries
- or both

★ A_4 flavour symmetry extension

A_4 Group

- It is the finite group of even permutations of 4 objects

A_4 Group

- It is the finite group of even permutations of 4 objects
- It has 12 elements in 4 equivalence classes

A_4 Group

- It is the finite group of even permutations of 4 objects
- It has 12 elements in 4 equivalence classes
- It has 4 irreducible representations (irreps):
  - Three 1-dimensional irreps:

$$\mathbf{1}, \quad \mathbf{1}', \quad \mathbf{1}''$$

  - One 3-dimensional irrep:

$$\mathbf{3}$$

A_4 Group

- It is the finite group of even permutations of 4 objects
- It has 12 elements in 4 equivalence classes
- It has 4 irreducible representations (irreps):
  - Three 1-dimensional irreps:

$$\mathbf{1}, \quad \mathbf{1}', \quad \mathbf{1}''$$

  - One 3-dimensional irrep:

$$\mathbf{3}$$

★ Suitable for describing 3 families!

A_4 Group Character Table

Class	$\chi^{(\mathbf{1})}$	$\chi^{(\mathbf{1}')}$	$\chi^{(\mathbf{1}'')}$	$\chi^{(\mathbf{3})}$
C_1	1	1	1	3
C_2	1	ω	ω^2	0
C_3	1	ω^2	ω	0
C_4	1	1	1	–1

where

$$\omega \equiv \sqrt[3]{1} = e^{i2\pi/3}$$

$$\omega + \omega^2 + 1 = 0$$

Irrep Products

Invariant terms:

$$\mathbf{1} = \mathbf{1} \times \mathbf{1} \qquad \mathbf{1} = \mathbf{1}' \times \mathbf{1}''$$

Product decomposition:

$$\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}$$

$$\begin{aligned}
(x_1, x_2, x_3) \times (y_1, y_2, y_3) &= x_1 y_1 + x_2 y_2 + x_3 y_3 && \sim \mathbf{1} \\
&+ x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3 && \sim \mathbf{1}' \\
&+ x_1 y_1 + \omega x_2 y_2 + \omega^2 x_3 y_3 && \sim \mathbf{1}'' \\
&+ (x_2 y_3, x_3 y_1, x_1 y_2) && \sim \mathbf{3} \\
&+ (x_3 y_2, x_1 y_3, x_2 y_1) && \sim \mathbf{3}
\end{aligned}$$

Irrep Products

Invariant terms from three-**3** product:

$$
\begin{aligned}
(x_1, x_2, x_3) \times (y_1, y_2, y_3) \times (z_1, z_2, z_3) &= x_1 y_2 z_3 + x_1 y_3 z_2 \\
&+ x_2 y_1 z_3 + x_2 y_3 z_1 \\
&+ x_3 y_1 z_2 + x_3 y_2 z_1
\end{aligned}
$$

Quantum Numbers

Fields	L	ℓ^c	ϕ_1	ϕ_2	ϕ_3	η_1	η_2	η_3	ξ
$SU(2)_L$	**2**	**1**		**2**			**3**		**3**
Y	–1	2		–1			2		2
A_4	**3**	**3**	**1**	$\mathbf{1}'$	$\mathbf{1}''$	**1**	$\mathbf{1}'$	$\mathbf{1}''$	**3**

Charged Lepton Masses [⇛]

$$\begin{aligned}\mathcal{L} \supset\ & h_1 \ \phi_1^0 \ (ee^c + \mu\mu^c + \tau\tau^c) \\ + \ & h_2 \ \phi_2^0 \ (ee^c + \omega\mu\mu^c + \omega^2\tau\tau^c) \\ + \ & h_3 \ \phi_3^0 \ (ee^c + \omega^2\mu\mu^c + \omega\tau\tau^c)\end{aligned}$$

Charged Lepton Masses [$\Rrightarrow$]

$$
\begin{aligned}
\mathcal{L} \supset\ & h_1\ \phi_1^0\ (ee^c + \mu\mu^c + \tau\tau^c) \\
+\ & h_2\ \phi_2^0\ (ee^c + \omega\mu\mu^c + \omega^2\tau\tau^c) \\
+\ & h_3\ \phi_3^0\ (ee^c + \omega^2\mu\mu^c + \omega\tau\tau^c)
\end{aligned}
$$

Once ϕ_i^0 get VEVs:

$$
\begin{aligned}
\mathcal{L} \supset\ & h_1\ v_1\ (ee^c + \mu\mu^c + \tau\tau^c) \\
+\ & h_2\ v_2\ (ee^c + \omega\mu\mu^c + \omega^2\tau\tau^c) \\
+\ & h_3\ v_3\ (ee^c + \omega^2\mu\mu^c + \omega\tau\tau^c)
\end{aligned}
$$

Charged Lepton Mass Matrix

Already diagonal in the flavour basis

$$M_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

$$\begin{aligned} m_e &= h_1 v_1 + h_2 v_2 + h_3 v_3 \\ m_\mu &= h_1 v_1 + \omega h_2 v_2 + \omega^2 h_3 v_3 \\ m_\tau &= h_1 v_1 + \omega^2 h_2 v_2 + \omega h_3 v_3 \end{aligned}$$

Neutrino Masses: η [$\Rrightarrow$]

$$\begin{aligned}\mathcal{L} \supset\ & \lambda_1\ \eta_1^0\ (\nu_e\nu_e + \nu_\mu\nu_\mu + \nu_\tau\nu_\tau) \\ +\ & \lambda_2\ \eta_2^0\ (\nu_e\nu_e + \omega\nu_\mu\nu_\mu + \omega^2\nu_\tau\nu_\tau) \\ +\ & \lambda_3\ \eta_3^0\ (\nu_e\nu_e + \omega^2\nu_\mu\nu_\mu + \omega\nu_\tau\nu_\tau)\end{aligned}$$

Neutrino Masses: η [$\Rrightarrow$]

$$\begin{aligned}\mathcal{L} \;\supset\; & \lambda_1 \;\; \eta_1^0 \;\; (\nu_e\nu_e + \nu_\mu\nu_\mu + \nu_\tau\nu_\tau) \\ + \; & \lambda_2 \;\; \eta_2^0 \;\; (\nu_e\nu_e + \omega\nu_\mu\nu_\mu + \omega^2\nu_\tau\nu_\tau) \\ + \; & \lambda_3 \;\; \eta_3^0 \;\; (\nu_e\nu_e + \omega^2\nu_\mu\nu_\mu + \omega\nu_\tau\nu_\tau)\end{aligned}$$

Once η_i^0 get VEVs:

$$\begin{aligned}\mathcal{L} \;\supset\; & \lambda_1 \;\; \langle\eta_1^0\rangle \;\; (\nu_e\nu_e + \nu_\mu\nu_\mu + \nu_\tau\nu_\tau) \\ + \; & \lambda_2 \;\; \langle\eta_2^0\rangle \;\; (\nu_e\nu_e + \omega\nu_\mu\nu_\mu + \omega^2\nu_\tau\nu_\tau) \\ + \; & \lambda_3 \;\; \langle\eta_3^0\rangle \;\; (\nu_e\nu_e + \omega^2\nu_\mu\nu_\mu + \omega\nu_\tau\nu_\tau)\end{aligned}$$

Neutrino Masses: η

Once η_i^0 get VEVs:

$$\begin{aligned}\mathcal{L} \supset\ & \lambda_1 \ \langle\eta_1^0\rangle \ (\nu_e\nu_e + \nu_\mu\nu_\mu + \nu_\tau\nu_\tau) \\ + \ & \lambda_2 \ \langle\eta_2^0\rangle \ (\nu_e\nu_e + \omega\nu_\mu\nu_\mu + \omega^2\nu_\tau\nu_\tau) \\ + \ & \lambda_3 \ \langle\eta_3^0\rangle \ (\nu_e\nu_e + \omega^2\nu_\mu\nu_\mu + \omega\nu_\tau\nu_\tau)\end{aligned}$$

Neutrino Masses: η

Once η_i^0 get VEVs:

$$\begin{aligned}\mathcal{L} \supset\ & \lambda_1 \langle\eta_1^0\rangle \left(\nu_e\nu_e + \nu_\mu\nu_\mu + \nu_\tau\nu_\tau\right)\\ +\ & \lambda_2 \langle\eta_2^0\rangle \left(\nu_e\nu_e + \omega\nu_\mu\nu_\mu + \omega^2\nu_\tau\nu_\tau\right)\\ +\ & \lambda_3 \langle\eta_3^0\rangle \left(\nu_e\nu_e + \omega^2\nu_\mu\nu_\mu + \omega\nu_\tau\nu_\tau\right)\end{aligned}$$

$$\begin{aligned}\mathcal{L} \supset\ & a \left(\nu_e\nu_e + \nu_\mu\nu_\mu + \nu_\tau\nu_\tau\right)\\ +\ & b \left(\nu_e\nu_e + \omega\nu_\mu\nu_\mu + \omega^2\nu_\tau\nu_\tau\right)\\ +\ & c \left(\nu_e\nu_e + \omega^2\nu_\mu\nu_\mu + \omega\nu_\tau\nu_\tau\right)\end{aligned}$$

Neutrino Mass Matrix: η

$$\begin{aligned}
\mathcal{L} \supset\ & a\ \left(\nu_e\nu_e + \nu_\mu\nu_\mu + \nu_\tau\nu_\tau\right) \\
+\ & b\ \left(\nu_e\nu_e + \omega\nu_\mu\nu_\mu + \omega^2\nu_\tau\nu_\tau\right) \\
+\ & c\ \left(\nu_e\nu_e + \omega^2\nu_\mu\nu_\mu + \omega\nu_\tau\nu_\tau\right)
\end{aligned}$$

Neutrino Mass Matrix: η

$$\mathcal{L} \supset a \left(\nu_e\nu_e + \nu_\mu\nu_\mu + \nu_\tau\nu_\tau\right)$$
$$+\ b \left(\nu_e\nu_e + \omega\nu_\mu\nu_\mu + \omega^2\nu_\tau\nu_\tau\right)$$
$$+\ c \left(\nu_e\nu_e + \omega^2\nu_\mu\nu_\mu + \omega\nu_\tau\nu_\tau\right)$$

$$M_\nu \ni \begin{pmatrix} a+b+c & 0 & 0 \\ 0 & a+\omega b+\omega^2 c & 0 \\ 0 & 0 & a+\omega^2 b+\omega c \end{pmatrix}$$

Neutrino Masses: ξ [$\Rrightarrow$]

$$\mathcal{L} \supset \lambda \left(\xi_1^0 \nu_\mu \nu_\tau + \xi_2^0 \nu_e \nu_\tau + \xi_3^0 \nu_e \nu_\mu \right)$$

Neutrino Masses: ξ [$\Rrightarrow$]

$$\mathcal{L} \;\supset\; \lambda \;\left(\xi_1^0 \nu_\mu \nu_\tau + \xi_2^0 \nu_e \nu_\tau + \xi_3^0 \nu_e \nu_\mu\right)$$

Once ξ_i^0 get VEVs:

$$\mathcal{L} \;\supset\; \lambda \;\left(\langle\xi_1^0\rangle \nu_\mu \nu_\tau + \langle\xi_2^0\rangle \nu_e \nu_\tau + \langle\xi_3^0\rangle \nu_e \nu_\mu\right)$$

Neutrino Masses: ξ [$\Rrightarrow$]

$$\mathcal{L} \;\supset\; \lambda \;\left(\xi_1^0 \nu_\mu \nu_\tau + \xi_2^0 \nu_e \nu_\tau + \xi_3^0 \nu_e \nu_\mu\right)$$

Once ξ_i^0 get VEVs:

$$\mathcal{L} \;\supset\; \lambda \;\left(\langle \xi_1^0 \rangle \nu_\mu \nu_\tau + \langle \xi_2^0 \rangle \nu_e \nu_\tau + \langle \xi_3^0 \rangle \nu_e \nu_\mu\right)$$

$$\mathcal{L} \;\supset\; \left(d\, \nu_\mu \nu_\tau + e\, \nu_e \nu_\tau + f\, \nu_e \nu_\mu\right)$$

Neutrino Mass Matrix: ξ

$$\mathcal{L} \supset \left(d\,\nu_\mu \nu_\tau + e\,\nu_e \nu_\tau + f\,\nu_e \nu_\mu \right)$$

Neutrino Mass Matrix: ξ

$$\mathcal{L} \supset (d\,\nu_\mu\nu_\tau + e\,\nu_e\nu_\tau + f\,\nu_e\nu_\mu)$$

$$M_\nu \ni \begin{pmatrix} 0 & f & e \\ f & 0 & d \\ e & d & 0 \end{pmatrix}$$

Neutrino Mass Matrix

$$M_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix}$$

Neutrino Mass Matrix

$$M_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix}$$

Simplifying assumptions:

$$b = c \qquad\qquad d = e = f$$

Neutrino Mass Matrix

$$M_\nu = \begin{pmatrix} a+2b & d & d \\ d & a-b & d \\ d & d & a-b \end{pmatrix}$$

Simplifying assumptions:

$$b = c \qquad\qquad d = e = f$$

Neutrino Mass Matrix

$$M_\nu = \begin{pmatrix} a+2b & d & d \\ d & a-b & d \\ d & d & a-b \end{pmatrix}$$

Simplifying assumptions:

$$b = c \qquad\qquad d = e = f$$

Predictions:

$$\theta_{23} = \pi/4 \qquad\qquad \theta_{13} = 0$$

Notation

Parameters

$$a = |a|\, e^{i\phi_a} \qquad b = |b|\, e^{i\phi_b} \qquad d = |d|\, e^{i\phi_d}$$

Phase differences:

$$\phi_1 \equiv \phi_a - \phi_d \qquad \phi_2 \equiv \phi_d - \phi_b$$

Atmospheric Neutrino Mass Splitting

If we consider

$$\Delta m^2_{\mathsf{SOL}} \simeq 0$$

then

$$b,\ d \in \mathbb{R}$$

and

$$\Delta m^2_{32} = 6bd \equiv \Delta m^2_{\mathsf{ATM}}$$

If, on the contrary,

$$\Delta m^2_{\mathsf{SOL}} \neq 0$$

then

$$\Delta m^2_{\mathsf{ATM}} \simeq \mathrm{Sign}[\cos(\phi_2)]6|b||d|$$

Solar Neutrino Mass Splitting

$$\Delta m^2_{21} = \sqrt{T_1^2 + T_2^2 + T_3^2} \equiv \Delta m^2_{\mathsf{SOL}}$$

where

$$T_1 \equiv 6\sqrt{2}|b||d| \sin(\phi_2)$$

$$T_2 \equiv 2\sqrt{2}|d| \Big(2|a| \cos(\phi_1) + |b| \cos(\phi_2) + |d|\Big)$$

$$T_3 \equiv -3|b|^2 + |d|^2 - 6|a||b| \cos(\phi_1 + \phi_2) + 2|a||d| \cos(\phi_1) - 2|b||d| \cos(\phi_2)$$

Therefore

$$|T_i| \leq \Delta m^2_{\mathsf{SOL}} \qquad \forall i$$

Inequalities

Normalizing by $|\Delta m^2_{\text{ATM}}|$, then

$$\sqrt{2}|\sin(\phi_2)| \leq \alpha$$

$$\frac{\sqrt{2}}{3|b|}\Big|2|a|\cos(\phi_1) + |b|\cos(\phi_2) + |d|\Big| \leq \alpha$$

$$\frac{1}{6|b||d|}\Big| -3|b|^2 + |d|^2 - 6|a||b|\cos(\phi_1+\phi_2)$$
$$+2|a||d|\cos(\phi_1) - 2|b||d|\cos(\phi_2)\Big| \leq \alpha$$

where

$$\alpha \equiv \Delta m^2_{\text{SOL}}/|\Delta m^2_{\text{ATM}}| \quad [\Rrightarrow]$$

Solar mixing angle

$$t_{2\mathrm{SOL}} \equiv \tan(2\theta_{12}) = \frac{2\sqrt{2}d}{3b - d}$$

Solar mixing angle

$$t_{2\mathrm{SOL}} \equiv \tan(2\theta_{12}) = \frac{2\sqrt{2}d}{3b-d}$$

Best value:

$$\tan^2\theta_{12} = 1/2$$

for

$$b = 0, \qquad b = 2d/3$$

Solar mixing angle

$$t_{2\mathsf{SOL}} \equiv \tan(2\theta_{12}) = \frac{2\sqrt{2}d}{3b - d}$$

Best value:

$$\tan^2\theta_{12} = 1/2$$

for

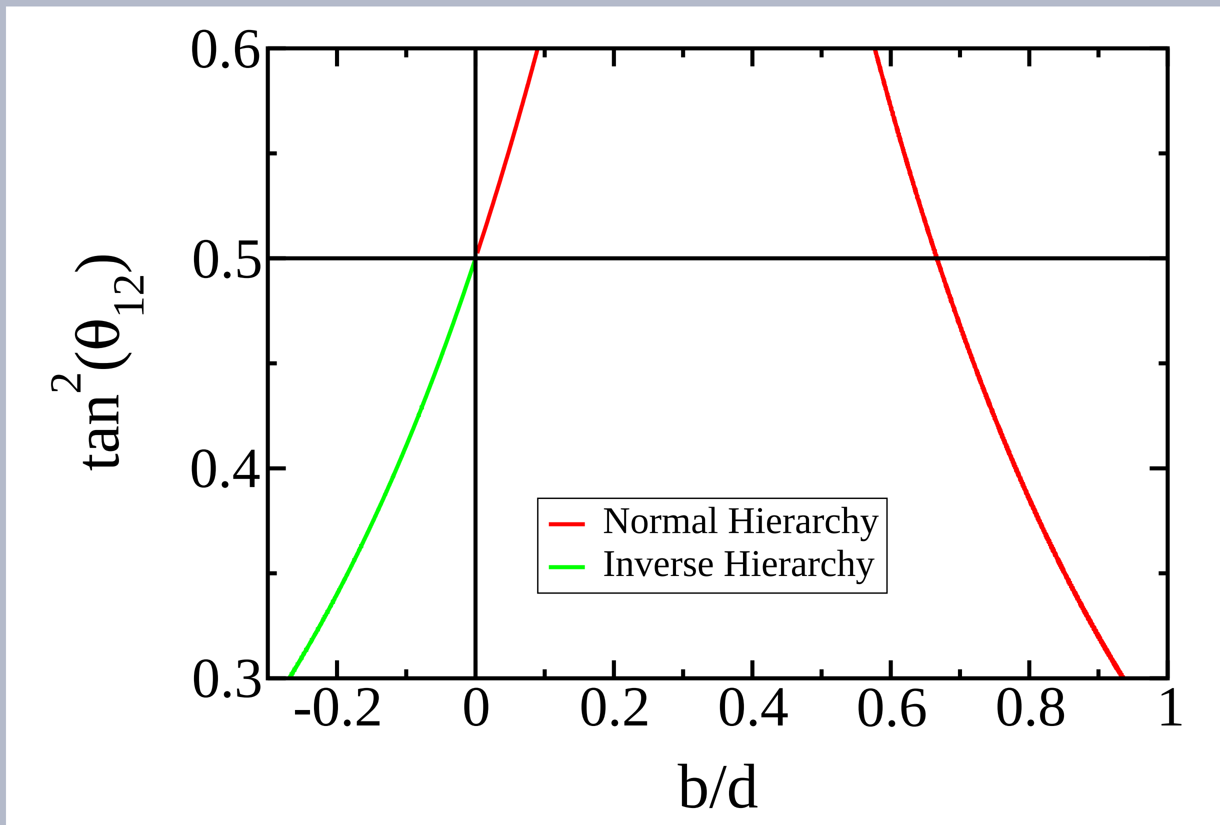
$$b = 0, \qquad b = 2d/3$$

Performing a series expansion of $\tan^2\theta_{12}$ around the two solutions...

Solar mixing angle

$$\tan^2\theta_{12} \simeq \frac{1}{2} + \frac{b}{d}$$

$$\tan^2\theta_{12} \simeq \frac{1}{2} - \frac{1}{d}\left(b - \frac{2}{3}d\right)$$



Neutrinoless Double Beta Decay

- Amplitude proportional to:

$$\langle m_\nu \rangle = m_{ee} = a + 2b$$

Neutrinoless Double Beta Decay

- Amplitude proportional to:

$$\langle m_\nu \rangle = m_{ee} = a + 2b$$

- We need to express a and b in terms of observables

Case-1: Real parameters

Case-1: Real parameters

- Solve this system of equations

$$\left.\begin{aligned} \Delta m^2_{\mathsf{SOL}} \equiv \Delta m^2_{21} &= |2a+b+d|\sqrt{(d-3b)^2+8d^2} \\ \Delta m^2_{\mathsf{ATM}} \equiv \Delta m^2_{32} &= 6bd \\ t_{2\mathsf{SOL}} \equiv \tan(2\theta_{12}) &= \frac{2\sqrt{2}d}{3b-d} \end{aligned}\right\}$$

Case-1: Real parameters

- Solve this system of equations

$$\left.\begin{array}{c}\Delta m^2_{\text{SOL}} \equiv \Delta m^2_{21} = |2a+b+d|\sqrt{(d-3b)^2+8d^2} \\ \Delta m^2_{\text{ATM}} \equiv \Delta m^2_{32} = 6bd \\ t_{2\text{SOL}} \equiv \tan(2\theta_{12}) = \dfrac{2\sqrt{2}d}{3b-d}\end{array}\right\}$$

- Express the parameters: a, b and d in terms of observables: Δm^2_{SOL}, Δm^2_{ATM} and $t_{2\text{SOL}}$

Case-1: Real parameters

- Solve this system of equations

$$\left.\begin{aligned}\Delta m^2_{\mathrm{SOL}} \equiv \Delta m^2_{21} = |2a+b+d|\sqrt{(d-3b)^2+8d^2} \\ \Delta m^2_{\mathrm{ATM}} \equiv \Delta m^2_{32} = 6bd \\ t_{2\mathrm{SOL}} \equiv \tan(2\theta_{12}) = \frac{2\sqrt{2}d}{3b-d}\end{aligned}\right\}$$

- Express the parameters: a, b and d in terms of observables: $\Delta m^2_{\mathrm{SOL}}$, $\Delta m^2_{\mathrm{ATM}}$ and $t_{2\mathrm{SOL}}$
- Substitute a and b in

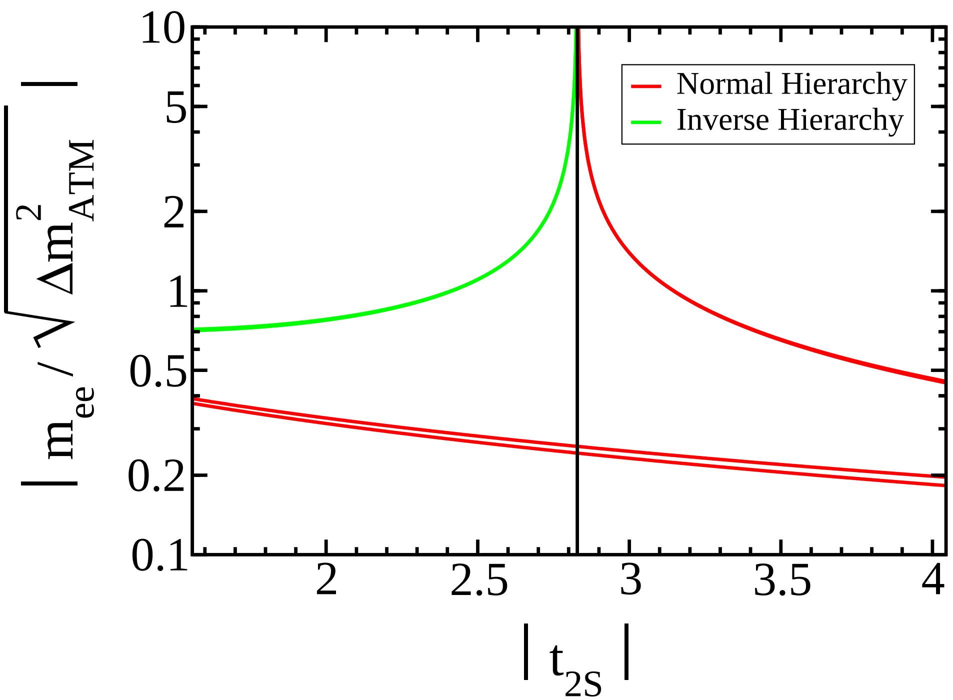
$$\langle m_\nu \rangle = m_{ee} = a + 2b$$

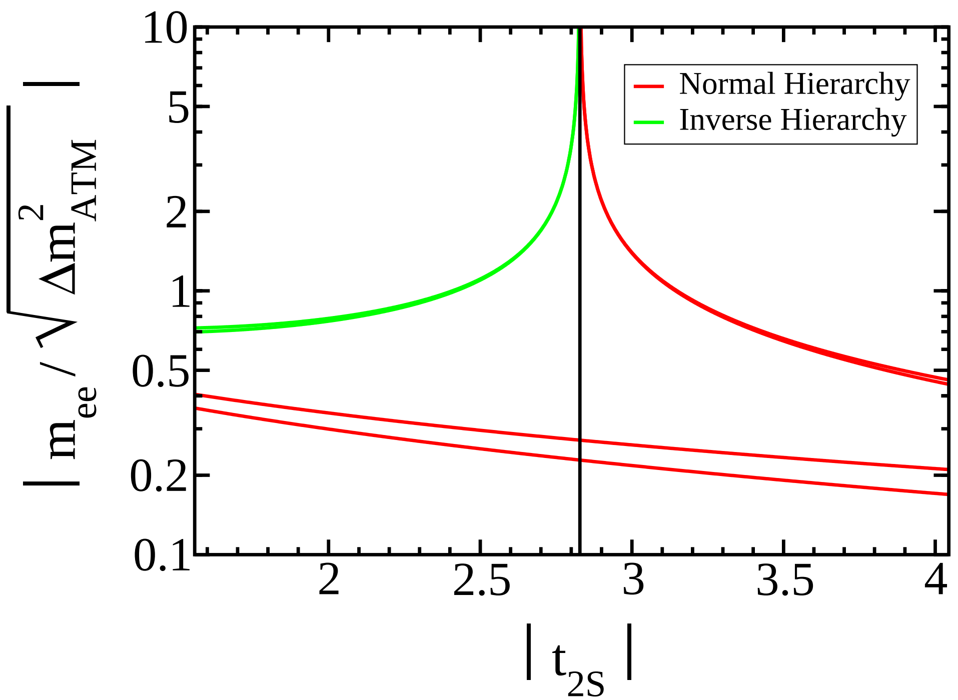
Effective mass parameter

$$\left|\frac{m_{ee}}{\sqrt{\Delta m^2_{\mathrm{ATM}}}}\right| = \mathrm{Sign}[\Delta m^2_{\mathrm{ATM}}]\frac{1}{\sqrt{2\sqrt{2}t_{2\mathrm{SOL}}+t^2_{2\mathrm{SOL}}}}$$

$$\pm\mathrm{Sign}[2\sqrt{2}t_{2\mathrm{SOL}}+t^2_{2\mathrm{SOL}}]\frac{\alpha\sqrt{2\sqrt{2}t_{2\mathrm{SOL}}+t^2_{2\mathrm{SOL}}}}{4\sqrt{1+t^2_{2\mathrm{SOL}}}}$$

Effective mass parameter





Left: $\alpha = 0.022$

Right: $\alpha = 0.065$

Vertical line: $\tan^2\theta_{12} = 1/2$

Case-1: Complex parameters

Case-1: Complex parameters

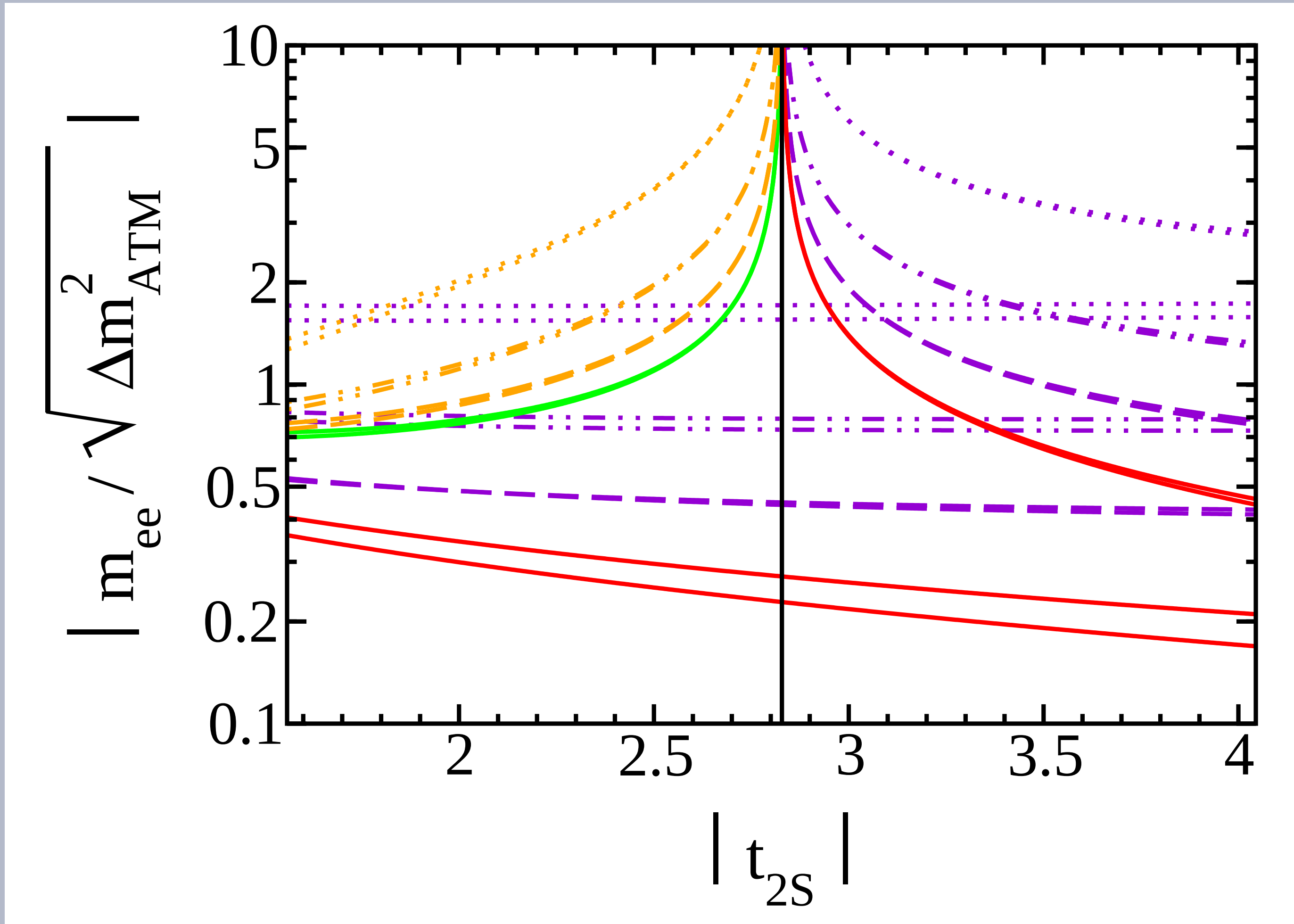
We use the approximations:

$$\sin\phi_2 \simeq 0$$

and

$$2|a|cos\phi_1 + |b|cos\phi_2 + |d| \simeq 0$$

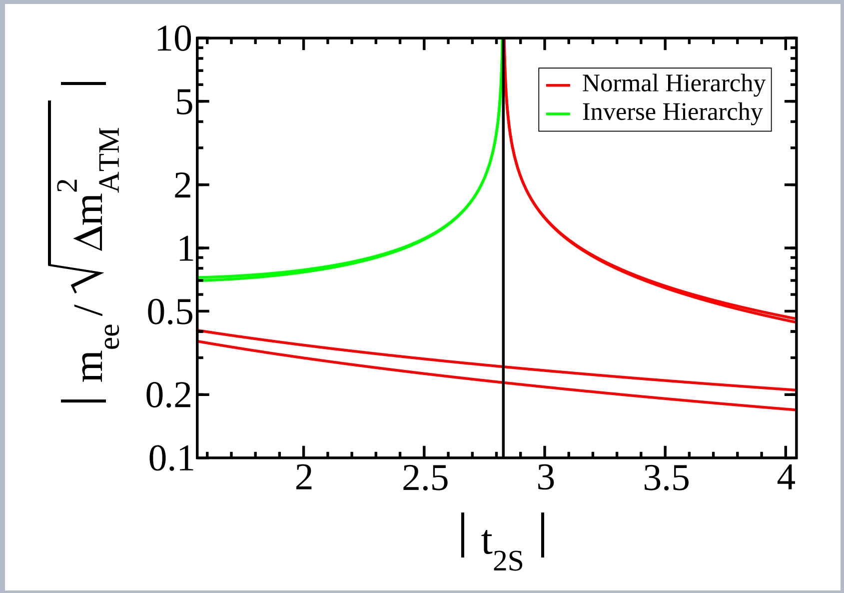
Effective mass parameter

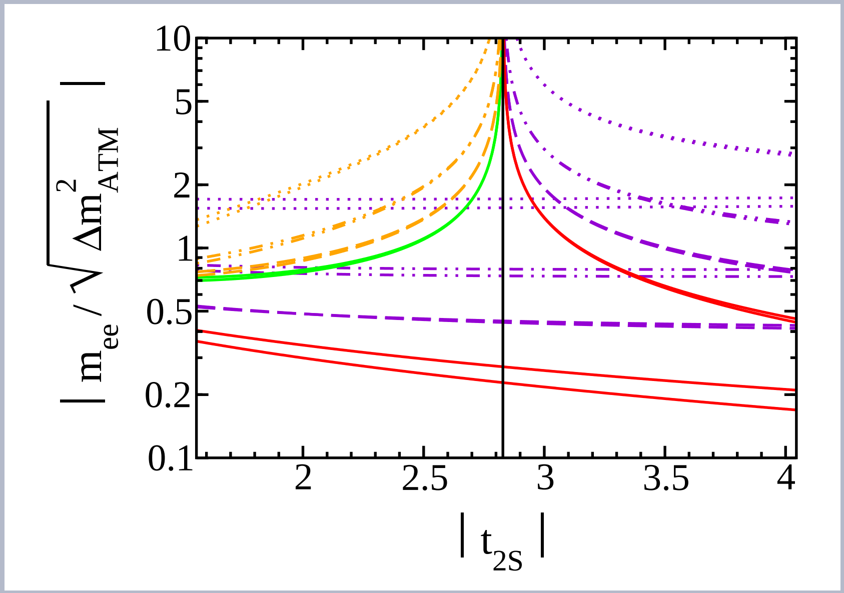


$\alpha = 0.065$

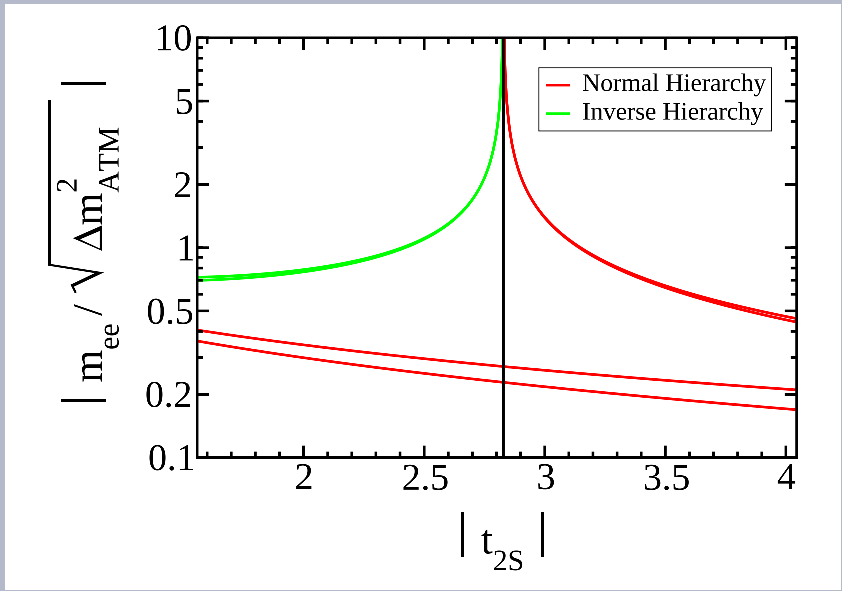
Different values of: $\cos(\phi_1) \in [-1, 1]$

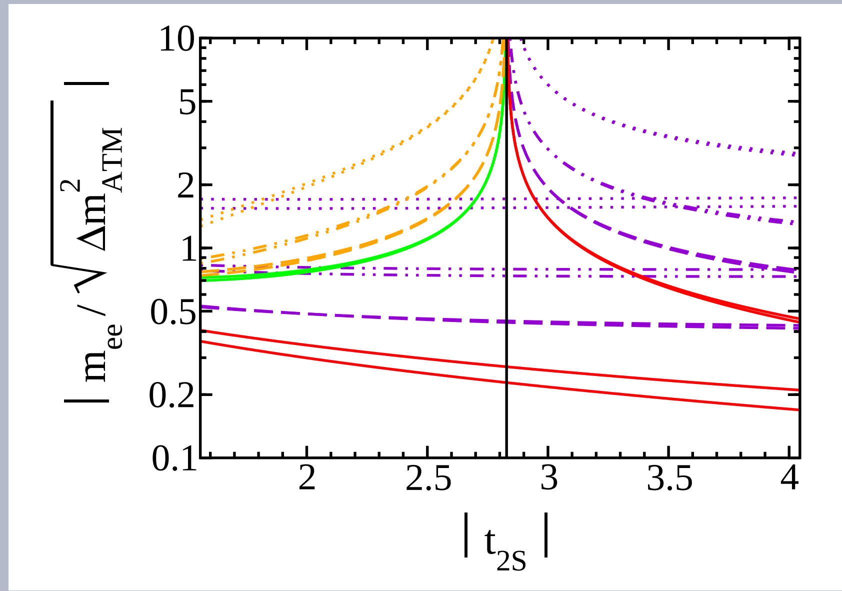
Effective mass parameter





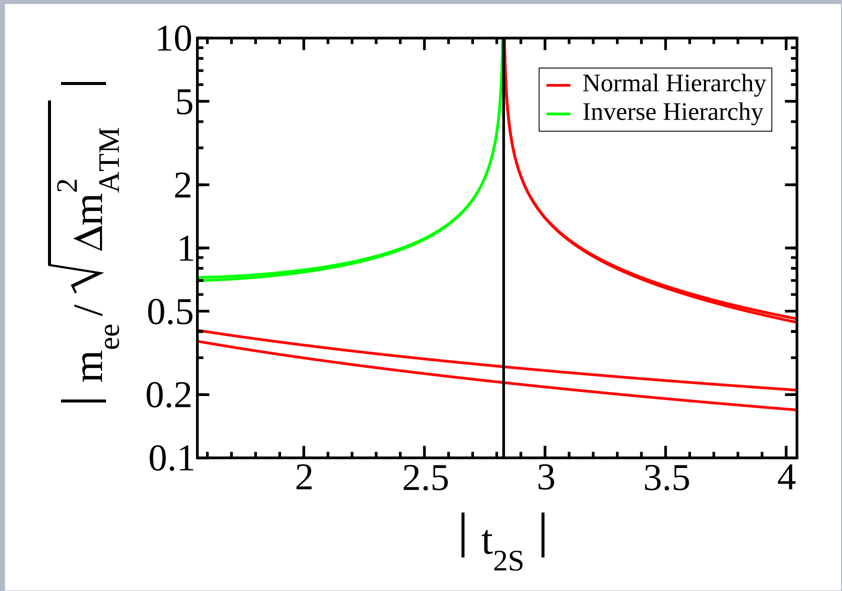
Effective mass parameter

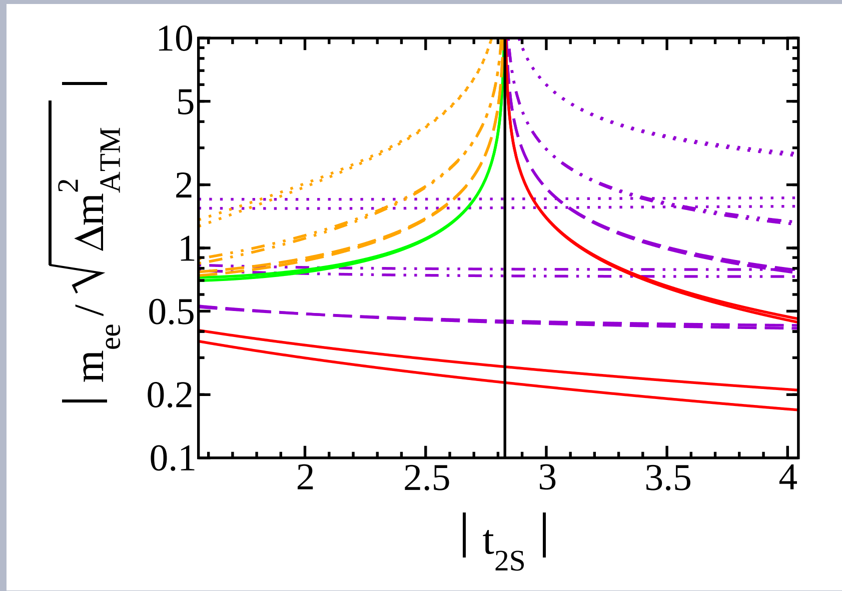




- The lower bounds are in fact exactly the same as obtained in the real case

Effective mass parameter





- The lower bounds are in fact exactly the same as obtained in the real case
- The maximum degree of destructive interference between the three neutrino–exchange contributions occurs in the real case with appropriate CP parities

Lower bounds values

Given the currently allowed experimental 3σ range

$$\tan^2\theta_{12} \in [0.30,\ 0.61]$$

Lower bounds values

Given the currently allowed experimental 3σ range

$$\tan^2\theta_{12} \in [0.30,\, 0.61]$$

we can set lower bounds for $|m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}|$

$$0.17 < |m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}| \quad \text{for} \quad \text{NH}$$

$$0.70 < |m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}| \quad \text{for} \quad \text{IH}$$

Lower bounds values

Given the currently allowed experimental 3σ range

$$\tan^2\theta_{12} \in [0.30,\ 0.61]$$

we can set lower bounds for $|m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}|$

$$0.17 < |m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}| \qquad \text{for} \quad \text{NH}$$

$$0.70 < |m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}| \qquad \text{for} \quad \text{IH}$$

★ Lower bound even for NH!

In the future...

If $\tan^2\theta_{12} \leq 1/2$ could ever be established

In the future...

If $\tan^2\theta_{12} \le 1/2$ could ever be established , then we would have

$$0.23 < \left|m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}\right| < 0.41 \qquad \text{for} \quad \text{NH}$$

$$0.70 < \left|m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}\right| \qquad \text{for} \quad \text{IH}$$

In the future...

If $\tan^2\theta_{12} \leq 1/2$ could ever be established , then we would have

$$0.23 < |m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}| < 0.41 \qquad \text{for} \quad \text{NH}$$

$$0.70 < |m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}| \qquad \text{for} \quad \text{IH}$$

★ We might be able to distinguish between both neutrino mass hierarchies!

In the future...

If $\tan^2\theta_{12} \leq 1/2$ could ever be established , then we would have

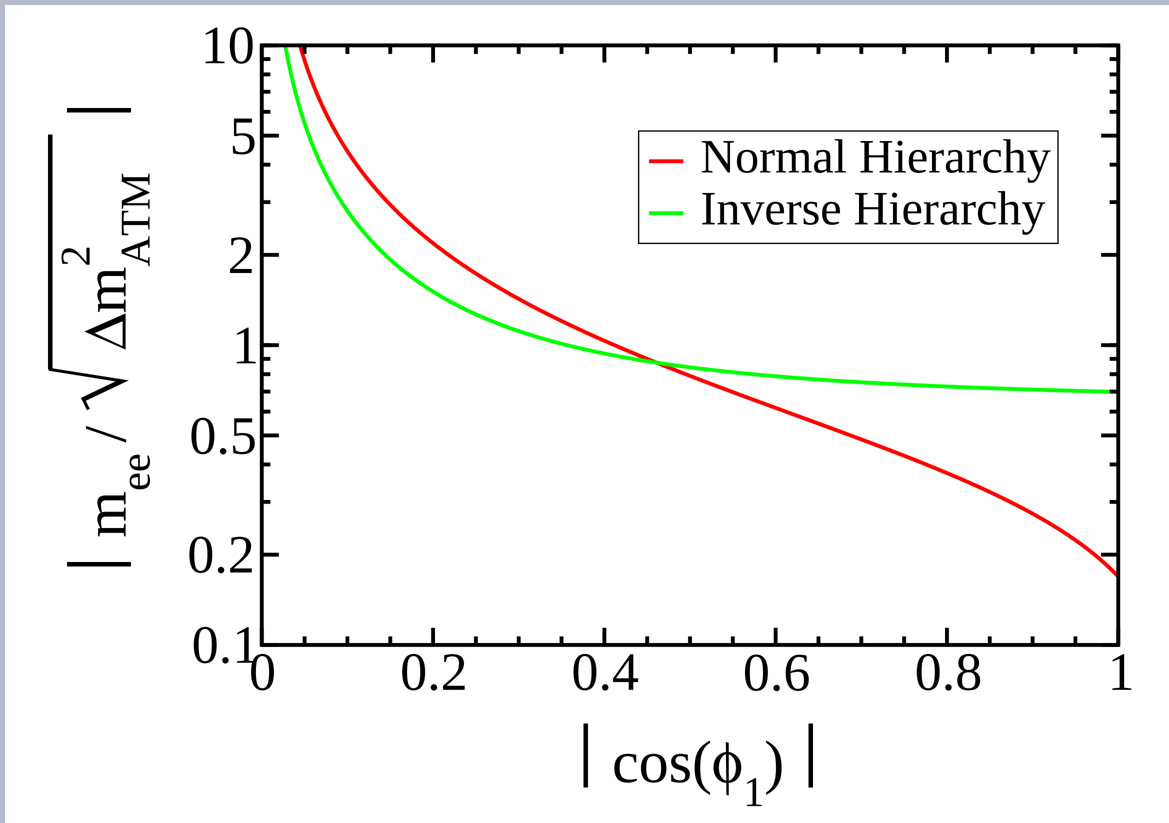
$$0.23 < \left| m_{ee}/\sqrt{\Delta m^2_{\rm ATM}} \right| < 0.41 \qquad \text{for} \quad \text{NH}$$

$$0.70 < \left| m_{ee}/\sqrt{\Delta m^2_{\rm ATM}} \right| \qquad \text{for} \quad \text{IH}$$

- ★ We might be able to distinguish between both neutrino mass hierarchies!
- It can also be seen that, up to order α corrections, m_{ee} can only be zero if $\tan^2\theta_{12} = 1$, now strongly rejected experimentally

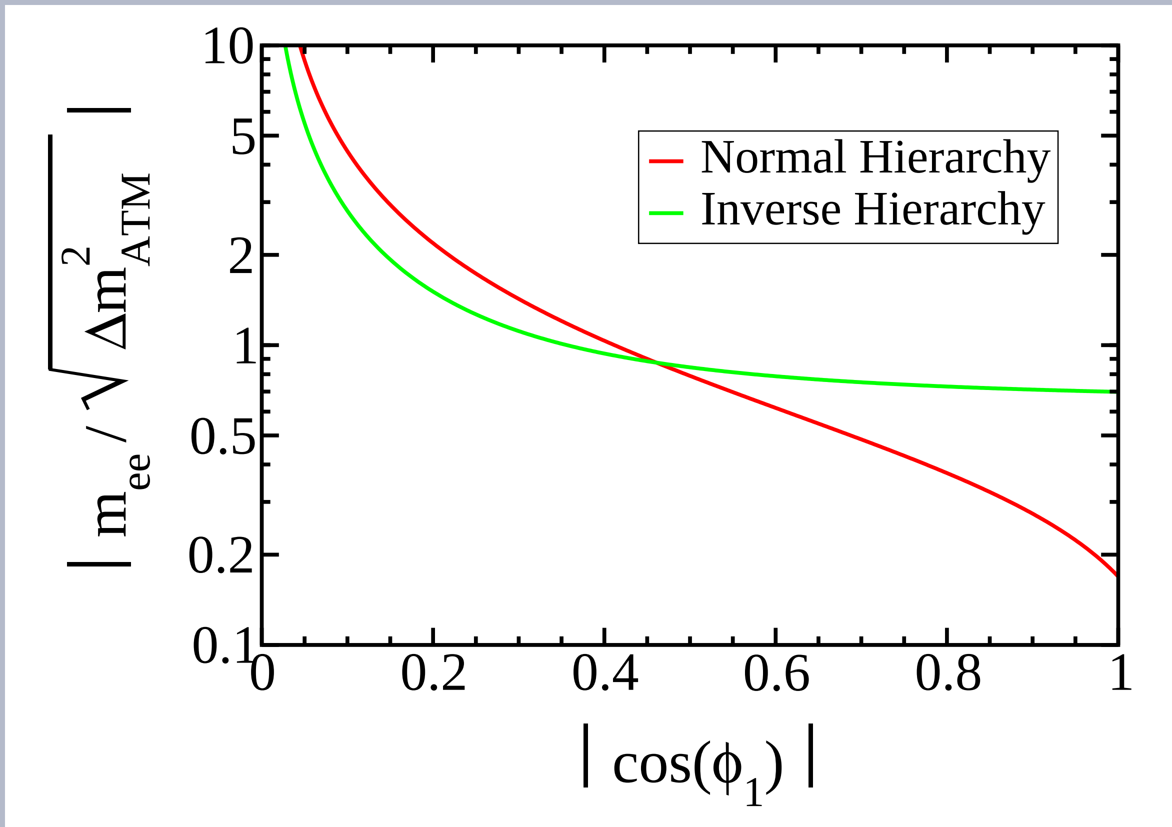
Lower bound phase dependence

The lower bounds on $|m_{ee}/\sqrt{\Delta m^2_{\text{ATM}}}|$ depends on the Majorana phase ϕ_1



Lower bound phase dependence

The lower bounds on $|m_{ee}/\sqrt{\Delta m^2_{\mathrm{ATM}}}|$ depends on the Majorana phase ϕ_1



★ Lower bounds become weaker for $\phi_1 = 0$

Conclusions

- There is a lower bound for the $0\nu\beta\beta$ amplitude even in the case of normal hierarchical neutrino masses

Conclusions

- There is a lower bound for the $0\nu\beta\beta$ amplitude even in the case of normal hierarchical neutrino masses
- The lower bound is robust as it holds irrespective of whether CP is conserved or not

Conclusions

- There is a lower bound for the $0\nu\beta\beta$ amplitude even in the case of normal hierarchical neutrino masses
- The lower bound is robust as it holds irrespective of whether CP is conserved or not
- In the latter case we show explicitly how the lower bound is sensitive to the value of the Majorana phase

Conclusions

- There is a lower bound for the $0\nu\beta\beta$ amplitude even in the case of normal hierarchical neutrino masses
- The lower bound is robust as it holds irrespective of whether CP is conserved or not
- In the latter case we show explicitly how the lower bound is sensitive to the value of the Majorana phase
- Neutrinoless double beta decay may be within reach of the next generation of high sensitivity experiments

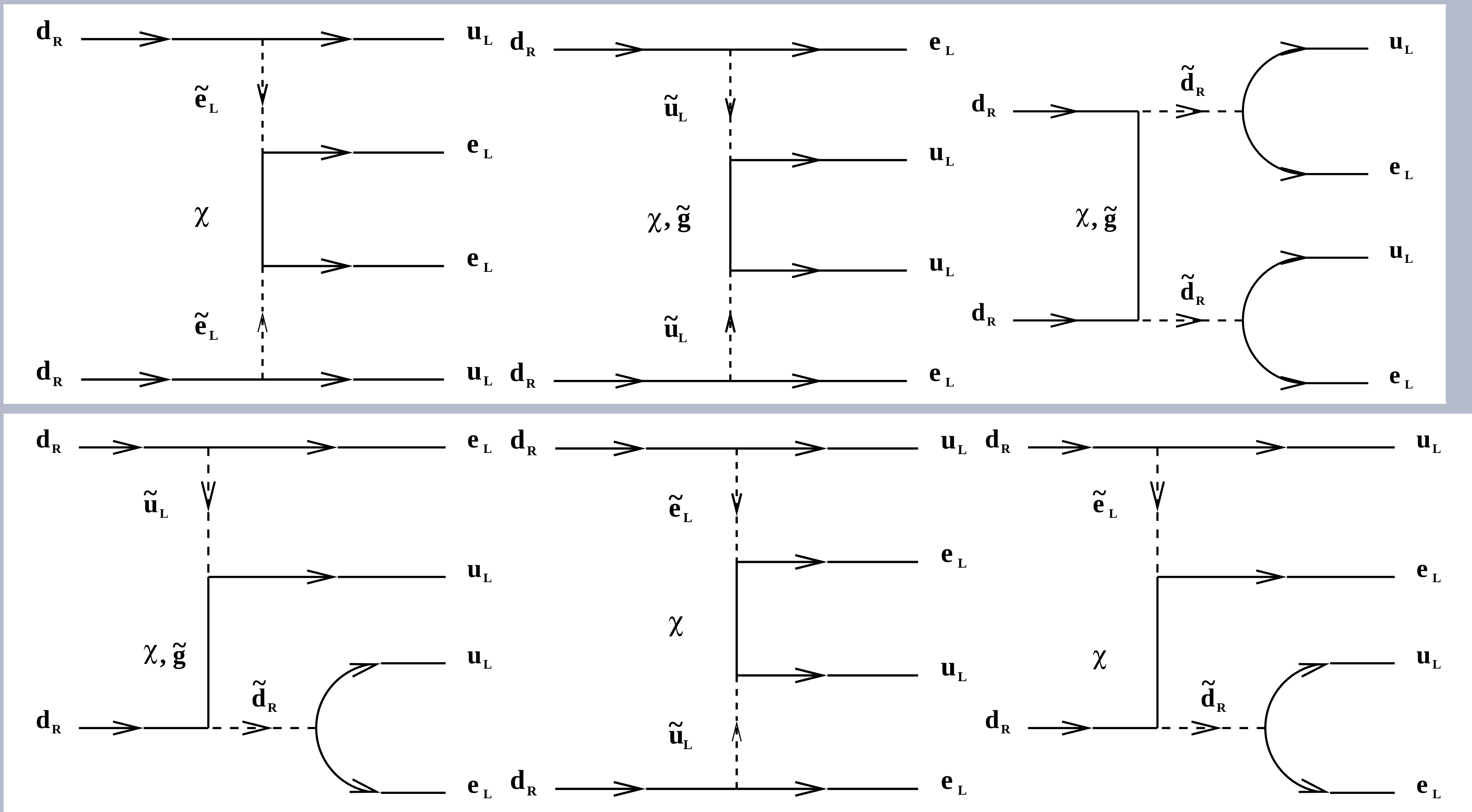
The End

Allowed 3-ν parameter values

parameter	best fit	2σ	3σ	4σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	8.1	7.5–8.7	7.2–9.1	7.0–9.4
Δm^2_{31} $[10^{-3}\text{eV}^2]$	2.2	1.7–2.9	1.4–3.3	1.1–3.7
$\sin^2\theta_{12}$	0.30	0.25–0.34	0.23–0.38	0.21–0.41
$\sin^2\theta_{23}$	0.50	0.38–0.64	0.34–0.68	0.30–0.72
$\sin^2\theta_{13}$	0.000	$\leq$ 0.028	$\leq$ 0.047	$\leq$ 0.068

[$\Lleftarrow$]

Other $0\nu\beta\beta$ Mechanisms



. . .

[⇚]

Charged Lepton Masses

Under $SU(2)$:

$$\mathcal{L} \supset h \, \phi \, L \, \ell^{c} \quad \sim \quad \mathbf{2} \times \mathbf{2} \times \mathbf{1}$$

Charged Lepton Masses

Under $SU(2)$:

$$\mathcal{L} \supset h \; \phi \; L \; \ell^c \quad \sim \quad \mathbf{2} \times \mathbf{2} \times \mathbf{1}$$

$$= h \; \phi^0 \; \ell \; \ell^c \quad \sim \quad \mathbf{1}$$

Charged Lepton Masses

Under A_4:

$$\mathcal{L} \supset h \; \phi^0 \; \ell \; \ell^c$$

Charged Lepton Masses

Under A_4:

$$
\begin{aligned}
\mathcal{L} &\supset h \;\; \phi^0 \;\; \ell \;\; \ell^c \\
&= h_1 \;\; \phi^0_1 \;\; \ell \;\; \ell^c \qquad \sim \qquad \mathbf{1} \times \mathbf{3} \times \mathbf{3} \\
&+ h_2 \;\; \phi^0_2 \;\; \ell \;\; \ell^c \qquad \sim \qquad \mathbf{1}' \times \mathbf{3} \times \mathbf{3} \\
&+ h_3 \;\; \phi^0_3 \;\; \ell \;\; \ell^c \qquad \sim \qquad \mathbf{1}'' \times \mathbf{3} \times \mathbf{3}
\end{aligned}
$$

Charged Lepton Masses

Under A_4:

$$\begin{aligned}
\mathcal{L} \supset\; & h \;\; \phi^0 \;\; \ell \;\; \ell^c \\
=\; & h_1 \;\; \phi_1^0 \;\; \ell \;\; \ell^c \quad \sim \quad \mathbf{1} \times \mathbf{3} \times \mathbf{3} \\
+\; & h_2 \;\; \phi_2^0 \;\; \ell \;\; \ell^c \quad \sim \quad \mathbf{1}' \times \mathbf{3} \times \mathbf{3} \\
+\; & h_3 \;\; \phi_3^0 \;\; \ell \;\; \ell^c \quad \sim \quad \mathbf{1}'' \times \mathbf{3} \times \mathbf{3}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L} \supset\; & h_1 \;\; \phi_1^0 \;\; (e e^c + \mu\mu^c + \tau\tau^c) \quad \sim \quad \mathbf{1} \times \mathbf{1} \\
+\; & h_2 \;\; \phi_2^0 \;\; (e e^c + \omega\mu\mu^c + \omega^2\tau\tau^c) \quad \sim \quad \mathbf{1}' \times \mathbf{1}'' \\
+\; & h_3 \;\; \phi_3^0 \;\; (e e^c + \omega^2\mu\mu^c + \omega\tau\tau^c) \quad \sim \quad \mathbf{1}'' \times \mathbf{1}'
\end{aligned}$$

Neutrino Masses: η

Under $SU(2)$:

$$\mathcal{L} \supset \lambda \; \eta \; L \; L \qquad \sim \qquad \mathbf{3} \times \mathbf{2} \times \mathbf{2}$$

Neutrino Masses: η

Under $SU(2)$:

$$\mathcal{L} \supset \lambda \;\; \eta \;\; L \;\; L \qquad \sim \qquad \mathbf{3} \times \mathbf{2} \times \mathbf{2}$$

$$= \lambda \;\; \eta^0 \;\; \nu \;\; \nu \qquad \sim \qquad \mathbf{1}$$

Neutrino Masses: η

Under A_4:

$$\mathcal{L} \supset \lambda \; \eta^0 \; \nu \; \nu$$

Neutrino Masses: η

Under A_4:

$$\begin{aligned} \mathcal{L} &\supset \lambda \;\; \eta^0 \;\; \nu \;\; \nu \\ &= \lambda_1 \;\; \eta_1^0 \;\; \nu \;\; \nu \qquad \sim \quad \mathbf{1} \times \mathbf{3} \times \mathbf{3} \\ &+ \lambda_2 \;\; \eta_2^0 \;\; \nu \;\; \nu \qquad \sim \quad \mathbf{1}' \times \mathbf{3} \times \mathbf{3} \\ &+ \lambda_3 \;\; \eta_3^0 \;\; \nu \;\; \nu \qquad \sim \quad \mathbf{1}'' \times \mathbf{3} \times \mathbf{3} \end{aligned}$$

Neutrino Masses: η

Under A_4:

$$
\begin{aligned}
\mathcal{L} \supset \ & \lambda \ \eta^0 \ \nu \ \nu \\
= \ & \lambda_1 \ \eta_1^0 \ \nu \ \nu \quad \sim \quad \mathbf{1} \times \mathbf{3} \times \mathbf{3} \\
+ \ & \lambda_2 \ \eta_2^0 \ \nu \ \nu \quad \sim \quad \mathbf{1}' \times \mathbf{3} \times \mathbf{3} \\
+ \ & \lambda_3 \ \eta_3^0 \ \nu \ \nu \quad \sim \quad \mathbf{1}'' \times \mathbf{3} \times \mathbf{3}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L} \supset \ & \lambda_1 \ \eta_1^0 \ \left(\nu_e\nu_e + \nu_\mu\nu_\mu + \nu_\tau\nu_\tau\right) \quad \sim \quad \mathbf{1} \times \mathbf{1} \\
+ \ & \lambda_2 \ \eta_2^0 \ \left(\nu_e\nu_e + \omega\nu_\mu\nu_\mu + \omega^2\nu_\tau\nu_\tau\right) \quad \sim \quad \mathbf{1}' \times \mathbf{1}'' \\
+ \ & \lambda_3 \ \eta_3^0 \ \left(\nu_e\nu_e + \omega^2\nu_\mu\nu_\mu + \omega\nu_\tau\nu_\tau\right) \quad \sim \quad \mathbf{1}'' \times \mathbf{1}'
\end{aligned}
$$

[⇐]

Neutrino Masses: ξ

Under $SU(2)$:

$$\mathcal{L} \supset \lambda \; \xi \; L \; L \qquad \sim \qquad \mathbf{3} \times \mathbf{2} \times \mathbf{2}$$

Neutrino Masses: ξ

Under $SU(2)$:

$$\mathcal{L} \supset \lambda \;\; \xi \;\; L \;\; L \qquad \sim \qquad \mathbf{3} \times \mathbf{2} \times \mathbf{2}$$

$$= \lambda \;\; \xi^0 \;\; \nu \;\; \nu \qquad \sim \qquad \mathbf{1}$$

Neutrino Masses: ξ

Under A_4:

$$\mathcal{L} \supset \lambda \ \xi^0 \ \nu \ \nu \quad \sim \quad \mathbf{3} \times \mathbf{3} \times \mathbf{3}$$

Neutrino Masses: ξ

Under A_4:

$$\mathcal{L} \supset \lambda \;\; \xi^0 \;\; \nu \;\; \nu \qquad \sim \qquad \mathbf{3} \times \mathbf{3} \times \mathbf{3}$$

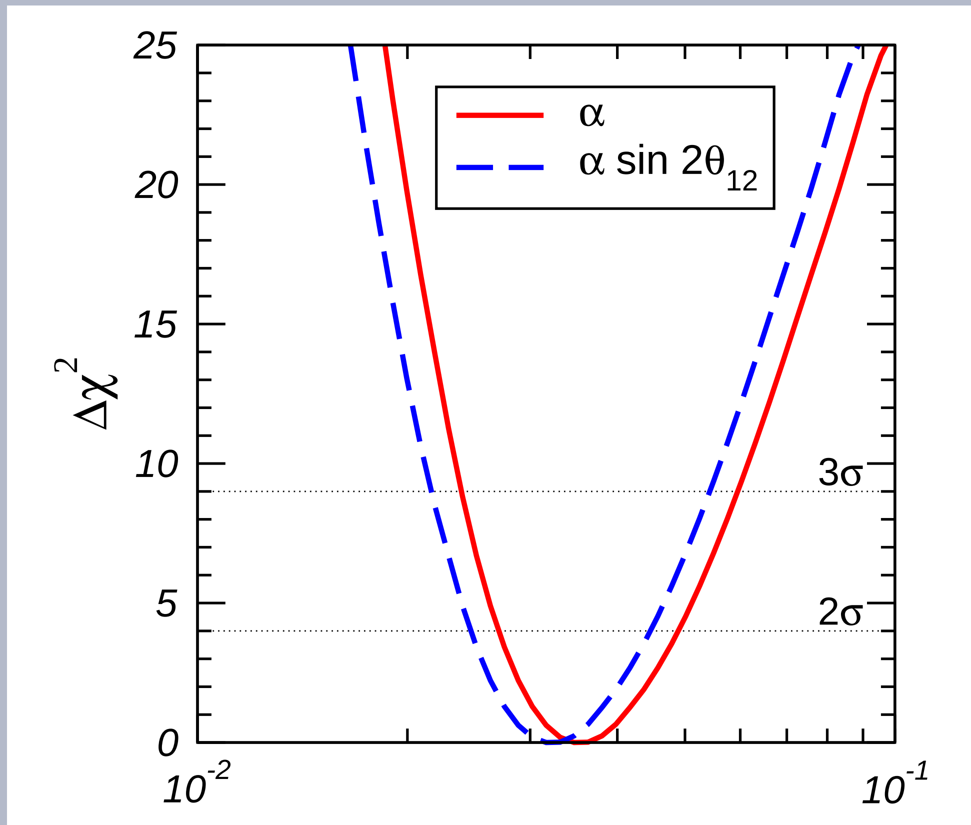
The invariant term is then

$$\mathcal{L} \supset \lambda \;\; (\xi_1^0 \nu_\mu \nu_\tau + \xi_2^0 \nu_e \nu_\tau + \xi_3^0 \nu_e \nu_\mu)$$

[⇚]

Current allowed values of α

$\Delta\chi^2$ from global oscillation data as a function of $\alpha \equiv \Delta m^2_{\text{SOL}}/|\Delta m^2_{\text{ATM}}|$ and $\alpha \sin 2\theta_{12}$:



[M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **6**, 122 (2004)]

Best fit value: $\alpha = 0.035$ [⇚]